Abstracts

International conference

«Torus Actions in Geometry, Topology, and Applications»

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List of speakers

Oleg Smolyanov

(Moscow State University)

Diffusion and quantum dynamics on Riemannian manifolds

One discusses representations of solutions of equations describing the diffusion and quantum dynamics of (quasi-)particles with time and position dependent (even anisotropic) mass in a compact Riemannian manifold. Such processes are related to problems of nanotechnology, theory of semiconductors and also to some problems in biology. The equations which describe these processes are evolutionary partial differential equations with elliptic operators of second order, whose coefficients depend on time and position. The Riemannian manifold is assumed to be embedded in an Euclidean space and the solutions are represented by Feynman and Feynman-Kac formulas.

The Feynman formula is a representation of a solution of an evolutionary equation by a limit of integrals over Cartesian products os a space E; the Feynman-Kac formula is a representation of the same solution by an integral over a space of functions, of the real variable, taking values in E. The Feynman formulas approximate integrals from Feynman-Kac formulas. In proofs some methods from [1]—[5] (see also [6]) are used.

Some conjectures and open problems are formulated.

[1] Gadella, M. and Smolyanov O.G., Feynman formulas for particles with position-dependent mass, *Dokl. Akad. Nauk*, **418**(6) (2008), 727–730; translation in *Dokl. Math.*, **77**(1) (2008), 120–123.

[2] Obrezkov, O.O., Smolyanov, O.G., and Truman, A., A generalized Chernoff theorem and a randomized Feynman formula, *Dokl. Akad. Nauk*, **400**(5) (2005), 596–601; translation in *Dokl. Math.*, **71**(1), 105–110.

[3] Ratiu T., Smolyanov O.G. Hamiltonian and Feynman aspects of secondary quantization (Russian), *Dokl. Akad. Nauk*, **450**(2) (2013), 150–153; English translation *Dokl. Math.*, **87**(3) (2013), 289–292.

[4] Sidorova, N. The Smolyanov surface measure on trajectories in a Riemannian manifold, *Infin. Dimens. Anal. Quantum Probab. Relat. Top.*, **7**(3) (2004) 461–471.

[5] Smolyanov, O.G., von Weizsäcker, H., and Wittich, O., Chernoff's theorem and discrete time approximations of Brownian motion on manifolds, *Potential Anal.*, **26**(1) (2007), 1–29.

[6] Plyashechnik, A.S., Feynman formulas for second-order parabolic equations with variable coefficients, *Russ. J. Math. Phys.*, **20**(3) (2013), 377–379.

DMITRI TALALAEV

(Moscow State University)

2-knot quasiinvariants and 2-dimensional quantum integrable systems

I'll speak about the Zamolodchikov tetrahedron equation, the associated cohomology construction and the role of some cocycles in the problem of classifying the 2-knots, i.e. the isotopy classes of embeddings of the 2-sphere into R^4. The involved technic refers to the theory of 3-d statistical models and related 2-d integrable quantum mechanical systems.

VIKTOR BYKOVSKIY

(Institute of Applied Mathematics RAS, Khabarovsk)

Elliptic systems of sequences and functions

Let $A, B : \mathbb{Z} \to \mathbb{C}$ $(A \not\equiv 0,\ B \not\equiv 0)$ and

$$C_1^{(0)}, \ldots, C_k^{(0)}, D_1^{(0)}, \ldots, D_k^{(0)} : \mathbb{Z} \to \mathbb{C},$$

$\exists$

$$C_1^{(1)}, \ldots, C_k^{(1)}, D_1^{(1)}, \ldots, D_k^{(1)} : \mathbb{Z} \to \mathbb{C},$$

that $\forall m, n \in \mathbb{Z}$

$$A(m+n)B(m-n) = \sum_{j=1}^{k} C_j^{(0)}(m) D_j^{(0)}(n)$$

and

$$A(m+n+1)B(m-n) = \sum_{j=1}^{k} C_j^{(1)}(m) D_j^{(1)}(n)$$

Then A and B (with minimal $R(A,B) = k -$ rank) elliptic systems of sequences.
The talk will be devoted to the description of all such A and B of rank 2.
The work is supported by RSF grant no. 14-11-00335.

SVJETLANA TERZIĆ

(University of Montenegro, Faculty of Natural Sciences and Mathematics)

The action of T^3 on $G_{4,2}$ and $\mathbb{C}P^5$ and its generalization to $(2n,k)$-manifolds

The initial examples for developing the theory of $(2n,k)$-manifolds are the Grassman manifold $G_{4,2}$ and the complex projective space $\mathbb{C}P^5$ endowed with the canonical action of the compact torus T^3. The first part of the talk will be devoted to these examples, where among the other, we provide complete description of the topology and differentiable structure of the corresponding orbit spaces. In the second part of the talk we extract crucial properties from the mentioned examples and introduce the notion of $(2n,k)$-manifolds by generalizing these properties to the action of a compact torus T^k on a closed manifold M^{2n}.

The talk is based is joint work with Victor M. Buchstaber.

Evgeny Smirnov

(Higher School of Economics and Independent University of Moscow)

Spherical double flag varieties

Classical Schubert calculus deals with orbits of a Borel subgroup in $\mathrm{GL}(V)$ acting on a Grassmann variety $\mathrm{Gr}(k, V)$ of k-planes in a finite-dimensional vector space V. These orbits (Schubert cells) and their closures (Schubert varieties) are very well studied both from the combinatorial and the geometric points of view.

One can go one step farther, considering the direct product of two Grassmannians and a Borel subgroup in $\mathrm{GL}(V)$ in acting diagonally on this variety. In this case, the number of orbits still remains finite, but their combinatorics and geometry of their closures become much more involved. However, something still can be said about them. I will explain how to index the closures of a Borel subgroup in $\mathrm{Gr}(k, V) \times \mathrm{Gr}(l, V)$ combinatorially and construct their desingularizations, which are similar to Bott-Samelson desingularizations for ordinary Schubert varieties, and speak about the analogues of these results for direct products of partial flag varieties for reductive groups of type different from A_n, due to P.Achinger, N.Perrin, and myself.

Ivan Limonchenko

(Moscow State University)

Homotopy theory of some toric spaces and combinatorics of Golod complexes

The notion of a Golod ring was introduced firstly by E.S.Golod for Noetherian local rings and is now a classical object of study in commutative algebra (homology of local rings). It appears in toric topology as a property of a Stanley–Reisner ring (or face ring) $k[K]$ of a simplicial complex K over a ring of integers or a field k and has already found a topological interpretation there. Due to V.M.Buchstaber and T.E.Panov theorem on the cohomology ring of a moment-angle complex $\mathcal{Z}_K$ and the results of A.Berglund and M.Jollenbeck, it is just the case when multiplication in the ring $H^*(\mathcal{Z}_K; k)$ is trivial (if k is a field). For some special classes of simplicial complexes it was shown in the works by J.Grbic and S.Theriault, D.Kishimoto and K.Iriye, J.Grbic, T.Panov, S.Theriault and J.Wu, that their face rings $k[K]$ are Golod ones and in all those cases (if integral homology groups of all induced subcomplexes in K are torsion free) the corresponding moment-angle complexes have homotopy types of wedges of spheres.

In 2007 A.Berglund and M.Jollenbeck introduced the notion of a minimally non-Golod simplicial complex, that is $k[K]$ is not Golod itself but deleting of any vertex from K turns the face ring into a Golod one.

In this talk we will consider mainly the case when K is a polytopal triangulated sphere. We will discuss some recent results motivated by a conjectured correspondence between a face ring being Golod or minimally non-Golod and the homotopy type of a moment-angle complex being a wedge of spheres or a connected sum of products of two spheres.

The work is supported in part by RSF, grant no. 14-11-00414.

Aleksander Gaifullin

(Steklov Mathematical Institute RAS, Moscow)

The analytic continuation of the volume of the simplex and the bellows conjecture in the Lobachevsky spaces

The standard object in the hyperbolic geometry is the function computing the volume of an n-dimensional simplex in the Lobachevsky space from the set of its dihedral angles. Another function closely related to the previous one computes the volume of an n-dimensional simplex from the set of the hyperbolic cosines of its edge lengths. In 1973 Aomoto showed that the latter function can be analytically continued to a multivalued analytic function Φ on $\mathbb{C}^{n(n+1)/2}$, and described the ramification locus of Φ. We obtain further results on the function Φ. Namely, let $C_n \subset \mathbb{C}^{n(n+1)/2}$ be the subset consisting of all vectors that can be realized as the vectors of the hyperbolic cosines of the edge lengths of an n-dimensional simplex in the Lobachevsky space. We show that if n is even, then any branch of Φ on the set C_n is real, and if n is odd, then the difference between any two branches Φ on the set C_n is purely imaginary. As a corollary of this result, we obtain the proof of the bellows conjecture in the odd-dimensional Lobachevsky spaces, which claims that the volume of any flexible polyhedron is constant during the flexion.

Mikiya Masuda

(Osaka City University, Japan)

Cohomology of toric origami manifolds

Delzant proves in 1988 that there is a bijective correspondence between symplectic toric manifolds and what are now called Delzant polytopes; so all the geometrical information of a symplectic toric manifold M is encoded by the corresponding Delzant polytope P and the (equivariant) cohomology ring of M is explicitly described in terms of P.

The notion of a toric origami manifold was introduced by Cannas da Silva-Guillemin-Pires in 2010 as a generalization of a symplectic toric manifold, where the 2-form on a toric origami manifold is allowed to degenerate along a hypersurface. They extend the result of Delzant by showing that there is a bijective correspondence between toric origami manifolds and origami templates, where an origami template is a collection of Delzant polytopes satisfying a certain compatibility condition. The topology of toric origami manifolds is rather more complicated than that of symplectic toric manifolds and it is unknown how to describe the (equivariant) cohomology of a toric origami manifold in terms of the associated origami template. In this talk I will discuss the cohomology of toric origami manifolds. This is a joint work with Anton Ayzenberg, Seonjeong Park and Haozhi Zeng ([1]).

[1] A. Ayzenberg, M. Masuda, S. Park, H. Zeng, Cohomology of toric origami manifolds with acyclic proper faces, arXiv:1407.0764

Iskander Taimanov

(Sobolev Mathematical Institute, Novosibirsk)

Blowing up solutions of the modified Novikov-Veselov equation and minimal surfaces

A construction of blowing up solutions to the modified Novikov-Veselov equation is proposed. It is based on the Moutard transformation of two-dimensional Dirac operators and its geometrical interpretation via surface geometry. An explicit example of such a solution constructed by using the Enneper minimal surface is discussed in detail.

Nikolay Tyurin

(JINR, Dubna, NRU Higher School of Economics, Moscow)

(Special) Lagrangian fibrations of the flag variety F^3

The flag variety F^3 is the phase space of the Gelfand - Zeytlin system, therefore it carries the corresponding fibration by lagrangian submanifolds. This fibration is not regular since F^3 is not toric: if we would like to exploite this fibration in view of the theory of special lagrangian fibrations of Fano varieties proposed by D. Auroux it is useless. On the other hand F^3 admits pseudotoric structure which helps in the construction of special fibration. At the same time these two approaches are similiar at certain point: they give lagrangian spheres in F^3 which are hamiltonian equivalent.

Sergei Matveev (joint talk with Vladimir Turaev)

(Chelyabinsk State University)

Dijkgraaf-Witten invariants over Z_2 of 3-manifolds

In 1990 R. Dijkgraaf and E. Witten derived numerical topological invariants of closed manifolds from cohomology classes of finite groups. The DW invariants have been extensively studied in the literature. A computation of the DW-invariant of a manifold requires a summation of several terms whose number depends exponentially on the first Betti number of the manifold. We show that the DW-invariant over Z_2 of a 3-manifold M is closely related to the Arf invariant of certain quadratic function on $H^1(M; Z_2)$. For orientable 3-manifolds the corresponding explicite formula is especially simple. Using it, we compute DW-invariants of all twice orientable Seifert manifolds.

The authors were partially supported by Laboratory of Quantum Topology of Chelyabinsk State University (Russian Federation government grant 14.Z50.31.0020).

Shizuo Kaji

(University of Southampton, UK)

External products in equivariant homology

The external product is a homomorphism

$$H_*(X) \otimes H_*(Y) \to H_*(X \times Y).$$

We generalise this to any fibre product, or more generally, homotopy pullback square. We see it specialises to interesting constructions in string topology and group cohomology.

This is joint work with Haggai Tene.

Dmitry Gugnin

(Moscow State University)

On Cohomology of Symmetric Products and I.G.Macdonald's Theorem

Let us denote by $\mathrm{Sym}^n X$ the n-th symmetric product X^n/S_n of a topological space X. The famous result of A.Dold'58 [1] states that if two connected CW complexes X and Y has equal integral homology $H_i(X;\mathbb{Z}) = H_i(Y;\mathbb{Z}), 1 \le i \le q$, then $H_i(\mathrm{Sym}^n X;\mathbb{Z}) = H_i(\mathrm{Sym}^n Y;\mathbb{Z}), 1 \le i \le q$, for all $n > 1$.

From this moment suppose that all spaces X has finitely generated integral homology in each dimension. Then from A.Dold's result it is easy to check that integral cohomology $H^*(\mathrm{Sym}^n X;\mathbb{Z})$ is also finitely generated in each dimension. Then one has ring isomorphisms

$$H^*(\mathrm{Sym}^n X;\mathbb{Z}) \otimes \mathbb{Q} \cong (H^*(\mathrm{Sym}^n X;\mathbb{Z})/\mathrm{Tor}) \otimes \mathbb{Q} \cong H^*(\mathrm{Sym}^n X;\mathbb{Q}).$$

If for two given CW complexes X and Y their rational cohomology rings are equal, $H^*(X;\mathbb{Q}) = H^*(Y;\mathbb{Q}) = A^*$, then by simple classical Transfer Theorem one has equality $H^*(\mathrm{Sym}^n X;\mathbb{Q}) = H^*(\mathrm{Sym}^n Y;\mathbb{Q}) = S^n A^*$, where $S^n A^* := (A^{\otimes n})^{S_n}$.

The first main result of my talk is the following theorem (the arxiv preprint will appear very soon).

Theorem 1(G.,2015). *Let X and Y are connected CW complexes such that $H^*(X;\mathbb{Z})/\mathrm{Tor} \cong H^*(Y;\mathbb{Z})/\mathrm{Tor}$. Then there exists an isomorphism of rings $H^*(\mathrm{Sym}^n X;\mathbb{Z})/\mathrm{Tor} \cong H^*(\mathrm{Sym}^n Y;\mathbb{Z})/\mathrm{Tor}$ for all $n > 1$.*
Moreover, for any additive basis

$$a_{i,j} \in H^i(X;\mathbb{Z})/\mathrm{Tor}, i > 0, 1 \le j \le \mathrm{rank}(H^i(X;\mathbb{Z})/\mathrm{Tor}),$$

and integral multiplication table $a_{i,j}a_{k,l} = c^{s,t}_{i,j;k,l}a_{s,t}$, there is an explicit algorithm for constructing some additive basis of $H^*(\mathrm{Sym}^n X;\mathbb{Z})/\mathrm{Tor}, n > 1$, and computing the multiplication table for this basis.

First and the most difficult step for proving this theorem is the Integrality Lemma, proved by the author in [3], which states the integrality of some special rational cohomology classes in $H^*(\mathrm{Sym}^n X;\mathbb{Q})$.

It is not hard to prove that for any k-dimensional topological manifold M^k and any $n > 1$ the space $\mathrm{Sym}^n M^k$ is a manifold without boundary **iff** $k = 2$. For $k = 1$ the space $\mathrm{Sym}^n M^1$ is an n-dimensional manifold with boundary, and for $k > 2$ the space $\mathrm{Sym}^n M^k$ is not even a homology manifold (in some points it has local homology distinct from the local homology of $\mathbb{R}^{nk}$).

So, one of the most interesting cases is $\mathrm{Sym}^n M^2$ for 2-dimensional manifolds M^2. It is easy to check that $\mathrm{Sym}^n M^2$ is orientable **iff** M^2 is orientable. The classical fact from algebraic geometry states that $\mathrm{Sym}^n M_g^2, n > 1$, has a canonical structure of smooth projective variety for any compact Riemann surface M_g^2 of arbitrary genus $g \geq 0$. It is easy to prove that $\mathrm{Sym}^n(\mathbb{C}P^1) = \mathbb{C}P^n$. So, the case $g = 0$ is trivial.

Suppose $g > 0$. The famous result of I.G.Macdonald'62 [2] states the following.

Theorem (I.G.Macdonald,1962). *Suppose M_g^2 is an arbitrary compact Riemann surface of genus $g > 0$. Then the ring $H^*(\mathrm{Sym}^n M_g^2; \mathbb{Z})$ has no torsion and is isomorphic to free graded commutative algebra over $\mathbb{Z}$ on $2g$ generators of degree 1 and one generator of degree 2 factorized by some concrete integral relations (I.G.Macdonald's ideal).*

But, as was noticed by the author in 2012, the proof of I.G.Macdonald's theorem relies on the following proposition, which in general is false.

Proposition. *Suppose that a connected compact polyhedron X has no torsion in integral cohomology $H^*(X; \mathbb{Z})$. Suppose also that $H^*(X; \mathbb{Q})$ is equal to the free graded commutative algebra $\Lambda_{\mathbb{Q}} \langle a_1, a_2, \ldots, a_k \rangle \otimes \mathbb{Q}[b_1, b_2, \ldots, b_l]$ over $\mathbb{Q}$ for odd-dimensional generators $a_1, a_2, \ldots, a_k$ and even-dimensional generators $b_1, b_2, \ldots, b_l$, factorized by some integral relations (integral polynomials in variables $a_1, a_2, \ldots, a_k$, $b_1, b_2, \ldots, b_l$). Then the integral cohomology ring $H^*(X; \mathbb{Z})$ is equal to the corresponding free ring $\Lambda_{\mathbb{Z}} \langle a_1, a_2, \ldots, a_k \rangle \otimes \mathbb{Z}[b_1, b_2, \ldots, b_l]$ over $\mathbb{Z}$ factorized by the same relations.*

However, in the special case of I.G.Macdonald's theorem, this proposition turns out to be true. The verification of the I.G.Macdonald's theorem, which follows from Theorem 1 above and additional reasoning, is the second main result to be presented on my talk.

The work was supported by the Russian Scientific Foundation (project 14-11-00414).

[1] A. Dold, *Homology of symmetric products and other functors of complexes,* Ann. of Math. 1958. Vol. 68. P. 54-80.

[2] I. G. Macdonald, *Symmetric products of an algebraic curve,* Topology. 1962. Vol. 1. P. 319-343.

[3] D. V. Gugnin, *Topological applications of graded Frobenius homomorphisms II,* Trudy Moscovskogo matematicheskogo obshestva. 2012. Vol. 73. Iss. 2. P. 207-228. (in Russian) Engl. trans.: Transactions of the Moscow Math. Soc. 2012. Vol. 73. P. 167-182.

Anatoliy Vershik

(St.Petersburg Mathematical Insitute RAS, SPGU, IPPI)

The exit-boundary of the group and graphs; De Finetti theorem for the free group

It will be explained what doest it mean exit-boundry of markov process and how to calculate it for classical graphs and groups. For group $BbbZ$ (Pascal graph) this calcualtion reduces to De Finetti's theorem using ergodic method. For free group and $SL(2, R)$ [e.g. arboreal Pascal graph or slow tree] this problem had been never posed and can be solved in the smilar way. I will try to tell about connections of this problem with random walks, harmonic analysis and theory of locally-semi-simple algebras.

Dmitry Millionshchikov

(Moscow State University)

Invariant complex structures on nilmanifolds: algebraic constraints

We study algebraic constraints on nilpotent Lie algebra $\mathfrak{g}$, which corresponds to some real nilmanifold M which arise because of the presence of a left invariant complex structure J on M. Some of these constraints can be expressed in terms of corresponding descending central series of $\mathfrak{g}$. Various examples of positive graded Lie algebras with complex structures have been constructed, in particular, we construct an infinite family $\mathfrak{D}(n)$ of such algebras that we have for their nil-index $s(\mathfrak{D}(n))$:

$$s(\mathfrak{D}(n)) = \left[\frac{2}{3}\dim\mathfrak{D}(n)\right].$$

Anthony Bahri

(Rider University, USA)

New approaches to the cohomology of polyhedral products

Geometric and algebraic approaches to the computation of the cohomology of polyhedral products will be described. The methods arise from homotopy theory.

A report on joint work with Martin Bendersky, Fred Cohen and Sam Gitler.

Ivan Arzhantsev

(NRU Higher School of Economics, Moscow)

Embeddings of commutative linear algebraic groups of corank one

Let K be an algebraically closed field of characteristic zero, $G_m = (K\backslash\{0\}, *)$ be its multiplicative group, and $G_a = (K, +)$ be its additive group. Consider a commutative linear algebraic group $G = G_m^r \times G_a$. We study equivariant G-embeddings, i.e. normal G-varieties X containing G as an open orbit. We prove that X is a toric variety and all such actions of G on X correspond to Demazure roots of the fan of X. In these terms, the orbit structure of a G-variety X is described.

This is a joint work with Polina Kotenkova.

Andrey Vesnin

(Sobolev Institute of Mathematics, Novosibirsk)

On Small Covers of Right-Angled Coxeter Orbifolds

The method to construct closed orientable hyperbolic 3-manifolds whose fundamental group is the minimal index torsion-free subgroup in the right-angled Coxeter group was suggested by the author (1987). Arising manifolds are small covers in sense of Davis and Januszkiewicz (1991). We will survey results on small covers of right-angled hyperbolic Coxeter orbifolds. The initial list of polyhedra, the homeomorphism problem, volumes and hyperellipticity of manifolds will be discussed.

Stephen Theriault

(University of Southampton, UK)

Moment-angle manifolds and Panov's problem

We answer a problem posed by Panov, which is to describe the relationship between the wedge summands in a homotopy decomposition of the moment-angle complex corresponding to a disjoint union of k points and the connected sum factors in a diffeomorphism decomposition of the moment-angle manifold corresponding to the simple polytope obtained by making k vertex cuts on a standard d-simplex.

This establishes a bridge between two very different approaches to moment-angle manifolds.

Hiroaki Ishida

(RIMS, Kyoto, Japan)

Torus invariant transverse Kaehler forms and moment maps

A transverse Kaehler form on a complex manifold is a positive (1,1)-form whose kernel coincides with the subspace tangent to the leaves of a foliation.

I will talk about torus invariant transverse Kaehler forms and moment maps, especially in case of complex manifolds with maximal torus actions.

Hiraku Abe

(Osaka City University Advanced Mathematical Institute, Japan)

The cohomology of Hessenberg varieties and representations of symmetric groups

Hessenberg varieties are defined as subvarieties of a full flag variety. This class of subvarities contains, as some special cases, Peterson varieties and the toric varieties associated with Weyl chambers. In this talk, we will discuss a relation between cohomology rings of regular nilpotent Hessenberg varieties (e.g. Peterson varieties) and regular semi-simple Hessenberg varieties (e.g. the toric varieties associated with Weyl chambers) in terms of representations of a symmetric group. This is a joint work with Megumi Harada, Tatsuya Horiguchi, and Mikiya Masuda.

Vladimir Vershinin

(Universite de Montpellier, France)
(Sobolev Institute of Mathematics, Novosibirsk)
(Laboratory of Quantum Topology, Chelyabinsk State University)

Lie algebras of pure braid groups of closed surfaces

We consider the Lie algebra associated with the descending central series filtration of the pure braid group of oriented closed surface of arbitrary genus. R. Bezrukavnikov gave a presentation of this Lie algebra over the rational numbers. We show that his presentation remains true for this Lie algebra itself, i.e. over the integers. We study also the graded Lie algebra of the descending central series of the pure mapping class group of a 2-sphere. A simple presentation of this Lie algebra is obtained.

Jelena Grbic

(University of Southampton, UK)

Homotopy Rigidity of the Functor $\Sigma\Omega$

The main problem of this talk is the study of the homotopy rigidity of the functor $\Sigma\Omega$. Our solution to this problem depends heavily on new decompositions of looped co-H-spaces. I shall start by recalling some classical homotopy theoretical decomposition type results. Thereafter, I shall state new achievements and discuss how new functorial decompositions of looped co-H-space arise from an algebraic analysis of functorial coalgebra decompositions of tensor algebras.

This is a joint work with Jie Wu.

Petr Grinevich (joint talk with S. Abenda)

(Landau ITP, Chernogolovka)

Totally positive Grassmannians and rational M-curves

The totally positive matrices are actively studied from 1930's. In particular they naturally arose in the study of claster algebras. In the soliton theory the points of totally positive Grassmannians correspond to the real regular solutions of the KP2-hierarchy. We show that all these solutions can be constructed in terms of reational degenerations of the M-curves.

Valentin Ovsienko

(Université de Reims Champagne–Ardenne, France)

Cluster algebras and cluster superalgebras

The notion of cluster algebras of Fomin-Zelevinsky will be explained from the point of view of symplectic geometry. We then define a class of commutative superalgebras generalizing classical cluster algebras. The main example is the space of supersymmetric second order difference equations with very special monodromy condition.

Dmitri Bolotov

(ILTPE, Kharkov, Ukraine)
On characterization of flat foliations

We show that a codimension one C^2 -foliation of nonnegative Ricci curvature on a closed manifold M, whose leaves have finitely generated fundamental group is flat if and only if M is $K(\pi, 1)$ - manifold. In particular the result is true for C^2 -foliations of nonnegative sectional curvature without restrictions on topology of leaves.

Pjotr Beben

(University of Southampton, UK)
Configuration Spaces and Polyhedral Products

We use configuration space models for spaces of maps into certain subcomplexes of product spaces (including polyhedral products) to obtain a single suspension splitting for the loop space of certain polyhedral products, and show that the summands in these splittings have a very direct bearing on the topology of polyhedral products, and moment-angle complexes in particular.

Sonja Hohloch

(University of Antwerpen, Belgium)
From compact semi-toric systems to Hamiltonian $\mathbb{S}^1$-actions and back

Roughly, a semi-toric integrable Hamiltonian system (briefly, a semi-toric system) on a compact 4-dimensional manifold consists of two commuting Hamiltonian flows one of which is periodic. Thus the flow parameters induce an $\mathbb{S}^1 \times \mathbb{R}$-action on the manifold. Under certain assumptions on the singularities, semi-toric systems have been classified by Pelayo & Vũ Ngọc by means of 5 invariants.

Every semi-toric system induces a Hamiltonian $\mathbb{S}^1$-action on the manifold by 'forgetting' the $\mathbb{R}$-valued flow parameter. Effective Hamiltonian $\mathbb{S}^1$-actions on compact 4-manifolds have been classified by Karshon by means of so-called 'labeled directed graphs'.

In a joint work with S. Sabatini and D. Sepe, we linked Pelayo & Vũ Ngọc's classification of semi-toric systems to Karshon's classification of Hamiltonian $\mathbb{S}^1$-actions. More precisely, we show that only 2 of the 5 invariants are necessary to deduce the Karshon graph of the underlying $\mathbb{S}^1$-action.

In an ongoing work with S. Sabatini, D. Sepe and M. Symington, we study how to 'lift' an effective Hamiltonian $\mathbb{S}^1$-action on a compact 4-manifold to a semi-toric system.

In this talk, we give an introduction to semi-toric systems and Hamiltonian $\mathbb{S}^1$-actions and sketch parts of our constructions.

Theodore Voronov

(School of Mathematics, University of Manchester, UK)

Microformal geometry: “thick morphisms” of supermanifolds, adjoints of nonlinear operators and homotopy algebras

We introduce a generalization of smooth maps of manifolds (or supermanifolds) called “thick morphisms”. Such morphisms are defined via formal canonical relations between cotangent bundles and make a formal category, a “thickening” of the usual category of smooth manifolds with the same class of objects. They induce pull-backs of smooth functions, which are formal nonlinear mappings with remarkable properties.

In particular, we shall explain how this new construction makes it possible to obtain an analog of the adjoint operator for the case when the initial operator is nonlinear. (We consider maps of vector spaces or fiberwise maps of vector bundles $\Phi : E_1 \to E_2$.) This gives a “nonlinear pushforward map” of the spaces of functions on the dual bundles $\Phi_* : \mathbf{C}^\infty(E_1^*) \to \mathbf{C}^\infty(E_2^*)$. (Since the mapping of functions is itself nonlinear, the functions should be even or “bosonic”; there is a parallel “fermionic” construction.)

Time permitting, we shall give an application to homotopy Poisson algebras and homotopy Lie (bi)algebroids.

(See preprints: arXiv:1409.6475 [math.DG] and arXiv:1411.6720 [math.DG].)

Alexei Ustinov

(Institute of Applied Mathematics RAS, Khabarovsk)

On Voronoi — Minkowski 3D continued fractions

There exist two geometric interpretations of classical continued fractions admitting a natural generalization to the multidimensional case. In one of these interpretations, which is due to Klein, a continued fraction is identified with the convex hull (the Klein polygon) of the set of integer lattice points belonging to two adjacent angles (1895–1896). The second interpretation, which was independently proposed by Voronoi and Minkowski, is based on local minima of lattices, minimal systems, and extremal parallelepipeds (1896). The vertices of Klein polygons in plane lattices can be identified with local minima; however, beginning with the dimension 3, the Klein and Voronoi —Minkowski geometric constructions become different.

The talk will be devoted to wrongly forgotten Voronoi — Minkowski 3-dimensional continued fractions.

Gaiane Panina

(St. Petersburg Department of the Steklov Mathematical Insitute)

On combinatorics of compactifications of $M_0^n(\mathbb{R})$

$M_0^n(\mathbb{R})$ is the space of configurations of n distinct labeled points on the circle modulo the action of $PSL(2, \mathbb{R})$. Equivalently, it is the space of real points of the moduli space of algebraic curves of genus zero with n labeled punctures. We shall discuss the combinatorics of (1) Deligne-Knudsen-Mumford compactification of $M_0^n(\mathbb{R})$; (2) A family of compactifications generated by simple games (in other terminology, a simple game means an Alexander self-dual complex).

The latter will be compared with Bier spheres.

EVGENY FOMINYKH

(Chelyabinsk State University)

Infinite families of 3-manifolds with known values of Matveev's complexity

A complexity theory of 3-manifolds gives an useful approach for their classification. Recall that the Matveev's complexity of a compact 3-manifold equals k if this manifold has an almost simple spine with k true vertices and has no almost simple spines with fewer true vertices. The problem of calculating the complexity of 3-manifolds is very difficult. In this talk I will present infinite families of hyperbolic 3-manifolds with totally geodesic boundary with known complexity. The talk is based on joint work with A. Vesnin and V. Turaev.

IVAN DYNNIKOV

(Steklov Mathematical Institute RAS, Moscow)

Annuli with Legendrian boundary (joint work with M.Prasolov)

In a recent work we discovered a strong connection between combinatorics of grid diagrams of knots and contact topology. In particular, we reduced the problem of efficient classification of links based on monotonic simplification of grid diagrams to some general questions about Legendrian knots. In the talk I will discuss the following question related to the subject. Let A be an annulus embedded in the tree-space so that A is tangent to the contact structure at all boundary points. This means, in particular, that A is cobounded by two knots K_1, K_2 that are Legendrian and have the same topologycal type. Is it true that K_1 and K_2 are always Legendrian equivalent? By using grid diagrams we prove a weaker statement and suggest a method to disprove the original one.

SHINTARO KUROKI

(The University of Tokyo, Japan)

A necessary and sufficient condition for the extension of an axial function of GKM graph

An (m, n)-type GKM graph $(\Gamma, \mathcal{A})$ is an m-valent graph labeled by some function $\mathcal{A} : E(\Gamma) \to Lie(T^n)$, called an axial function, where $E(\Gamma)$ is the set of oriented edges of Γ, $Lie(T^n)$ is the Lie algebra of the n-dimensional torus T^n, and $n \leq m$ (if $n = m$, $(\Gamma, \mathcal{A})$ is called a torus graph).

An (m, n)-type GKM graph is related to $2m$-dimensional manifolds with T^n-actions with some conditions. In this talk, I will define a free $\mathbb{Z}$-module $\mathcal{O}(\Gamma, \mathcal{A})$ and introduce the following theorem: an (m, n)-type GKM graph extends to an (m, ℓ)-type GKM graph for some $n \leq \ell \leq m$ if and only if the rank of $\mathcal{O}(\Gamma, \mathcal{A})$ is greater than or equal to ℓ.

I will also apply this theorem to the extension of a torus action on some manifold.

Nikolay Erokhovets

(Moscow State University)

Combinatorics of flag simple polytopes

The talk is based on the joint work with V.M. Buchstaber.

A convex n-polytope P is called *simple* if any vertex of P is contained in exactly n facets. A simple polytope P is called *flag* if any set of it's pairwise intersecting facets $F_{i_1}, \dots, F_{i_k} : F_{i_p} \cap F_{i_q} \neq \varnothing$ has nonempty intersection $F_{i_1} \cap \dots \cap F_{i_k} \neq \varnothing$.

Let P be a simple polytope and $G(P)$ is a graph with vertices– facets of P and edges – pairs of intersecting facets of P. For any subgraph $\mathcal{G} \subset G(P)$ there is an operation of *graph-truncation* that gives a polytope $P_{\mathcal{G}}$.

Example 1. Let $P = I^3$ be a 3-cube, and $\mathcal{G}$ be two 3-circles corresponding to triples of facets intersecting in opposite vertices of the cube. Then $P_{\mathcal{G}}$ is combinatorially equivalent to the dodecahedron.

Proposition 2. *Polytope $P_{\mathcal{G}}$ is simple if and only if for any vertex $\boldsymbol{v} = F_{i_1} \cap \dots \cap F_{i_n} \in P$ the induced subgraph $\mathcal{G}|_{\{F_{i_1},\dots,F_{i_n}\}}$ is a disjoint union of points, edges and 3-circles.*

Theorem 3. *If P is flag, then $P_{\mathcal{G}}$ is flag. If $n = 3$, then $P_{\mathcal{G}}$ is flag if and only if it is simple, for any missing face $F_i \cap F_j \cap F_k = \varnothing$ of P one of the edges $F_i \cap F_j$, $F_j \cap F_k$, and $F_k \cap F_i$ belongs to $\mathcal{G}$, and for any triangular face F_i of P there is a vertex $F_i \cap F_j \cap F_k \in P$ such that $F_i \cap F_j$, $F_i \cap F_k \notin \mathcal{G}$, and $F_j \cap F_k \in \mathcal{G}$.*

Let $p_k(P)$ be a number of k-angle 2-faces of P.

Corollary («The Eberhard theorem for flag polytopes»)**.** *For every sequence $(p_k | 4 \leqslant k \neq 6)$ of nonnegative integers satisfying $2p_4 + p_5 = 12 + \sum_{k \geqslant 6}(k-6)p_k$, there exists integer p_6 and a flag simple 3-polytope P^3 with $p_k = p_k(P^3)$ for all $k \geqslant 4$.*

Theorem 4 (Compare to [V12])**.** *Combinatorial simple 3-polytope is flag if and only if it can be obtained from cube I^3 by a sequence of edge-truncations and cuttings off of pairs of consequent edges of 2-faces that are k-kons with $k \geqslant 5$.*

Theorem 5 (Compare to [LN13])**.** *Any two combinatorial flag simple 3-polytopes can be connected by a sequence of edge-truncations and inverse operations. Moreover, if a flag 3-polytope has at most 14 facets, then after one edge-truncation it can be obtained from the cube I^3 by a sequence of edge-truncations.*

Corollary. *A 2-truncation of a dodecahedron is a 2-truncated cube.*

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[LN13] F. Lutz, E. Nevo, *Stellar theory for flag simplicial complexes*, arXiv: 1302.5197.

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Alexander Zheglov

(Moscow State University)

Commuting differential operators of rank two, automorphisms of the first Weyl algebra and a noncommutative analogue of the Mordell conjecture

The talk is based on the joint work with A.E. Mironov.

Let A_1 be the first Weyl algebra $\mathbb{C}[x][\partial_x]$. Consider a generic polynomial equation in two variables $f(X,Y)=0$ that has a solution in A_1. Each such solution is a pair of commuting ordinary differential operators in A_1. A noncommutative analogue of the Mordell conjecture is a conjecture proposed by Yu. Berest, which says that the orbit space of the group action of $Aut(A_1)$ on the set of solutions of this equation is infinite if the geometric genus of the corresponding Riemann surface is 1, and is finite otherwise. This conjecture has an intimate connection with the famous Dixmier conjecture for A_1.

We construct a family of examples of hyperelliptic curves of any genus having infinite orbit spaces, and prove the Berest conjecture for genus 1 curves. As a by-product this proof clarifies the description of self-adjoint commuting ordinary differential operators with polynomial coefficients for the Krichever-Novikov operators of rank two, genus one.

Christophe Wacheux

(EPFL, Lausanne, Switzerland)

Toward a classification of semi-toric integrable Hamiltonian systems: the use of local models

An integrable Hamiltonian system (IHS) on a symplectic manifold of dimension $2n$ is c-almost-toric if the flow of $n-c$ components of the moment map yield a global Hamiltonian action of an $(n-c)$-torus. Moreover, one asks that all critical points be non-degenerate in some sense and without hyperbolic components.

When $c=0$ (toric case), we have a complete classification by Atiyah, Guillemin & Sternberg theorem and the Delzant theorem.

When $c=1$ (semi-toric case), both these theorem fails to be true. Yet, in dimension $2n=4$, Vu Ngoc and Pelayo gave a classification "à la Delzant", that includes a semi-toric version of Atiyah–Guillemin & Sternberg theorem.

To give a version of this theorem in any dimension, we rely on local models of the singular Lagrangian foliation first introduced by Eliasson, and apply them to our specific case.