Superconducting Quantum Arrays

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Abstract—Superconducting Quantum Arrays (SQAs) based on integration of quantum cells each consisting of two Josephson-junction parallel arrays (Differential Quantum Cell (DQC)) are analyzed for applications in broadband radio frequency systems. These SQAs are capable of providing highly linear magnetic signal to voltage transfer with high dynamic range. Both detail study of the quantum cells with realistic parameters and analysis of their characteristics including voltage response linearity are presented and discussed. A prototype of an active electrically small antenna is implemented based on an SQA containing 560 DQCs. We demonstrated the SQA voltage response swing as high as ~100 mV at transfer factor of ~6.5 mV/µT.

Index Terms—Josephson junctions, arrays, quantum cells, dynamic range, voltage response linearity, broadband systems, active antenna.

I. INTRODUCTION

The term “Superconducting Quantum Arrays” (SQA) has been recently suggested to denote special arrays designed to achieve both highly linear magnetic signal to voltage transfer function and high dynamic range [1]. SQA is a uniform periodic structure composed of identical superconducting cells with a linear voltage response to applied magnetic signal. These cells are electrically connected in series into a 2D array or can form a 3D series/parallel configuration shown in Fig. 1. SQA is characterized by independent operation of individual cells and collective behavior of an entire array generating an output signal. The dynamic range of the linear output signal increases with the number of cells N proportionally to √N. Indeed, in view of the independence of fluctuations of the cell output voltage, the spectral density of low-frequency (at the signal frequency) fluctuations of the circuit output voltage for cells connected in series is \( S_{V}^{(0)}(0) = N S_{V}^{(0)}(0) \), where \( S_{V}^{(0)}(0) \) is the spectral density of low-frequency fluctuations of the output voltage of one cell. Thus, the rms output signal fluctuations \( V_{f} = [NS_{V}^{(0)}(0)]^{1/2} \), decrease with \( N \) as \( 1/\sqrt{N} \). One can design a high-performance broadband amplifier by integrating a superconducting quantum array with a broadband input line to apply an input magnetic signal to all array cells (see, for example, the designs proposed in [5, 6]). Moreover, active electrically small antennas (ESAs) can be developed on the basis of SQAs to implement simultaneously broadband reception and amplification of electromagnetic signals. Practical implementation of these active ESAs is significantly simplified due to the absence of input RF line. Integration with a superconducting transformer (concentrator) of magnetic flux makes it possible to form an active ESAs of transformer type [7, 8]. At the same time, SQAs in the form of 2D arrays of superconducting cells with nonsuperconducting electric connection of the cells can be used directly as active ESAs of a transformer-less type [1].

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Fig. 1. Conceptual design of (a) a 2D Superconducting Quantum Array (SQA) and (b) a 3D multi-chip SQA with parallel connection of the chips.
Two types of elementary cells with linear voltage response to magnetic signal were proposed as a basic block for SQAs or quantum cell: bi-SQUID [9] and the so-called differential cell [10-12]. The latter delivered better performance for SQAs.

This paper is to study performance characteristics of SQAs based on differential quantum cells (DQCs) through the detailed analysis of DQC with realistic values of coupling inductances between Josephson junctions, linearity characteristics of magnetic flux to voltage conversion and output capability. Experimental data for a prototype of SQA-based active electrically small antenna are presented as well.

II. DIFFERENTIAL QUANTUM CELL

Differential Quantum Cell consists of two elementary parallel arrays of \( K = 10...15 \) Josephson junctions, differentially connected and oppositely biased by some magnetic flux \( \delta \Phi \) (see Fig. 2a). Such an implementation exploits a close-to-parabolic form of the main peak sides and thereby delivers better performance for SQAs.

In (2) when \( \delta \Phi = \Phi_4 \) and \( \delta \Phi = \Phi_b \), the analysis of parallel arrays of \( K = 10...15 \) Josephson junctions, the mutually shifted vertex positions results in a linear dependence, the differential connection of two parallel arrays oppositely biased by a certain magnetic flux \( \delta \Phi \) results in a linear voltage response of the quantum cell (solid line)

\[
V(\Phi) = V_z(\Phi + \delta \Phi) - V_1(\Phi - \delta \Phi),
\]

as well as voltage responses \( V'_1(\Phi - \delta \Phi) \) and \( V'_2(\Phi + \delta \Phi) \) of the cell shoulders (dash lines). The presented responses are given in terms of characteristic voltage of Josephson junctions \( V_c \).

If one considers a parallel array in the limiting case of zero inductances between the Josephson junctions, the analysis of its voltage response can be done analytically [11]. It shows that the array response well approaches the one for the limiting circuit size when number \( N \) of Josephson junctions exceeds 10...15. This number is even more accurate for the arrays with nonzero coupling inductances, since contribution of distant junctions become substantially weaker as coupling becomes smaller [10]. Basing on this fact, we studied practical arrays containing 10 Josephson junctions.

A. Parabolic Approximation

The DQC linearity is limited by the terms of next order of smallness in the description of the array response slopes \( V'_{\Phi}(\Phi) \) and \( V''(\Phi) \), which are terms of the 4th and 6th orders:

\[
V_{\Phi,k}(\Phi) = C_k + \Phi^* + \Phi_k + \Phi_6 \cdot \Phi + \text{const},
\]

where \( C_k, a_k, l_k \) are constants, \( \Phi^* \), \( \Phi_k \), and \( \Phi_6 \) - the vertex coordinates, which are positive for the right side relation \( V_{\max}(\Phi) \) and negative for the left side relation \( V_{\min}(\Phi) \). This allows us to write differential voltage responses as follows:

\[
\begin{align*}
V(\Phi) &= V_1(\Phi - \delta \Phi) - V_2(\Phi + \delta \Phi) \\
&= [k(\Phi - \delta \Phi)^2 - 2a_4(\Phi - \delta \Phi)^3 + 3a_6(\Phi - \delta \Phi)^4] \cdot \Phi \\
&- 8[a_4(\Phi - \delta \Phi)^2 + 5a_6(\Phi - \delta \Phi)^3] \cdot \Phi^3 \\
&- 12a_4(\Phi - \delta \Phi)^4 \cdot \Phi^5.
\end{align*}
\]

The 4th and 6th order terms do not generate nonlinear terms in (3) when \( \delta \Phi = \Phi_4 \) and \( \delta \Phi = \Phi_b \), correspondingly.

It seems impractical to realize a parallel array of Josephson junctions with normalized coupling inductances \( l = 2\pi l_i^0 / \Phi_0 \), where \( \Phi_0 = h / 2e \) is the magnetic flux quantum, and \( l_i^0 \) is the Josephson-junction critical current, \( l_i^0 \sim 100 \mu A \) - a typical value for niobium process with critical current density \( 4.5 \text{ ka/cm}^2 \). We performed numerical simulation of the parallel arrays with \( l_i^0 \) coupling inductances ~0.3 and analyzed how close the voltage responses can be approximated by a parabolic shape. Our study shows that at low (but realistic) normalized coupling inductances \( l \sim 0.3...0.7 \) the voltage response is close to parabolic law.

Fig. 3 presents such a parabolic approximation for voltage responses of an idealized parallel array with zero coupling inductances \( l = 0 \) (Fig. 3a) and a realistic array of 10 junctions coupled by inductances with normalized value \( l = 0.5 \) (Fig. 3b,c). The presented fits satisfy three requirements: (i) to fit the widest region of the responses by eq. (2) with (ii) the closest positions of the 4th and 6th order term vertexes which (iii) are to be located in the center of the region. As compared to the idealized parallel array, the voltage response of the realistic array has significantly wider main peak which can be approximated by (2) in two overlapping regions as shown in Fig. 3b,c. Some increase in bias current \( I_b \) above the maximum critical current of the array \( I_c \) (up to \( \sim 1.06 I_c \)) allows widening the first range to \( \Phi = 0 \) (see Fig. 3b). In these regions, factor \( k \) of the main parabola has close values \(-0.22 \) and \(-0.29 \) (in terms of \( V_k(\Phi_0)^2 \)), and different signs of the other terms in (2): \( a_4 = -0.26 \), \( a_6 = 0.81 \) and \( a_4 = 0.11 \), \( a_6 = -0.05 \) (in terms of \( V_k(\Phi_0)^2 \) and \( V_k(\Phi_0)^3 \)), correspondingly.

In the first region of the approximation, positions of the 4th and 6th order term vertexes are sensitive to the current biasing. At \( I_b = 1.06 I_c \), they well approach each other. For the second region, the fit parameters are practically insensitive to the change in the bias current and the high-order term vertexes...
leads to a gradual degradation of the close-to-
cell characterized by the response (3), the output signal
B. contribute to the array response.
of junctions which can interact with each other and, therefore,
response shape due to a progressive reduction in the number
parabolic law and therefore gives no improvement in the
linearity of the quantum cell response. The insensitivity to the
centre which was found earlier as the optimal one for idealized
oscillation frequency.

Further increase in the coupling inductances (above \( l \sim 0.8 \)) leads to a gradual degradation of the close-to-
parabola response shape due to a progressive reduction in the number
of junctions which can interact with each other and, therefore,
contribute to the array response.

B. Response Linearity

If one applies a sinusoidal input signal \( \Phi = A \sin(\omega t) \) to the
cell characterized by the response (3), the output signal \( V(\Phi) \)
consists of a fundamental tone with amplitude
\[
B_1 \approx 4\delta(\Phi - \Phi^*) A
\]
and two harmonics with amplitudes as follows:
\[
B_2 = [10A_6(\Phi_0 - \delta\Phi)]^3 + 2A_6(\Phi_0 - \delta\Phi)] A^3
\]
\[
+ (15/4) A_6(\Phi_0 - \delta\Phi) A^5,
\]
\[
B_3 = -(3/4) A_6(\Phi_0 - \delta\Phi) A^5.
\]
In accordance with the one-tone analysis technique, the
linearity of the magnetic flux to voltage transformation can be expressed by formula
\[
Lin = B_1 / \max\{B_2, B_3\}.
\]

This is an intricate problem to perform an accurate
numerical analysis of the flux-to-voltage transformation with a
high accuracy and be able to observe a linearity reaching in
excess of \( 10^3 \). In [12], we reported on linearity dependences
on both current and magnetic biases calculated as follows. At
first, we thoroughly calculated the shoulder responses \( V_{R.I.t} \).
Next, we varied magnetic frustration \( \delta\Phi \) in discrete steps from
about zero to \( 2\Phi_0 \), each time calculating parameters of the
approximating relation (3) over interval \( \delta\Phi - A \) to \( \delta\Phi + A \),
where \( A \) is an amplitude of input magnetic flux. Then, the
linearity (shown in Fig. 4 of Ref. [12]) was calculated using
formulas (4)-(7). However, the observed small change of the
fit parameters with both signal amplitude and \( \delta\Phi \) (within the
two approximation regions shown in Fig. 3b,c) significantly
affect the linearity near its maxima. This fact reveals
shortcoming of the used calculation method when analyzing
high linearity values. This motivated us to use different
calculation technique providing the most adequate analysis of
the response linearity.

Here, we present the optimized data for the voltage
response linearity calculated using a precise numerical
simulation of the realistic quantum cell and Fourier analysis of
its output signal when applying a sinusoidal input signal. The
analysis confirms the found in [12] optimal value \( I_0 = 1.06 I_C \)
for the current biasing at \( l = 0.5 \). Fig. 4 shows the differential
cell response linearity vs magnetic frustration \( \delta\Phi \) at different
amplitudes of the applied magnetic signal (expressed in per
cents of \( \delta\Phi \)). One can see three maxima in the linearity at
\( \delta\Phi \approx 0.4\Phi_0, 1.71\Phi_0 \) and \( 1.35\Phi_0 \). The second and third peaks can
be associated with centres of the ranges of parabolic approximations (at \( \delta\Phi \approx \Phi_0 \) shown in Figs. 3b and 3c.

Fig. 3. Approximation of the main peak side of the voltage response of parallel array of 10 Josephson junctions by (3) in case of (a) \( l = 0 \) (vertex positions \( \Phi^* =
0.87, \Phi_0 = 0.52, \Phi_0 = 0.51 \)); (b) \( l = 0.5 \) for the first domain \( \Phi^* = 2.47, \Phi_0 = 0.86, \Phi_0 = 0.85 \); and (c) for the second domain \( \Phi^* = 2.2, \Phi_0 = 1.36, \Phi_0 = 1.57 \).

Voltage response is shown by dash line, bold solid line shows fit of the curve, and thin solid line shows the quadratic parabola. This figure is adopted from [12].

Fig. 4. Linearity of the output signal of the practicable differential cell at optimal current biasing \( I_0 = 1.06 I_C \) versus magnetic frustration \( \delta\Phi \) of the cell
shoulders for different amplitudes of the input sinusoidal signal exploiting
30% (1), 50% (2), and 70% (3) of the total swing of the cell response.
Normalized coupling inductance \( l = 0.5 \), each parallel array contains \( N = 10 \)
Josephson junctions. Inset is an overlay of the linearity dependence (in same
scale) for an idealized cell (\( l = 0 \)).
The first maximum in Fig. 4 reveals an additional region of the parabolic approximation with positions of the vertexes \( \Phi_4 \) and \( \Phi_5 \) near \( 0.4\Phi_0 \), the region is located inside the much wider region shown in Fig. 3b and therefore was not found earlier [12]. This additional region and the region shown in Fig. 3c give the lack of convergence of the high-order-term vertex positions and therefore the corresponding side maxima in Fig. 4 shifts with the input signal amplitude, as it follows qualitatively from formulas (5), (6). In contrast to these peaks, the middle maximum in Fig. 4 does not shift with the signal amplitude due to the fact that the 4-th and 6-th parabola vertexes approach each other at amplitude due to the fact that the 4-th and 6-th parabola vertexes approach each other at 

\[
\Phi_4 \approx 0.71\Phi_0
\]

and can be as high as \(-95\) dB and \(-80\) dB for the signals over 30% and 60% of the total response swing, correspondingly. Moreover, the requirement for accuracy of the magnetic frustration \( \delta\Phi \) seems not too stringent.

One should point also to additional limitations of linearity characteristics of practical SQAs which can be caused by spread in the cell parameters and possible inequality in magnetic frustrations of the cell shoulders. To minimize these factors, both a symmetric pattern of the SQA shoulders and serial connection of two identical control lines applying magnetic biasing to the shoulders can be useful. For available Hypres niobium process [13], a relatively small spread of Josephson-junction critical currents is tolerable. The affect of junction spread decreases with the number of junctions in DQC shoulders and number of the cells forming the SQA.

C. Load Impact

If a load with impedance \( R_c \) is connected to the differential cell output as shown on inset in Fig. 5, some part of the bias currents \( I_b \) applied to the DQC shoulders (parallel arrays) flows through the load, and the current varies with output voltage. This causes substantial changes in the cell response. As it is evident from Fig. 5, these changes appear even at \( R_c = 100 R_N \) and become strongly pronounced at \( R_c = 10 R_N \), where \( R_N \) is normal resistance of the cell shoulder. Maximum distortion of the response takes place, when signal \( \Phi \) approaches \( -\delta\Phi \) or \( +\delta\Phi \), i.e., the extreme points of the linear response.

The effective currents \( I_R, I_L \) flowing through the cell shoulders (arrays A2 and A1 on inset in Fig. 5) change with output voltage \( V = V_R - V_L \) in antiphase as follows:

\[
I_R = I_b - V/R_c, \quad I_L = I_b + V/R_c \quad (8)
\]

The maximum reduction in each of the currents takes place when total flux applied to the shoulder \( (\Phi_R = 0 + \delta\Phi \) or \( \Phi_L = 0 - \delta\Phi \), correspondingly) is close to \( 2\delta\Phi \), i.e., far from the peak point of the shoulder response, and therefore does not practically affect the response. However, the increase of the currents strongly affects shoulder responses. In fact, each current peaks at the peak point of its shoulder response (when flux \( \Phi_R \) or \( \Phi_L \) approaches 0, correspondingly) and produces a significant decrease of the shoulder response peak. This fact points at possible solution of the problem through some decrease in the bias currents allowing balancing the increase in \( I_R \) and \( I_L \) at the point of interest. As follows from (8), the optimal decrease in the biasing can be found as \( \Delta I_b \approx V_{\text{max}}/R_c \), where \( V_{\text{max}} \) is amplitude of the cell response. It means that the decrease in \( I_b \) should reduce maximal value of the currents \( I_R, I_L \) down to \( I_b \) for idealized cell (\( l = 0 \)) or to 1.06 \( I_b \) for the realistic DQC with coupling inductances \( l = 0.5 \).

Our numerical simulations show that the decrease in the current biasing of the cell shoulders allows us to balance the load impact within the load impedance \( R_c \) down to about \((15...10)R_N \). This means that output current can reach \(5 \) to \(7\% \) of the bias current \( I_b \). Increasing number \( K \) of Josephson junctions composing the parallel arrays, one gets decrease in \( R_N \) and hence, rise in both \( I_b \) and allowable output current. For SQA-based devices, the desired values of impedance and output current can be achieved by connecting in parallel several 2D SQAs as shown schematically in Fig. 1b.

III. SQA AS AN ACTIVE ANTENNA

The implementation of a transformer-less active electrically small antenna (ESA) based on a two-dimensional SQA with non-superconductive connection of DQCs allows one to significantly increase the number of cells in SQA (occupying the same area as a transformer-based ESA) and thus obtain a much larger peak-to-peak amplitude of the antenna response.

The array uniformity ensures a uniform distribution of the magnetic component of electromagnetic wave in the array and, therefore, provides an identical magnetic signal at the input of each cell (maybe, except for the periphery rows, which can be excluded from the output signal formation). Fig. 6 schematically shows a mechanism of conversion of the perpendicular magnetic flux applied to the entire area of the superconducting shoulder of DQC into a parallel magnetic flux applied to an elementary parallel circuit. Screening Meissner currents flow through the edges of the superconducting films \( S1 \) and \( S2 \), which form the cell, and connect over the inner surfaces of these films in the region of their overlap (where Josephson junctions are formed) and thus induce a magnetic flux to be applied to a parallel circuit of Josephson junctions. In fact, magnetic field \( B \) in the gap between the superconductor films satisfies Maxwell’s equation \( \text{rot}(B) = \mu_0 \mu_j \), and hence \( B = \mu_0 j_0 \lambda \), where \( j_0 \) is the current density at the film surface and \( \lambda \) is depth of
penetration. Therefore, magnetic flux applied to every section of the Josephson-junction array equals to
\[ B \cdot (d + 2\lambda) \cdot dx = \mu_0 j_0 \lambda (d + 2\lambda) \cdot dx, \]
where \( d \) is thickness of an isolator film, and \( dx \) is a distance between Josephson junctions.

A prototype of the transformer-less ESA based on DQC-based SQA was fabricated using HYPRES niobium process with critical current density 4.5 kA/cm² [13]. The antenna prototype occupies an area of 3.3 \( \times \) 3.3 mm² on a 5 \( \times \) 5 mm² chip and contains 560 DQCs connected in series. The circuit is designed in the form of two differentially connected series circuits, each containing 560 elementary parallel arrays (the DQC shoulders) composed of twelve Josephson junctions.

Fig. 7(a) presents typical traces of the voltage responses of the series circuits composed of the twelve-junction parallel arrays (the cell shoulders). A set of the voltage responses of the antenna prototype measured at different magnetic frustrations of the two series circuits composing the antenna is shown in Fig. 7(b). The total peak-to-peak voltage response of this antenna reaches almost 100 mV, and the steepness of the magnetic signal conversion into output voltage is \( \frac{dV}{dB} \approx 6500 \) \( \mu \)V/\( \mu \)T. In our measurements, a magnetic field was applied using an external multi-turn coil.

Our experimental setup did not allow us to apply two-tone technique to measure linearity of the antenna voltage response. However, our prior testing of the transformer-based antenna prototype based on DQC type cells showed linearity of about 70 dB [1]. This was measured at signal frequency 300 kHz within \( \approx 30\% \) to \( \approx 80\% \) of the linear region of the voltage response.

IV. CONCLUSION

The performed study of the differential quantum cell (DQC) with realistic parameters including analysis of attainable characteristics confirms the DQC applicability as a basic cell for a high performance superconducting quantum arrays. These SQAs can be used as front-end circuits for broadband radio frequency systems capable of providing highly linear magnetic signal to voltage transfer with high dynamic range. Prototype of active electrically small antenna of transformer-less type has been implemented on the base of SQA containing 560 DQCs. The antenna prototype demonstrated its transfer factor of \( \approx 6.5 \) mV/\( \mu \)T and the voltage response swing of almost 100 mV.

Such superconducting devices can be used in broadband receiving systems with direct digitization of input signal [14-20] for communications, radar, and signal intelligence. The SQA arrays are also beneficial for many applications, in which SQUIDs and SQUID arrays are being used [21-23].

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