Nonlinear enhancement and saturation phenomena in high intensity focused ultrasound beams

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Simulation of focused ultrasound beams has been performed in weakly dissipative medium for a wide range of source linear focusing gains and source pressure amplitudes, including the strongly nonlinear regime with shocks. The effects of nonlinear enhancement of focusing gain and saturation are studied. Our theoretical model is based on the Khokhlov – Zabolotskaya (KZ) nonlinear parabolic equation. An asymptotic frequency domain numerical algorithm which enables modeling of strongly distorted waves with shocks using a relatively small number of harmonics is employed. Acoustic beams with an initially Gaussian amplitude shading are considered. The results of simulations for various parameters of the acoustic field are presented and compared with known approximate analytic results. The effect of focusing gain on nonlinear enhancement of acoustic energy concentration and saturation levels is discussed.

INTRODUCTION

Amplification factor of acoustic focusing systems strongly depends on the combined effects of diffraction and nonlinear propagation. Nonlinear enhancement of peak positive pressure and intensity amplification factor at focus compared to the linear one can be observed for moderate source output because of better focusing and relative diffraction phase shift of higher harmonics [1 - 4]. For higher source output, nonlinear saturation phenomenon is caused by effective absorption of acoustic energy at the shocks on the way to focus [5 - 7]. Various approximate expressions have been obtained to predict nonlinear enhancement of focusing gain and saturation level at the focus for axisymmetric ultrasound beams in a weakly dissipative medium [2, 3, 5, 6]. However, there is no general analytic result that includes the combined effect of nonlinearity and diffraction in the focal area of the beam. Numerical results are available for some chosen specific parameters [4, 7]. In the present work numerical simulations of focused ultrasound beams have been performed over a wide range of source pressure amplitudes and linear focusing gains [8 - 10].

THEORETICAL APPROACH

Propagation of an intense axisymmetric focused acoustic beam in a nonlinear medium is governed by the KZ (Khokhlov- Zabolotskaya) equation with the boundary condition given at the source as a focused beam with initially Gaussian amplitude shading and harmonic initial waveform[3].

NUMERICAL ALGORITHM

Numerical simulations are performed in the frequency domain using an operator splitting procedure. A modified spectral approach is used for modeling the nonlinear term[8]. The approach is based on a known high-frequency asymptote of a shock wave spectrum and it enables us to model nonlinear waves with shocks using a relatively small number of harmonics[4, 8-10]. For additional acceleration of the code a nonuniform convergent spatial grid is used based on the known linear analytic solution for the focused Gaussian beam [3, 10]. All dimension parameters of the beam can be reduced to two dimensionless parameters N (nolinearity) and G (linear focusing gain):

$$N = \frac{\varepsilon\,\omega_0 p_0 F}{c_0^3 \rho_0}\,; \qquad G = \frac{\omega_0 a^2}{2 c_0 F}\,;$$

Here p_0 - is the source pressure amplitude, c_0 - is the sound speed, ω_0 – is the fundamental frequency, ε - is the coefficient of nonlinearity, ρ_0 - is the ambient density of the fluid, a – is the source radius, and F – is the geometrical focal length.

RESULTS

The strongest nonlinear enhancement of focusing gain compared to its linear value ($N = 0$) is obtained for the positive peak pressure: 2 - 2.6 times (Fig. 1); is less pronounced for the peak-to-peak pressure: 1.4 - 1.8 times; and for the intensity: 1.1 - 1.35 times. The focusing gain for the peak negative pressure always decreases monotonically with source amplitude. The effect of nonlinear enhancement is more pronounced for higher linear focusing gain.

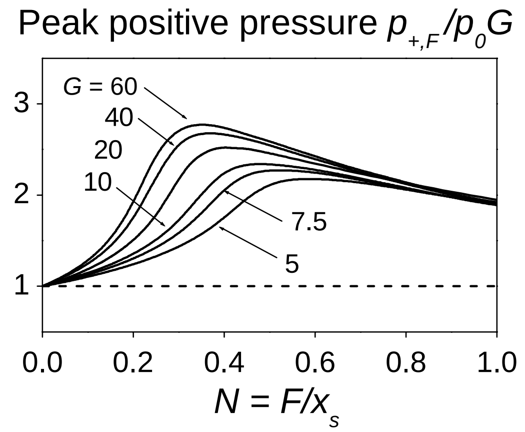

FIGURE 1. Nonlinear enhancement of focusing gain for moderate acoustic pressure amplitudes.

Figure 2 illustrates nonlinear saturation, caused by effective absorption of acoustic energy at the shocks for higher source amplitude. Saturation level is found to depend weakly on the linear gain G and it is about three times higher for the peak positive pressure, 2.2 times higher for the peak-to-peak pressure, 1.5 higher for the negative pressure, and 5 times higher for the intensity compared to the values, predicted by approximate analytic theory[1,5,6].

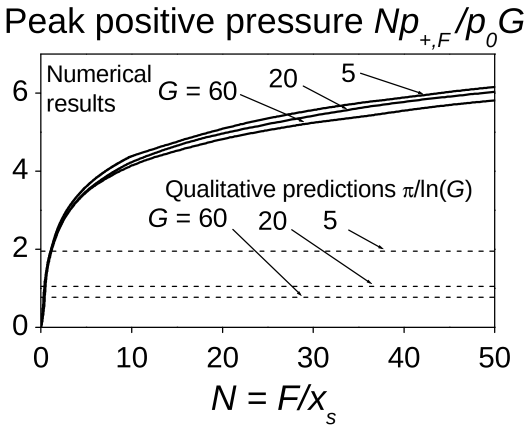

FIGURE 2. Nonlinear saturation in focused beam for higher acoustic pressure.

CONCLUSIONS

Numerical simulation of focused acoustic beams is performed over a wide range of linear gains (G) and source amplitudes (N). It is shown that nonlinear enhancement of focusing gain is different for different characteristics of the acoustic field and for different values of linear gain. Enhancement of the focusing gain is found to be more pronounced for the peak positive pressure and for higher linear gains. The levels of nonlinear saturation at the focus are obtained for very high source amplitudes. The results of simulations give lower enhancement and higher saturation levels compared to the approximate analytic predictions[1,2,5,6].

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REFERENCES

1. K.A. Naugolnykh, L.A. Ostrovsky, *Nonlinear Wave Processes in Acoustics,* Cambridge University Press, 1998, pp. 163-164.
2. L.A. Ostrovskii and A.M. Sutin, *Akust. Zh.*, **22**, 1300-1303 (1975).
3. M.F. Hamilton, V.A. Khokhlova, O.V. Rudenko, *Journ. Acoust. Soc. Am.*, **101**, 1298-1308 (1997).
4. V.A. Khokhlova, M.A. Averkiou, S.J. Younghouse, M.F. Hamilton, and L.A. Crum, In: *Proc. of 16th ICA/135th ASA*, 2875-2876 (1998).
5. K.A. Naugol’nikh and E.V. Romanenko, *Akust. Zh.*, **3**, 294-296 (1959).
6. D.R. Bacon, *Ultr. in Med. & Biol.*, **10**, 189-195 (1984).
7. T. Christopher and E.L. Carstensen, *Ultr. in Med. & Biol.*, **22**, 1103-1116 (1996).
8. Yu.A. Pishchal’nikov, O.A. Sapozhnikov, and V.A. Khokhlova, *Acoust. Phys.*, **42**, 362-367 (1996).
9. F.P. Curra, P.D. Mourad, V.A. Khokhlova and L.A. Crum, *IEEE Trans. Ultrason., Ferroelect., Freq. Contr.*, **47**, 1077-1089 (2000).
10. V.A. Khokhlova, S.S. Kashcheeva, M.A. Averkiou, and L.A. Crum, In: *Proc. of 15th ISNA*, 151-154 (2000).