

## Comparison of Turbulence Models in the Calculation of Supersonic Separated Flows

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**Abstract:** Air-jet engines are designed with the next generation of calculations on a wide range of speed and altitude, which leads to the use of new technical solutions. Gap generator nozzle ejectors, diffusers with a sudden expansion flow of self-regulation provide a wide range of design parameters. The currents in these devices is accompanied by flow separation from the walls and on certain modes of shock-wave structure vibrations. Calculation of supersonic separated flows is at present some methodological difficulties. Practical application of turbulence models implemented in modern computer packages, the subject of this article.

**Key words:** Base pressure • Base drag • Separated flows • Flow with sudden expansion • Model of turbulence • Shock-wave structure

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### INTRODUCTION

The most detailed review of turbulence models in the application to the calculation of the real air-breathing jet engines is given in the monograph [1]. Like a number of other review articles, this work examines mainly subsonic and transonic flows. The following models of turbulence have been pre-selected for testing in this article: standard  $k$ - $\epsilon$  model of Launder-Sharma,  $k$ - $\epsilon$  Realizable, RNG  $k$ - $\epsilon$ , standard SST  $k$ - $\omega$  model and Transition SST  $k$ - $\omega$  model, which are implemented in Ansys Fluent software package from version 6 and in Ansys CFX from version 11.

Although these models are widely used for a variety of engineering problems, they are still the subject of active research. Basic relations and values of empirical constants of the above turbulence models can be found in [2, 3].

The basis of all two-parameter models of turbulence using Reynolds averaged Navier-Stokes [1], is the Boussinesq hypothesis [2] on the turbulent (eddy) viscosity. It assumes proportionality of Reynolds stress tensor to arithmetic mean of strain rate tensor. This means that the turbulence affects the averaged turbulent flow the same way as the molecular viscosity affects the laminar flow. For simple cases, such as flat boundary layers, in the absence of large pressure gradients, the Boussinesq

assumption approximately satisfied. In complex flows, for example, with a large curvature of the streamlines, or separation and subsequent addition of the boundary layers, the Boussinesq hypothesis does not apply. This creates problems with the calculation of strongly swirling flows and flows where the effects of streamlines curvature are very important. Separated supersonic turbulent base flows in the tracts of future jet engines just refer to the above case.

**Realizable  $k$ - $\epsilon$  model:**  $k$ - $\epsilon$  realizable model proposed in [4]. The term “realizable” means that the model satisfies certain mathematical constraints on the normal stresses, consistent with the physics of turbulent flows (in the calculation of high-gradient streams negative values of eddy viscosity are excluded). An improved method for calculating the turbulent viscosity is introduced in this model and the equation for the dissipation rate is derived from the exact transport equation rms of the fluctuating component of the vorticity.

Compared to the standard version of  $k$ - $\epsilon$  model,  $k$ - $\epsilon$  realizable model more accurately predicts the distribution of the dissipation rate of flat and round jets and also provides a better prediction of the boundary layers characteristics in a large pressure gradients, separated and recirculating flows.

**Rng K-[Epsilon] Model:** In the standard k-[epsilon] model turbulent viscosity is determined by a single characteristic linear turbulence scale, but in reality all scales of motion contribute to turbulent diffusion. RNG k-[epsilon] model is developed by Yakhot *et al.* [5] using Re-Normalisation Group (RNG) mathematical methods, to take account of the turbulent motion different scales existence. The method shows higher accuracy in the simulation of flows in rotating cavities [1].

**K-[Omega] Turbulence Model:** K-[omega] model similar to the k-[epsilon], but it uses equation for the turbulent energy dissipation rate [omega] instead of dissipation equation. The variable [omega] determines the characteristic linear turbulence scale and k determines the energy of turbulence. This model describes well the near-wall flows, including those with the large pressure gradients. It has, however, problems in the calculation of jet streams. The most important of these is extreme sensitivity of the k-[omega] model to the boundary conditions in the external flow and the initial conditions of the turbulence level.

**Standard Sst K-[Omega] Model and Transition Sst Model:** K-[omega] SST (Shear Stress Transport) turbulent model was introduced in 1993 by Menter [6] and immediately became very popular. This model is essentially a union of the two models (k-[epsilon] away from the walls and the k-[omega] in the wall region). Researchers who use standard SST model, typically find that it shows good results in mixing layers at medium pressure gradients.

In areas with high normal stresses such as congestive flow standard SST k-[omega] model generates too high turbulence levels, which leads to a significant qualitative change in the calculated flow pattern [5].

Transition SST model allows us to describe turbulence more accurately due to the introduction of additional transport equations.

**The Results of Turbulence Models Testing:** The stream flowing from the Laval nozzle into a cylindrical coaxial channel with a sudden expansion is a good model of more complex structures that are used in the perspective jet engines, because it contains all the characteristic elements of these trends: the area with the recirculating flow, interaction with the walls of the mixing layers, separation of the boundary layer. The flow can also be non-stationary during certain stages.

The main task of the calculation of supersonic flows with a sudden expansion of the flow is to determine average values of the static pressure in the areas of recirculating flow, known as the base pressure, depending on the total pressure upstream of the nozzle. In this article we compared the turbulence models with the experimental data for the jets flowing from the nozzles with  $Ma = 2$  and  $3$ , half-angles  $[\theta] a = 8$  and  $15$  degrees, diameter of the critical cross section  $d = 10.6$  mm to coaxial cylindrical bore with diameter  $d=85$  mm and  $4.6$ ,  $6$  and  $13.8$  calibers long. The calculations were performed by relaxation method. Full pressure was set upstream of the nozzle, then intermediate result was recorded and then total pressure was increased by  $5$  atm, while the results obtained in the previous step were used as initial conditions. Thus, we obtain the dependence of bottom pressure from the total pressure upstream of the nozzle. Convergence of the difference grid and the effect of local grid refinement at the walls were investigated.

Figure 1 shows a comparison of simulation results with experimental data. It can be seen that there is an area of the ambiguity on the  $P_b-P_o$  plot corresponding to low frequency fluctuations. The upper and lower curves on the graph mark maximum and minimum values of  $P_b$  for one oscillation cycle at given total pressure upstream the nozzle  $P_o$ . Arrows indicate the hysteresis of the dependence of  $P_b$  on  $P_o$  during increase or discharge of pressure upstream of the nozzle.

The calculations showed that the modes with the open bottom area, when the separated supersonic flow does not interact with the walls of the channel and gas flows from the surrounding environment the bottom region, k-[epsilon] realizable model demonstrates better accuracy. It allows the use of relatively coarse difference mesh. Increase in the number of cells from  $40,000$  to  $160,000$  in the axisymmetric formulation of the problem does not significantly change the results. On modes with a closed bottom area, when the main stream accumulates on the walls of the channel and there is no reverse flow, k-[epsilon] realizable model systematically lowers the value of bottom pressure. This is due to the fact that under conditions of high pressure gradients k-[epsilon] realizable model overstates level of turbulence and, consequently, the value of the turbulent viscosity in the recirculation zones. As a result, ejection capability of the mixing layer improves at the boundary plane and the balance set at a lower value of the pressure in the bottom region.

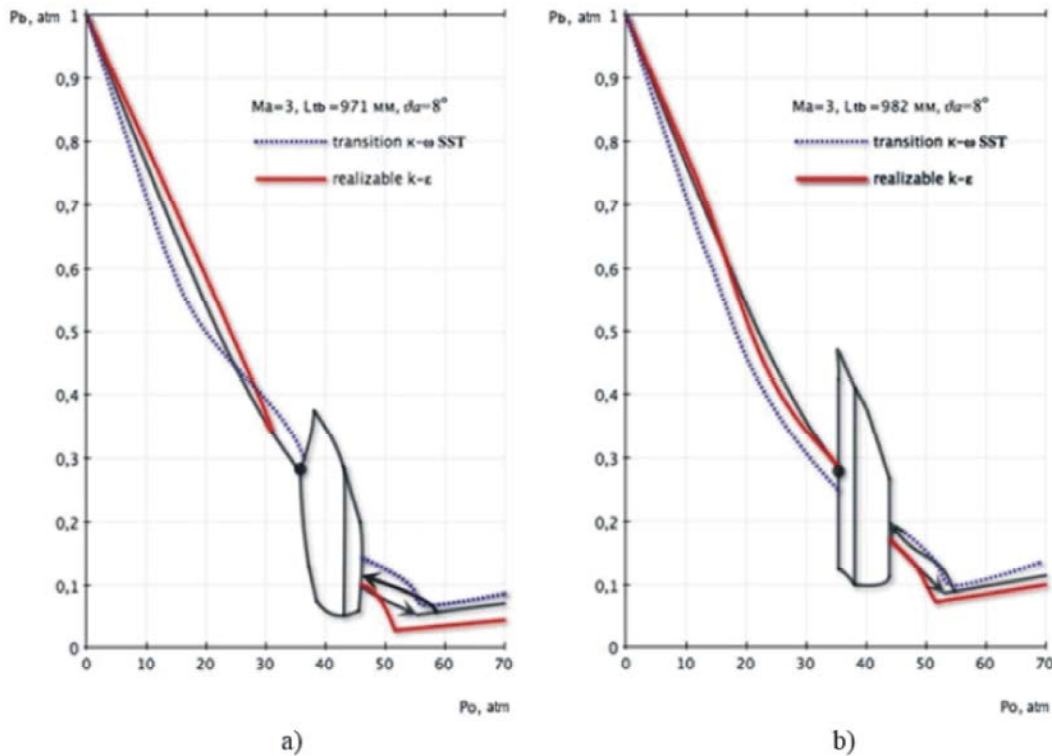


Fig. 1: Comparison of theoretical and experimental results (black solid line) for the gas outflow into a cylindrical bore.  $M_a=3$ ,  $[\theta] \alpha = 8^\circ$

Transition SST turbulence model, on the contrary, by using a relatively coarse regular difference grid lowers the value of the bottom pressure on modes with an open bottom area and overstates in the modes with attachment of the jet to the wall of the channel. Refinement of the grid near the walls of the nozzle and the channel allows considerably better calculate the return flow in the annulus between the main stream and the channel walls and in the area of turning of the jet mixing layer at the point of accumulation on the wall. The error decreases significantly, but still remains systematic.

The rest of turbulence models show the worst results.

### CONCLUSION

The paper considers various models of turbulence. The benefits of each turbulence model in the solution of certain problems were identified during the research.

**Findings:** Testing has shown that the k- $[\epsilon]$  realizable and transition SST turbulence models give the best results in the calculation of supersonic flows, typical for advanced jet engines.

These turbulence models cannot be directly used for the calculation of supersonic flows in the unsteady conditions [10, 11], as in their derivation uses the average flow parameters over time. However, there are low-frequency vibrations, which belong to quasi-stationary class [12], i.e. whose period is significantly longer than the characteristic time of the occurring gas-dynamic processes. The possibility of using the above turbulence models for the calculation of such fluctuations requires further study [13].

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