Search for conditions of compressive fracture of hard brittle ceramics at impact loading

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ABSTRACT

In this paper we discuss three different experimental configurations to diagnosing the modes of inelastic deformation and to evaluating the failure thresholds at shock compression of hard brittle solids. One of the manifestations of brittle material response is the failure wave phenomenon, which has been previously observed in shock-compressed glasses. However, based on the measurements from our “theory critical” experiments, both alumina and boron carbide did not exhibit this phenomenon. In experiments with free and pre-stressed ceramics, while the Hugoniot elastic limit (HEL) in high-density B4C ceramic was found to be very sensitive to the transverse stress, it was found relatively less sensitive in Al2O3, implying brittle response of the boron carbide and ductile behavior of alumina. To further investigate the effects of stress states on the shock response of brittle materials, a “divergent flow or spherical shock wave” based plate impact experimental technique was employed to vary the ratio of longitudinal and transversal stresses and to probe conditions for compressive fracture thresholds. Two different experimental approaches were considered to generate both longitudinal and shear waves in the target through the impact of convex flyer plates. In the ceramic target plates, the shear wave separates a region of highly divergent flow behind the decaying spherical longitudinal shock wave and a region of low-divergent flow. Experiments with divergent shock loading of alumina and boron carbide ceramic plates coupled with computer simulations demonstrated the validity of these
1. Introduction

Mechanical behaviors of brittle materials, either static or dynamic, are often dominated by fracture under compression in the presence of confining stresses. At low-stress or no confinement, brittle materials lose their load carrying capacity (shear strength) by axial cracking at some threshold stress (failure threshold). Increasing levels of confining stresses tend to suppress axial cracking, increase the failure threshold to higher stress values, and cause brittle to ductile transition in materials. At very high levels of confining stresses, this transition mechanism seems to prevent loss of shear strength. Heard and Cline (1980) observed brittle to ductile transition in AlN, and BeO leading to the onset of ductile-like inelastic behavior at a very high confinement stress. Below the brittle–ductile transition, the compressive strengths of ceramics were observed to rapidly increase with increasing confinement. Further increase in the confinement above the transition threshold does not increase the shear strength and the strength stays nearly constant. The confining stress level corresponding to brittle to ductile transition seems to be different for different materials. For instance, it is 1.25 for alumina and 0.5 GPa for BeO and AlN.

In the plate impact configuration under one-dimensional strain, below the elastic limit, the lateral confining stress \( \sigma_2 \) increases \( \sigma_2 = \sigma_1/(1 - \nu) \) with increasing shock stress. \( \sigma_1 \) is the longitudinal stress and \( \nu \) is Poisson’s ratio. The lateral stress levels depend on material’s Poisson’s ratio, the fracture threshold, and the shock stress level required for the brittle–ductile transition. The fracture mechanism changes from an axial cracking based fracture to a ductile yielding above the transition level. The planar plate impact experimental configuration often is not conducive to interpret and/or unambiguously determine the “inelastic” compressive behavior of brittle material (Kanel and Bless, 2002). In this paper, we discuss three possible ways of studying both the mode (either brittle or ductile) of inelastic response of ceramic materials and the brittle to ductile transition threshold conditions.

The possibility for studying the mode of inelastic compressive response is based on the different sensitivity of the yield strength and the failure threshold to the variation of lateral stress. Chen and Ravichandran (1996, 2000) in their Split Hopkinson Pressure Bar experiments provided a controlled confinement by installing a shrink-fit metal sleeve on the lateral surface of the cylindrical ceramic specimen. This technique of varying the transversal stress was recently applied in our planar impact tests of alumina by Zaretsky and Kanel (2002).

One manifestation of brittle deformation (response) is the failure wave phenomenon, which was observed in shock-compressed glasses (Kanel, 2006; Bless and Brar, 2007). The failure wave is a network of cracks nucleated on the impacted surface and propagated into the elastically stressed body. Although a possibility of the existence of failure waves in ceramics as reported by Bourne et al. (1997, 1998a, b), their existence needs additional proof. In this paper, we present results obtained from alternative impact experimental configurations using laminated targets to probe failure waves in ceramic materials.

Most impact and penetration application problems are associated with multiaxial deformation and stress conditions. However, most of the fundamental data to understand and interpret the failure phenomena were associated with the limiting case of one dimensional plate impact test configuration. If, however, the material inelastic response under uniaxial shock compression is found to be ductile, under an arbitrary impact loading, the response of a brittle solid either ductile or brittle, is not always predictable. In such cases, a comprehensive material description requires knowledge of conditions of the brittle–ductile transition in the material under a multiaxial loading condition.

Taking advantage of the “ideal” one dimensional strain based plate impact configuration technique, it is desirable to expand the capabilities of this technique towards obtaining the wider range of confinement and multiaxial stress states \((\sigma_1,\sigma_2)\) characterized by varying the \(\sigma_1/\sigma_2\) ratio. The one dimensional strain loading condition does not allow one to vary the ratio \(\sigma_1/\sigma_2\), whereas in the divergent
configuration this ratio grows with strain. This paper presents a combined experimental and analytical approach for generating divergent flows in a plate by impacting with a convex impactor (flyer plate) with a surface curvature of radii ranging from 200 to 600 mm in a 58-mm, 4.2-m length gas gun. Another way of generating spherical (or divergent) waves is using an explosive facility in which a convex shape impactor could be formed during an explosive launch.

To shed some light on “Brittle–Ductile Transition” phenomena under a wide range of stress ratios \( \sigma_1/\sigma_2 \), we have attempted to interpret the results from divergent experiments on brittle materials through a series of computer simulations of 1-D spherically- and 2-D axis-symmetric divergent shocks in ceramics.

2. Search for the failure waves in shock-compressed ceramics

As stated above, the failure wave is a network of cracks that are nucleated on the impacted surface and propagate into the elastically stressed body. Our recent investigations (Kanel et al., 2002) confirmed that the failure wave in shock-compressed glass is a mechanical wave that obeys the Rankine–Hugoniot conservation equations: the small increase in amplitude of the longitudinal stress across the failure wave results in the corresponding increments of particle velocity and density. The propagation velocity of the failure wave is lower than the speed of sound; it is not directly related to the material compressibility but is determined by the speed of the crack growth. To probe further the fracture wave phenomena in ceramics, the two different experimental configurations (see Fig. 1) employed earlier in the experimental study of failure waves in glass (Kanel et al., 2002) were considered.

When an elastic compression wave passes through each layer, failure waves are initiated at the surfaces; therefore, the amplitude of the leading elastic wave attenuates due to relaxation caused by the failure wave. The reduction in the leading elastic wave amplitude continues at each interface passed by the elastic wave until the glass failure threshold is reached. Finally, if the number of plates in the pile is large enough, a precursor wave is formed whose amplitude gradually approaches the failure threshold. Thus, comparing the planar impact response of a layered plate with that of a single thick plate seems to be an effective experimental method to study the failure wave formation and to determine the failure threshold.

In the experiments with a single thick sample plate (Fig. 1a), the Hugoniot Elastic Limits (HELs) of the target materials were determined from the VISAR (Barker and Hollenbach, 1974) signals. In the experiments with alumina samples, the VISAR recorded the velocities of the sample free surface; in the case of boron carbide the VISAR measured particle velocities at the interface between the sample and a PMMA window.

The experiments were performed with hot pressed alumina (96% purity, 3.92 ± 0.02 g/cm\(^3\) density) and boron carbide (97% purity, 2.48 ± 0.02 g/cm\(^3\) density) supplied by Microceramica Ltd. (Carmiel, Israel). The mean grain size of ceramics was around 10 \( \mu \)m; the intergranular space was filled by smaller particles. The samples were cut from thicker plates using diamond tools with a surface roughness of 2 \( \mu \)m or less. The longitudinal \( c_l \) and the shear \( c_s \) sound speeds were: \( c_l = 10.72 \pm 0.22 \) km/s and

![Fig. 1. Scheme of the experimental arrangements for recording failure waves in shock-compressed brittle ceramics.](image-url)
$c_s = 6.34 \pm 0.05 \text{ km/s for alumina and } c_l = 13.7 \pm 0.3 \text{ km/s and } c_s = 8.75 \pm 0.1 \text{ km/s for boron carbide. The flye}\text{r thickness was 2 mm and the impact velocity was } 1.9 \pm 0.05 \text{ km/s.}

Fig. 2 shows the difference between the measured particle velocities in two plate impact experiments on soda lime glass: one with a single thick plate and the other with layers of thin plates (Kanel et al., 2002). Initiation of the failure wave at each pile interface generates the two-wave structure in the elastic compression wave signal. Fig. 3 presents the results of two experiments with thick alumina plates together with the velocity records obtained after two experiments with piles of thin alumina plates. Reproducibility of the data is quite good. Unlike the similar experiments with glasses, no evidence of the failure wave was recorded. Instead of an expected decrease of the HEL in the pile with respect to the single plate, the data show a modest increase. The time interval between the elastic precursor front and the “plastic” compressive wave is larger for the single plate than that for the pile of the same total thickness. Partially, this discrepancy is caused by the presence, in the pile sample, of the thin gaps between the plates because of their rough surfaces. As a result, the total time of propagation of the elastic precursor front through the pile is a sum of the time of wave propagation through the plates and the time of closing these gaps. Assuming that the surface velocity for closing the four sample gaps (Fig. 3) is about 300 m/s, the 50-ns difference between the “pile” and the “bulk” elastic wave traveling times yields a 5-μm gap width. The velocity of second (“plastic”) compressive wave through the pile with closed gaps coincides with that in a bulk sample. As a result, the delay in arrival of the elastic precursor front, caused by the gaps, decreases the time interval between the elastic precursor front and the “plastic” compressive wave in the free surface velocity history.

Fig. 2. A comparison of VISAR signals between a single plate and a pile of plates for soda lime glass at 6.3 GPa peak stress.

Fig. 3. VISAR signals for alumina: 6 mm thick single plate and four plates of 1.55 mm thick each.
The difference in HEL values from experiments with bulk ceramic plate and the pile is caused by decay of elastic precursor wave in alumina (Furnish and Chhabildas, 1998). In the case of pile, the decay is interrupted by the gaps. Due to ramped waveform behind the elastic front, closing of a gap occurs with acceleration. As a result, the stress at precursor front immediately after the gap is higher than that before so that the precursor decay re-starts from a higher stress. Thus, the effective way of the precursor decay in the pile is less than that for single plate of the same thickness and, as a result of different decays, the HEL values recorded in the free surface velocity histories are also different.

The particle velocity profiles recorded in impact experiments on boron carbide pile of 6.2-mm total thickness and bulk B₄C plate of 6.07-mm thickness are shown in Fig. 4. The difference in the wave arrivals is, again, the result of surface roughness. The recorded waveforms oscillate in a manner similar to that observed by Kipp and Grady (1990) in their experiments with B₄C backed by LiF windows. Although there are many reasons to expect brittle behavior of B₄C during shock compression, no evidence for the existence of a failure wave in B₄C is provided by the profiles. On the contrary, instead of the expected decrease, the amplitude of the elastic precursor wave obtained with the pile sample shows some increase with respect to that obtained in the bulk plate.

It seems reasonable to assume that a failure wave may appear in a material body, the surface state for which differs much from that inside the body. Probably, the other necessary condition is a specific relationship between the stress components. Anyway, whereas observation of the failure wave in glasses allows conclusions about their brittle response for uniaxial shock compression over a certain stress range, the absence of the appearance of a failure waves in shock-compressed ceramics not necessary means that their response is ductile.

3. Varying the shock-compressed state by pre-stressing

Fig. 5 illustrates the difference for a lateral pre-stressing on the Hugoniot elastic limit of ductile and brittle materials. In ductile materials, the inelastic deformation is plastic yielding whose onset is associated with the Von Mises or Tresca criteria. In the case of 1D-strain loading both criteria coincide, with the result:

$$\sigma_1 - \sigma_2 = Y_d.$$  \hspace{1cm} (1)

Here, the yield stress, $Y_d$, is implied to be constant and, respectively, the onset of ductile yielding is independent of pressure. In brittle materials, the onset of inelastic deformation corresponds to initiation of brittle compressive fracture. It may be described by the Griffith’s failure criterion

$$\left(\sigma_1 - \sigma_2\right)^2 = Y_{br}(\sigma_1 + \sigma_2),$$  \hspace{1cm} (2)

which is based on the assumption that the compressive failure starts when the highest local tensile stress acting on the longest crack of most vulnerable orientation reaches a critical value. In contrast
to the case of ductile response, the stress difference \((\sigma_1 - \sigma_2)\) at failure onset grows with increasing pressure (mean stress). Actually, the Griffith’s failure criterion (2) presents a minimum estimation of the compressive failure threshold. Usually the dependencies \(\sigma_1(\sigma_2)\) at the failure threshold are stronger than that predicted by this criterion (McClintock and Argon, 1966).

Intersection of the uniaxial compression beam \(\sigma_1 = \sigma_2 (1 - \nu)/\nu\) with one of the lines in Fig. 5 presenting either the ductile yielding criterion (1) or the criterion of compressive fracture (2) determines the HEL value. One can see that applying additional transversal confining stress \(\pi\) shifts the intersection point toward higher values of longitudinal stress \(\sigma_1\). The shift is much greater for the brittle response than that of ductile one. Estimating the sensitivity of the HEL value to small variations of the transversal stress \(\pi\) gives (Zaretsky and Kanel, 2002):

\[
\frac{d\sigma_{\text{HEL}}^{\text{duct}}}{d\pi} = \frac{(1 - \nu)}{(1 - 2\nu)},
\]

(3)

\[
\frac{d\sigma_{\text{HEL}}^{\text{brit}}}{d\pi} = \frac{(1 - \nu)(3 - 2\nu)}{(1 - 2\nu)}.
\]

(4)

As apparent from Eqs. (3) and (4) the sensitivity of the Hugoniot elastic limit to the confining stress for brittle response is \((3 - 2\nu)\) times higher than that for the ductile one. Due to this, measuring the sensitivity of the HEL to small variations of transversal stress \(\pi\) allows one to verify whether the criterion of brittle failure or ductile yielding is applicable, and also the mode of inelastic deformation produced in the material by planar impact loading.

The transverse stress was varied experimentally by using shrink-fit sleeves as suggested by Chen and Ravichandran (1996, 2000). The samples, hot-pressed \(\text{Al}_2\text{O}_3\) and \(\text{B}_4\text{C}\), were precisely cut disks of 25 ± 0.002-mm diameter and 5-mm thickness. The shrink-fit steel rings of outer diameter 45 mm were machined from rods of normalized 4340 steel with the Young’s modulus \(E = 200\) GPa and \(\nu = 0.28\). The inside diameter of the rings was cut by a CNC turning machine to a diameter smaller than the diameter of the samples by \(\delta = 0.1 ± 0.005\) mm. Prior to the insertion of the ceramic discs, the rings were heated to 600 °C. The confining stress \(\pi\) produced by the ring was estimated by use of the known solution for an axi-symmetric boundary value problems and the yield stress of the ring material which was measured on the witness sample. More experimental details may be found elsewhere (Zaretsky and Kanel, 2002). The stress state of the steel ring falls just a little outside its elastic range; the confining stress was found equal to \(\pi = 0.3\) GPa for alumina samples and \(\pi = 0.32\) GPa for the boron carbide samples.

The corresponding experimental arrangement is shown schematically in Fig. 6. The study was based on comparison of waveforms in free and pre-stressed samples under the same impact conditions. The planar impact loading of alumina samples was carried out using 1-mm copper flyer plates launched by a 57-mm gas gun facility with a velocity of 500 ± 10 m/s. The impactor-sample misalignment was controlled in each shot by three flush pins and did not exceed 0.5 mrad. The boron carbide samples were shock-loaded by aluminum flyer plates of 2-mm thickness accelerated to a

![Fig. 5.](image-url)

**Fig. 5.** Different influence of a lateral pre-stressing on the Hugoniot elastic limit for ductile and brittle materials.
velocity of 1900 ± 50 m/s by explosive facilities. At the chosen ratio of sample thickness to diameter (5 and 25 mm, correspondingly), shock compression at the sample axis is uniaxial during the complete time of recording. The lateral surface of the sample or the ring may influence the stress history at the sample axis 0.5–1 μs later.

Fig. 7 presents the results of two experiments with free samples of alumina ceramic having sound speeds \( c_l = 10.03 ± 0.01 \) km/s and \( c_s = 5.93 ± 0.02 \) km/s and density \( ρ = 3.740 ± 0.005 \) g/cm\(^3\). The free surface velocity profiles \( u_{fs}(t) \) were reproducible and similar to that measured for alumina ceramics earlier by Munson and Lawrence (1979). The transition from elastic to inelastic response occurs at \( u_{fs} = 285 \) m/s and corresponds to a HEL = 5.35 GPa and to a yield stress \( Y = 2\sigma_{HEL}(c_s/c_l)^2 = 3.74 \) GPa. The velocity ramp behind the elastic wave front is associated with strain hardening and stress relaxation. The distinct spall signal characterized by the velocity pull-back \( Δu_{fs} \), Fig. 7, is present on both velocity profiles. Using the measured \( Δu_{fs} \) and \( c_l \) values the alumina spall strength estimates are 0.40 and 0.52 GPa for these two shots.

Fig. 8 compares the results of planar impact tests with free and pre-stressed alumina samples. Each shown free surface velocity profiles for the average of two shots indicating a scatter that did not exceed ±5 m/s. In the case of plastic yielding, the expected longitudinal stress increment caused by 0.3-GPa lateral pre-stressing should be equal to 0.43 GPa. The latter corresponds to a 23-m/s increase of the sample free surface velocity at the HEL. In the case of brittle inelastic response, the HEL stress and corresponding sample free surface velocity, Eqs. (3) and (4), should be 2.5 times greater and the expected free surface velocity increment should be close to 60 m/s. The measured difference between the average \( u_{fs}(t) \) data for confined and unconfined alumina ceramics for the first 0.05 μs after arrival of the elastic wave was 10–15 m/s. Accounting for the ± 3 m/s uncertainty of the VISAR measurements of free surface velocity and scatter from shot to shot, we conclude that under uniaxial shock compression alumina behaves as a ductile material.

The results of similar experiments with boron carbide samples of two different densities are shown in Fig. 9a and b. The velocity histories for samples of higher density (Fig. 9a) demonstrate a distinct
increase of the HEL value caused by small transversal pre-stressing. The pre-stressing has also changed the shape of the waveform. In the case of free samples, the elastic compression wave is followed by a gradual velocity increase towards the foot of the second compression wave. In the pre-stressed samples the gradual increase has been replaced by a plateau. Such change of profile shape with pre-stressing leads to difficulties in calculating the HEL from the waveforms. Defining the HEL as the intersection point of the upward extrapolation of the elastic shock discontinuity with the leftward extrapolation of the intermediate part of the profile yields interface velocities at the HEL equal to \( u_{\text{HEL}} = 700 \pm 10 \) m/s and to \( u_{\text{HEL}} = 775 \pm 10 \) m/s for the free and the pre-stressed samples, respectively. Using Barker and Hollenbach (1970) data on shock Hugoniot of PMMA yields for the \( r_{\text{HEL}} \) of free and pre-stressed B\(_4\)C samples the values \( r_{\text{HEL}} = 13.5 \pm 0.15 \) GPa and \( r_{\text{HEL}} = 15.1 \pm 0.2 \) GPa, respectively. So, the 0.32-GPa pre-stressing of the boron carbide samples leads to the average increase of the \( r_{\text{HEL}} \) by 1.6 \pm 0.35 GPa. Eq. (4) gives the HEL increment \( \Delta r_{\text{HEL}} \approx 1.06 \pm 0.03 \) GPa for brittle response, whereas the expected increment for ductile response is, in accordance with Eq. (3), 2.7 times less. Thus, the sensitivity of the HEL for boron carbide ceramic to variations of lateral stress exceeds essentially that expected for ductile compressive response. It is even greater than that expected for brittle response, Eq. (4). The discrepancy, 1.6 GPa vs. 1.06 GPa, appears to be due to the use of the Griffith’s criterion which underestimates the values of the compressive failure threshold. The difference between the velocity profiles of free and the pre-stressed samples decreases to zero after a lapse of about 150 ns.

![Fig. 8](image1)

**Fig. 8.** Comparison of the free surface velocity histories for pre-stressed alumina samples (495 and 505 m/s; solid line) and free alumina samples (505 and 513 m/s; dashed line).

![Fig. 9](image2)

**Fig. 9.** Velocity histories of sample–PMMA interfaces obtained on free and pre-stressed samples of boron carbide of 2.492 \pm 0.015 g/cm\(^3\) density (a) and 2.370 \pm 0.005 g/cm\(^3\) density (b) impacted by aluminum flyer plates having velocities of 1.9 \pm 0.05 km/s.

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For the low-density samples of boron carbide radial pre-stressing produces an opposite effect to that for the dense samples. As seen in Fig. 9b, both free and pre-stressed samples demonstrates the same HEL = 8.6 GPa. The parameters behind the elastic front in the pre-stressed porous sample are even lower than that for the free sample. In order to understand the difference in response of ceramics of different porosity we have to account for the difference between compaction and dilatancy under compression. In the case of a material with theoretical density compaction does not occur, whereas the compressive fracture produces some porosity as a result of the appearance and growth of cracks. The latter is known as a dilatancy or bulking phenomenon. Thus the porosity of dense brittle material may only be increased with the onset of compressive fracture. Lateral pre-stressing impedes the cracks opening, thus, the failure threshold should increase. Compression of initially porous brittle material is accompanied by two simultaneous processes: by the cracking which increases the porosity, and by the compaction which causes the porosity to decrease. As a result, the HEL value of porous brittle material is controlled either by compressive failure threshold or by the compaction threshold, depending on which process starts earlier. It should be noted that a hydrostatic pressure cannot cause fracture of a dense material while the compaction of porous material under hydrostatic compression is always possible. The transversal pre-stressing is responsible for some initial surplus of hydrostatic compression, which should decrease the compaction threshold of a porous material. Obviously, this effect is responsible for the different response to pre-stressing of boron carbide with different densities. Probably, some residual porosity is also responsible for the small effect of pre-stressing in the experiments on alumina.

It is apparent from Fig. 9 that the lateral pre-stressing leads to similar changes of waveforms: for both sample densities ramped growth of the velocity ahead inelastic compression front is replaced by a plateau or even by the velocity decay. Also, there is some trend to increase, with pre-stressing, the time interval between elastic and inelastic compression wave. These changes may be a result of different modes and rates of the precursor decay in the free and pre-stressed samples. Comparison of the waveforms shown in Fig. 9a and b demonstrates different steepness of second compression wave depending on porosity of the material. The rise-time of second compression wave in the low-density boron carbide samples is between 5 and 10 ns while in the case of high-density boron carbide it was between 40 and 70 ns. Since the rise time of a steady shock wave is controlled by the plastic viscosity, the difference indicates an effectively smaller viscosity for porous ceramic.

The experiments with pre-stressed ceramic samples show unambiguously the ductile response of alumina and the brittle response of boron carbide under planar impact loading. In our experiments with hot-pressed alumina the lateral stress at the HEL was varied between 1.6 (free samples) and 1.9 GPa (pre-stressed samples). That means the transition from brittle compressive failure to ductile yielding occurs in the alumina somewhere between the confining pressure of 1.25 Gpa, which, as was shown by Heard and Cline (1980), alumina remains brittle, and 1.6 GPa. In the case of the hot-pressed boron carbide the highest transversal stress of 2.97 GPa was apparently insufficient to induce the brittle to ductile transition.

If the response of a brittle material is brittle under planar shock wave loading, its behavior for impact or penetration will also be brittle. However, if the material remains ductile under uniaxial shock compression, we need to know the conditions of the brittle–ductile transition in order to describe the material behavior. It would be therefore desirable to expand the capabilities of the shock-wave experiments in order to obtain measurements over a wider range of stressed states at varying ratios of longitudinal to transversal stresses.

4. Use of divergent impact loading for study of the compressive fracture of brittle materials

4.1. Preliminary analysis of divergent flows generated by impact of convex flyer plate

As mentioned above, arbitrary impact loading is accompanied by a wide-range in variation of the ratio $\sigma_2/\sigma_1$ between principal transversal, $\sigma_2$, and longitudinal, $\sigma_1$ stresses. On the contrary, the uniaxial material flow produced by planar impact is characterized by the constant ratio, $\sigma_2/\sigma_1$. Varying the $\sigma_2/\sigma_1$ ratio can be realized in the pressure-shear impact experiments (Abou-Sayed et al., 1976; Sundaram and Clifton, 1998). However, it would be difficult to find an elastic material for anvils in...
order to study the behavior of hard ceramics in this way and under high-velocity impact loading. A divergent flow, characterized by the variable stress ratio, \( \sigma_2/\sigma_1 \), suggests a solution of the problem of laboratory material testing under loading conditions close to those of real impact conditions. Earlier measurements under conditions of divergent shock loading using explosive facilities were performed by Tranchet and Collombet (1995) for spherical samples and by Kanel et al. (1998) for thin-wall cylindrical samples. Their experiments, however, were too complicated for implementation and were not optimized for observation of the compressive fracture events.

Fig. 10 shows possible load paths produced for spherical divergent flow of brittle materials. Under such loading, in spite of the decay of both \( \sigma_r \) and \( \sigma_\theta \) stress components, the stress states at the shock front remain at the same ray \( \sigma_r = \sigma_\theta (1 - \nu)/v \) similar to uniaxial elastic compression. However, while in planar impact the uniaxially-compressed state is completely reversible within the elastic domain and the stress states do not leave the ray \( \sigma_r = \sigma_\theta (1 - \nu)/v \), in divergent flow the stress states leave this ray immediately behind the shock discontinuity as a result of the expansion of each spherical layer. The result of such departure from the planar impact path is the growth of the difference \( \sigma_r/\sigma_\theta \) as the distance from the shock front increases. Possible material state trajectories in the principal stress space are shown in Fig. 10b by arrows. The divergent character of the flow turns the material trajectory towards the line corresponding to compressive fracture. The stress states at the fracture line, unachievable under planar impact loading, may be achieved behind the divergent compression wave; at some distance from the shock front the hoop stress may even become tensile.

Thus, in divergent flow brittle failure may occur later behind the shock wave even if the initial shock compression is purely elastic. When brittle failure happens under compression, the load-carrying ability of the material should drop and the deviatoric stress in the vicinity of the failure site should relax. It may be expected that an unloading signal accompanying this event will appear in the waveform.

The divergent shock loads may be created by detonation of high explosives that may provide high symmetry of divergent loading, but this approach is expensive, associated with too high of initial peak stresses, too rapid decay of the shock, and very limited capabilities to vary load parameters. A more promising way is to generate nearly spherical shocks in plane samples using impactors with a spherical impact surface of relatively large radius of curvature. Such a method of generating divergent flow in a plane target, as shown in Fig. 11, requires detailed consideration.

Collision of a spherical impactor with a plane target creates divergent shock wave in the latter. The radius of curvature of the shock wave in the target, \( R_S \), is smaller than the impactor radius: \( R_S < R_i \). The value of \( R_S \) is estimated in following way. During the initial phase of collision the shock waves in the target and in the impactor are emitted from the impactor–target interface bounded by the intersection line \( P \) of radius \( r(t) \). For a spherical impactor with radius \( R_i \) the intersection line moves in accordance with the relationship

\[ r(t) = R_i \left(1 - \frac{t}{C_0 m} \right)^{1/2} \]

Fig. 10. (a) Scheme and (b) load paths for spherical shock compression of brittle materials.

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\[ r(t)^2 + (R_i - u_i t)^2 = R_i^2, \]  
\[ (5) \]

Assuming a spherical shape for the elastic shock wave in the target gives:

\[ r(t)^2 + (R_S - c_t t)^2 = R_S^2, \]  
\[ (6) \]

where \( c_t \) is the velocity of propagation of elastic shock wave. Combining Eqs. (5) and (6) one has for the radius of the shock front

\[ R_S = \frac{2u_i R_i - u_i^2 t + c_t^2 t}{2c_t}. \]  
\[ (7) \]

The initial \((t = 0)\) radius of curvature of the shock front at the impact axis is

\[ R_S = R_i u_i / c_t. \]  
\[ (8) \]

At some instant \( t_c \) the velocity of the expanding intersection line (point \( P \) in Fig. 11) may become subsonic with respect to the target material. As a result, at later time, the shock wave in the target outruns the intersection line. The time \( t_c \) at which this occurs may be found by equating the time derivative of \( r(t) \) from Eq. (5)

\[ \frac{dr}{dt} = \frac{R_i - u_i t}{\sqrt{2u_i R_i t - u_i^2 t^2}} \]  
\[ (9) \]

to the speed of sound, \( dr/dt = c_t \). This yields

\[ t_c = \frac{R_i}{u_i} \left( 1 - \sqrt{\frac{c_t^2}{u_i^2 + c_t^2}} \right). \]  
\[ (10) \]

At the time of transition from the supersonic to subsonic regime of collision the shock front in the target becomes perpendicular to the impact surface. Substitution of \( t_c \) into Eq. (5) gives us the radius \( R_{Sc} = R_S \) of the elastic shock wave in the target at which it breaks away from the line of impactor/target contact:

\[ R_{Sc}^2 = R_S^2 - \frac{u_i^2}{c_t^2 + u_i^2}. \]  
\[ (11) \]
For ceramic targets $c_l \geq 10 \text{ km/s}$, whereas the impact velocity $u_i$ typically must not exceed 1 km/s in order to produce a shock stress below the HEL. For this ratio, the relationships (11) and (8) give approximately the same value of $R_s$, which means that the shape of the divergent shock wave in the target is very close to spherical. The loss of contact between the shock front and the intersection line due to the change of the speed of the line motion from supersonic to subsonic should cause instability of the collision process.

It is useful for further analysis to have a dependence of the stress at the elastic shock front as a function of its radius. This dependence may be found within the acoustic approximation. The conservation relations for spherical stress wave are (Wilkins, 1999; Fowles, 1970):

\[
\frac{\partial u}{\partial h} + \rho_0 \frac{r_0^2}{r^2} \left[ \frac{2u}{\rho r} - \frac{\partial V}{\partial t} \right] = 0, \tag{12}
\]

\[
\frac{\partial \sigma_r}{\partial h} + \rho_0 \frac{r_0^2}{r^2} \left[ 2 \frac{\sigma_r - \sigma_\theta}{\rho r} + \frac{\partial u}{\partial t} \right] = 0, \tag{13}
\]

where $h$ is Lagrangian coordinate. The derivative of the stress and the particle velocity along the shock wave trajectory $L$ are

\[
\frac{d u}{d h} \bigg|_L = \frac{\partial u}{\partial h} + \frac{1}{c_l} \frac{\partial u}{\partial t}, \tag{14}
\]

\[
\frac{d \sigma_r}{d h} \bigg|_L = \frac{\partial \sigma_r}{\partial h} + \frac{1}{c_l} \frac{\partial \sigma_r}{\partial t}. \tag{15}
\]

After accounting for the conservation laws (12) and (13) and putting $r = r_0$ immediately behind the shock discontinuity, Eqs. (14) and (15) transform to

\[
2 \frac{d \sigma_r}{d h} = \frac{1}{c_l} \frac{\partial \sigma_r}{\partial t} - 2 \rho_0 \frac{\sigma_r - \sigma_\theta}{\rho r} - \rho_0 c_l \left[ \frac{2u}{\rho r} - \frac{\partial V}{\partial t} \right], \tag{16}
\]

\[
2 \frac{d \sigma_\theta}{d h} = \frac{1}{c_l} \frac{\partial \sigma_\theta}{\partial t} - \rho_0 \frac{\sigma_r - \sigma_\theta}{\rho r} \left( 2 \frac{c_s}{c_l} u - \sigma_r + \sigma_\theta \right), \tag{17}
\]

From Eqs. (16), (17), and accounting for the Rankine-Hugoniot equation $\sigma_r = \rho_0 c_i u$, we obtain

\[
2 \frac{d \sigma_r}{d h} = \frac{1}{c_l} \frac{\partial \sigma_r}{\partial t} + 2 \rho_0 \frac{\sigma_r - \sigma_\theta}{\rho r} - \rho_0 c_l \left[ \frac{2u}{\rho r} - \frac{\partial V}{\partial t} \right]. \tag{18}
\]

For spherical flow in solids it may be shown that for an acoustic approach

\[
\frac{\partial \sigma_r}{\partial t} = -\rho \rho_0 c_i^2 \frac{\partial V}{\partial t} + 4G \frac{u}{r}, \tag{19}
\]

where $G$ is the shear modulus. After substitution Eq. (19) into (18) we have

\[
2 \frac{d \sigma_r}{d h} = (\rho_0^2 c_i^2 - \rho_0 c_i) \frac{\partial V}{\partial t} + 2 \rho_0 \frac{\rho c_s u c_i}{c_l} - \rho_0 c_i u - \sigma_r + \sigma_\theta), \tag{20}
\]

where $c_s$ is the shear wave speed. At small compressions, $\rho \approx \rho_0$, we may rewrite (20) as

\[
\frac{d \sigma_r}{d h} = \frac{1}{r} \left( 2 \rho c_s u \frac{c_i}{c_l} - \rho_0 c_i u - \sigma_r + \sigma_\theta \right). \tag{21}
\]

Taking into account that $\sigma_\theta = \left( K - \frac{2}{3} \right) \frac{\Delta V}{V} = \sigma_r - 2 \rho_0 c_s^2 \frac{u}{c_l}$, we finally obtain

\[
\frac{d \sigma_r}{d h} = -\frac{\sigma_r}{r} \tag{22}
\]

Or

\[
\sigma_r = \sigma_r \frac{r}{r}. \tag{23}
\]

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with $\sigma_r$ being the radial stress behind the spherical shock front of radius $r$. Thus, the acoustic approach provides the decay of the shock stress inversely proportional to the shock front radius. Computer simulations of elastic spherical shock wave confirmed this conclusion.

Thus, estimations show that impact of a convex flyer plate generates divergent shock wave in the target whose shape is very close to spherical. This, however, does not mean spherical symmetry of the flow over the whole shock-compressed area. The radial stresses behind the shock front are found to be larger at the impact axis than at the wave front periphery.

As mentioned above the stresses produced by such loading should not exceed the target elastic limit. This condition, however, generates both longitudinal and shear waves simultaneously at the impacted surface; the geometry of such flow is shown in Fig. 12. The shear wave propagates with the velocity $c_s < c_l$. The slow moving shear waves reduce divergence flows and saturate or reduce the shear stress $(\sigma_r - \sigma_\theta)$ levels.

The stress states produced by impact of the convex impactor (copper) in the plane-parallel alumina sample were analyzed with use of the commercially available AUTODYN 2-D code (Century Dynamic Inc., 2005). Fig. 13 shows an example of the AUTODYN simulation results. The waveforms for divergent impact loading demonstrate gradual increase of the longitudinal stress and the shear stress behind the compression wave while the pressure is almost constant. The growth of the shear stress is halted at the expected arrival time of the shear wave at the observation point. After that the shear stress decreases slowly. The flow generated in planar alumina samples by convex copper impactors was found to be nearly spherical between the longitudinal and shear wave fronts within $\pm 30^\circ$ from the impact axis.

The shear wave shown in Fig. 12 separates the domain 1 of highly divergent flow behind the spherical longitudinal shock wave from the domain 2 of low-divergent flow. Roughly speaking, the divergence of flow in region 1 is characterized by the curvature of the longitudinal wave, which is about one inverse centimeter in the experiments discussed below. Divergence of flow in region 2 is much lower, a few tenths of a inverse centimeter.

Fig. 14 presents the time–distance diagrams outlining possible regions of initiating the compressive fracture in such two-domain flows. It is assumed that the shock compression itself does not produce any fracture; the fracture threshold is reached as a result of the increase in deviatoric stresses during divergent flow. For purely spherical flow (both the impactor and the target are spherical) the compressive fracture threshold should be reached in a shock-compressed sample first on its impact surface. In the case of a plane target impacted by a convex flyer plate the existence of the two flow domains bounds the possible locus of fracture initiation; the fracture is initiated between the longitudinal and shear waves. The separation with propagation distance between the two waves increases the duration of the divergent phase of the flow associated with growing deviatoric stresses. On the other hand, the decay of the longitudinal shock wave decreases the rate of growth of the stress

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difference and, thus, increases the time needed in order to reach the failure threshold. As a result, the failure initiation should first occur in the immediate region ahead of the shear wave. When the failure occurs it should produce an unloading signal which, in turn, should be displayed in the wave profile.

4.2. Experimental configurations

Divergent impact conditions were produced in two experimental configurations: using both gas gun and explosive facilities. In all cases the radius of curvature of shock waves in the targets was in the range of 1–3 cm.

Fig. 13. The histories of radial stress, pressure (mean stress) and shear stress generated in a 5-mm alumina plate by impact of a convex copper flyer plate having a 300-mm radius of curvature. Impact velocity is 260 m/s. The observation point is at 3-mm distance from the impacted surface on the impact axis. The dotted vertical line at ~0.48 μs corresponds to shear wave arrival at the observation point.

Fig. 14. The time–distance diagrams for initiating a compressive fracture under divergent impact loading. Dashed lines show possible trajectories along which the loading state for the material intersects the fracture threshold in the cases of spherical impact (a) and impact by a convex flyer plate (b).

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A schematic of the experiments using launching of an impactor plate by a gas gun facility is shown in Fig. 15a. The 58-mm, 4.2-m length gas gun of the Laboratory of Dynamic Behavior of Materials at Ben Gurion University launched copper flyer plates with a spherical impact surface. The convex impactor with a surface curvature of radii ranging from 200 to 600 mm were prepared of copper discs (5-mm thickness) that were glued to the front surface of PMMA disks (15-mm thickness, both the disks of 56-mm diameter), which, in turn, were glued to the front edge of the hollow aluminum sabot. Further machining of the copper head of the assembly by precision CNC techniques provided the projectile-impactor concentricity. Actually, the impactor was a spherical segment of given radius of curvature and about 40-mm diameter extended (0.4–0.5 mm) from the plane disk. The 10-mm plane impactor shoulders were used for shortening the velocity and trigger pins. Uncertainty of the impactor surface curvature near the projectile axis did not exceed +30 mm for the impactor radius $R_i = 200$ mm and +90 mm for the impactor radius $R_i = 600$ mm. In all shots the shoulder-sample misalignment did not exceed 2 mrad.

In the case of use of the explosive facilities shown in Fig. 15b, the convex shape of impactors was formed during explosive launch. The shape of the impactor plate was recorded using a mechanical streak camera. In the experiments discussed below, we used copper impactors of 5.5-mm thickness and aluminum impactors of 7-mm thickness. The radii of curvature of the impactors were 365 and 195 mm and the impact velocities were 555 and 1475 m/s, respectively. In order to decrease the peak stress of shock compression, the ceramic sample plates of 8–10 mm thickness were covered by aluminum, Teflon, or PMMA base plates, which were of 4.1–5.9-mm in thickness.

If the rear surface of a spherical sample is free, the longitudinal stress at the surface is always zero. The transversal stress at the surface becomes tensile immediately after initiation of the surface motion. Since the tensile strength of brittle ceramics is low, the start of the spherical expansion of the surface layer of the sample should result in its tensile fracture prior to the initiation of its fracture in compression. The sample surface layer fractured in tension becomes unsuitable for transmission of the tensile signals from the deeper sample region: the fractured surface layer cannot be decelerated. This means that manifestations of unloading waves accompanying the compressive fracture will not appear in the longitudinal velocity histories recorded at the sample free surface; the sample surface should be protected against the tensile fracture by a window. Respectively, the velocity history of the sample/window interface should be used for studying the compressive failure.

In the gun experiments the VISAR was used for monitoring the velocity of the interface between alumina samples and sapphire windows. One surface of the sapphire window of 6-mm thickness and 20-mm diameter was gold-sputtered in order to provide sufficient reflectivity of the interface. The sputtered window surface was glued to the rear surface of the alumina sample. The apparent velocities were corrected using the derivative of sapphire refractive index on compression.
$k_0 = 0.7864$ (Setchell, 2002). The difference between literature data on sapphire refractive index $n_0 = 1.771763^{12}$ ($\lambda = 532$ nm) and that for actual VISAR light ($\lambda = 514$ nm) was neglected, as well as the influence of the wave curvature on the index correction. The latter is justified by negligibly small difference $\mu_0 - k_0 = n_0 - 1 - k_0$ to which the curvature correction is proportional (Wackerle et al., 1987).

In the explosive experiments we recorded the free surface velocity histories of copper witness plates of 2.6-mm thickness placed behind the ceramic samples. The dynamic impedance of copper is very close to that of the tested ceramics; so we expect that copper witness plates will minimally distort the flow in ceramic samples and that their free surface velocity histories will reproduce well the waveforms arriving at the rear surfaces of the samples.

The materials tested were two kinds of alumina and boron carbide ceramics. 99-% purity hot-pressed alumina (Microceramica Ltd., Karmiel, Israel) was tested in the gun experiments. Its density was in the range of 3.896–3.910 g/cm³, the longitudinal sound speed was $c_l = 10.58 \pm 0.05$ km/s and the shear sound speed was $c_s = 6.23 \pm 0.02$ km/s. Poisson’s ratio determined from the sound velocities was $\nu = 0.235 \pm 0.004$. The HEL of this alumina has been found to be 5.84 ± 0.09 GPa, which corresponds to a yield strength $Y = 4.1 \pm 0.2$ GPa. The samples were discs of 25-mm diameter and 5-mm thickness.

In the explosive experiments, the behavior of Russian B6 alumina ceramic and hot-pressed B$_4$C ceramic were studied. B6 is a cold pressed and sintered alumina ceramic of 1800 HV hardiness, which contains 96% Al$_2$O$_3$. In accordance with measurements of water absorption, the B6 ceramic had no porosity. Its density was $\rho = 3.85$ g/cm³, and the longitudinal sound speed was $c_l = 10.15$ km/s. The Hugoniot elastic limit of this material was measured and preliminarily found equal to 6.5 GPa. Tested samples were plates ground to dimensions of $80 \times 80 \times 10$ mm$^3$. The boron carbide ceramic was hot-pressed, contained grains of 1–10 $\mu$m size, had a density of 2.52 g/cm$^3$ and a longitudinal sound speed of $13.62 \pm 0.1$ km/s. The HEL of this material was measured to be 14 GPa. The final samples were in the form of plates, 80 mm × 80 mm × 8 mm.

4.3. Results of measurements

Fig. 16 presents the results of experiments on B6 alumina samples subjected to divergent impacts. In the shot with corresponding to the divergent impact experiment marked 1 in Fig. 16 the radial stresses near the input surface of ceramic sample exceeded the HEL. The stresses in shots 2 and 3 remained below the HEL. All waveforms for divergent impact loading demonstrate a gradual increase of velocity behind the longitudinal compression wave and a sudden decrease of slope near the expected arrival time of the shear wave. The waveforms do not display an unambiguous signature of compressive fracture.

![Fig. 16](image-url) Divergent tests on 10 mm thickness B6 alumina. The velocity histories of copper witness plates were recorded.
Fig. 17 summarizes results of the divergent impact experiments with B₄C ceramic plates over a wide range of peak stresses. The waveform measured at maximum peak stress radically differs from that at low stresses: instead of an increase, the free surface velocity measurements at the highest level show a decrease of stress behind the compression wave front. At lower peak stresses, the stress decrease behind the shock wave is replaced by a stress increase. In alumina, the stress growth behind the shock wave was practically linear; however, in the B₄C waveforms are nonlinear. We associate these features with a slow fracture process in shock compressed boron carbide.

Fig. 18a presents typical particle velocity profiles in gun experiments with convex copper flyer plates having a 600-mm radius of curvature. A fast initial ramp is followed by a slow velocity increase corresponding to the divergent flow accompanied by an increase of the stress difference \(\sigma_1 - \sigma_2\). The distinct failure signatures (shown by arrows) are apparent in both profiles. In the absence of failure, the interface velocity should continue to grow up to the instant of the shear wave arrival at the interface. The arrows show arrivals of the failure initiation signal at the interface. Dashed lines show the time for the expected arrival of the shear wave (I), and the arrival of the shock wave at the rear surface of the sapphire window (II).

Another type of velocity profile displaying two distinct failure signatures (marked with arrows) is shown in Fig. 18b. The solid line shows result of a computer simulation with no failure. The right arrow corresponds to the failure event taking place at approximately 2.2 mm from the sample axis. The interface velocity profile obtained from an AUTODYN simulation of this impact experiment (without failure) is shown in Fig. 18b in order to emphasize the arrival of the shear wave at the interface, which suspends growth of the stress difference and interface velocity. The first velocity decrease and recovery corresponds to rapid fracture initiated on the alumina sample axis at the distance of about 0.8 mm from the sample-sapphire interface. The second velocity downfall seems to correspond to failure initiation aside the sample axis.

4.4. Data analysis and interpretation

In Figs. 19 and 20 the results of explosive experiments are shown in normalized form; measured values of the free surface velocity have been normalized by the \(u_s\) value, where \(u_s\) is the free surface velocity just behind the shock front. Estimated initial radii of curvature of the shock waves were between 2 and 2.5 cm in all shots. At low impact stresses the wave profiles clearly demonstrate qualitative and quantitative similarities and, for both materials, practically coincide in normalized velocities.

The shear wave distinctly separates the regions of high- and low-divergent flows, which are characterized by different stress gradients. In the low-divergent region behind the shear wave the velocities (and obviously the stresses) are nearly constant, whereas in the high-divergent region there is practically a linear transition between the decaying spherical wave and the low-divergent state.
Fig. 18. (a) Typical VISAR records of the velocity of an alumina-sapphire interface obtained by impact loading of plane-parallel alumina samples by convex copper impactors. (b) Comparison of measured and calculated particle velocity histories of an alumina plate impacted by a convex impactor with a 300-mm radius of curvature at 208 m/s impact velocity.

Fig. 19. Normalized free surface velocities of divergent tests on 10 mm thickness B6 alumina shown earlier in Fig 16.

Fig. 20. Normalized free surface velocities of divergent tests on 10 mm thickness B6 alumina shown earlier in Fig 17.
The similarity of the waveforms may be considered as indirect evidence of purely elastic behavior without fracture at these lowest shock stresses.

At impact stresses above the HEL the inelastic deformation processes begin immediately at the impact surface. Corresponding waveforms differ from those measured at low stresses. In the case of alumina, the ductility results in a lesser slope for the whole upper part of the waveform. In the case of brittle boron carbide, the gradual growth behind the shock front is replaced by the velocity decay as a result of its compressive fracture. In shots with boron carbide at intermediate stresses the steepness of the initial parts of waveforms behind the shock front gradually decrease with increasing the stress, as well as the normalized velocity values at the time of arrival of the shear wave.

Even when fracture events were not recorded, results of measurements may be used to outline the range of stressed states which are below the failure criterion. Since the measured waveforms demonstrate growing radial stresses behind the compression wave, it is plausible to assume for rough estimations that spherical extension occurs at constant mean stress. In this case it may be shown that

$$\dot{S}_0 = -2C_0 \frac{u_p}{r}, \quad \dot{S}_r = -2S_0,$$

where $S_r$ and $S_0$ are the radial and hoop deviatoric stresses, correspondingly. Fig. 21 presents the estimated trajectories of stressed states for outer layers of the alumina samples. Initial states on these trajectories correspond to measured free surface velocity immediately behind the elastic compression wave. The time interval $\Delta t = 0.65 \mu s$ for the estimated spherical extension is the delay time between longitudinal and shear wave arrivals at the sample surface. The flow radius $r = 31–35$ mm was estimated, accounting for thicknesses of the base plates and the samples.

The estimated stressed state in shot 1 is beyond the Von Mises criterion of ductile yielding, which means that plastic deformation should occur during divergent extension. Data of shots 2 and 3, in general, are in agreement with the data of Heard and Cline (1980). In order to obtain more information about behavior of ceramics in these conditions, computer simulations are necessary.

Computer simulations of the gun experiments with alumina were performed with the 2-D axisymmetric Lagrangian solver AUTODYN 2-D code. The elastic-perfectly plastic response of alumina was described by a von Mises yield criterion with $Y_0 = 4.3$ GPa. The simplified equations governing the material compressive failure (Ashby and Sammis, 1990; Chen and Ravichandran, 2000) are

$$|\sigma_r - \sigma_\theta| \leq \sigma_f, \quad \sigma_f = \sigma_{f0} + f \sigma_\theta. \quad (22)$$

The post-failure behavior of the comminuted material is given by

$$|\sigma_r - \sigma_\theta| \leq \sigma_c, \quad \sigma_c = \sigma_{c0} + f \sigma_\theta. \quad (23)$$

and has the form of the Mohr–Coulomb friction law. Here, $\sigma_f$ is the compressive failure threshold, $\sigma_{f0}$ is the post-failure inter-particle friction in the comminuted material, and $\sigma_{f0}, f, \sigma_{c0}$, and $c$ are material
When the failure criterion (22) is satisfied, compressive cracking of the material is initiated. The degree of material fracture is described by a damage parameter $D$ varied between zero (no fracture) and one (complete fracture), while the fracture increment $dD$ is assumed to be proportional to the increment of inelastic deformation $de$,

$$dD = kD de$$  \hspace{1cm} (24)

The damage-induced degradation of material properties is described by replacing the failure threshold, $r_f$, by the local threshold $r_D$ of damaged material:

$$r_D = \left( \frac{1}{C0} \right)^D r_f + Dr_c$$  \hspace{1cm} (25)

The search for failure parameters, $r_{f0}, f, r_c, c$, and $k_D$, was conducted in several stages, with the goal of obtaining an optimal set suitable for all the experiments by reproducing, as well as possible, the velocity profiles obtained in each shot.

Fig. 22 provides a comparison of a VISAR record with calculated waveforms using various sets of parameters. The difference between the best-fit parameter and the average may be accepted as an estimate of the uncertainty for determining the various parameters.

To summarize, both the alumina compressive failure threshold (failure initiation in compression) and the trajectory of the comminuted alumina particles are described in principal stress space by the linear equations with equal slopes

$$\sigma_f = 3.45 + 1.86\sigma_\theta$$  \hspace{1cm} [GPa]  \hspace{1cm} (26)

$$\sigma_c = 1.3 + 1.86\sigma_\theta$$  \hspace{1cm} [GPa]  \hspace{1cm} (27)

The parameter, $k_D$, relating the alumina comminuting level $D$ and the inelastic strain value is found equal to $k_D = 270 \pm 20$ and corresponds to relatively fast kinetics of cracking in alumina.

The failure threshold described by Eq. (26) intersects the ductile yield line, $\sigma_1 = 4.3 + \sigma_2$ at the coordinates $\sigma_1 = 5.33$ GPa, $\sigma_2 = 1.01$ GPa. The latter implies that under radial compressions larger than 1.01 GPa, the response of alumina should become ductile. However, the static data of Heard and Cline (1980) indicated that the alumina remained brittle up to a radial compression of about 1.25 GPa. The disagreement appears to be related to the oversimplified description of the ductile behavior of alumina. A 5% increase of the yield strength of alumina shifts the radial stress towards the value found by Heard and Cline (1980).

5. Discussions and conclusions

To fundamentally evaluate inelastic deformation and failure thresholds in hard brittle materials, such as $Al_2O_3$ and $B_4C$ type armor ceramics, this paper considered three different plate impact
experimental configurations. The first configuration utilized a layered ("pile") target assembly, the second employed pre-stressed target plate, and the third included a convex flyer plate. We identified three parameters that influence inelastic deformation and failure threshold: surfaces, transverse stress, and ratio of shear and longitudinal stresses. The "pile" experiment provides critical conditions to evaluate the effects of surfaces. The pre-stress experiment allows investigating the effect of transverse stress on the failure threshold. The divergent flow experiment permits conditions to vary the stress ratio.

One of the main motivations to consider these "theory critical" test configurations is to probe and establish whether the failure wave phenomenon observed in "amorphous" glasses is present in "polycrystalline" ceramic materials or not. Based on the analyses of the measured time resolved wave profiles, there is no evidence for the existence of failure waves in shock-compressed alumina and boron carbide ceramics. Several investigators, based on their experimental studies on several glasses, reported that failure waves nucleated or initiated at surfaces and it is a surface state induced phenomenon. Therefore, the failure wave that appears in a material due to surface states (conditions) differs significantly from the failure processes inside the bulk material. The glasses indeed contain numerous crack nuclei at surfaces, whereas the bulk material is very homogeneous. This is not the case for polycrystalline ceramics. Another necessary condition for failure wave propagation is a specific relationship between the stress components; transverse stresses in uniaxially compressed material suppress propagation of failure waves in ceramics but not in glasses. The observation of failure waves in glasses confirms that the failure process below HEL is driven by brittle cracking under uniaxial shock compression. The absence of failure wave signatures in glasses when compressed well above HEL (Kanel et al., 2005) suggests brittle to ductile transition. Whereas it is not possible to confirm whether the elastic–plastic transition in shock compressed ceramics is ductile or brittle in the absence of any "signature" in the wave profiles.

Experiments with pre-stressed ceramic samples unambiguously showed ductile response of alumina and brittle response of boron carbide under one-dimensional shock compression. The modeling of ductile response of some ceramics will require determination of parameters associated with brittle–ductile transition. The planar plate impact techniques were modified to generate divergent loading conditions in the targets in order to vary the ratio of longitudinal and transversal stresses as well as to obtain experimental data for brittle–ductile transition. Under the modified technique, a convex flyer plate was employed for generating quasi-spherical shock waves. The shear wave separates a region of highly divergent flow behind the decaying spherical longitudinal shock wave from a region of low-divergent flow. Because of the existence of these two regions seems to constrain the occurrence of compressive fracture at a material point within the time interval between arrivals of the longitudinal and shear waves. An alternative way to control compressive fracture is perhaps to develop an idealistic one-dimensional spherical or cylindrical impact loading. Unfortunately, this technique will require expensive ceramic samples of exact geometries as well as it would be very difficult to control or tailor certain experimental parameters or variables.

In summary, the analyses of convex flyer based plate impact experiments on alumina and boron carbide ceramic target plates through computer simulations indicated that the divergent impact loading is a realistic and promising way of varying the stressed state and thus determining the failure conditions for brittle materials. When the strength or constitutive models for brittle solids do not account for the physics associated with different failure modes, it may not be possible to calibrate or obtain a consistent set of values for the model parameters using shock wave data.

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