Simulation of snow water equivalent by mathematical models of different complexity

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Introduction

Snow water equivalent (SWE) is the most important hydrological characteristic of the snow cover, because it provides the information on the amount of water that can eventually contribute to river runoff during the snowmelt. Basic principles of snow accumulation and melt, and thus the SWE simulation were elaborated about 50 years ago [1–3, 36, 37]. However, new models which profit from the progress in knowledge and technological development (data acquisition, computing) are developed permanently. Current models of snow cover evolution strive to simulate also some internal properties of the snowpack such as snow stratigraphy, temperature distribution in the snowpack, mass and energy fluxes, etc. [e.g. 6, 19, 23]. Some models attempt to include the effects of vegetation on snow-related processes [18, 26] or consider snowdrift [14, 24, 25, 31]. Except point simulation of snow cover evolution, distributed models have been developed to simulate SWE for river basins [10, 30, 33, 39]. Koivusalo [20] has recently provided an overview of snow cover modeling for hydrology.

Most snow models similarly using the threshold air temperature (or temperatures) to determine the portion of precipitation falling as snow generally simulate snow accumulation. Algorithms of snow melting can be roughly divided into three big groups. The simplest ones (index models) are based on the relationships between snowmelt and the conventionally measured meteorological variables, e.g. air temperature (the degree-day model), wind speed, etc. These models attempt to describe the complex process of snowmelt by means of simpler relationships. Algorithms of the second group strive to solve the complex energy balance of the snow cover (the energy-based models). Some models that are denoted as the «energy-based» still employ empirical relationships for snowmelt (e.g. using the air temperature to simulate melt contributions from various energy fluxes). Other energy-based models use more sophisticated physical approach in snowmelt modeling, although in details none of them can avoid empirical relationships. This is due to computational complexity or unavailability of necessary input data. Algorithms of the third group use generalized equations based on direct empirical relationships between snowmelt and selected meteorological characteristics. They are not used so often as the algorithms of the first two groups and, strictly speaking, they may not be called «mathematical models».

The most often used are the index models. Temperature index (degree-day) models proved their efficiency and are used in operational streamflow forecasting for a long time [9, 22, 27, 38, etc.]. World Meteorological Organization [40] has compared 11 different models used in several countries. Most of them were the degree-day models and were designed to work at a basin scale. Melloh [28] reviewed 7 major operational models that are based on the investigations of the U.S. Army Corps of Engineers [36, 37] and the U.S. National Weather Service [4, 5]. Except the degree-day, they have also the option of using the energy-balance of the snowpack to calculate the snowmelt.

Along with the development of the snow models, numerous studies attempted to inter-compare their results [7, 11, 13, 16]. The comparison is based on using the same input data and comparing modeled outputs with actually measured values [15]. Bengtsson and Singh [8] highlighted that the complexity of the model should reflect basin conditions. A simple degree-day model can be suitable for large basins in which runoff permanently increases during snowmelt, but the model has to be distributed related to land cover and topography. Also for small-forested basins where most of streamflow is of groundwater origin, the degree-day model combined with a conceptual runoff model can reproduce runoff well. In catchments in which the overland flow is an important runoff component, runoff fluctuates during a day. In such conditions, a high-resolution snow model is required to simulate the runoff.

The objective of this paper was to compare the ability of snow models with different complexities to simulate snow cover characteristics (mainly SWE) under different meteorological and landscape conditions.

Study area and data

The climatic data that served as the inputs into tested models and the snow data that were used to validate them were measured at three sites in the Jalovecký creek catchment, north Slovakia (Figs 1 and 2).

The first site called Ondrašová, is situated in the Lipov valley at altitude 570 m a.s.l. It represents the snow formation conditions of the wide mountain valley (shallow snow cover of shorter duration). This site has the best data. Climatic data are measured there with different frequencies. Precipitation is measured daily, cloudiness and wind speed three times per day. Air temperature (at 2 m and 5 cm above the surface), air humidity, soil temperatures at
depths of 5, 10, 20, 50 and 100 cm, scalar wind speed, sunshine duration and snow depth are measured continuously. Snow characteristics at the nearby snow course (depth at 20 points, water equivalent at 3 points, snow structure at one pit, snow temperature at the same pit at several depths) were measured with varying frequency.

Two other sites (Červenec) are situated in the Western Tatra Mountains. They represent the snow formation conditions at high altitudes at the open area and in the forest. Meteorological station is situated at the open area (site Červene — open area). It provided continuous measurements of the air temperature (at 2 m and 15 cm above the surface), air humidity, wind speed, global radiation, soil surface temperature and soil temperature at the depth of 15 cm. Precipitation is measured by the raingauge (weekly) and storage gauge (monthly). The readings of the gauges were recalculated into daily precipitation depths according to station Ondrašová. Snow course data (varying frequency of measurements) comprised 60 measurements of snow depth (SD) and 3 measurements of snow water equivalent (SWE). Snow structure and snow temperature at several depths were measured in the snow pit.

Discharge of the Jaloveck creek measured at the outlet of the mountain part of the catchment was used as additional data to identify the snowmelt events.

Winters 2006 and 2007 had different climatic and snow characteristics. Winter 2006 was cold and long at the lower elevations, although in mountains the maximum SWE was just «normal». Winter 2007 was mild with little snow at lower elevations, but SWE values were above-average in the mountains.

Snow cover models

The temperature and temperature-wind models represented Index snow cover models. These models are lumped, i.e. they do not take into account snow layering and the snowpack is represented as one layer. Two lumped models represented the energy-based models and one distributed model, e.g. the model that simulates multi-layered snowpack. The lumped energy-based models were represented by the combined extended approach by Braun [12] – further denoted as EXT, and the UEB model [35]. The multi-layer energy-based models were represented by the SPONSOR [34], which participated recently in a large model intercomparison project [15].

The models were run in a daily time step. Basic simulated output was snow water equivalent (SWE) which was simulated by all the models. The UEB (snowpack and snow surface temperatures) and SPONSOR (snow depth, snow temperature, layers) models simulated additional outputs. The modeling strategy was aimed at proper simulation of both maximum SWE and the timing of snowmelt, because these two characteristics are the most important in snow hydrology for flood runoff forecasting. After satisfactory simulation of SWE, other measured and simulated characteristics (snow surface temperatures, snow depths) were compared.

Index models. Two index models were used – the temperature index model and the temperature-wind index model. Snow accumulation in both models was calculated identically. The snow was accumulated if the air temperature was below the threshold value determining the beginning of the snowmelt. The type of falling precipitation depended on the air temperature. Fraction of snow was calculated according to the following equation [33]:

\[ P_{\text{snow}} = \frac{T_{\text{rel}} + T_{\text{snow}} - T}{T_{\text{trans}}} \]

where \( P_{\text{snow}} \) is the fraction of snow on the total precipita-
tion, $T_{R/S}$ [°C] is the air temperature at which 50% of snow is falling as snow, $T_{trans}$ [K] is one half of the transition range from snow to rain.

The following equations were used to calculate the snowmelt [33]:

- temperature index: $M = c_0(T - T_{0,m})$,
- temperature-wind index: $M = (c_1 + c_2 u)(T - T_{0,m})$,

where $M$ [mm] is snowmelt, $c_0$ and $c_1$ [mm/°C·day] are the air temperature dependent melt factors, $T$ [°C] is measured air temperature, $T_{0,m}$ [°C] is the threshold air temperature for the beginning of snowmelt, $c_2$ [mm/(°C·m·s⁻¹·day)] is the wind dependent melt factor, $u$ [m/s] is wind speed.

**Energy-based models.** Three energy-based models were used in the study. Braun [12] based his first model on the extended combination approach (EXT). Snow accumulation in the EXT model was simulated the same way as in the index models above. Snowmelt was simulated differently for the days with and without precipitation. During the days with precipitation above 2 mm, the snowmelt is composed of radiation melt, melt from sensible heat, melt from latent heat and melt from energy given by precipitation. The air temperature is the main parameter to calculate these melt components. Detailed information can be found in [33]. Snowmelt during the days without precipitation is calculated by the equation analogous to equation (1).

Snowpack in the UEB model [35] is characterized by three state variables, water equivalence $W$ [m], energy content $U$ [kJ/m] and the age of snow surface which is used only for albedo calculations. Time evolution of $U$ and $W$ is determined by solving the following energy and mass equations:

$$\frac{dU}{dt} = Q_{sn} + Q_{li} + Q_{p} - Q_{g} + Q_{le} - Q_{h} - Q_{m},$$

$$\frac{dW}{dt} = P_r + P_s - M_r - E,$$

where $Q_{sn}$ is net shortwave radiation, $Q_{li}$ is incoming long-wave radiation, $Q_{p}$ is advection heat from precipitation, $Q_{g}$ is ground heat flux, $Q_{le}$ is outgoing long-wave radiation, $Q_{h}$ is sensible heat flux, $Q_{le}$ is latent heat flux due to sublimation/condensation and $Q_{m}$ is advected heat removed by meltwater (all in kJ/m²·hr). $P_r$ is the rainfall rate, $P_s$ is the snowfall rate, $M_r$ is the meltwater outflow from the snowpack and $E$ is the sublimation from the snowpack (all in mm/hr).

Measured precipitation is partitioned into rain and snow using threshold air temperatures similar as in equation (1). The influence of wind on snow redistribution is expressed by the drift factor used to multiply the snowfall rate. The model considers also the influence of forest cover on the energy balance of the snowpack. The SOLEI model [29] calculated solar radiation for UEB.

The SPONSOR model [34] is the updated land surface model using the multi-layer snow scheme that includes physical properties of each layer. The density of each snow layer is taken into account during its evolution. After the snowfall a new snow layer appears. Its physical properties change according to meteorological conditions and previous evolution of the snow layers according to certain criteria [17, 21, 32, etc.]. Snow layers are united once their properties are close enough. Basic characteristics of the models are given in Table 1.

**Results and discussion**

The results of simulations with different models together with selected input data (precipitation, air temperature) are shown in Fig. 3 and summarized in Table 2.

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**Fig. 2.** Snow profiles Ondrašová (a), Červenec-open area (b), and Červenec-forest (c).

Рис. 2. Местоположение профилей в Ондрашова (a), Червениц — открытый участок (b), Червениц-лес (c)
The results given by the temperature and temperature-index models were very similar. Wind-induced snowmelt was less significant than the melt caused by the air temperature. Fixed values of the melt factors did not allow to simulate sudden changes of SWE (e.g. at Ondrašová in winter 2006). Fixed value of threshold air temperature $T_{0,m}$ (beginning of snowmelt) sometimes resulted either in delayed simulation of snowmelt or underestimation of SWE before the beginning of the main phase of snowmelt.

With regard to simulation of snow accumulation, it seems that the threshold temperature $T_{R/S}$ should also vary. Fixed value of $T_{R/S}$ results either in underestimated snow accumulation during the accumulation phase of winter or overestimation of SWE during the melting phase (the model simulates snow accumulation despite it does not occur in reality). At Červenec the snow accumulation was not very sensitive to the threshold air temperature $T_{R/S}$. For a wide range of $T_{R/S}$ the simulated SWE during the accumulation phase of winter 2006 was similar. It could mean that the daily precipitation (input data) was too low to simulate measured SWE. This is especially evident for Červenec-forest in winter 2006. In reality it would either point out to lower measured precipitation, e.g. due to wind losses, or higher measured SWE due to snowdrift.

**Energy-based models.** Extended combined approach (EXT) allowed better simulation of SWE variability at Ondrašová in winter 2006 than the index models while the simulation of melting was similar (very close to measured values). Otherwise, the results were comparable with those provided by the temperature index model. UEB provided much better results for the Ondrašová (2006) and Červenec-forest than EXT and the index models. Snow surface temperature was simulated by the UEB with varying efficiency. In the forest, the simulated and measured snow surface temperatures were very similar; at Ondrašová and Červenec-open area the simulations were not so successful.

SWE simulations by SPONSOR for Ondrašová and winter 2006 were better than the simulations of other models.

### Table 1

<table>
<thead>
<tr>
<th>Model</th>
<th>Approach</th>
<th>Simulated output</th>
<th>Input data</th>
<th>Number of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature index (TI)</td>
<td>Index—SWE</td>
<td>$P, T$</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Temperature—wind index (TWI)</td>
<td>Index—SWE</td>
<td>$P, T, WS$</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Extended combined (EXT)</td>
<td>Energy balance—SWE</td>
<td>$P, T, WS, RH, Q_{si}^{<strong>}, Q_{li}^{</strong>}$</td>
<td>6 basic of 19 possible parameters</td>
<td></td>
</tr>
<tr>
<td>UEB</td>
<td>Energy balance—SWE, SST, SPT</td>
<td>$P, T, WS, RH, Q_{si}^{<strong>}, Q_{li}^{</strong>}$</td>
<td>7 process-related, type of soil, type of vegetation</td>
<td></td>
</tr>
<tr>
<td>SPONSOR</td>
<td>Energy balance—SWE, SD, SST, SPT</td>
<td>$P, T, WS, RH, Q_{si}^{<strong>}, Q_{li}^{</strong>}$</td>
<td>7 process-related, type of soil, type of vegetation</td>
<td></td>
</tr>
</tbody>
</table>

*SWE — snow water equivalent, SD — snow depth, SST — snow surface temperature, SPT — snowpack and soil temperature, $P$ — precipitation, $T$ — air temperature, $WS$ — wind speed, $E$ — saturation vapour pressure, $RH$ — relative humidity, $Q_{si}$— incoming solar radiation, $Q_{li}$— incoming long-wave radiation.

**If not measured, $Q_{si}$ and $Q_{li}$ are calculated.

### Table 2

<table>
<thead>
<tr>
<th>Model</th>
<th>Simulated output</th>
<th>Evaluation of results</th>
</tr>
</thead>
<tbody>
<tr>
<td>TI</td>
<td>SWE</td>
<td>Ondrašová 2006 — problem with short snowmelt episode; Červenec-forest underestimated in winter 2006</td>
</tr>
<tr>
<td>TWI</td>
<td>SWE</td>
<td>As TI.</td>
</tr>
<tr>
<td>EXT</td>
<td>SWE</td>
<td>Similar to TI, better simulation for Ondrašová 2006.</td>
</tr>
<tr>
<td>UEB</td>
<td>SWE</td>
<td>The best simulations for Ondrašová 2006 and Červenec-forest; otherwise similar to TI</td>
</tr>
<tr>
<td></td>
<td>SST</td>
<td>Very good for Červenec-forest, worse for Ondrašová and Červenec-open area</td>
</tr>
<tr>
<td></td>
<td>SPT</td>
<td>Comparable with measurements, especially in the forest</td>
</tr>
<tr>
<td>SPONSOR</td>
<td>SWE</td>
<td>Better for Ondrašová than TI, very good for Červenec-open area (2006), overestimated for Červenec-open area (2007), no simulations for other sites</td>
</tr>
<tr>
<td></td>
<td>SD</td>
<td>Very good for Ondrašová and Červenec-open area 2006, overestimated for Červenec-open area 2007</td>
</tr>
<tr>
<td></td>
<td>SST</td>
<td>Similar to UEB</td>
</tr>
<tr>
<td></td>
<td>SPT</td>
<td>Worse than the results of UEB</td>
</tr>
</tbody>
</table>
Fig. 3. Selected input data (P — precipitation, T — air temperature) and results of simulations at Ondrašová (a), Červenec-open area (b), and Červenec-forest (c)

1 — measured values; calculated by models: 2 — UEB, 3 — SPONSOR, 4 — TI, 5 — EXT; 6 — measured runoff in the Jalovecky creek. SST — snow surface temperature, SPT — snow and soil temperature, SD — snow depth, SWE — snow water equivalent; TI — temperature index model, EXT — the model using the extended combined approach (energy balance)

Рис. 3. Примеры входящих параметров (P — осадки, T — температура воздуха) и результаты моделирования для долины (Ондровава) (a), открытого участка (Червенец) (b) и горного леса Червенец (c)

1 — измеренные значения; 2 — 5 — полученные по моделям: 2 — UEB, 3 — SPONSOR, 4 — TI, 5 — EXT; 6 — измеренный сток в ручье Jalovecky. SST — температура поверхности снежного покрова, SPT — температура почвы и снежной толщи, SD — толщина снежного покрова, SWE — водный эквивалент снежного покрова; TI — модель индексированной температуры, EXT — модель, использующая расширенный комбинированный подход (энергетический баланс)
models, although UEB provided similar results. Snow surface temperatures from SPONSOR for Ondrašová were better than the UEB simulations. On the other hand, UEB provided better simulations of integrated soil and snow temperatures (SPT). Measured SPT data were calculated as mean values of soil temperatures at the depth of 10 cm and snow temperatures measured every 10 cm in the snow pits. At Červeneč-forest there are no soil temperature measurements. Measured SPT data were therefore calculated only from snow pit measurements.

SWE simulations of SPONSOR and UEB at Červeneč-open area were similar (see Fig. 3, b). SPONSOR simulated Snow depths (SD) very well except for the winter 2007 at Červeneč-open area where the simulated SD and SWE were significantly overestimated, maybe due to problems of interpretation of the precipitation type.

Runoff data from the Jalovecký creek (the gauge is situated at catchment outlet at altitude 800 m a.s.l., catchment altitudes vary from 800 to 2178 m a.s.l., mean 1500 m a.s.l.) indicate that the first runoff event at the beginning of April 2006 and in January 2007 (an unusual occurrence) were caused by snowmelt (maybe combined with rainfall) at lower altitudes.

Conclusion
The study confirmed that the index models can provide acceptable estimates of snow water equivalents under different conditions of snow cover formation. They can therefore be used in practical applications where very little input data is available. However, if the snow water equivalents rapidly change, the energy-based models could give better results. The more sophisticated energy-balance models provide also other snow characteristics (e.g. snow temperatures, snow depths) which can help in the research oriented studies. However, in all applications the models should be carefully validated using measured snow characteristics. The problem of model validation is beyond the objectives of this study as well as the discussion on the pros and cons of simple versus sophisticated models. However, the study highlighted the importance of availability of suitable measured data for both model development and validation. High quality meteorological and especially snow data can be an issue even at research stations.

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ВОСПРОИЗВЕДЕНИЕ ВОДНОГО ЭКВИВАЛЕНТА СНЕЖНОГО ПОКРОВА МАТЕМАТИЧЕСКИМИ МОДЕЛЯМИ РАЗНОЙ СТЕПЕНИ СЛОЖНОСТИ

Представлен краткий обзор развития моделирования накопления и таяния снежного покрова и проведено сравнение результатов, полученных по нескольким моделям разной степени сложности. Модели опробовались для трёх участков в северной Словакии — долины (абсолютная высота 570 м), открытого пространства в горах (1500 м) и горного леса (1420 м). Для моделирования использовались данные двух контрастирующих зим 2006 и 2007 гг. За исключением вневзволнных эпизодов снеготаяния, простоявшая модель (градусы/дни) показала сходные результаты воспроизведения водного эквивалента снежного покрова с полученными по основанной на энергетическом потоке модели EXT (расширенный комбинированный подход).

В целом наилучшее воспроизведение водного эквивалента снежного покрова было получено по более сложным, основанным на энергетическом потоке моделям UEB и SPONSOR. Последние также позволяли получить температуру поверхности снежного покрова и температурный профиль от почвы через снежную толщу. Наибольшая точность была достигнута с использованием модели UEB для залёгшего участка. Воспроизведение температурного профиля по модели UEB было несколько лучше, чем по модели SPONSOR, а полученная с использованием модели SPONSOR толщина снежного покрова в большинстве случаев хорошо совпадала с измеренными величинами. Результаты свидетельствуют о необходимости детальных данных по снежному покрову и метеопараметрам для проверки и создания моделей.