Cross-modulation coupling of incoherent soliton modes in photorefractive crystals

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A problem of nonlinear wave propagation through a photorefractive crystal (PRC) with drift nonlinearity has been evaluated. A class of spatially localized solitonlike solutions with finite energy has been found. Solutions of this class can be considered as multicomponent solitons, combined by two or more mutually incoherent self-consistent components bonded by cross-modulation coupling. Light field spatial distributions of the components look like zero and higher order modes of their common waveguide formed in PRC due to its nonlinearity. We have shown such a soliton is stable on distances of several diffraction lengths and its spatial structure is robust to collisions and appreciable (more than 10% in intensity) stochastic perturbations of the intensity distributions. With taking into account saturation of PRC nonlinearity, parameters of all the soliton components (their amplitudes and widths) change quasiperiodically as the soliton propagates. The components do not exchange energy while a small fraction of the energy is emitted on few diffraction lengths. We discuss also a possibility of excitation of incoherent solitonlike multielectron states in conjugated polymers, ferromagnetics, and superconductors. These states could correspond to some mutually incoherent and self-consistent wave packets composed of electronic wave functions and propagating along conjugated chains or prominent atomic planes. [S1063-651X(98)09804-3]

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I. INTRODUCTION

One of the most exciting problems of modern laser physics is an investigation of self-organization in systems consisting of a nonlinear medium and a light field. For the simplest case of a medium with spatially localized (local) nonlinear response—the case of so-called ''Kerr-type'' nonlinearity [1], stable self-consistent solutions (solitons) of problems of such kind have been studied rather well. The concepts of one-component [2] and two-component (''vector'') [3] solitons as self-consistent spatially localized solutions (eigenfunctions) of many nonlinear problems have solidly clamped in quite different fields of modern physics. Fiber optics and optics of ultrashort laser pulses [1,4,5], nonlinear optics and laser spectroscopy [6–8], physics of one-dimensional (1D) chains and two-dimensional (2D) atomic planes in ferromagnetics [9,10], high-temperature superconductors (HTSC) [11], conjugated polymers [12–14], etc. may be listed among these fields. From the model point of view, results, obtained in recent investigations of solitons, multisolitons, and stable soliton pairs in photorefractive crystals (PRCs), are of great importance. The reason for such great activity in this field is a very strong optical nonlinearity of PRCs. Perceptible manifestations of nonlinear effects can be observed here under illumination by laser beams with intensities about only some mW/cm^2 (Ref. [15]). Starting from the pioneer papers [16–18], related to PRCs with drift (local) nonlinear response [19], so-called ''bright'' [20–22], ''dark'' [22,23], ''gray'' [24], vector [25] and ''vortex'' [26] solitons, multisolitons [27], some questions of such soliton propagation, interactions [28,29], spatial dimensionality [30,31], and stability [32–34] were intensively studied. It was shown, for example, that in PRC one can ''write'' a stable solitonlike waveguide, which is able to capture relatively week (in intensity) light beams [27,35]. Moreover, when the wavelengths of intensive light beams are not thrown in the photorefractive sensitivity range, such beams can be captured in waveguides written in PRCs by low-intensity spatial solitons too. The stable pairs of two incoherent spatial solitons of any of the types listed above [36,37] were observed. All the obtained results open wide prospects for controlling light by light in transmitting and processing of optical information.

The main goal of the paper is the presentation of a class of spatially localized stable solitonlike light beams, propagating through PRC with drift (local) nonlinear response. A particular case of such a solution—a specific kind of two-component vector soliton—has been considered before for the case of nonlinear media with Kerr-type nonlinearity [38,39]. The multicomponent screening soliton of this class has a finite energy and corresponds to two or more bonded and mutually incoherent light beams—the components. In relation to the character of light field distribution, the components correspond to some zero and higher order modes, confined within the nonlinear waveguide formed in PRC by themselves. Interaction of the bonded modes is of pure cross-modulation (reactive) character and is not accompanied by the energy exchange.

II. MODEL

A basis of our model is a steady-state solution of a well-known system of material equations [19] for the internal electric field $E_{sc}(x,z)$. In the 2D case without taking into account a photovoltaic effect and saturation of PRC nonlinearity, this system solution can be written in a very simple form [40]

$$E_{sc}(x,z) = -\frac{E_0}{I_0}\, I(x,z). \tag{1}$$

Here I_0 describes the so-called dark conductivity; $I(x,z)$ is the radiation intensity. We suppose the external static electric field $E_0 \sim 10$ kV/cm is applied to PRC in the transverse direction (along the x axis), the optical radiation propagates along the longitudinal z axis, $I(x,y)\ll I_0$, and spatial scale of changing light field is not less than the wavelength. It means our analytical consideration is limited by the local component of PRC nonlinearity without taking into account its saturation, i.e., by the simplest case of steady-state screening of the external electric field E_0 when PRC can be described as a nonlinear medium with Kerr-type nonlinearity [40]. Therefore, all types of multicomponent solitons considered here are so-called screening solitons [22,24].

We describe a propagating light beam with the complex amplitude $A(x,z)$ by a standard shortened wave equation [1]

$$i\,\frac{\partial A}{\partial z}=\frac{1}{2k}\,\frac{\partial^2 A}{\partial x^2}+k\,\frac{\delta\eta}{\eta}\,A, \tag{2}$$

written in the paraxial approximation with no regard for absorption. Here k is the wave number; $\delta\eta=-(r_{\mathrm{eff}}\eta^3/2)E_{sc}(x,z)$ is the nonlinear addition to the refractive index η; r_{eff} is the effective electro-optical constant. In Eq. (2), we neglect the uniform (in the x direction) change of the refractive index induced by E_0. Equations (1) and (2) form the simplest version of the self-consistent problem, which takes into account the redistribution of $I(x,z)$ and $E_{sc}(x,z)$ in the Kerr-type approximation of PRC nonlinearity. The 2D model describes experiments with so-called ''slit'' beams [41], which are widely used in practical study of photorefractive solitons due to a large anisotropy of PRC nonlinearity. The role of saturation effects will be discussed later (Secs. III and IV) when we proceed to our computer calculation results. In this case, Eq. (1) is modified by introducing an additional denominator in the form $1+I(x,z)/I_0$ (see, for example, [24]).

III. TWO-COMPONENT SOLITONS

In this section, we summarize and classify some known results concerned with the possible spatial structure of two-component (vector) solitons. It should be noticed that these results have been mainly obtained in a time domain, i.e., in connection with problems of ultrashort pulse propagation through a nonlinear optical fiber [38,39,42–44]. We also present here some results related with the interaction between two two-component solitons in the Kerr-type approximation of PRC nonlinearity as well as with influence of PRC nonlinearity saturation on the soliton propagation.

Let us first find two-component solitonlike solutions of the system, combined with Eqs. (1) and (2). We will be interested in the case of separating variables

$$A(x,z)=Y_1(x)\exp(-i\nu_1 z)+Y_2(x)\exp(-i\nu_2 z). \tag{3}$$

Here the real functions $Y_{1,2}(x)$ determine spatial distributions of two mutually incoherent components of light field along the x axis; the positive constants $\nu_{1,2}$ define nonlinear phase shifts of the components along the z axis. That means we try to find steady-state (on z) distributions of the intensity. With taking into account the mutual incoherence of the components $Y_{1,2}$, substitution of Eq. (3) into Eqs. (1) and (2) results in the system of equations for spatial profiles of the components' amplitudes in the Kerr-type approximation of PRC nonlinearity

$$\frac{d^2Y_{1,2}}{dx^2}+2k\left[\frac{\alpha_0E_0}{I_0}\,(Y_1^2+Y_2^2)-\nu_{1,2}\right]Y_{1,2}=0, \tag{4}$$

where $\alpha_0=\frac{1}{2}kr_{\mathrm{eff}}\eta^2$. To normalize Eq. (4), let us introduce the dimensionless coordinates $\xi=x/x_0$, $\zeta=z/L_d$, and the amplitudes of the components $\rho_{1,2}(\xi)=Y_{1,2}(\xi)\sqrt{R/I_0}$. Here x_0 is determined by a specific transverse scale of the problem, for example, by the width of one of the light beams; $L_d=kx_0^2$ is the diffraction length corresponding to x_0; $R=L_d/L_r$; $L_r=|\alpha E_0|^{-1}$ is the nonlinear refraction length. Further, most of the illustrative material of our paper will be presented as 3D dependencies of absolute values of dimensionless amplitudes $|\rho_i|$ $(i=1,2,...)$ of the components on the plane of dimensionless spatial coordinates (ξ,ζ). After normalization, Eq. (4) transfers into the system of equations regarding the normalized amplitudes

$$\frac{d^2\rho_{1,2}}{d\xi^2}\pm2[(\rho_1^2+\rho_2^2)-\beta_{1,2}]\rho_{1,2}=0, \tag{5}$$

where $\beta_{1,2}=L_d\nu_{1,2}$; the signs ''+'' and ''−'' correspond to the cases of focusing $(\delta\eta>0)$ and defocusing $(\delta\eta<0)$ nonlinearities. Both of the cases can be realized under a corresponding choice of PRC orientation and E_0 direction [19]. It is easy to check the system (5) conserves the same character in the case of counterpropagating light beams.

Depending on the sign, the system (5) has well-known particular solutions looking like one-component bright [20,21],

$$\rho_1=\pm\sqrt{2\beta_1}/\cosh(\sqrt{2\beta_1}\xi),\quad \rho_2=0\quad\text{and}\quad \rho_1=0,$$
$$\rho_2=\pm\sqrt{2\beta_2}/\cosh(\sqrt{2\beta_2}\xi), \tag{6}$$

or dark [22],

$$\rho_1=\pm\sqrt{\beta_1}\,\tanh(\sqrt{\beta_1}\xi),\quad \rho_2=0\quad\text{and}\quad \rho_1=0,$$
$$\rho_2=\pm\sqrt{\beta_2}\,\tanh(\sqrt{\beta_2}\xi), \tag{7}$$

photorefractive screening solitons. Solutions (6) and (7) are stable [45], while in any real experiment the needed 1D diffraction and self-interaction can be realized only by using slit beams with uniform field distribution in direction orthogonal to ξ and ζ [41]. In doing so, one must take into account a possibility of beam filamentation, which can develop on PRC

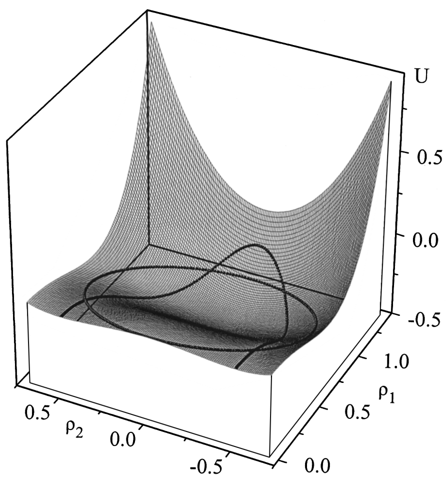

FIG. 1. Trajectories of screening soliton (11), composed by two mutually incoherent bright and dark components, and a two-component soliton (13) (open and closed solid lines, correspondingly): self-focusing nonlinearity; ρ_1, ρ_2, and U are dimensionless amplitudes of the soliton components and the potential energy; $\beta_1 = 1$, $\beta_2 = 0.25$.

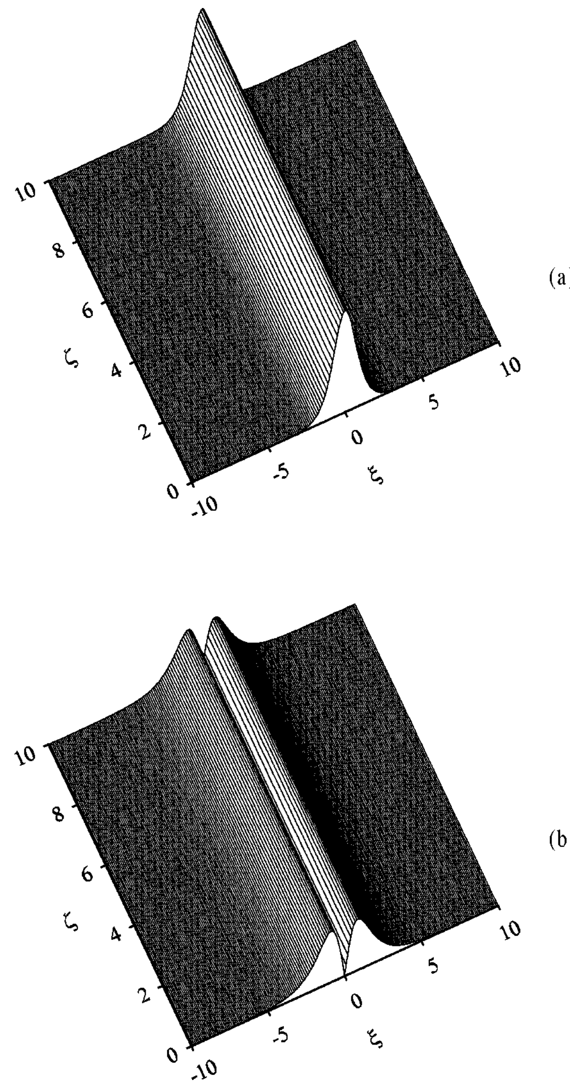

FIG. 2. Spatial distributions $|\rho_1(\xi)|$ (a) and $|\rho_2(\xi)|$ (b) of a two-component soliton (13) propagating along the ζ axis: ξ and ζ are the dimensionless spatial coordinates; $\beta_1 = 1$, $\beta_2 = 0.25$.

length due to modulation instability [46]. Stability of 2D solitons is achieved by saturation of PRC nonlinearity [47,48].

To classify two-component self-consistent solutions of more general character, one may use the following way. Let us consider an analogy between the dependencies $\rho_{1,2}(\xi)$, which are determined by the system (5), and nonlinear oscillations of two coupled oscillators in the common dimensionless potential

$$U(\rho_1,\rho_2) = \pm[\tfrac{1}{2}(\rho_1^2+\rho_2^2)^2 - (\beta_1\rho_1^2+\beta_2\rho_2^2)]. \tag{8}$$

As earlier, the signs ''+'' and ''−'' correspond to the cases of focusing and defocusing nonlinearities, correspondingly. On the plane (ρ_1,ρ_2), the dimensionless potential (8) has some local extrema corresponding to specific points of its phase portrait. However, suggesting (for definiteness) that $\beta_1 > \beta_2$ and considering the self-focusing case, it is easy to check that only two absolute minima $U(\rho_1,\rho_2)$ with the coordinates $\rho_1 = \pm\sqrt{\beta_1}$, $\rho_2 = 0$ are the points of stable equilibrium focuses, and the only local maximum $\rho_1 = \rho_2 = 0$ is a nodal point. All other specific points are of a saddle character (Fig. 1). In the degenerate case $\beta_1 = \beta_2 = \beta$, all the points with the coordinates $\rho_1^2 + \rho_2^2 = \beta$, which are located on the bottom of a ''valley'' of the potential (8), correspond to an indifferent equilibrium.

Except the one-component bright and dark solitons (6) and (7), there are also some known two-component solutions of Eq. (5) that can be written by elementary functions. For example, if $\beta_1 = \beta_2 = \beta$, such solutions of Eq. (5) are well-known trivial stable pairs—Manakov solitons [3]—formed by two incoherent bright (the self-focusing case)

$$\rho_1 = [\sqrt{2\beta}/\cosh(\sqrt{2\beta}\xi)]\cos(\alpha),$$

$$\rho_2 = [\sqrt{2\beta}/\cosh(\sqrt{2\beta}\xi)]\sin(\alpha) \tag{9}$$

or dark (the self-defocusing case)

$$\rho_1 = \sqrt{\beta}\tanh(\sqrt{\beta}\xi)\cos(\alpha), \quad \rho_2 = \sqrt{\beta}\tanh(\sqrt{\beta}\xi)\sin(\alpha) \tag{10}$$

solitons. Such stable pairs, which have been first realized as a stable pair of laser pulses with orthogonal polarizations in nonlinear waveguides [49], have been observed recently in PRC [35]. In dimensionless coordinates (ρ_1,ρ_2,U), both the solutions are the projections (on axis $\rho_{1,2}$) of corresponding one-component solitons (6) and (7) with trajectory planes rotated around U symmetry axis by the angle α.

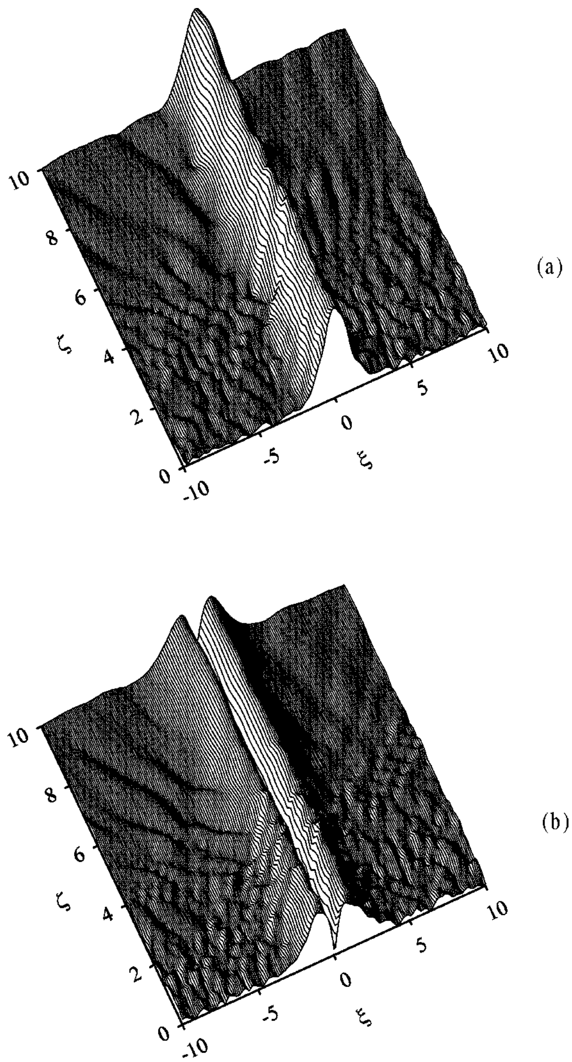

FIG. 3. Stability of spatial distributions $|\rho_1(\xi)|$ (a) and $|\rho_2(\xi)|$ (b) of a two-component soliton (13): light field components are perturbated by a Gaussian noise (10% in intensity); ξ and ζ are the dimensionless spatial coordinates; $\beta_1=1$, $\beta_2=0.25$.

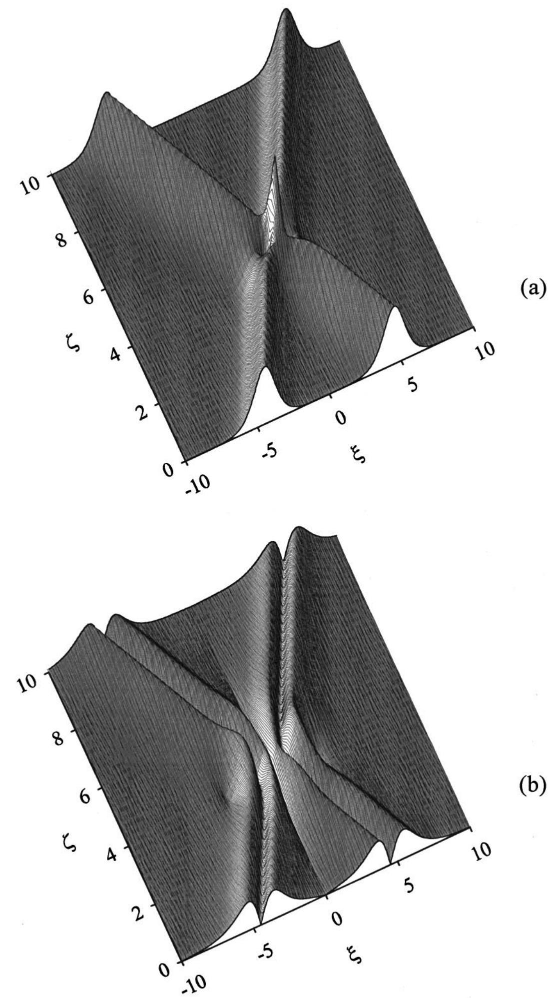

FIG. 4. Stability of spatial distributions $|\rho_1(\xi)|$ (a) and $|\rho_1(\xi)|$ (b) of a two-component soliton (13): crossing with the same two-component soliton; ξ and ζ are the dimensionless spatial coordinates; $\beta_1=1$, $\beta_2=0.25$.

When $\beta_1>\beta_2$, well-known [42–44,50,51] stable pairs can be formed by incoherent bright and dark solitons both in the self-focusing case

$$\rho_1=\pm\sqrt{2\beta_1-\beta_2}/\cosh[\sqrt{2(\beta_1-\beta_2)}\xi],$$

$$\rho_2=\pm\sqrt{\beta_2}\tanh[\sqrt{2(\beta_1-\beta_2)}\xi], \tag{11}$$

and in the self-defocusing one

$$\rho_1=\pm\sqrt{\beta_1}\tanh[\sqrt{2(\beta_1-\beta_2)}\xi],$$

$$\rho_2=\pm\sqrt{2\beta_2-\beta_1}/\cosh[\sqrt{2(\beta_1-\beta_2)}\xi]. \tag{12}$$

The latter solution exists only when $\beta_1<2\beta_2$. Such stable pairs, which have been firstly predicted for the problem of propagation of two ultrashort laser pulses with orthogonal polarizations through nonlinear fibers [42,43], have been also observed in PRC [5,36]. In the focusing case, the soliton trajectory starts ($\xi\to-\infty$) from the unstable equilibrium point $(0,\pm\sqrt{\beta_2})$ [the local maximum on the bottom of the potential (8) valley] and comes ($\xi\to+\infty$) to the symmetrical point $(0,\mp\sqrt{\beta_2})$ (an open trajectory in Fig. 1). On the plane (ρ_1,ρ_2) the trajectory looks like a half-ellipse with half-axes $\sqrt{2\beta_1-\beta_2}$ and $\sqrt{\beta_2}$ and, depending on the sign of its asymptotes, lies either in the half-plane $\rho_1\geqslant 0$ (Fig. 1) or in the half-plane $\rho_1\leqslant 0$.

All the listed above two-component solitonlike solutions of Eq. (5) are self-consistent pairs of bright and dark solitons. Moreover, if such a pair includes a dark soliton, the pair energy is infinitely large. That is why some types of known two-component solitons for PRC are of mainly methodical interest. For the case of self-focusing nonlinearity of PRC, we predict a possible type of two-component soliton with a finite energy. It should be noticed that this self-consistent solution of the nonlinear Schrödinger equation has been known as a solution of the problem of propagation of two ultrashort laser pulses with orthogonal polarizations through a nonlinear waveguide with Kerr-type nonlinearity [36,39]. While an analytical expression for the two-component solution, which is under discussion here, is absolutely the same as, for example, that in [36] because of the approximations used (see Sec. II), computer calculations show that a solution of this type can be realized in PRC with saturation ($I_0 \sim I$) as well. However, we postpone discussion of this statement to the end of this section. In the same manner as for the solution (9), this soliton trajectory starts from and returns to the point (0,0) on the plane (ρ_1, ρ_2) (a closed trajectory in Fig. 1). However, in this solution, the component $\rho_2(\xi)$ is odd. At the same time, in contrast to the pair (11), the function $\rho_2(\xi)$ exponentially decreases with $\xi \to \pm\infty$. For $\beta_1 = 4\beta_2$, this new (of course, only for the case of photorefractive media) two-component solution can be written in an explicit form:

$$\rho_1 = \pm\sqrt{6\beta_2}/\cosh^2(\sqrt{2\beta_2}\xi),$$

$$\rho_2 = \pm\sqrt{6\beta_2}\,\sinh(\sqrt{2\beta_2}\xi)/\cosh^2(\sqrt{2\beta_2}\xi). \tag{13}$$

Figure 2 illustrates spatial profiles of two mutually incoherent symmetrical $\rho_1(\xi)$ [Fig. 2(a)] and asymmetrical $\rho_2(\xi)$ [Fig. 2(b)] components of the soliton (13) and their stable propagation on the distance $\zeta = 10$, which on a real experimental scale corresponds to a PRC length of about 5 cm. Numerical calculations show that, propagating through PRC without ρ_2, the symmetrical component ρ_1 transforms into a one-component bright soliton. At the same time, the asymmetrical component ρ_2 without ρ_1 gradually converts into a pair of one-component bright solitons moving apart (along ξ) and opposite in phase.

While it is known that vector solitons of such kind must be stable [45], it is important to estimate their stability range. Figure 3 illustrates stability of the soliton (13) structure on significant (10% in intensity) stochastic perturbations of input profiles of both field components $\rho_{1,2}$ by a Gaussian noise. Our numerical calculations show that increase of the noise level (up to 20% in intensity) results (in some runs) in the soliton (13) decay into a pair of bright solitons (6), which consequently have a larger stability regarding such perturbations. The next two figures illustrate structural stability of the soliton (13) regarding collisions (crossing) with the same two-component (Fig. 4) and one-component bright (Fig. 5) solitons.

Computer simulation enables us to follow the transforming spatial structure of such a soliton with changing β_1/β_2. We revealed that a limiting case of solution (13) for $\beta_1/\beta_2 \to 1$ looks like a pair of two-component bright solitons $[\rho_1^{(\alpha)}, \rho_2^{(\alpha)}]$ and $[\rho_1^{(\beta)}, \rho_2^{(\beta)}]$ infinitely separated along ξ (and, correspondingly, noninteracting) in the form of Eq. (9). Each such soliton consists of two mutually incoherent components $\rho_1^{(\alpha,\beta)}$ and $\rho_2^{(\alpha,\beta)}$. At the same time, corresponding components of both solitons $\rho_{1,2}^{(\alpha)}$ and $\rho_{1,2}^{(\beta)}$ are coherent in pairs. The components $\rho_1^{(\alpha)}$ and $\rho_1^{(\beta)}$ are in phase, whereas $\rho_2^{(\alpha)}$ and $\rho_2^{(\beta)}$ are opposite in phase. Increasing β_1/β_2 results in gradual approach of the solitons to one another along ξ. In the case of their full spatial coincidence (the limit $\beta_1/\beta_2 \to \infty$), the opposite-phase components $\rho_2^{(\alpha)}$ and $\rho_2^{(\beta)}$ ''quench'' each other through interference and the one-component bright soliton (6) forms. Obviously, such an analogy is very conditional because the shapes of two-component bright solitons of the pair gradually change with their closing due to cross-modulation interaction.

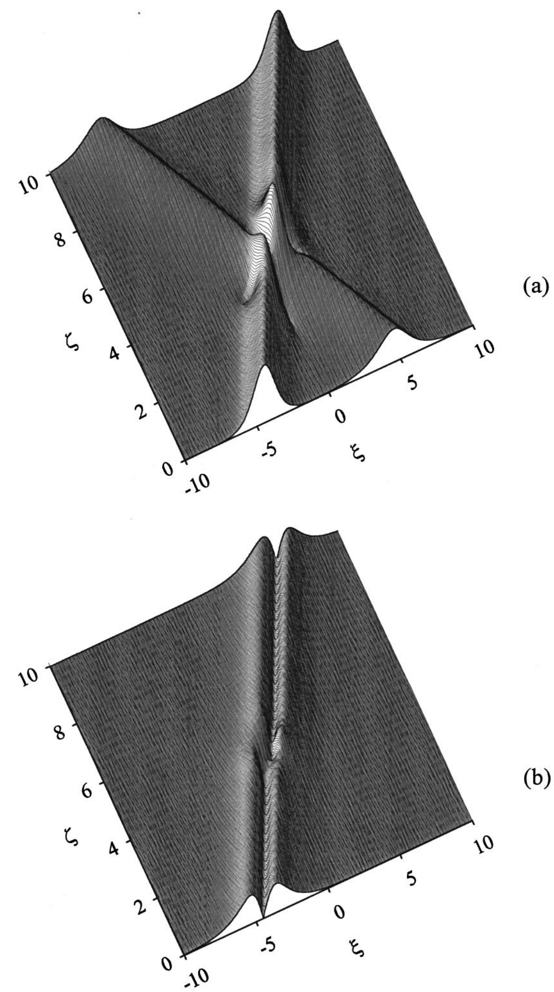

FIG. 5. Stability of spatial distributions $|\rho_1(\xi)|$ (a) and $|\rho_2(\xi)|$ (b) of two-component soliton (13): crossing with bright soliton (6); ξ and ζ are the dimensionless spatial coordinates; $\beta_1 = 1$, $\beta_2 = 0.25$.

So, for the case of photorefractive nonlinearity, the solution (13) is a new stable and realizable two-component spatial soliton related to the solutions with separating variables at least for the case when $I_0 \gg I$. However, results of our computer calculation of a much more realistic situation $I_0 \sim I$, when we took saturation of PRC nonlinearity into account, did not change this conclusion in principle. Figure 6 illustrates evolution of spatial profiles $\rho_{1,2}(\xi)$ of both components of two-component soliton (13) as they propagate through PRC. It is easy to see that the amplitudes and widths of both components change quasiperiodically. The components do not exchange by the energy while a small fraction of the energy is emitted on a distance about some diffraction lengths.

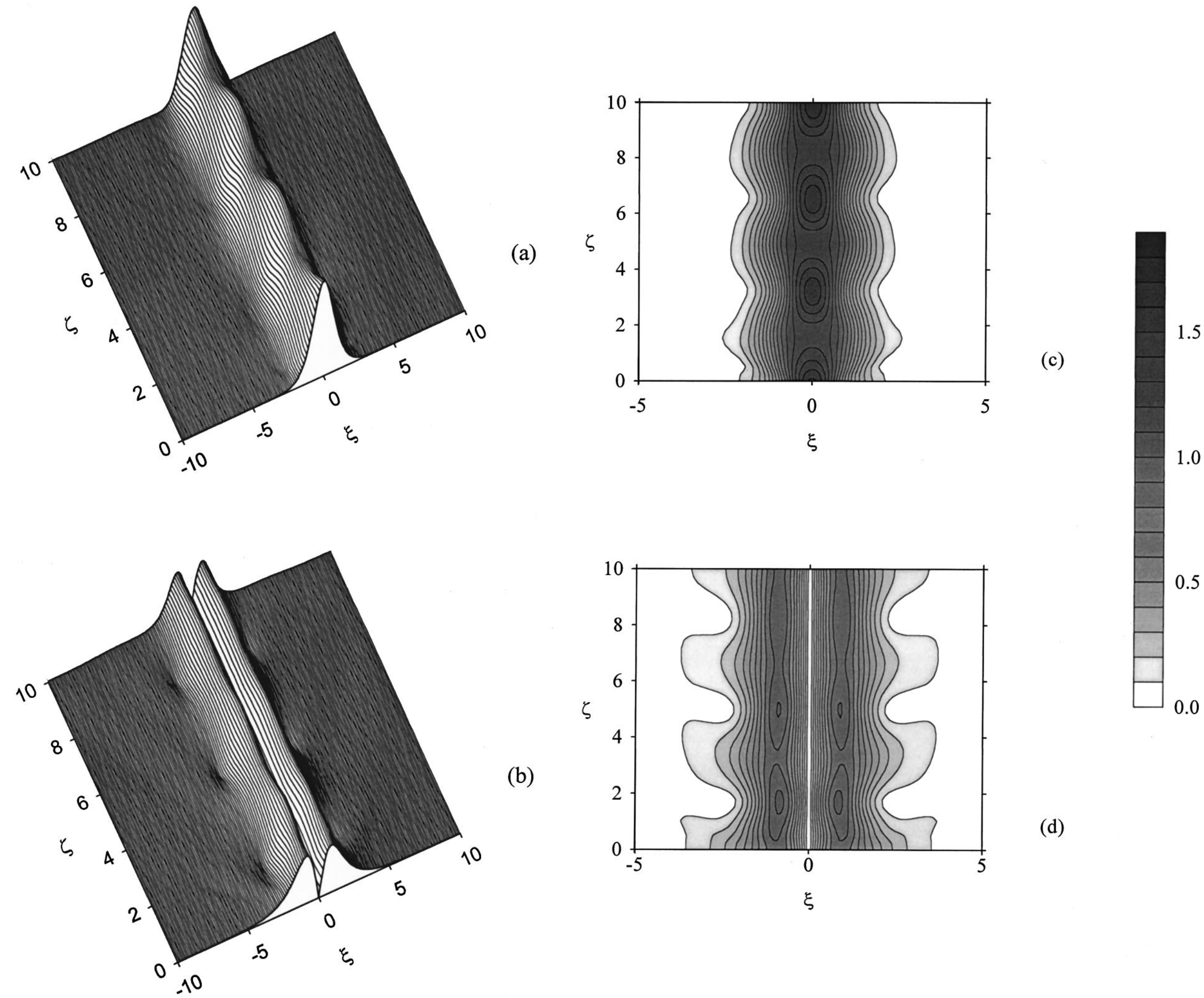

FIG. 6. Spatial distributions (a), (b) and isolines (c), (d) of $|\rho_1|$ (a), (c) and $|\rho_2|$ (b), (d) components of two-component soliton (13): saturation parameter $(\rho_1^2+\rho_2^2)_{\xi=0}/I_0=1$; ξ and ζ are the dimensionless spatial coordinates; $\beta_1=1$, $\beta_2=0.25$.

IV. SELF-CAPTURING OF HIGH-ORDER MODES

In this section, we present analytical and numerical results related to multicomponent solitons of a new class. We consider here the cases of photorefractive drift nonlinearity without and with saturation.

The two-component solution (13) describes cross-modulation capturing and propagating of two modes of zero and first order inside a stable nonlinear waveguide, written in PRC. In this section, we show that there are much more complicated stable solutions of the same type, i.e., there are multicomponent solitons composed of more than two incoherent self-consistent light components. All such components have a finite energy and spatially localized profiles, looking like profiles of high-order eigenmodes of their common nonlinear waveguide. As far as we know, multicomponent solitons of this type have not been discussed before even for the case of Kerr-type nonlinearity. In our opinion, the main reason for the lack of discussion is connected with some difficulties in their practical realization. Because two-component solitons of this type include only two components, one can simply choose their polarizations to be orthogonal [38,39,42,43]. In case of multicomponent solitons composed of three or more components, one must use another way to exclude their interference. One can use, for example, the soliton components at different carrier frequencies and a nonlinear medium with rather slow nonlinear response. In the case where the medium is not able to follow by the light field beats resulting from the interference, the situation will be fully equivalent to the case of mutually incoherent light field components. Namely, this case, which was considered some years ago to describe spatially localized electronic pairs in high-temperature superconductors [52] (so-called ''bisoliton'' model of high-temperature superconductivity) and in molecular chains [53], can be practically realized in PRC.

It is easy to check that a nonlinear waveguide, formed in a PRC by nonlinear interaction of the soliton (13) components, can be characterized by the following distribution of the refractive index:

$$\Delta\eta \propto \Delta\eta_{\max}/\cosh^2(\xi/\xi_0). \tag{14}$$

Here $\Delta\eta_{\max}=6\beta_2$ and $\xi_0=1/\sqrt{2\beta_2}$ describe the maximal value of nonlinear addition to η and the nonlinear waveguide width. Let us suppose that all the solutions of the considered class write (inside PRC) a nonlinear waveguide with a spatial profile analogous to Eq. (14). However, we will consider now $\Delta\eta_{\max}$ and ξ_0 as parameters, which have to be determined. Then, in the first step, the problem will be reduced to one of two rather well studied problems: calculating of localized states inside a potential well with shape corresponding to Eq. (14) [54] or finding of eigenmodes of an optical waveguide with specified refractive index profile, given by Eq. (14) [55]. In the second step, we will require the solitons, which will be found on the first step, write inside PRC a nonlinear waveguide with namely the used (on the first step) profile of the refractive index (14). This requirement will separate self-consistent solutions from all the solutions, which will be found in the first step, and enable us to determine the constants of corresponding expansion in a basis of the waveguide eigenfunctions, i.e., of its spatial modes. This approach has much in common with the approach developed by Snyder *et al.* [47].

Even in the first step, such a problem does not generally have an elemental solution and its eigenfunctions can be represented only by hypergeometric functions. However, a character of solutions is significantly simplified for discrete relations between $\Delta\eta_{\max}$ and ξ_0 [54,55]. In such cases, the eigenfunctions can be expressed through so-called Legendre associated polynomials [56] and the system of equations, which must be solved, can be written in the form

$$\frac{d^2\rho_i^{(n)}}{d\xi^2}+2\left[\frac{\beta_0 n(n+1)}{\cosh^2(\sqrt{2\beta_0}\xi)}-\beta_i\right]\rho_i^{(n)}=0, \tag{15}$$

where $n=1,2,...$ and $i=1,2,...,n$. The condition of self-consistent multicomponent soliton $[\rho_1^{(n)},\rho_2^{(n)},\ldots,\rho_n^{(n)}]$ can be determined as

$$\sum_{i=1}^{n}[\rho_i^{(n)}]^2=\frac{\beta_0 n(n+1)}{\cosh^2(\sqrt{2\beta_0}\xi)}. \tag{16}$$

Without stopping here on routine calculations, let us write only some first multicomponent solutions of this class for $n=1$, 2, 3, and 4

$$\rho_1^{(1)}=\sqrt{2\beta_0}/\cosh(\sqrt{2\beta_0}\xi), \quad \beta_1^{(1)}=\beta_0; \tag{17}$$

$$\rho_1^{(2)}=\sqrt{6\beta_0}/\cosh^2(\sqrt{2\beta_0}\xi), \quad \beta_1^{(2)}=4\beta_0,$$

$$\rho_2^{(2)}=\sqrt{6\beta_0}\,\sinh(\sqrt{2\beta_0}\xi)/\cosh^2(\sqrt{2\beta_0}\xi), \quad \beta_2^{(2)}=\beta_0; \tag{18}$$

$$\rho_1^{(3)}=\frac{3}{2}\sqrt{5\beta_0}/\cosh^3(\sqrt{2\beta_0}\xi), \quad \beta_1^{(3)}=9\beta_0,$$

$$\rho_2^{(3)}=\sqrt{30\beta_0}\,\sinh(\sqrt{2\beta_0}\xi)/\cosh^3(\sqrt{2\beta_0}\xi), \quad \beta_2^{(3)}=4\beta_0,$$

$$\rho_3^{(3)}=\left[\frac{1}{2}\sqrt{3\beta_0}/\cosh(\sqrt{2\beta_0}\xi)\right][4-5/\cosh^2(\sqrt{2\beta_0}\xi)], \quad \beta_3^{(3)}=\beta_0; \tag{19}$$

$$\rho_1^{(4)}=\sqrt{35\beta_0/2}/\cosh^4(\sqrt{2\beta_0}\xi), \quad \beta_1^{(4)}=16\beta_0,$$

$$\rho_2^{(4)}=\frac{3}{2}\sqrt{35\beta_0}\,\sinh(\sqrt{2\beta_0}\xi)/\cosh^4(\sqrt{2\beta_0}\xi), \quad \beta_2^{(4)}=9\beta_0,$$

$$\rho_3^{(4)}=\sqrt{5\beta_0/2}[6-7/\cosh^2(\sqrt{2\beta_0}\xi)]/\cosh^2(\sqrt{2\beta_0}\xi), \quad \beta_3^{(4)}=4\beta_0,$$

$$\rho_4^{(4)}=\frac{1}{2}\sqrt{5\beta_0}\,\sinh(\sqrt{2\beta_0}\xi)[4-7/\cosh^2(\sqrt{2\beta_0}\xi)]/\cosh^2(\sqrt{2\beta_0}\xi), \quad \beta_4^{(4)}=\beta_0. \tag{20}$$

It is easy to check that the first solution (17) is nothing more than the bright soliton (6), the second one (18) corresponds to the above-described two-component soliton (13). However, the third (19) and the fourth (20) ones are already next-order solitonlike solutions, including the third and the fourth incoherent light field components. The solitons (19) and (20) include components with spatial profiles corresponding to the second symmetrical and asymmetrical modes of the nonlinear waveguide, written in PRC by all the soliton components. Figure 7 shows spatial profiles of all the components of Eq. (19) and illustrates their stable propagation along the ζ axis. In the scale of a real experiment, the distance $\zeta=10$ corresponds to a PRC length about 5 cm. As previously, computer simulation of the more realistic situation $I_0\sim I$, when saturation of PRC nonlinearity has been taken into account, does not change this conclusion in general. Figure 8

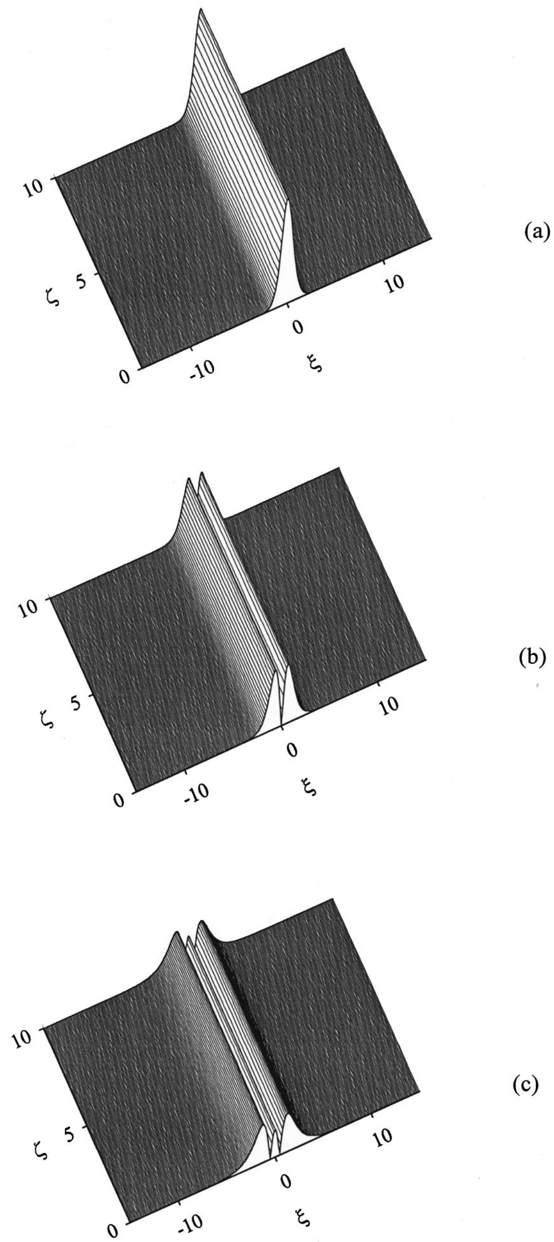

FIG. 7. Spatial distributions $|\rho_1^{(3)}(\xi)|$ (a), $|\rho_2^{(3)}(\xi)|$ (b), and $|\rho_3^{(3)}(\xi)|$ (c) of three-component soliton (19) propagating along the ζ axis: ξ and ζ are the dimensionless spatial coordinates; $\beta_1^{(3)}=9$, $\beta_2^{(3)}=4$, $\beta_3^{(3)}=\beta_0=1$.

illustrates evolution of spatial profiles $\rho_{1-3}^{(3)}(\xi)$ of all the components of soliton (19) as the soliton propagates through PRC. While the amplitudes and widths of all the components change quasiperiodically, it is easy to see that the soliton (19) conserves its structural stability.

In fact, there is one more multicomponent solution that can be considered as a kind of trivial generalization of Manakov soliton (9) (Ref. [3]) to the n-component $(i=1,2,...,n)$ case. If we consider the case when $\beta_i^{(n)}\equiv\beta_0$, the system (15) can be reduced to a very simple form:

$$\frac{d^2\rho_i^{(n)}}{d\xi^2}+2\beta_0\left[\frac{2}{\cosh^2(\sqrt{2\beta_0}\xi)}-1\right]\rho_i^{(n)}=0. \qquad (21)$$

That means all the soliton components are proportional to each other [compare with Eq. (9)]

$$\rho_i^{(n)}=a_i^{(n)}\frac{\sqrt{2\beta_0}}{\cosh(\sqrt{2\beta_0}\xi)} \qquad (22)$$

and we must only require that

$$\sum_{i=1}^{n}[a_i^{(n)}]^2=1 \qquad (23)$$

to obtain one more self-consistent solution.

To conclude the section, let us note one more specific feature of multicomponent solitons that could be important in many applications. In contrast to the two-component solitons (9), the solitons of this class give a possibility to vary $\Delta\eta_{\max}$, i.e., to control all the parameters of the photoinduced waveguide by changing the number of soliton components.

V. CONCLUSIONS AND FINAL REMARKS

To summarize, let us briefly list our main results and conclusions.

Starting from the simplest model of PRC with drift nonlinearity (Kerr-type approximation of PRC nonlinear response), we found a class of multicomponent solitonlike solutions, corresponding to stable propagation of some self-consistent components—mutually incoherent spatially localized light beams with finite energy and specific spatial structure. In relation to the shape of light field distribution, the components look like zero and higher order modes, confined within a nonlinear waveguide formed in PRC by themselves. A particular case of this class solution—specific case of two-component soliton, corresponding to our second order solution—has been known before for the case of Kerr-type nonlinearity [42,43]. However, we have shown here that there are much more complicated solutions of the same type because these solutions include more than two mutually incoherent components. We checked that the solitons of this class are not only physically realizable but conserve their structural stability on PRC length about 10 diffraction lengths. In the case of Kerr-type nonlinearity, interaction of the components is of pure cross-modulation (reactive) character and is not accompanied by the energy exchange. The simulation of a more realistic situation, $I_0\sim I$, when saturation of PRC nonlinearity was taken into account, did not change this conclusion in general. In our opinion, such multicomponent solitons could be important to many applications because they offer new possibilities in the changing relation between the maximal increment of the refractive index $\Delta\eta_{\max}$ and the nonlinear waveguide spatial scale ξ_0.

A possibility to neglect by the interference of soliton components, which is necessary to realize new multicomponent solitons, can be realized when the components at different carrier frequencies ω_i ($i=1,2,...$ is the number of the component) and a nonlinear medium with rather slow rise time T of nonlinear response are used. In this case, one must only satisfy the condition $\Delta\omega_{ij}T\gg1$, where $\Delta\omega_{ij}=|\omega_i-\omega_j|$. This is namely the case considered above by the example of photorefractive nonlinearity.

A new class of multicomponent solitonlike solutions of

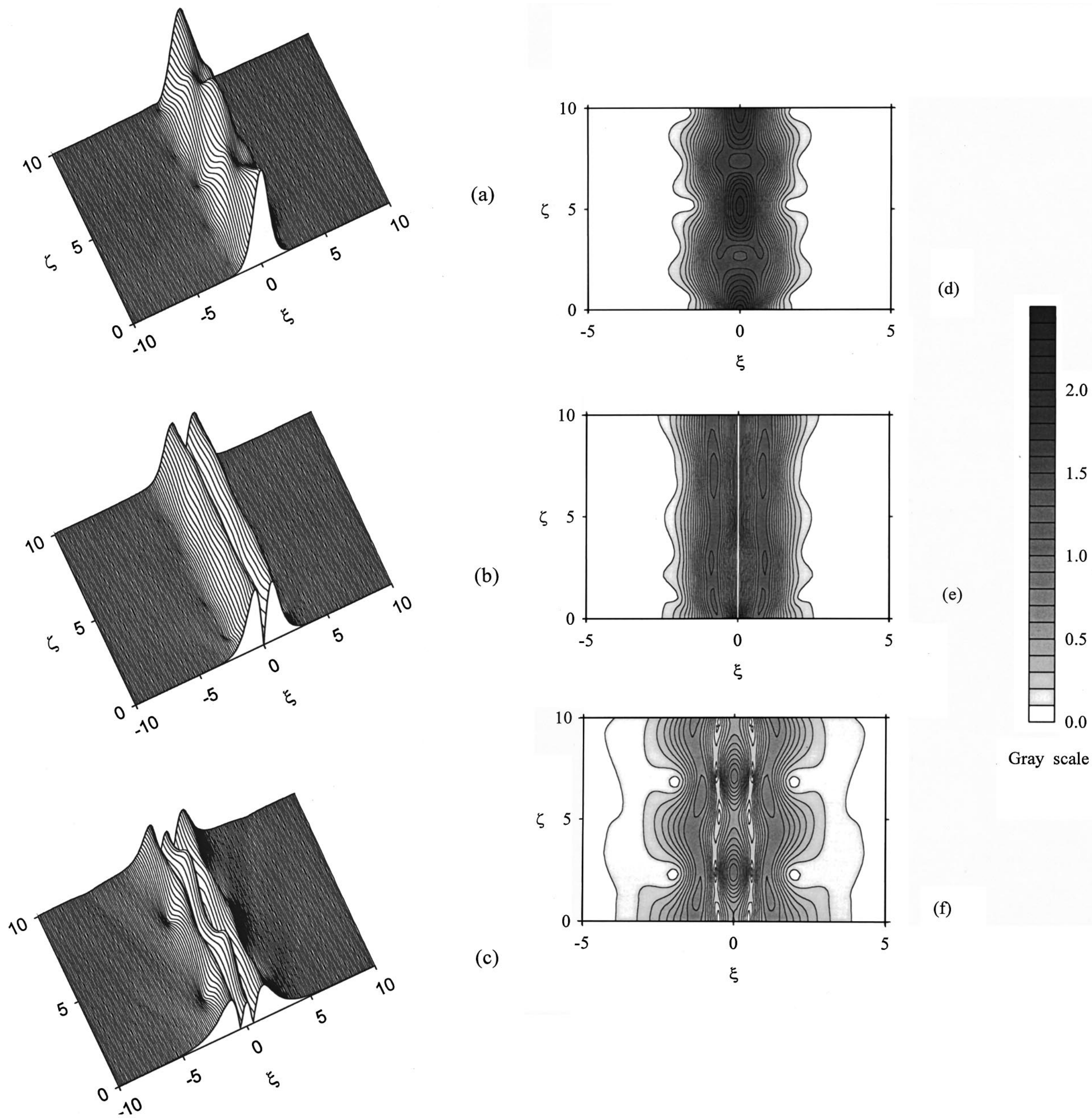

FIG. 8. Spatial distributions (a), (b), (c) and isolines (d), (e), (f) of $|\rho_1^{(3)}|$ (a), (d), $|\rho_2^{(3)}|$ (b), (e), and $|\rho_3^{(3)}|$ (c), (f) components of three-component soliton (19): saturation parameter $\{[\rho_1^{(3)}]^2+[\rho_2^{(3)}]^2+[\rho_3^{(3)}]^2\}_{\xi=0}/I_0=0.5$; ξ and ζ are the dimensionless spatial coordinates; $\beta_1^{(3)}=9$, $\beta_2^{(3)}=4$, $\beta_3^{(3)}=\beta_0=1$.

nonlinear Schrödinger equation should be of rather general character. In modern physics, this equation has become universal because it properly takes into account the first (cubic) term in the expansion of nonlinear polarization in a standard wave equation. A spatiotemporal analogy is also of great importance; it enables one to extend all the main specific features of the spatial multicomponent solitons found on a time axis, i.e., to extend solutions of many spatial problems to problems connected with propagation of ultrashort light pulses. In many cases, namely, by this equation, one can describe propagation of stable spatially localized wave packets of electron wave functions with taking into account, for example, an electron-phonon interaction [52,53]. Such multicomponent solutions of the nonlinear Schrödinger equation also may be interesting in connection with problems of 1D chains in ferromagnetics [9,10] and conjugated polymers [12–14], as well as of 2D atomic planes in high-temperature superconductors [11]. Here, an idea about some incoherent but bounded and stable electron wave packets, which propagate along 1D conjugated atomic chains or 2D isolated atomic planes, can be very fruitful. The packets (components of a multicomponent soliton) will have mutually orthogonal spatial distributions. The needed incoherence of a soliton's components may be supplied by a fast phase relaxation or different carrier frequencies of the wave packets. Moreover, the formation of such incoherent but bounded electron wave packets under excitation of coherent wave packets (superconducting pairs) by a picosecond laser pulse with a photon energy of ~ 2 eV may explain a rather strange experimental result, which we recently obtained in cooled Y-Ba-Cu-O films. In this experiment, we observed an energy gap in the spectrum of Y-Ba-Cu-O electronic states during more than 3 ns after fast heating of the superconducting film by a picosecond laser pulse [57].

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