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Status of the Experiments on Measurement of the Newtonian Gravitational Constant

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Abstract—Due to the weakness of gravity, the accuracy of the Newtonian gravitational constant G is essentially below the accuracy of other fundamental constants. The current value of G, recommended by CODATA in 2006, based on all results available at the end of 2006, is $G = (6.67428 \pm 0.00067) \times 10^{-11}$ m^3 kg^{-1} s^{-2} with a relative error of 100 ppm. The accuracy of the best experimental results is 15–40 ppm, although the scatter of the results is large enough. Therefore new experiments at a level of accuracy of 10–30 ppm are rather topical. One of the problems of improving accuracy of G is a precision measurement of the period of eigen oscillations of a torsion balance. The nonlinear behavior of the torsion balance with five degrees of freedom has been studied. It was shown that swing modes are excited by the acting environmental noise. A coupling of the swing modes to the torsional mode has been revealed. Methods of suppressing the effect of mode couplings have been considered.

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1. INTRODUCTION

The Newtonian gravitational constant G together with Planck's constant $\hbar$ and the speed of light c are the fundamental constants of nature which represent fundamental limits: c is the maximum speed, $\hbar$ is the minimum angular momentum and G is the gravitational radius of unit mass (the minimum radius of a sphere for relativistic gravitational collapse).

While the absolute values of the fundamental constants c and $\hbar$ are known with high accuracy and their "constancy" is not put to doubt, the situation with the gravitational constant G is absolutely different. Due to the weakness and nonshieldability of the gravitational interaction, the accuracy of experimental determination of G is essentially below that of other fundamental constants. Measurements of the gravitational constant are connected with absolute measurements of three physical values: time, mass and length, and consequently it is necessary to perform absolute measurements at high technology level in order to have a reliable estimation of G.

The first device for measurement of mutual gravitational attraction of small laboratory bodies, the horizontal torsion balance, has been made at the end of the 18th century by Henry Cavendish. Hundred years after Newton's discovery of the law of gravity, in 1797–1798, he performed an experiment and determined the value of the gravitational constant, $G = (6.67 \pm 0.07) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, with a relative uncertainty of 10^4 ppm (part per million, i.e., $\times 10^{-6}$). Cavendish also determined the mass and mean density of the Earth. The significance of the Cavendish's experiment was not only restricted to determination of the G value. The main thing is that he has proved the validity of the universal law of gravitation for small laboratory bodies.

The modern experimental facilities for measurement of the gravitational constant are complicated devices performed at high technology level, but their main part is also a horizontal torsion balance. After 2000, five new results on measurement of G with a relative error less than 50 ppm have been published. However, these results also did not cover each other within confidential intervals. In 2006, the Committee on Data for Science and Technology (CODATA) recommended for the Newtonian gravitational constant a value of $G = (6.67428 \pm 0.00067) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, with an uncertainty of 100 ppm. This new value of G is based on the data accessible by the end of 2006.

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Since Cavendish's first laboratory measurement over 200 years ago, the reduction in uncertainty of G has been only two orders of magnitude. The progress in measurement of G is occurring slowly enough: the error value decreases by approximately a factor of 10 per century, and the knowledge of the absolute value of G is still rather poor. New experiments on measurement of the Newtonian gravitational constant at the accuracy level of 10–30 ppm are topical and desirable.

2. THE PRINCIPLE OF CAVENDISH-TYPE EXPERIMENT

The torsion balance, or torsion pendulum, is a traditional tool for performing high-precision gravitational experiments. The principle of Cavendish-type experiments on measurement of the Newtonian gravitational constant is as follows. A bar with two equal test masses is suspended on a thin torsion wire in the gravitational field of big source (attracting) masses (the bar and the torsion wire form the torsion balance). Due to the gravitational interaction of the test and source masses, the torsion balance is twisted by some angle which is monitored by an optical lever (Fig. 1).

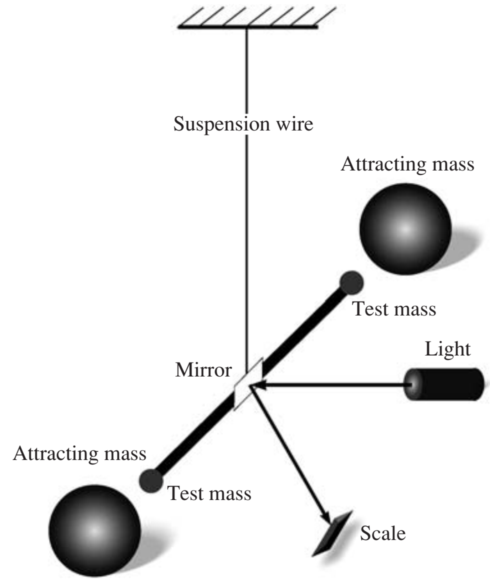

Fig. 1. The principle of Cavendish-type experiments.

In most of modern experiments, the gravitational constant is determined by the so-called *time-of-swing* method. The *time-of-swing* method, developed by Heyl [1], is based on detecting the change in the eigen oscillation frequency of the torsion balance in the gravitational field of the source masses in two different configurations, referred to as "near" and "far" positions. Since in both configurations the torsion balance oscillates in the gravitational field produced by the source masses, the gravitational torsion constant (the "gravitational rigidity") $K_g(\varphi)$ is added to the elastic torsion constant of the wire K_e. As a result, the squared oscillation frequency becomes

$$\omega^2(\varphi) = \frac{K_e + K_g(\varphi)}{J}\,, \qquad (1)$$

where J is the of moment of inertia of the torsion bar relative to the wire axis; φ is the angle between the pendulum axis and that of the source masses.

The gravitational rigidity is a derivative of the moment of interaction force between the source masses and the torsion balance,

$$K_g(\varphi) = \frac{\partial\Gamma}{\partial\varphi} = GC_g(\varphi), \qquad (2)$$

where $C_g(\varphi)$ is the coupling coefficient determined by the geometric dimensions of the torsion balance and attracting masses.

The frequency ω is measured in the process of the experiment. The moment of inertia J and the coupling constant $C_g(\varphi)$ are calculated on the basis of weighing and measuring of geometrical dimensions of the torsion balance and source masses. Hence in Eq. (1) there are two unknown parameters (if we substitute Eq. (2) to Eq. (1)): the elastic torsion constant K_e and the gravitational constant G. Therefore to determine G it is necessary to measure the frequency ω at least for two different configurations of the source masses, the "near" and "far" positions. So, the gravitational constant G can be obtained as

$$G = J\frac{(\omega^2)_1 - (\omega^2)_2}{(C_g)_1 - (C_g)_2} = J\frac{\Delta\omega^2}{\Delta C_g}, \qquad (3)$$

where the indices 1 and 2 indicate two different positions of the attracting masses.

3. THE MODERN HISTORY OF G DETERMINATION

The modern history of G determination covers 30 to 35 years and has started from three experiments performed in the 70-s of the last century. These were the experiment of *Observatoire de Recherches de la Meteorologie Nationale* (France), reported in 1972 [2], the experiment of *Sternberg Astronomical Institute of Moscow University*, reported in 1979 [3], and the experiment of the *National Bureau of Standards* (USA), reported in 1982 [4]. The CODATA system of values of fundamental constants of 1986 contained the G value with a relative uncertainty of 128 ppm, which was mainly based on the value obtained by Luther and Towler [4], but with

The world's best experiments on measurement of G and CODATA values

Authors, year of publication		G, $\times 10^{-11}$ $m^3\ kg^{-1}\ s^{-2}$	STD, $\times 10^{-11}$ $m^3\ kg^{-1}\ s^{-2}$	ppm
Ponticis, 1972	[2]	6.6714	0.0006	90
Sagitov, Milyukov, et al., 1979	[3]	6.6745	0.0008	120
Luther and Towler, 1982	[4]	6.6726	0.0005	75
CODATA, 1986	[5]	**6.67259**	**0.00085**	**128**
Michaelis et al., 1995	[6]	6.7154	0.0006	90
Karagioz, Izmailov, 1996	[7]	6.6729	0.0005	75
Bagley, Luther, 1997	[8]	6.6740	0.0007	105
CODATA, 1998	[11]	**6.673**	**0.010**	**1500**
Jun Luo et al., 1999	[10]	6.6699	0.0007	105
Fitzgerald, Armstrong, 1999	[9]	6.6742	0.0007	105
Gundlach, Merkowich, 2000	[12]	6.674215	0.000092	14
Quinn, Speake et al., 2001	[13]	6.67559	0.00027	41
Schlamminger et al., 2002	[14]	6.67407	0.00022	33
CODATA, 2002	[15]	**6.6742**	**0.0010**	**150**
Armstrong, Fitzgerald, 2003	[16]	6.67387	0.00027	41
Schlamminger et al., 2006	[17]	6.67425	0.00010	16
CODATA, 2006		**6.67428**	**0.00067**	**100**

its uncertainty doubled, which reflects the fact that, historically, measurements of G have been difficult to carry out and the result of Luther and Towler was possibly not final [5].

In the 90-s of the last century, a number of laboratory experiments on measurement of the Newtonian gravitational constant were done with relative uncertainties of about 100 ppm and less (Michaelis, 1995 [6]; Karagioz, 1996 [7]; Bagley, 1997 [8]; Fitzgerald, 1999 [9]; Jun Luo, 1999 [10]). These results are partly summarized in Table. Nevertheless, the discrepancies between the values of the gravitational constant obtained in these experiments remained large enough. In particular, the value of $G = 6.7146$ obtained in the *Physikalish Technische Bundesanstalt* (Germany) [6], was more than by 40 standard deviations (i.e., more than 5000 ppm) above the value of G recommended by CODATA in 1986. As a result of such a scatter of G values, CODATA had to increase the uncertainty significantly and in 1998 recommended the value $G = (6.673 \pm 0.010) \times 10^{-11}$ m^3 kg^{-1} s^{-2} [11], with a relative error of 1500 ppm. i.e., the uncertainty of G knowledge increased by almost a factor of ten!

In the following years (2000–2002), three new results, with relative errors smaller than 50 ppm, were published. These are the experiments of the *University of Washington* (USA), 2000, with a relative error of 14 ppm [12], the *University of Bermingham* (Great Britain), 2001, with a relative error of 41 ppm [13], and the *University of Zurich* (Switzerland), 2002, with a relative error of 33 ppm [14]. Although the situation with G has been considerably improved since the 1998 adjustment, these new results are not in complete agreement, as can be seen from Table and Fig. 2. These new G values are not crossed inside the confidence intervals. Based on weighted means of results after 1998, all of which round to $G = 6.6742 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, as well as their uncertainties, the relatively poor agreement of the data, and the historic and apparently continuing difficulty of assigning an uncertainty to a measured value of G that adequately reflects its true reliability, CODATA has taken as the 2002 recommended value $G = (6.6742 \pm 0.0010) \times 10^{-11}$ m^3 kg^{-1} s^{-2} with a relative uncertainty of 150 ppm [15].

In 2003, a new result on G measurement was reported by Armstrong and Fitzgerald from the *Measurement Standards Laboratory* (New Zealand),

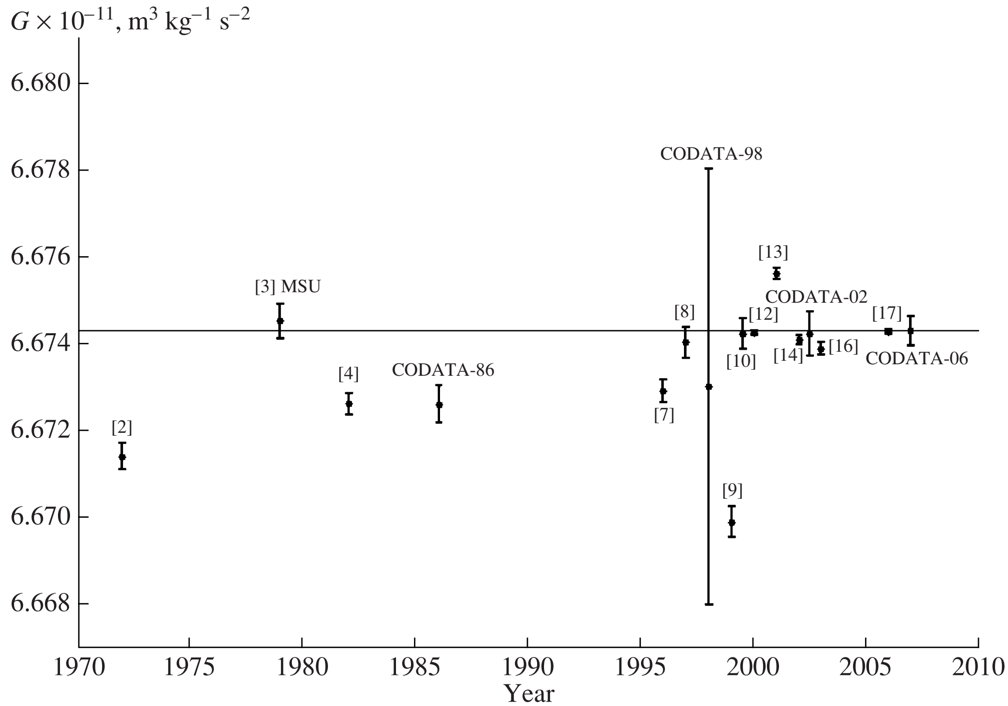

Fig. 2. The results of the world's best experiments on G measurement and CODATA values. The dash line is the CODATA-2006 value.

with an uncertainty of 40 ppm [16]. This uncertainty is smaller by a factor of about 2.5 than the uncertainty of their previous result [9], but its standard deviation interval does not intersect formally with the CODATA-2002 one. Finally, the latest result of G determination was published in 2006. It is the experiment of Schlamminger et al, from the *University of Zurich* (Switzerland) [17]. This result, with an uncertainty of 16 ppm, practically coincides with the CODATA-2002 value.

The current value of the Newtonian gravitational constant, recommended by CODATA in 2006, based on all results available by the end of 2006, is $G = (6.67428 \pm 0.00067) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, with a relative uncertainty of 100 ppm[1] .

Looking at Fig. 2, we can conclude that some of the values of the 30-year measurements of G do not coincide with each other and with the latest CODATA value. Until now there is no convincing explanation of such a large scatter in the G values obtained in different experiments, although some hypotheses have been discussed. One of them was expressed by Kuroda [18]. The *time-of-swing* method of G determination assumes that the elastic torsion constant K_e remains the same at each orientation of the source masses, but this assumption has been put to doubt. Kuroda has shown that damping of the torsion balance is caused by losses in the suspension fiber (inelasticity of the fiber), and, for a Heyl-type experiment, the measured value of G will be biased upward by a factor of $(1 + 1/\pi Q)$, where Q is the quality factor of torsion oscillations.

Thus, the intricate situation which has developed in the recent decades with G measurements makes topical the performance of new experiments at a relative uncertainty of 10 to 30 ppm.

4. THE HISTORICAL EXPERIMENT OF MOSCOW UNIVERSITY

The results of measurement of the Newtonian gravitation constant at the *Sternberg Astronomical Institute of Moscow University* has been published in 1979 [3]. It used the *time-of-swing* method. Changing of the gravitational configurations has been achieved not by rotating the source masses but by changing their position along the axis of the torsion beam. The scheme of the experimental setup of Moscow University is shown in Fig. 3.

Measurements of the gravitational constant have been performed in four series of experiments. The first series has been carried out from March to May 1975, the second one from March to June 1976, the third one from January to March 1977 and the fourth one from October 1977 to January 1978. The first series consisted of three measurements of G, the second and

[1] http://www.codata.org

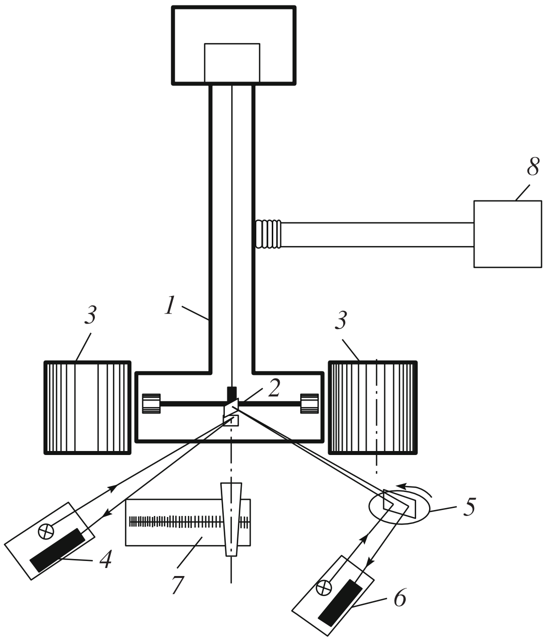

Fig. 3. The scheme of the experimental setup of SAI MSU: (*1*) copper vacuum chamber; (*2*) torsion balance; (*3*) source masses; (*4, 5, 6*) optical lever systems; (*7*) collimator for distance measurements; (*8*) vacuum pump.

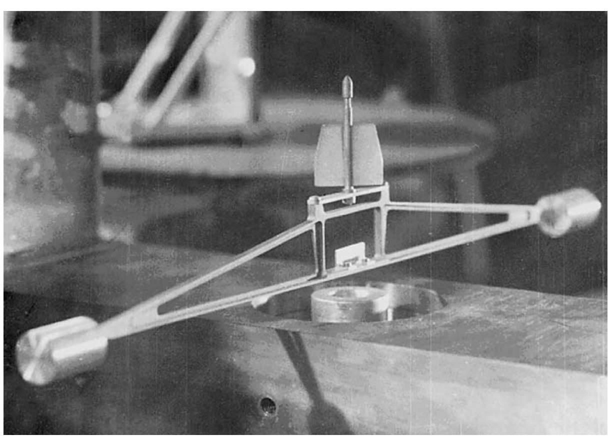

Fig. 4. Torsion balance No. 1, suspended in experimental setup.

third ones of five measurements, and in the fourth one ten measurements of G were made. To exclude possible systematic errors in the experimental results, between the series of experiments the torsion balance was changed (two types of torsion balances were used, Fig. 4) and repeated weighing of the source masses and the torsion balance were conducted as well as repeated measurements of the geometrical dimensions of the torsion balance.

In each measurement of the first three series, one value of the gravitational constant G was calculated. In the last, fourth series, the attracting masses occupied consecutively four fixed positions, therefore in each such experiment it was possible to determine three independent values of G.

The t-criterion has approved, that on a 95% significance level all four samples belong to one general population with some average value. Therefore the final result was found as an average in a sample containing 43 values of G and is equal to $G = (6.6745 \pm 0.0008) \times 10^{-11}$ m^3 kg^{-1} s^{-2}.

The t-criterion has also approved the assumption that there is no time variation of G (the fourth series of measurements was conducted 2.5 years after the first series). On the basis of the measured value of G, the masses and average density of the Earth, the Moon and the Sun have been determined in the metric system of units.

Finally, we would like to emphasize the following fact. The SAI MSU value, $G = (6.6745 \pm 0.0008) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, was published in 1979. CODATA-1986 has recommended the value of $G = (6.67259 \pm 0.00085) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, which does not coincide with SAI MSU value. And 27 years after this experiment, in 2006, CODATA recommended value of G which practically coincides with this “old” value. From the position of modern experience, the SAI MSU value, due to the carefulness of preparation and carrying out of the experiment, despite the imperfection of measuring technology of that time, has been apparently the least subject to systematic errors. Thus, the 27-year-old value of G is relevant and significant now.

5. TORSION BALANCE IS THE PRINCIPAL TOOL OF THE EXPERIMENT

New technological approaches and optimization of the configuration of experimental setups have shown that the gravitational constant can be measured at an accuracy level of 15 to 40 ppm (Table 1). Nevertheless, there are a number of problems which should be solved in the experiments of this high level. One of such problems is the precision of measuring the period of eigen oscillations of the torsion balance. The torsion balance is a complicated system with many degrees of freedom, and due to nonlinear couplings between them, new oscillations in so-called coupled modes appear. This leads to perturbations of the basic torsional mode and, as a consequence, to a great uncertainty in determining its period.

Degrees of freedom of the torsion balance. The detailed structure of the torsion balance which is used in the *Huazhong University of Science and Technology* (China) experiment on measurement of G is shown in Fig. 5 [20]. The test mass, a rectangular bar, is suspended from point O by a tungsten fiber of

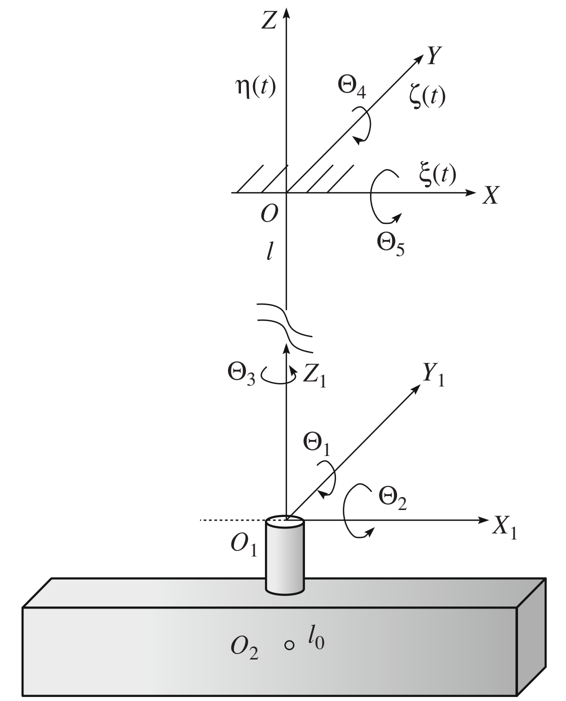

Fig. 5. The torsion balance and the coordinate systems are chosen to describe its motion.

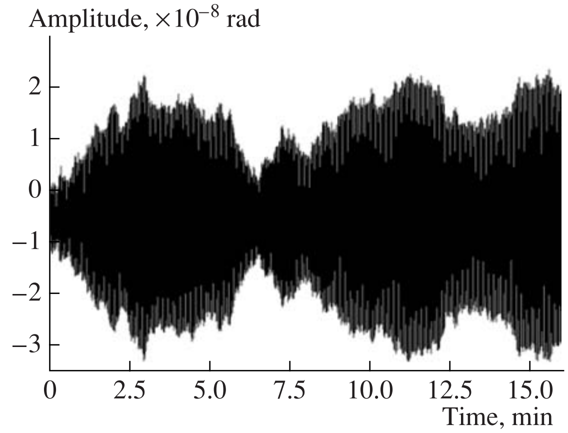

Fig. 6. Swing oscillations on the ZY plane, excited by seismic noise with an amplitude of about 1 mGal.

length l. To describe the dynamics of the torsion balance, we define two coordinate systems: a stationary Cartesian coordinate system $OXYZ$, with the origin at O located at the suspension point of the torsion balance, and another coordinate system, $O_1X_1Y_1Z_1$, fixed rigidly with the test body—the torsion bar. Its origin is at O_1 the point of attachment of the fiber to the torsion bar. The center of mass of the torsion balance is located at point O_2, at a distance l_0 from point O_1 in the vertical direction. The suspension point is driven by fluctuation forces (seismic noise), which cause random displacements of the suspension point $\xi(t)$, $\eta(t)$ and $\zeta(t)$, accordingly along X, Y and Z directions. Other fluctuation forces, acting directly on the torsion bar (e.g., random fluctuations of a residual gas in the vacuum chamber, temperature variations etc.), can provide the torsion balance rotation $\upsilon(t)$.

In such a setting of the problem, the system has five degrees of freedom, to be designated as Θ_1, Θ_2, Θ_3, Θ_4, and Θ_5, respectively. Here the parameters Θ_1 and Θ_4 represent rotation angles around the axes Y_1 and Y and describe swing oscillations in the plane XZ. The parameters Θ_2 and Θ_5 represent rotation angles around the axes X_1 and X and describe swing oscillations in the plane YZ. The angle Θ_3 represents rotation about the Z-axis and describes the principal torsion oscillations.

Hence, the set of equations (4)–(8), describing the motion of the torsion balance, consists of five nonlinear equations, also containing fluctuating terms:

$$(J_y + ml_0^2)\ddot{\Theta}_1 + \frac{\dot{\Theta}_1}{\tau_1}(J_y + ml_0^2) + mgl_0\Theta_1$$
$$= -mll_0\ddot{\Theta}_4 - ml_0\xi_0(t) + (J_y - J_x)\ddot{\Theta}_2\Theta_3; \quad (4)$$

$$(J_x + ml_0^2)\ddot{\Theta}_2 + \frac{\dot{\Theta}_2}{\tau_2}(J_x + ml_0^2) + mgl_0\Theta_2$$
$$= -mll_0\ddot{\Theta}_5 - ml_0\ddot{\eta}_0(t) + (J_y - J_x)\ddot{\Theta}_1\Theta_3; \quad (5)$$

$$J_z\ddot{\Theta}_3 + \frac{J_z\dot{\Theta}_3}{\tau_3} + K_e\Theta_3 = (J_x - J_y)(\dot{\Theta}_1^2 - \dot{\Theta}_2^2)\Theta_3$$
$$+ 2(J_x - J_y)\dot{\Theta}_1\dot{\Theta}_2\Theta_3^2 - \frac{2}{3}(J_x - J_y)(\dot{\Theta}_1^2 - \dot{\Theta}_2^2)\Theta_3^3$$
$$-2J_z\ddot{\Theta}_1\Theta_2 + (J_x - J_y - 2J_z)\dot{\Theta}_1\dot{\Theta}_2 + K_e\vartheta(t); \quad (6)$$

$$ml^2\ddot{\Theta}_4 + \frac{\dot{\Theta}_4}{\tau_4}ml^2 + mgl\Theta_4$$
$$= mll_0\ddot{\Theta}_1 - ml\ddot{\xi}_0(t); \quad (7)$$

$$ml^2\ddot{\Theta}_5 + \frac{\dot{\Theta}_5}{\tau_5}ml^2 + mgl\Theta_5$$
$$= -mll_0\ddot{\Theta}_2 - ml\ddot{\eta}_0(t). \quad (8)$$

Here, m is the mass of the torsion balance; J_x, J_y, and J_z are the moments of inertia of the torsion bar relative to the axes X_1, Y_1 and Z_1, respectively; τ_i are the time constants for each degree of freedom, K_e is the torsion elastic constant, $\vartheta(t)$ is a fluctuation function, describing random action on the torsion degree of freedom.

Eqs. (4), (7) and (5), (8) describe swing oscillations in two perpendicular planes, ZX and ZY, while Eq. (6) describes torsion oscillations. An analytic solution of this set of equations was found in [19,

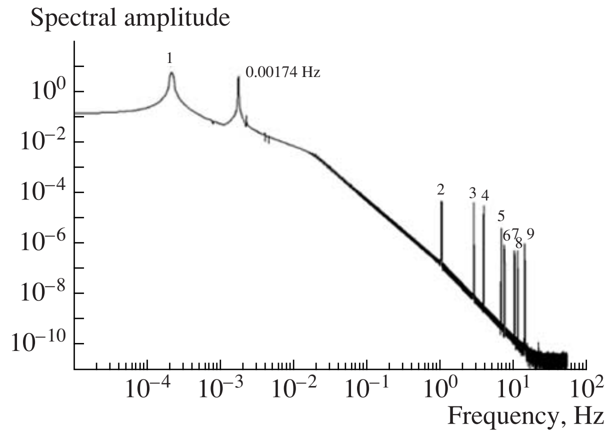

Fig. 7. Spectrum of torsion oscillations of an asymmetric torsion balance. The peak at 1.74×10^{-3} Hz is an eigenmode of torsion oscillations, the peaks marked with the indices 1 to 9 are coupled modes.

20]. Here, the set of equations has been solved by a numerical method, and simulation of motion of the torsion balance has been carried out.

Swing oscillations. We have studied the character of excitation of swing oscillations of the torsion balance. A numerical experiment has shown that oscillations in swing degrees of freedom are exited by random noise of seismic origin and occur with a time-variable magnitude (Fig. 6). A spectral analysis of swing oscillations has shown that the oscillations in each swing degree of freedom are beatings of all quasi-harmonic swing modes with a random amplitude changing in time. The swing frequencies are determined by the geometric parameters of the torsion balance. The random nature of swing oscillations is determined by the seismic noise that affects the suspension point. It was also shown that even with damping, due to the action of seismic noise, swing oscillations are a steady process.

Torsion oscillations and mode couplings. Oscillations in the torsion degree of freedom are described by Eq. (6). The terms in the right-hand side of this equation are nonlinear combinations of random quasi-harmonic swing modes. So, the right-hand side of this equation determines forced oscillations of the torsion balance, which are torsional mode couplings, or coupled modes. The frequencies of the coupled modes are simply linear combinations of swing-mode frequencies. The torsion balance represented in Fig. 5 has an asymmetric configuration which is characterized by inequality of all moments of inertia, i.e., $J_x \neq J_y \neq J_z$. The swing-mode frequencies of the asymmetrical torsion balance are also different. Some of them can be close to each other. Their linear combination will determine a low-frequency coupled mode which can disturb the torsion mode.

A numerical simulation of the torsion balance motion with five degrees of freedom, described by the set of equations (4)–(8), has been carried out. A spectral analysis of the torsional degree of freedom revealed a number of harmonics (Fig. 7). The most intensive mode with the frequency 1.74×10^{-3} Hz is the eigenmode of torsion oscillations. Other modes are torsional mode couplings. Coupled modes with the frequencies of 2×10^{-4} to 5×10^{-3} Hz are closest to the eigenmode and can disturb the latter. Therefore the problem of high-precision measurements of the torsional-mode frequency must be solved under the condition of maximal suppression of the coupled modes.

A traditional method of removing the effects of low-frequency torsional mode couplings is to employ a magnetic damper in the torsion system to overcome the seismic noise and consequently to suppress the intensity of swing modes. This method is called *amplitude suppression* and is used in the experimental setup mentioned above.

The other way which we have proposed for suppression of combined modes is called *frequency suppression*. The main idea of this method is to choose the geometry of the torsion balance in such a way as to “move” the frequencies of the coupled modes to a high-frequency range. In this case, the influence of the coupled modes can be reduced to minimum. In particular, a symmetric configuration of the torsion balance ($J_x = J_y \neq J_z$) leads to degeneracy of swing modes, i.e., the swing frequencies in the ZX plane coincide with those in the ZY plane.

CONCLUSIONS

There are a number of problems which must be paid attention in order to achieve the desirable accuracy of G measurement. One of them is stability of torsion oscillations. This stability also depends on many factors, such as nonlinearity and thermoelasticity of the torsion wire, electrostatic effects, effects of the geomagnetic field and so on. Among them there is the effect of coupling of the swing modes to the torsional mode. The torsion balance is a complicated system with many degrees of freedom, and due to nonlinear couplings between them, new oscillations appear in the so-called coupled modes. This leads to perturbations of the basic torsional mode, and, as a consequence, to a big uncertainty in determining its period. It was shown that this effect in the torsion balance behavior arises directly from a driven combination of the swing modes of the balance, and the swing modes are excited by the acting environmental noise. To successfully suppress these types of mode coupling in the torsion balance, amplitude or frequency suppression must be used in the experimental setup [19, 20].

Measurement of fine effects in experimental physics, especially in gravitational experiments, needs high technology and nontrivial methods. Nevertheless, any experiment, even carefully prepared and performed at the hottest features level, does not give an absolute and final "knowledge", but only sets a new limit on a testable effect. From this point of view, the technological development of gravitational experiments, including measurement of the Newtonian gravitational constant, is practically an "infinite" process, at least in the nearest future. We believe that new experiments, which are now in progress in some world gravitational centers, give the G value with a relative uncertainty within 10–20 ppm and will be a successive stage in the knowledge of the Newtonian gravitational constant.

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