Generation regimes of bidirectional hybridly mode-locked ultrashort pulse erbium-doped all-fiber ring laser with a distributed polarizer

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We report on the stable picosecond and femtosecond pulse generation from the bidirectional erbium-doped all-fiber ring laser hybridly mode-locked with a coaction of a single-walled carbon nanotube-based saturable absorber and nonlinear polarization evolution that was introduced through the insertion of the short-segment polarizing fiber. Depending on the total intracavity dispersion value, the laser emits conservative solitons, transform-limited Gaussian pulses, or highly chirped stretched pulses with almost 20 nm wide parabolic spectrum in both clockwise (CW) and counterclockwise (CCW) directions of the ring. Owing to the polarizing action in the cavity, we have demonstrated for the first time, to the best of our knowledge, an efficient tuning of soliton pulse characteristics for both CW and CCW channels via an appropriate polarization control. We believe that the bidirectional laser presented may be highly promising for gyroscopic and other dual-channel applications.

1. INTRODUCTION

Bidirectional ultrashort pulse (USP) ring lasers may be highly prospective for different dual-channel applications. Among them gyroscopy attracts much attention due to possible lock-in effect mitigation, which is of great concern for high-precision gyro development [1]. It is quite natural to extrapolate this idea to USP fiber lasers owing to their exceptional efficiency, compactness, and stability compared with USP dye or solid-state lasers. However, bidirectional operation of a mode-locked (ML) fiber ring laser was realized for the first time only in 2008 by Kieu and Mansuripur [2] after 41 years since the first demonstration of the He–Ne bidirectional ML laser by Buholz and Chodorow [3].

Up to now several schemes of bidirectional ML fiber lasers have been successfully demonstrated [2,4–9]. Some of them exploit the pure ring cavity design [2,4–6], but the others contain a four-port circulator for independent control of clockwise (CW) and counterclockwise (CCW) pulses [7–9]. It should be stressed that the vast majority of these lasers generate conservative solitons (CSs) with ~600 fs/1 ps duration in both CW and CCW directions. It is quite obvious due to the excellent intrinsic stability against different perturbations that is inherent to CSs. More recently Yao [10] has presented a bidirectional stretched-pulse erbium-doped fiber ring laser by means of the appropriate intracavity dispersion management that allowed significant pulse-width shortening down to ~200 fs in each channel. However, none of the schemes demonstrated by now enables broad tunability of countercirculating pulse characteristics, which, in turn, would be quite useful for gyroscopic applications.

It is of great importance that the colliding pulse mechanism of passive mode locking (so-called colliding pulse mode locking [CPML]) in bidirectional lasers implies a presence of a saturable absorber (SA) in the cavity for initiating USP generation in both directions of the ring [11]. Moreover, to favor CPML generation, CW and CCW pulses should meet each other in the SA, thus reducing total cavity losses [2,11]. It evidently means that SA thickness should be significantly less than a laser pulse...
length for the most stable CPML generation [2,11]. In addition, pulse fluence should be of the order of the SA saturation fluence to promote total loss reduction in the case of the simultaneous arrival of countercirculating pulses to the SA [11]. Modern saturable absorbers, such as semiconductor saturable absorber mirror (SESAM) [7] or carbon nanotubes [2,4,6,9,10] and graphene [8], easily satisfy these requirements being the best candidates for bidirectional ML fiber laser development.

Though, being compared to quantum well-based SESAMs, single-walled carbon nanotube-based saturable absorbers (SWCNT-SAs) benefit from much more easier and cost-effective fabrication techniques, the possibility of being used in both linear and ring cavities, ultrafast carrier relaxation time [12], and significantly enhanced noise characteristics [13]. Furthermore, SWCNTs can be naturally dispersed in different polymer matrices forming high optical quality thin films that can be further built in the laser cavity without breaking the all-fiber setup as well as requiring mechanical or thermal adjustments [14–16]. Unlike SESAMs, CNT-based saturable absorbers exhibit saturable behavior over a much wider spectral band (up to 500 nm and more) at the expense of the inherent distribution of SWCNTs diameters and chiralities [14,15,17], which is quite favorable for wideband photonic-switching applications [18]. Nonetheless, despite well-elaborated SWCNT-based film fabrication technologies, it is difficult to control such a crucial characteristic for laser mode locking as an absorber response time during the SWCNT-SA fabrication process. As a matter of fact, SWCNT-SA response time defined by a complicated tube–tube interactions (excited-state energy is transferred to metallic NTs with further quenching) [12] strictly depends on the particular sample fabrication process [19]. Moreover, a polarization dependence of saturable absorption cannot be controlled as well since SWCNTs orientations are randomly distributed. All these features make SWCNT-SA characteristics as well as corresponding ML laser performance absolutely unpredictable, thus impeding a realization of a reproducible USP generation especially in the bidirectional laser.

To overcome this drawback hybrid mode-locking mechanism may be employed which implies a coaction of a so-called “slow” SA, such as SWCNTs, and a “fast” one, such as nonlinear polarization evolution (NPE), based on the nonlinear Kerr effect in fibers [20]. In this case SWCNT-SA not only facilitates mode-locking start-up due to lower saturation energy but also provides a nonlinear filtering and spurious CW radiation suppression [21], significantly improving laser performance. High-quality USP shaping, in turn, primarily occurs due to the extremely fast NPE mechanism. Commonly NPE is introduced by means of a point fiber-pigtailed polarizer embedded into a cavity [21]. However, in this paper, we propose a distributed polarizer based on the short-segment Panda-type birefringent fiber possessing a polarizing effect [20] for NPE launch in the ring cavity, which additionally allows us to keep the all-fiber design of our bidirectional ML fiber ring laser. Moreover, apart from a strong polarization arrangement, polarizing action in the cavity is strictly favorable for providing an efficient control over CW and CCW pulse characteristics throughout the fine tuning of intracavity losses.

In this work we investigate generation regimes of the bidirectional colliding pulse hybridly ML erbium-doped all-fiber laser depending on the total cavity dispersion and demonstrate for the first time to the best of our knowledge fine control over CW and CCW pulse characteristics.

2. EXPERIMENTAL SETUP

The bidirectional hybridly ML laser setup is shown in Fig. 1. The ring cavity is based on the home-made erbium-doped active fiber (Er3+-fiber) pumped with a 980 nm, 200 mW laser diode through the 980/1550 wavelength division multiplexer. We used three single-mode step-index active fibers with strictly different group velocity dispersion (GVD) values: anomalous (#1), close to zero (#2), and normal (#3). Additionally, intra-cavity dispersion was carefully managed by means of the SMF-28 fiber ($\beta_2 = -21.2 \text{ ps}^2/\text{km}$ at 1560 nm) and SMF-LS fiber ($\beta_2 = +3.3 \text{ ps}^2/\text{km}$ at 1560 nm). Principal characteristics of Er-doped fibers employed are summarized in Table 1.

To realize CPML generation, a thin-film saturable absorber based on single-walled carbon nanotubes (CNT module) was placed in the ring cavity, as shown in Fig. 1. Arc-discharged SWCNTs were dispersed in the $\approx 50$ μm thick carboxymethyl-cellulose polymer film [14,15] that was further sandwiched between two FC/APC connectors (one or two successive units) forming common CNT module [20,22]. In this work two SAs were used: one of them was prepared with pure CNTs [22], but the other contained boron-nitride-doped CNTs (C:BNNTs) [22,23]. Meanwhile the concentration of B and N atoms in the resulting CNT lattice was very low (it was estimated to be less than 5%) to modify the CNT diameter and energy zone structure significantly. Thus, they should be considered as defects in the regular CNT mesh. SWCNT diameters estimated from the radial breathing mode frequency positions in corresponding Raman spectra were distributed around $1.4$ nm [24] as inherent to arc-discharged SWCNTs.
Table 1. Characteristics of Erbium-Doped Fibers

<table>
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<tr>
<th>Er-doped fiber ##</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
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<tr>
<td>GVD at 1560 nm [ps²/km]</td>
<td>-24.3</td>
<td>+6.6</td>
<td>+22.2</td>
</tr>
<tr>
<td>Small-signal absorption at 980 nm [dB/m]</td>
<td>11</td>
<td>43</td>
<td>14</td>
</tr>
<tr>
<td>Mode-field diameter at 1560 nm [µm]</td>
<td>10.6</td>
<td>4.5</td>
<td>5.3</td>
</tr>
<tr>
<td>Cut-off wavelength [µm]</td>
<td>1.34</td>
<td>0.9</td>
<td>1.0</td>
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Small-signal transmittance values ($T_0$) are close to each other and amount to $T_0 \approx 61\%$ for CNT-SA (two films installed) and $T_0 \approx 54\%$ for C:BNNT-SA (one film installed) at 1560 nm wavelength, respectively [22]. However, as it is shown in Fig. 2, saturation behavior measured with a homemade 1.56 µm probe Er-doped fiber laser with ≈120 fs pulse width at 38.1 MHz repetition rate is strictly different for given samples [22].

As a matter of fact, SWCNT-SAs have the ability of pulse shaping due to their saturable absorption behavior. Experimentally obtained dependence of losses $l((l = 1 - T = 1 - E_p/E_{p0})$ on the input pulse energy $E_p$ was fitted assuming a previously developed model of a slow saturable absorber [25] implying a probe laser pulse width to be much shorter than the absorber recovery time. In fact, this assumption is justified with a quite accurate approximation of the experimental data, as it is depicted in Fig. 2. As a result, modulation depth of the film transmission $\Delta T$ for C:BNNT-SA ($\Delta T = 14.9 \pm 0.6\%$) is more than 4 times (4.1) larger than that for the CNT-SA ($\Delta T = 3.6 \pm 0.2\%$) keeping almost the same saturation energy ($E_{Sat} = 21 \pm 3$ pJ) for C:BNNT-SA, $E_{Sat} = 23 \pm 4$ pJ for CNT-SA) [22]. Taking into account mode field diameter of the SMF28 fiber (≈10.6 µm) corresponding saturation fluence was calculated to be $F_{sat} = 24$ µJ/cm² for C:BNNT-SA and $F_{sat} = 26$ µJ/cm² for CNT-SA. In addition, being a crucial characteristic for a laser mode locking, the ground-state recovery time was indirectly estimated on the basis of careful analysis of the laser pulse width dependence on the pulse energy for a corresponding SWCNT-SA ML erbium-doped all-fiber ring soliton laser to be τ_s ≈ 700 fs for CNT-SA and τ_s ≈ 400 fs for C:BNNT-SA, respectively. The detailed information about this method is presented in [22].

An additional pulse shaping mechanism was realized through the fast NPE implementation in the laser cavity. In fact, the key element of the NPE is a saturator that introduces intensity-dependent losses. As in our previous work on the hybridly ML soliton fiber laser, here we apply a specially developed birefringent “Panda”-type silica-glass fiber [20] also possessing a polarizing effect (so-called PZ fiber) as an all-fiber intracavity polarizer. Its polarizing action is based on the cut-off wavelengths difference of the orthogonally polarized HE_{11} fiber modes (slow and fast ones) owing to the anisotropic $W$-type refractive-index profile [20].

The ratio of output powers of these modes (extinction ratio) exceeds 20 db in the 100 nm wide spectral band centered at 1560 nm for a 0.7 m long PZ fiber inserted into the ring, as it is demonstrated in the inset of Fig. 1. The GVD introduced by the PZ fiber was estimated from the measured refractive index profile to be $\beta_2(PZF) \approx -28.6$ ps²/km at 1560 nm wavelength.

To complete NPE setup, two polarization controllers (PCs) were embedded into the ring according to Fig. 1. It is worth noting that the NPE setup contributes to the intracavity loss variation as well, which is crucial for ML laser performance.

Laser radiation was put out through a 3 dB fused fiber coupler placed between the CNT module and PZ fiber.

Of course, the laser cavity is free of an isolator, which is a necessary condition for the bidirectional generation appearance. Meanwhile, CW and CCW outputs were supplied with isolators to prevent any parasitic reflection from a measurement setup.

It should be also stressed that a laser cavity itself as well as a pumping scheme are not symmetrical. However, as it will be shown later, this fact is not a drawback for CW and CCW pulse characteristics equalization.

During experiments pulse intensity autocorrelations were measured with an INRAD autocorrelator, while pulse spectra were registered with a high-resolution (0.01 nm) ANDO spectrometer. CW and CCW pulse trains were simultaneously observed by means of the fast photodetectors with bandwidths of $f_{PD} \approx 5$ GHz and a dual-channel 350 MHz TEKTRONIX 2465A analog oscilloscope. Radiofrequency spectra were detected by means of the 9 kHz/3 GHz ROHDE&SWARZ FSL3 spectrum analyzer with a 300 Hz resolution bandwidth (RBW).

3. EXPERIMENTAL RESULTS

A. Conservative Solitons

At first we realized bidirectional ML generation with 2.5 m long Er-doped fiber #1 and CNT-SA in the ring. Taking into account GVD values of different fibers composing a 6 m long ring, the total intracavity GVD value was estimated to be $DT = -0.14$ ps². As a matter of fact, such large anomalous GVD should support a soliton-type USP generation [20].

Stable self-starting single-pulse ML generation in both directions of the ring was obtained in a rather narrow pump power range from 60 to 88 mW. Figure 3 shows autocorrelation traces and spectra of CW and CCW pulses at 76 mW pump power, which were almost fully equalized through the appropriate PCs adjustment. It should be stressed here that central wavelengths

![Fig. 2. Saturation behavior of SWCNTs-based saturable absorbers upon 120 fs pulse excitation at 1560 nm wavelength.](image-url)
of countercirculating pulses were almost equal within the 0.2 nm resolution band of the ANDO spectrum analyzer. Pulse widths amounted to 795 and 798 fs while output average powers were measured to be 1.35 and 1.21 mW for CW and CCW channels, respectively. Obviously, as seen in Fig. 3, good approximation of experimental data with corresponding soliton functions \( P(\omega) \propto \text{sech}^2(\omega) \) together with well-resolved Kelly side-peaks (in logarithmic scale) is evidence in favor of the soliton-type pulse generation in both directions of the ring. However, the time-bandwidth product \( \text{TBP} = \Delta \nu \cdot \tau_p \) for all pulses obtained varied from 0.36 to 0.42, slightly exceeding the well-known TBP value of 0.315 inherent to fundamental solitons.

It is worth noting that during autocorrelation measurements we observed an interesting effect: maximizing of the autocorrelation signal via input pulse polarization adjustment for one channel led to its zeroing for another and vice versa. Thus, we can expect that pulses are crossed in the SA being almost orthogonally polarized, which promotes destructive pulse interaction reduction [11] and improves bidirectional laser performance.

This hypothesis possibly accounts for the outstanding tunability of CW and CCW pulse characteristics experimentally demonstrated in Fig. 4. In this case fine variation of the soliton pulse width and spectrum width (the inset) was realized via PCs adjustments, i.e., intracavity loss control for CW and CCW channels at a constant pump power of 81 mW. Obviously, loss increase is responsible for the pulse energy reduction and, as a result, corresponding soliton pulse width growth in accordance with the soliton area theorem. As it was expected both curves intersect at \( E_p \approx E_p^{\text{CCW}} \) condition proving an exceptional capability of the scheme developed to generate almost equalized countercirculating pulses despite essential cavity asymmetry. Taking into account this fact together with almost equal shortest pulse widths obtained for CW (524 fs) and CCW (502 fs) channels it is reasonable to expect almost equal NPE actions for CW and CCW channels in the CS generation regime. Furthermore, NPE action can be carefully controlled in both channels through appropriate PCs adjustments.

Average output power varied in this experiment from 1.09 to 2.28 mW for the CCW channel and from 0.84 to 2.02 mW for the CW one. Corresponding laser pulse width ranged from 887 to 502 fs for the CCW channel and from 1.28 ps to 524 fs for the CW one. Furthermore, it was possible to suppress any channel (CW or CCW) significantly through the appropriate PCs adjustment resulting in the stable unidirectional generation in the ring. Meanwhile, the contrast \( Q \) of laser outputs measured as the ratio of the highest to the lowest average output power \( Q = P_{\text{max}} / P_{\text{min}} \) was equal to 18.8 dB \( (P_{\text{max}}^{\text{CW}} = 1.73 \text{ mW}, P_{\text{min}}^{\text{CW}} = 23 \mu \text{W}) \) for the CW channel (rightmost point in Fig. 4) and 32.1 dB \( (P_{\text{max}}^{\text{CCW}} = 2.28 \text{ mW}, P_{\text{min}}^{\text{CCW}} = 1.4 \mu \text{W}) \) for the CCW one (leftmost point in Fig. 4) at the constant pump power of 81 mW.

We attribute the origination of these weak parasitic signals to the pulse scattering in the thin-film SWCNT-based SA since a discernible part of its nonsaturable losses stems from the light scattered by randomly distributed SWCNTs [26,27] and further caught by a single-mode fiber. The seed power is likely to be the same for both channels, although the difference in resulting signals (and measured contrasts as well) can be apparently ascribed to the nonsymmetrical cavity design.

Figure 5 shows weak signal spectra together with corresponding USP ones. First of all, well-resolved Kelly side-peaks further prove soliton-type USP generation in the ring. Moreover, intracavity GVD calculated on the basis of side-peaks positions in the spectra of both CW and CCW pulses \( (\Delta T = -0.12 \text{ ps}^2) \) [20] is very close to that one estimated using GVD of fibers composing the ring \( (\Delta T = -0.14 \text{ ps}^2) \). Obviously, as seen in Fig. 5, both central wavelengths and side-peaks positions almost coincide for USP and the corresponding weak signal (especially for CW channel suppressed), which is evident in favor of the scattering nature of weak parasitic signals. Of course, during its propagation along the ring weak radiation undergoes some evolution giving rise, in particular, to the Lorentzian spectra shapes, as it is demonstrated in Fig. 5(b).
As a matter of fact, parasitic radiation in one channel does not prevent the laser from a stable USP generation in the other. However, USP scattering in a SA can be a reason for the mutual influence of CW and CCW channels. Moreover, different contrasts of the bidirectional laser outputs can indicate unequal influence of CW and CCW channels on each other arising from the cavity asymmetry. To verify this assumption we independently recorded radiofrequency spectra of both CW and CCW pulse trains (without mixing them) in the case of almost equal outputs ($P_{\text{out}}^{\text{CW}} = 1.1$ mW, $P_{\text{out}}^{\text{CCW}} = 0.9$ mW at 60 mW pump power). As it is depicted in Fig. 6, there are low-intensity left-side-peaks near the fundamental repetition rate frequency ($f_{\text{rep}} = 33.56$ MHz (equal for both CW and CCW pulse trains)) in the countercirculating channels’ spectra. Of course, there are right-side-peaks located the same distance ($f_{\text{bn}}$) from the fundamental frequency apart from the parasitic radiation scattered by CNT-SA. Indeed, being naturally overlapped due to the colliding pulse ML mechanism, the weak scattered light and USP interact during their copropagation along the ring leading to the possible frequency pulling and, as a result, lock-in effect appearance, which could be a significant drawback for a dead-band free gyro development. Obviously, beat notes are naturally unstable resulting in some evolution of the beat note frequency $f_{\text{bn}}$ in time, as seen in Fig. 6. Moreover, the side-peak in the CW channel spectrum is more intense (at $\approx12$ dB), which is in consistency with our previous measurements of output contrast.

Taking into account corresponding power spectrum values [$S(f)$] it is possible to estimate beat note modulation depth ($m$) for each channel according to the following expression: $m = 2\sqrt{S(f_{\text{bn}})/S(f_{\text{rep}})}$. Modulation depth appears to be $m \approx 0.75\%$ for the CCW channel and $m \approx 3.1\%$ for the CW one. Furthermore, output contrasts ($Q$) for CW and CCW channels determined as a corresponding ratio of the laser power ($P_{\text{av}}^{\text{LR}}$) to the parasitic radiation power ($P_{\text{PR}}^{\text{av}}$) can be derived from the following relationship: $m \approx 2\sqrt{P_{\text{av}}^{\text{LR}}/P_{\text{av}}^{\text{PR}}} = 4/f_{\text{rep}}$ giving $Q \approx 4/m^2 = S(f_{\text{rep}})/S(f_{\text{bn}})$. Retrieved from Fig. 6, $Q$ amounts to $\approx49$ dB for the CCW channel and $\approx36$ dB for the CW one, respectively. Output contrasts in this case significantly exceed those of the unidirectional generation reported above. It is quite evident since laser generation in this case occurs in both channels. Thus, results obtained undoubtedly indicate the unequal mutual influence of CW and CCW channels that resulted from the cavity asymmetry.

Further we examined hybrid mode-locking features. To do this we collected the data for both CW and CCW pulse widths $\tau_p$ from Fig. 4 and plotted them with respect to the corresponding pulse energies $E_p$, as it is depicted in Fig. 7. It turns out that whole data obtained for both CW and CCW pulses can be carefully approximated with one decaying exponential function $\tau_p = \tau_0 + A \cdot \exp(-E_p/E_0)$ with an asymptote $\tau_0 = 470 \pm 35$ fs. It should be stressed that both $\tau_0$ and the shortest pulse width $\tau_{\text{min}}$ ($\tau_{\text{min}} = 502$ fs for the CCW channel) and $\tau_{\text{min}} = 524$ fs for the CW one) are close to the pulse width $\tau_p$ retrieved from a purely unidirectional hybridly ML soliton ring laser (with isolator embedded) built from the same elements and at the same pump power of 81 mW ($\tau_p = 490$ fs, $P_{\text{av}} = 3.3$ mW, $E_p = 75.6$ pJ) [20]. Moreover, they are sufficiently smaller than a pulse width ($\tau_p = 795$ fs) measured in the unidirectional laser without NPE action (the PZ fiber is substituted with the SMF28 one) at the same pump
power [22] and corresponding CNT-SA used. Thus, NPE plays a significant role in the pulse shaping process in both channels proving the hybrid nature of the ML generation in the given bidirectional fiber laser. In general, the operation regime realized should be considered as collision pulse hybrid mode locking (CPHML).

We have made a successful attempt to realize ML generation with C:BNNT-SA in the cavity and obtained the pulses’ characteristics together with their tuning behavior being very close to those observed in the laser with CNT-SA, which also confirms an exceptional role of the hybrid mode-locking mechanism in the soliton generation regime of the bidirectional laser developed.

### B. Gaussian Pulses

Further we have investigated bidirectional laser performance with 1 m long Er-doped fiber #2 in the ring. It should be noted that stable ML generation in this case was realized only with the CNT-SA used. Total intracavity GVD was calculated to be $D_T \approx -0.072 \text{ ps}^2$ on the basis of GVD of fibers composing a 4.4 m long ring.

Figure 8 shows autocorrelation traces and spectra of CW and CCW pulses measured at the pump power of 54 mW ($P_{\text{out}}^\text{CW} = 0.36 \text{ mW}$, $P_{\text{out}}^\text{CCW} = 0.48 \text{ mW}$). There are some noteworthy features arising from this data. First of all, despite noticeable anomalous intracavity GVD, there are no Kelly side-peaks in smooth pulse spectra depicted (compare them with spectra shown in Fig. 3). Moreover, pulse spectra also with autocorrelation traces for both CW and CCW pulses were carefully approximated with Gaussian functions rather than soliton ones. In addition, taking into account appropriate spectra FWHM and laser pulse widths, time-bandwidth products ($\text{TBP} = \Delta\nu \cdot \tau_p$) for CW and CCW pulses were calculated to be 0.49 and 0.46, respectively, being much closer to the fundamental Gaussian-pulse TBP value of 0.441 rather than soliton one (0.315). Thus, we claim the generation of almost bandwidth-limited Gaussian pulses in both CW and CCW directions of the ring. Since pulse spectra are quite narrow as compared to the spectra of common dispersion-managed stretched-pulse lasers [10] we should not expect significant pulse breathing in the cavity. On the contrary, pulses propagate along the cavity nearly keeping their widths.

Possibly, low intracavity peak power leading to the insufficient nonlinearity (self-phase modulation) action prevents CS pulse formation inside a cavity despite anomalous intracavity dispersion. Indeed, corresponding soliton order $N(N = \sqrt{\lambda L/\lambda N})$ estimated for CW and CCW pulses amounts to $N_{\text{CW}} \approx 0.33$ and $N_{\text{CCW}} \approx 0.43$, respectively, being sufficiently less than characteristic CS order ($N_{\text{CS}} \approx 1$). Moreover, being rather low for even soliton pulse formation, peak powers of both CW and CCW pulses are likely to be insufficient for discernible NPE action in both channels. Thus, the laser pulse width is primarily determined by CNT-SA performance ($\tau_p \sim \tau_s$) in the colliding pulse ML setup.

In contrast to the CS generation regime, we were not able to realize pulse characteristics tuning via intracavity PCs adjustments. Furthermore, we were unable to fully suppress any channel through PCs adjustments, which might show evidence in favor of some polarization locking for CW and CCW pulses inside a cavity [28].

### C. Stretched Pulses

The next step was using 3 m long Er-doped fiber #3 introducing significantly positive dispersion. Hence, to realize stable ML generation we had to use a C:BNNT saturable absorber and insert a 4 m long SMF-LS fiber for additional dispersion and nonlinearity management, which gave us slightly positive total 10.4 m long cavity GVD of $D_T \approx +0.001 \text{ ps}^2$.

Autocorrelation traces and spectra of CW and CCW pulses retrieved from the cavity at 45 mW pump power are depicted in Fig. 9. Obviously, such broad spectra with $\approx 20 \text{ nm}$ width (at $-10 \text{ dB}$ level) expect $\sim 200 \text{ fs}$ USP generation. Thus, output 2.2 ps pulses are sufficiently chirped and could be consequently compressed. It is worth noting that measured spectra strongly differ from Gaussian ones and are closer to parabola. In addition, almost equal output spectra of the same shape (related to the same compressed pulse widths) imply almost equal NPE actions in CW and CCW channels.

Recently we have published research on the hybridly ML dispersion-managed unidirectional laser with distributed polarizer that was made from the same components [29]. We observed $\sim 100 \text{ fs}$ stretched pulse generation with the same broad
parabolic-type spectrum implying some aspects of the self-similar pulse evolution in the positive dispersion Er-doped active fiber. Meanwhile, we experimentally proved pulse breathing behavior finding a “focus” point position inside a negative dispersion segment of the ring cavity with slightly positive total GVD of +0.018 ps² [29]. Moreover, we proposed the “focus” point to coincide with a C:BNNT-SA position in the ring providing additional stabilization of the USP generation. In the case of colliding pulse ML operation this assumption is also quite reasonable since C:BNNT-SA is disposed in the vicinity of the center of the negative dispersion cavity segment [10,29]. It is well known that slightly positive total GVD favors broadband stretched-pulse generation in the dispersion-managed laser cavity, which has been realized in the given bidirectional Er-doped all-fiber laser. Unfortunately, the stability and reproducibility of the generation regime realized are insufficient for its further use in gyroscopic or other dual-channel applications. Moreover, as in the case of Gaussian pulses we were unable to realize pulse characteristics tuning due to a strictly narrow interval of possible PCs adjustments and pump powers responsible for stable USP generation.

D. Temporal and Spectral Characteristics of the CW and CCW Pulse Trains

Finally, we have analyzed temporal and spectral features of CW and CCW pulse trains by means of the analog oscilloscope and radiofrequency spectrum analyzer with 300 Hz resolution bandwidth. Obviously, Fig. 10(a) confirms highly stable CW ML generation in both channels. Further analysis in a shorter time interval of 100 ns revealed mutual synchronization of CW and CCW pulse trains. Indeed, as seen in Fig. 10(b), the oscilloscope was triggered by the CW pulse train (up), but the CCW pulse train (down) appeared to be also synchronized (both pulse trains stayed on screen still). Evidently this fact implies equal pulse repetition rate frequencies  for both CW and CCW channels. To prove this assumption we further explored the radiofrequency spectrum of mixed CW and CCW trains with the highest available resolution of 300 Hz. As it undoubtedly follows from Fig. 10(c), repetition rate frequencies coincide within the 300 Hz resolution band.

As it is expected, the main reason for Δf_rep origination is intracavity GVD influence [6,10]. Indeed, Δf_rep that can be simply expressed as Δf_rep = Δλ|D_j|/(2π c f^2_rep / λ^2) is directly proportional to the wavelengths difference Δλ of countercirculating pulses and total intracavity dispersion D_j. The typical beat note signal of superimposed by an external delay line CW and CCW pulses is shown in Fig. 10(d) while its radio-frequency spectrum depicted in Fig. 10(e) displays characteristic side-peaks located 1.07 MHz (beat note frequency f_bn) apart from the fundamental repetition rate frequency of 43.6 MHz. Evidently, f_bn should be considered as a difference in spectrum positions of CW and CCW pulses in frequency domain, i.e., f_bn = Δλ · (c/λ^2). It is of great importance that no beat note signal was observed unless CW and CCW pulses were superimposed again. Moreover, using an external delay line for CW and CCW pulse superposition, we experimentally found the pulses’ crossing points inside a cavity; one of them appears to coincide with a CNT-SA position, which is an absolutely predictable result taking into account the colliding pulse ML mechanism responsible for stable bidirectional laser operation. The second one is located inside the erbium-doped active fiber.

According to the data from Fig. 10(e), the central wavelengths’ difference of CW and CCW pulses was estimated to be Δλ ≈ 10^-7 nm (at λ_0 ≈ 1560 nm and D_j = -0.12 ps²,}
\( f_{\text{ba}} = 1.07 \text{ MHz} \) giving rise to the \( \Delta f_{\text{rep}} \sim 10^{-5} \text{ Hz} \), which is significantly less than the lowest \( f_{\text{rep}} \) instability \( \sigma_{\text{r}}(f_{\text{rep}})/f_{\text{rep}} \sim 10^{-7}/10^{-9} \) at the averaging time interval \( \tau \sim 10^{-3}/10^{2} \) s \text{ mea}\text{sured for common free-running USP laser} [30]. Thus, we claim the coincidence of repetition rates of CW and CCW pulse trains that is a necessary condition for their superposition and, as a result, beat note signal observation. Moreover, it is evident that both \( f_{\text{rep}} \) exhibit the same evolution under a thermal or vibration influence, for instance, providing highly stable overlapping of CW and CCW pulses. Meanwhile, it should be stressed that temporal synchronization of CW and CCW pulses is based on the colliding pulse mechanism of laser mode locking. Indeed, \( \sim 1 \text{ ps pulse with } \sim 200 \mu \text{m spatial length is longer (in space) than a thin-film saturable absorber, which stabilizes the cross-point position of CW and CCW pulses inside a cavity assuring strictly reliable CW and CCW pulses' synchronization.} 

4. CONCLUSIONS

In conclusion, we have elaborately explored generation regimes of the bidirectional erbium-doped all-fiber ring laser hybridly mode-locked (ML) with a coaction of single-walled carbon nanotube-based saturable absorber and nonlinear polarization evolution that was introduced through the insertion of the short-segment polarizing fiber.

In the case of negative intracavity GVD of \( -0.12 \text{ ps}^2 \), we have observed stable CS generation in both CW and CCW channels with \( \pm 500 \text{ fs}/\text{1.4 ps pulse width and } \sim 1 \text{ mW output average power. Moreover, due to the polarizing action in the cavity we have demonstrated for the first time to the best of our knowledge an efficient tuning of CS characteristics (pulse width and spectrum width) for both CW and CCW channels via an appropriate intracavity loss control. In addition, we experimentally observed mutual influence of CW and CCW pulses originated from parasitic light scattering in the CNT-based saturable absorber. An increase of intracavity GVD up to \( -0.07 \text{ ps}^2 \) through positive dispersion Er-doped fiber insertion resulted in the generation of almost transform-limited Gaussian pulses with durations strictly determined by CNT-SA performance in the colliding pulse mode-locking setup. Near zero cavity dispersion \(( \pm 0.001 \text{ ps}^2 \) we have realized stretched-pulse generation with almost \( 20 \text{ nm wide parabolic spectrum in both clockwise and counterclockwise directions of the ring. Finally, we have experimentally proved that an equality of the CW and CCW pulse repetition rates together with the mutual synchronization of countercirculating pulse trains is crucial for beat note signal observation and, as a result, gyroscopic effects investigation. Thus, the difference in repetition rates of CW and CCW pulse trains was estimated to be \( \Delta f_{\text{rep}} \sim 10^{-5} \text{ Hz on the basis of experimental beat note signal analysis, which is significantly less than } f_{\text{rep}} \text{ instability measured for common free-running USP lasers. We believe that the colliding pulse hybridly ML bidirectional laser presented may be highly promising for future gyroscopic and other dual-channel applications.}

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