Slepton production in a background electromagnetic field

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Scalar-lepton production via the decay $\ell \to \tilde{\ell}\tilde{\gamma}$ induced by a constant homogeneous electromagnetic field is considered. Asymptotic estimations of the decay-width in different domains are analyzed.

Supersymmetric theories [1] proposing a symmetry of bosons and fermions are believed to be the most promising candidates for new physics beyond the standard model. This belief is based on purely theoretical motivations, because there are no phenomenological indications that supersymmetry (SUSY) might be relevant for a description of nature. One reason why SUSY has fascinated many physicists is that it gives rise to a special ultraviolet behaviour of supersymmetric field theories and provides a neat solution of the hierarchy problem [2]. Besides, in a local version of SUSY (supergravity) supersymmetric transformations are related to the space–time transformations of general relativity, thus enabling one to construct an acceptable quantum theory of gravity [3]. All this makes SUSY a very attractive idea and compels one to find experimental evidence for it.

Supersymmetric theories predict that all particles must appear in pairs of identical quantum numbers differing in spin by one half. For example, in a minimal supersymmetric version of the standard model [4] there are two spin-zero particles (sleptons) denoted usually by $\tilde{\ell}_R$ and $\tilde{\ell}_L$ for each generation of leptons ℓ. Apart from that, all gauge vector bosons are accompanied by neutral massive Majorana fermions such as the superpartner of the photon, the photino $\tilde{\gamma}$. In the limit of exact SUSY all ordinary particles and their partners have the same masses. However, in nature, SUSY, if it exists, is broken and the superpartners must be heavy enough to have avoided detection so far. Experimental observation of these will be one of the simplest ways to discover SUSY. Different suggestions have been given in the literature for detecting charged sleptons in e^+e^-, $p\bar{p}$, e^-p machines [5]. A considerable amount of work has already been carried out in both the experimental and theoretical directions but the situation is still unclear.

The purpose of this paper is to study the effect of slepton production via the decay $\ell \to \tilde{\ell}\tilde{\gamma}$ induced by a constant homogeneous electromagnetic field. The physics of quantum processes in external electromagnetic fields is of great interest because it has many applications in the investigations of charged particles' interactions with monocrystals [6], of astrophysics [7], cosmology [8], etc. The most powerful theoretical tool in this research is the Furry perturbative approximation making use of exact solutions of relativistic particles' wave equations in the presence of a background electromagnetic field $F_{\mu\nu}$. Within this method the total width of the decay $\ell \to \tilde{\ell}\tilde{\gamma}$ can be obtained by employing the well known Volkov wave functions [9]. The result of straightforward calculations can be written in the form

$$\Gamma(\ell \to \tilde{\ell}\tilde{\gamma}) = \tfrac{1}{2}\alpha m_\ell \times \int_0^1 du\{\pi[\mathrm{Ai}(z)\mathrm{Gi}'(z) - \mathrm{Ai}'(z)\mathrm{Gi}(z)]f_1(u) + \mathrm{Ai}(z)f_2(u) + \mathrm{Ai}'(z)f_3(u)\}, \qquad (1)$$

with the abbreviations

$$f_1(u)=\tfrac{1}{2}(1-\epsilon\zeta_\parallel)u+\tfrac{1}{2}(1+\epsilon\zeta_\parallel)(\lambda_2-\lambda_1+1-u)\,,$$
$$f_2(u)=\zeta_\perp[\chi u^2(1-u)]^{1/3}\,,$$
$$f_3(u)=-(1+\epsilon\zeta_\parallel)[\chi^2 u(1-u)^2]^{1/3}\,. \qquad (2)$$

Special (Airy type) mathematical functions entering into eq. (1) have the following integral representations [10]:

$$\mathrm{Ai}(z)=\frac{1}{\pi}\int_0^\infty \mathrm{d}t\cos(tz+\tfrac{1}{3}t^3)\,,\quad \mathrm{Ai}'(z)=\frac{\mathrm{dAi}(z)}{\mathrm{d}z}\,,$$
$$\mathrm{Gi}(z)=\frac{1}{\pi}\int_0^\infty \mathrm{d}t\sin(tz+\tfrac{1}{3}t^3)\,,\quad \mathrm{Gi}'(z)=\frac{\mathrm{dGi}(z)}{\mathrm{d}z}\,, \qquad (3)$$

with the argument

$$z=[\chi u^2(1-u)]^{-2/3}\times[\lambda_1 u+\lambda_2(1-u)-u(1-u)]\,. \qquad (4)$$

Our general expression (1) corresponds to a semi-classical approximation of the total width in the domain of comparatively weak external fields, when one can neglect corrections being proportional to the small parameters $|g_1|\ll 1$, $|g_2|\ll 1$

$$g_1=\tfrac{1}{4}e^2 m_\ell^{-4}F_{\mu\nu}F^{\mu\nu}\,,\quad g_2=-\tfrac{1}{4}e^2 m_\ell^{-4}F_{\mu\nu}\tilde{F}^{\mu\nu}\,, \qquad (5)$$

where $\tilde{F}^{\mu\nu}=\tfrac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}$. In such a case there is $F_{\mu\nu}$-dependence of $\Gamma(\ell\to\tilde{\ell}\tilde{\gamma})$ only through the parameter [#1]

$$\chi=em_\ell^{-3}\sqrt{-(F_{\mu\nu}p^\nu)^2}\,, \qquad (6)$$

which defines the background field strength in the rest frame of the initial lepton ℓ ($p_\nu^2=m_\ell^2$).

Polarization effects of the decay are described by the lepton's parallel $\tfrac{1}{2}\zeta_\parallel$ and orthogonal $\tfrac{1}{2}\zeta_\perp$ mean helicities related with the polarization four-vector s_μ in the following way:

$$\zeta_\parallel=e^2\chi^{-2}m_\ell^{-5}(s_\mu F^{\mu\nu}F_{\nu\lambda}p^\lambda)\,,$$
$$\zeta_\perp=-e\chi^{-1}m_\ell^{-3}(s_\mu\tilde{F}^{\mu\nu}p_\nu)\,. \qquad (7)$$

The multiplier ϵ in eq. (2) stands for ± 1 regarding what mode of the decay is considered:

[#1] For the electron it is the well known invariant of the quantum synchrotron radiation theory [11].

$$\epsilon=+1\,,\quad \text{for } \ell\to\tilde{\ell}_R\tilde{\gamma}\,,$$
$$=-1\,,\quad \text{for } \ell\to\tilde{\ell}_L\tilde{\gamma}\,. \qquad (8)$$

The masses of superparticles entering into (2), (4) through ratios

$$\lambda_1=(M_{\tilde{\ell}}/m_\ell)^2\,,\quad \lambda_2=(m_{\tilde{\gamma}}/m_\ell)^2\,, \qquad (9)$$

are model conditioned. Depending upon whether global or local SUSY is considered there exist two distinct types of mass phenomenology. Experimental information available at present allows only to give the lower limits $M_{\tilde{\ell}}>22$ GeV, $M_{\tilde{\gamma}}>0.5$ GeV [12]. On this account it is quite permissible to treat λ_1, λ_2 as free parameters assuming in addition $\lambda_1\gg 1$.

The general formula for total width (1) is rather complicated because the integration over the spectral variable

$$u=1-e\chi^{-1}m_\ell^{-3}\sqrt{-(F_{\mu\nu}q^\nu)^2} \qquad (10)$$

(with q^ν being the slepton's momentum) cannot be done analytically. For the sake of simplicity let us consider asymptotic estimations of the total width (1) in different domains. Supposing the slepton to be much heavier than the photino ($\lambda_1\gg\lambda_2$) one gets

$$\Gamma=\tfrac{1}{18}\alpha m_\ell\lambda_1(1+\epsilon\zeta_\parallel)\kappa_1/\sqrt{3}\exp(-\sqrt{3}/\kappa_1)\,,\quad \text{for } \kappa_1\ll 1\,,$$
$$\Gamma=\tfrac{1}{18}\alpha m_\ell\lambda_1(1+\epsilon\zeta_\parallel)\tfrac{1}{3}\Gamma(\tfrac{2}{3})(3\kappa_1)^{2/3}\,,\quad \text{for } \kappa_1\gg 1\,, \qquad (11)$$

with $\kappa_1=\chi\lambda_1^{-3/2}$, $\Gamma(\tfrac{2}{3})=1.354...$ (Γ is the Euler gamma function). In the opposite case of the photino being the heaviest particle ($\lambda_1\ll\lambda_2$) more tedious calculations lead to

$$\Gamma=\tfrac{1}{2}\alpha m_\ell\lambda_2(1+\epsilon\zeta_\parallel)\kappa_2\sqrt{\frac{\lambda_1}{3\lambda_2}}\exp\left(-\frac{1}{\kappa_2}\sqrt{\frac{3\lambda_1}{\lambda_2}}\right)\,,\quad \text{for } \kappa_2\ll(\lambda_1/\lambda_2)^{1/2}\,,$$
$$\Gamma=\tfrac{1}{2}\alpha m_\ell\lambda_2(1+\epsilon\zeta_\parallel)\tfrac{1}{3}\kappa_2^2\,,\quad \text{for } (\lambda_1/\lambda_2)^{1/2}\ll\kappa_2\ll 1\,,$$
$$\Gamma=\tfrac{1}{2}\alpha m_\ell\lambda_2(1+\epsilon\zeta_\parallel)\tfrac{1}{27}\Gamma(\tfrac{2}{3})(3\kappa_2)^{2/3}\,,\quad \text{for } \kappa_2\gg 1\,, \qquad (12)$$

with $\kappa_2=\chi\lambda_2^{-3/2}$.

The intermediate region of the slepton and the

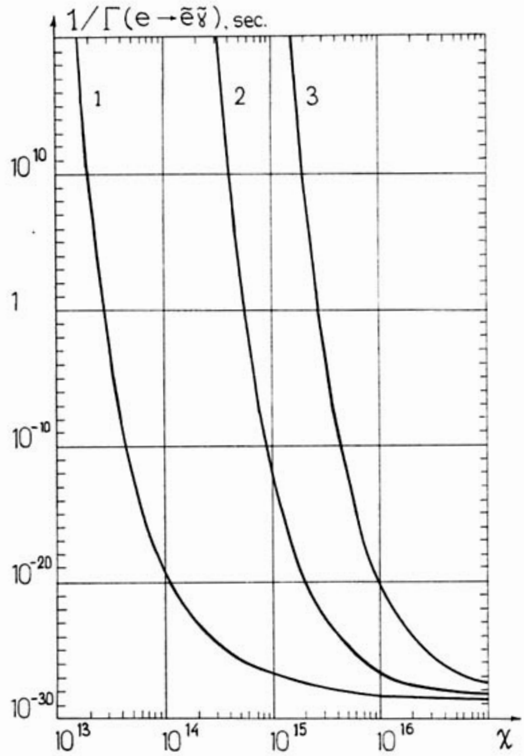

Fig. 1. Illustrative estimations showing how the time of decay depends upon the parameter χ. The curves represent the following choices: (1) $M_{\tilde{e}}=50$ GeV, $m_{\tilde{\gamma}}=0$ GeV, (2) $M_{\tilde{e}}=100$ GeV, $m_{\tilde{\gamma}}=100$ GeV, (3) $M_{\tilde{e}}=50$ GeV, $m_{\tilde{\gamma}}=500$ GeV.

photino having comparable masses can be characterized by the expression

$$\Gamma=\tfrac{1}{27}\alpha m_{\ell}\lambda(1+\epsilon\zeta_{\parallel})\kappa/\sqrt{2}\exp(-9/2\kappa)\,,$$
$$\text{for } \kappa\ll 1\,,$$
$$\Gamma=\tfrac{1}{27}\alpha m_{\ell}\lambda(1+\epsilon\zeta_{\parallel})\tfrac{1}{2}\Gamma(\tfrac{2}{3})(3\kappa)^{2/3}\,,$$
$$\text{for } \kappa\gg 1\,, \tag{13}$$

which is valid provided that $|\lambda_1-\lambda_2|\ll\lambda\equiv\frac{1}{2}(\lambda_1+\lambda_2)$.

From an inspection of formulas (11)–(13) one concludes that regardless of the mass ratio $m_{\tilde{\gamma}}/M_{\tilde{\ell}}$ it is very hard to produce a slepton by the reaction $\ell\to\tilde{\ell}\tilde{\gamma}$ in the domain of relatively weak fields ($F_{\mu\nu}\ll m_{\ell}^2/e$). This is due to the factor of exponential suppression which is typical for processes induced by an external field [13]. Unfortunately the region of greatest physical interest ($\chi\sim 10^{14}$) is not accessible now by experiment. The results of numerical computations in the case of the electron are shown in fig. 1. However, as a final remark let us stress that in the vicinity of astrophysical objects with extremely high electromagnetic fields this process might very well be observed. For instance, in theoretical papers devoted to pulsars [14] it is suggested that magnetic fields of sufficient order in the characteristic quantum electrodynamic value $m_{\ell}^2/e\sim 10^{13}$ G probably exist in nature. Further details of astrophysical applications will be considered in a forthcoming publication.

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