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**Динамика возмущенного вращательного
движения небесных тел**

$$\mathbf{G}: G, \theta, \rho, l, g, h$$

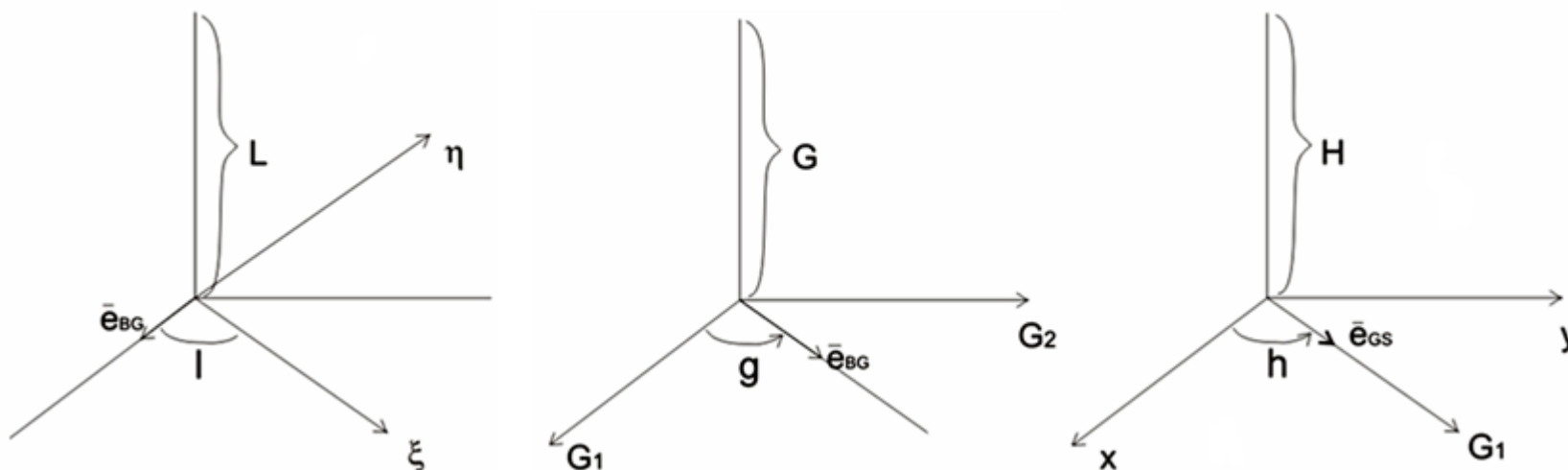

G: L, G, H, l, g, h

$$H = \mathbf{G} \cdot \mathbf{k}_s = G(\mathbf{k}_G \cdot \mathbf{k}_s) = Gg_{33} = G \cos \rho$$

Глава 11. Дифференциальные уравнения вращательного движения твердого тела в переменных Андуайе и Пуанкаре.

$$G, \quad \theta, \quad \rho, \quad l, \quad g, \quad h,$$

$$L = \mathbf{G} \cdot \mathbf{k}_b = G(\mathbf{k}_G \cdot \mathbf{k}_b) = Gb_{33} = G \cos \theta, \quad H = \mathbf{G} \cdot \mathbf{k}_s = G(\mathbf{k}_G \cdot \mathbf{k}_s) = Gg_{33} = G \cos \rho;$$



$$L = -Ep - Dq + Cr,$$

$$G = \sqrt{(A^2 + F^2 + E^2)p^2 + (F^2 + B^2 + D^2)r^2 +$$

$$+ 2(-AF - FB + ED)pq + 2(-AE + FD - EC)pr + 2(FE - BD - DC)qr},$$

$$H = p(A \sin \Theta \sin \Phi - F \sin \Theta \cos \Phi - E \cos \Theta) + q(-F \sin \Theta \sin \Phi + B \sin \Theta \cos \Phi - D \cos \Theta) +$$

$$+ r(-E \sin \Theta \sin \Phi - D \sin \Theta \cos \Phi + C \cos \Theta).$$

$$L = p_{\Phi} ,$$

$$G = \sqrt{p_{\Theta}^2 + p_{\Phi}^2 + \sec^2 \Theta (p_{\Psi} - p_{\Phi} \cos \Theta)^2} ,$$

$$H = p_{\Psi} .$$

$$p = G \left[\sin \theta (a \sin l - f \cos l) - e \cos \theta \right] ,$$

$$q = G \left[\sin \theta (-f \sin l + b \cos l) - d \cos \theta \right] ,$$

$$r = G \left[\sin \theta (-e \sin l - d \cos l) + c \cos \theta \right] .$$

$$p = \frac{G}{A} \sin \theta \sin l ,$$

$$q = \frac{G}{B} \sin \theta \cos l ,$$

$$r = \frac{G}{C} \cos \theta .$$

$$\omega = \sqrt{p^2 + q^2 + r^2} =$$

$$= G \sqrt{\sin^2 \theta \sin^2 l (a^2 + f^2 + e^2) + \sin^2 \theta \cos^2 l (f^2 + b^2 + d^2) + \cos^2 \theta (e^2 + d^2 + c^2) + \sin^2 \theta \sin 2l (e - af - bf) + \sin 2\theta \sin l (df - ea - ce) + \sin 2\theta \cos l (ef - db - cd)} ,$$

$$\omega = G \sqrt{\sin^2 \theta \left(\frac{\sin^2 l}{A^2} + \frac{\cos^2 l}{B^2} \right) + \cos^2 \theta \frac{1}{C^2}}$$

11.4. Доказательство каноничности преобразования от переменных Эйлера $\Psi, \Theta, \Phi, p_\Psi, p_\Theta, p_\Phi$ к переменным Андуайе L, G, H, l, g, h .

$$\mathbf{G}d\mathbf{\Omega} = p_\Psi d\Psi + p_\Theta d\Theta + p_\Phi d\Phi \qquad Ldl + Gdg + Hdh = \mathbf{G}d\mathbf{\Omega}$$

$$L = p_\Phi,$$

$$G = \sqrt{p_\Theta^2 + p_\Phi^2 + \cos ec^2 \Theta (p_\Psi - p_\Phi \cos \Theta)^2},$$

$$H = p_\Psi,$$

$$l = \Phi - \arccos \left\{ \frac{p_\Psi - p_\Phi \cos \Theta}{\sqrt{p_\Theta^2 + \cos ec^2 \Theta (p_\Psi - p_\Phi \cos \Theta)^2} \sin \Theta} \right\},$$

$$g = \arccos \left\{ \frac{p_\Psi p_\Phi - \left[p_\Theta^2 + p_\Phi^2 + \cos ec^2 \Theta (p_\Psi - p_\Phi \cos \Theta)^2 \right] \cos \Theta}{\sqrt{p_\Theta^2 + \cos ec^2 \Theta (p_\Psi - p_\Phi \cos \Theta)^2} \sqrt{p_\Theta^2 + p_\Phi^2 \cos ec^2 \Theta - c \tan^2 \Theta p_\Psi^2 - 2 \cos ec \Theta c \tan \Theta p_\Psi p_\Phi}} \right\},$$

$$h = \Psi - \arccos \left\{ \frac{p_\Phi - p_\Psi \cos \Theta}{\sqrt{p_\Theta^2 + p_\Phi^2 \cos ec^2 \Theta - c \tan^2 \Theta p_\Psi^2 - 2 \cos ec \Theta c \tan \Theta p_\Psi p_\Phi}} \right\}.$$

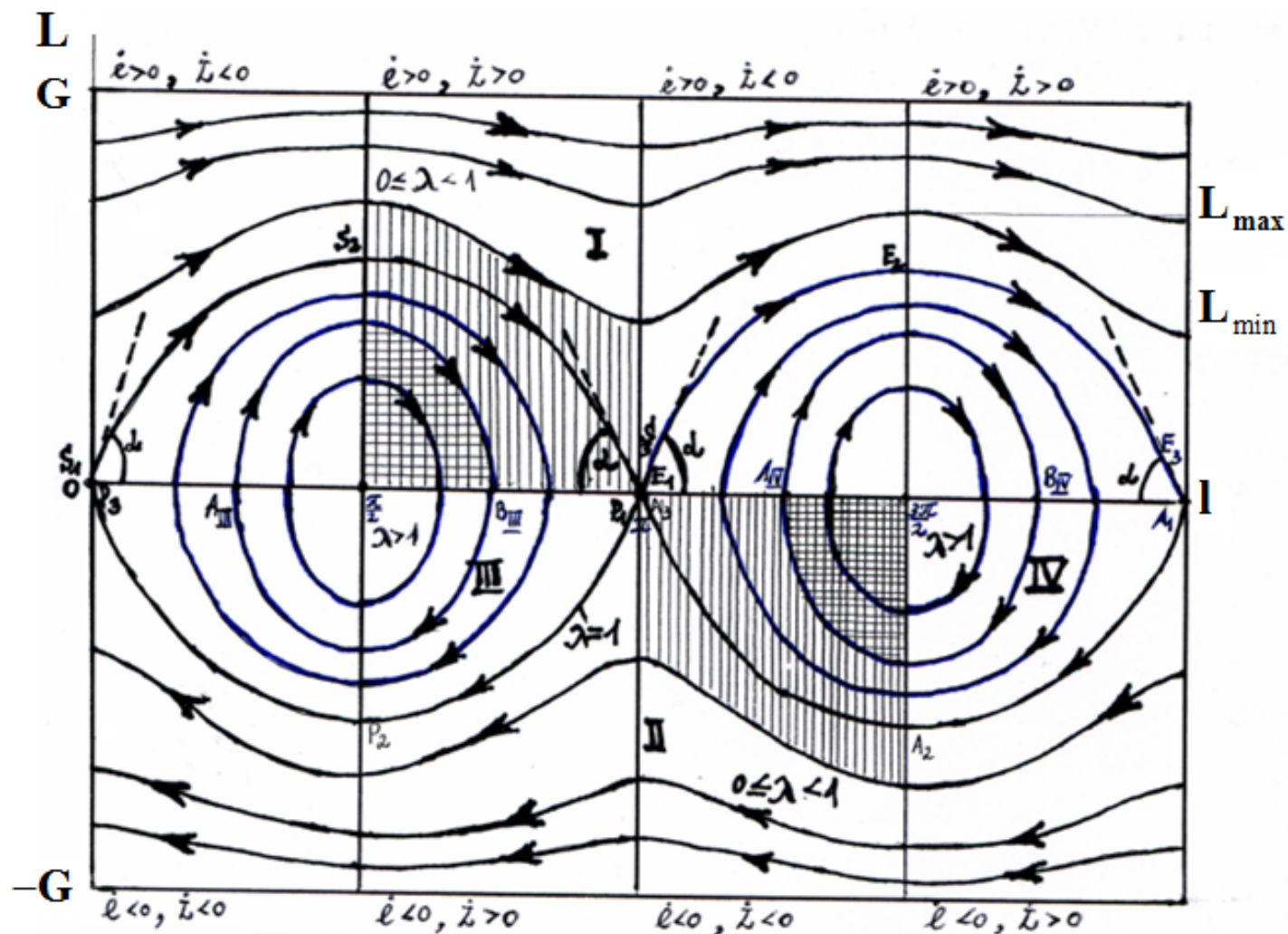
Фазовая плоскость. Области движения I и III.

$$L = L(l, \lambda, \kappa, G) = \frac{G\kappa}{\sqrt{\kappa^2 + \lambda^2}} \sqrt{\frac{1 - \lambda^2 + (\kappa^2 + \lambda^2) \sin^2 l}{1 + \kappa^2 \sin^2 l}}$$

$$\kappa^2 = \frac{C(B-A)}{A(C-B)} > 0,$$

$$\lambda^2 = \frac{A^2}{C^2} \kappa^2 \frac{p_0^2}{r_0^2} > 0$$

$$\lambda = \sqrt{1 - \frac{L_{\min}^2}{L_{\max}^2}}$$



11.5. Канонические уравнения вращательного движения твердого тела в переменных Андуайе.

$$\begin{aligned}\frac{dL}{dt} &= -\frac{\partial F}{\partial l}, & \frac{dl}{dt} &= \frac{\partial F}{\partial L}, \\ \frac{dG}{dt} &= -\frac{\partial F}{\partial g}, & \frac{dg}{dt} &= \frac{\partial F}{\partial G}, \\ \frac{dH}{dt} &= -\frac{\partial F}{\partial h}, & \frac{dh}{dt} &= \frac{\partial F}{\partial H},\end{aligned}$$

$$F = T - U$$

$$\begin{aligned}F &= \frac{1}{2}(G^2 - L^2)(a \sin^2 l + b \cos^2 l - f \sin 2l) + \frac{1}{2}cL^2 - \\ &- L\sqrt{G^2 - L^2}(d \cos l + e \sin l) - U(L, G, H, l, g, h, t).\end{aligned}$$

Уравнения движения в переменных Андуайе:

$$G, \theta, \rho, l, g, h$$

$$\frac{dG}{dt} = \frac{\partial U}{\partial g}$$

$$\frac{d\theta}{dt} = \frac{1}{2}G \sin \theta \left(\frac{1}{A} - \frac{1}{B} \right) \sin 2l + \frac{1}{G} \cot \theta \frac{\partial U}{\partial g} - \frac{1}{G} \csc \theta \frac{\partial U}{\partial l}$$

$$\frac{d\rho}{dt} = \frac{1}{G} \cot \rho \frac{\partial U}{\partial g} - \frac{1}{G} \csc \rho \frac{\partial U}{\partial h}$$

$$\frac{dl}{dt} = -G \cos \theta \left(\frac{1}{C} - \frac{\sin^2 l}{A} - \frac{\cos^2 l}{B} \right) + \frac{1}{G} \csc \theta \frac{\partial U}{\partial \theta}$$

$$\frac{dg}{dt} = G \left(\frac{\sin^2 l}{A} + \frac{\cos^2 l}{B} \right) - \frac{1}{G} \cot \theta \frac{\partial U}{\partial \theta} - \frac{1}{G} \cot \rho \frac{\partial U}{\partial \rho}$$

$$\frac{dh}{dt} = -\frac{1}{G} \csc \rho \frac{\partial U}{\partial \rho}$$

$$U = U(\theta, \rho, l, g, h, t)$$

11.7.1. Канонические уравнения в переменных Пуанкаре.

$$\begin{aligned}\frac{d\Lambda}{dt} &= -\frac{\partial F}{\partial \lambda}, & \frac{d\lambda}{dt} &= \frac{\partial F}{\partial \Lambda}, \\ \frac{d\Gamma}{dt} &= -\frac{\partial F}{\partial \gamma}, & \frac{d\gamma}{dt} &= \frac{\partial F}{\partial \Gamma}, \\ \frac{dZ}{dt} &= -\frac{\partial F}{\partial z}, & \frac{dz}{dt} &= \frac{\partial F}{\partial Z},\end{aligned}\quad F = F_0 - U$$

$$F_0 = \frac{1}{2} \left(\frac{\sin^2 \gamma}{A} + \frac{\cos^2 \gamma}{B} \right) \Lambda^2 + \frac{1}{2} \left(\frac{1}{C} - \frac{\sin^2 \gamma}{A} - \frac{\cos^2 \gamma}{B} \right) (\Lambda^2 - 2\Lambda\Gamma + \Gamma^2)$$

$$\Lambda = G, \quad \lambda = l + g + h,$$

$$\Gamma = 2G \sin^2 \frac{\theta}{2} = G(1 - \cos \theta), \quad \gamma = -l,$$

$$Z = 2G \sin^2 \frac{\rho}{2} = G(1 - \cos \rho), \quad z = -h.$$

$$\begin{aligned}
\Lambda &= \Lambda, & \lambda &= \lambda, \\
\xi &= \sqrt{2\Gamma} \cos \gamma, & \eta &= \sqrt{2\Gamma} \sin \gamma, \\
p &= \sqrt{2Z} \cos z, & q &= \sqrt{2Z} \sin z.
\end{aligned}$$

$$\begin{aligned}
\frac{d\Lambda}{dt} &= -\frac{\partial F}{\partial \lambda}, & \frac{d\lambda}{dt} &= \frac{\partial F}{\partial \Lambda}, \\
\frac{d\eta}{dt} &= -\frac{\partial F}{\partial \xi}, & \frac{d\xi}{dt} &= \frac{\partial F}{\partial \eta}, \\
\frac{dp}{dt} &= -\frac{\partial F}{\partial q}, & \frac{dq}{dt} &= \frac{\partial F}{\partial p},
\end{aligned}$$

$$F = F(\Lambda, \xi, \eta, p, q, \lambda, t).$$

$$\begin{aligned}
\Lambda &= G, & \lambda &= l + g + h, \\
\xi &= 2\sqrt{G} \sin \frac{\theta}{2} \cos l, & \eta &= -2\sqrt{G} \sin \frac{\theta}{2} \sin l, \\
p &= 2\sqrt{G} \sin \frac{\rho}{2} \cos h, & q &= -2\sqrt{G} \sin \frac{\rho}{2} \sin h.
\end{aligned}$$

$$\begin{aligned}
\Lambda &= \Lambda, & \lambda &= \lambda, \\
\xi &= \sqrt{2\Gamma} \cos \gamma, & \eta &= \sqrt{2\Gamma} \sin \gamma, \\
p &= \sqrt{2Z} \cos z, & q &= \sqrt{2Z} \sin z.
\end{aligned}$$

$$\begin{aligned}
\frac{d\Lambda}{dt} &= -\frac{\partial F}{\partial \lambda}, & \frac{d\lambda}{dt} &= \frac{\partial F}{\partial \Lambda}, \\
\frac{d\eta}{dt} &= -\frac{\partial F}{\partial \xi}, & \frac{d\xi}{dt} &= \frac{\partial F}{\partial \eta}, \\
\frac{dp}{dt} &= -\frac{\partial F}{\partial q}, & \frac{dq}{dt} &= \frac{\partial F}{\partial p},
\end{aligned}$$

$$F = F(\Lambda, \xi, \eta, p, q, \lambda, t).$$

$$\begin{aligned}
\Lambda &= G, & \lambda &= l + g + h, \\
\xi &= 2\sqrt{G} \sin \frac{\theta}{2} \cos l, & \eta &= -2\sqrt{G} \sin \frac{\theta}{2} \sin l, \\
p &= 2\sqrt{G} \sin \frac{\rho}{2} \cos h, & q &= -2\sqrt{G} \sin \frac{\rho}{2} \sin h.
\end{aligned}$$

$$U = U(\theta, \rho, l, g, h, t) = U(b_{ij}, \rho, h, t)$$

$$b_{11} = \cos(\mathbf{i}_b, \mathbf{i}_G) = (\mathbf{i}_b \cdot \mathbf{i}_G) = \cos l \cos g - \sin l \sin g \cos \theta,$$

$$b_{21} = \cos(\mathbf{i}_b, \mathbf{j}_G) = (\mathbf{i}_b \cdot \mathbf{j}_G) = \cos l \sin g + \sin l \cos g \cos \theta,$$

$$b_{31} = \cos(\mathbf{i}_b, \mathbf{k}_G) = (\mathbf{i}_b \cdot \mathbf{k}_G) = \sin \theta \sin l,$$

$$b_{12} = \cos(\mathbf{j}_b, \mathbf{i}_G) = (\mathbf{j}_b \cdot \mathbf{i}_G) = -\sin l \cos g - \cos l \sin g \cos \theta,$$

$$b_{22} = \cos(\mathbf{j}_b, \mathbf{j}_G) = (\mathbf{j}_b \cdot \mathbf{j}_G) = -\sin l \sin g + \cos l \cos g \cos \theta,$$

$$b_{32} = \cos(\mathbf{j}_b, \mathbf{k}_G) = (\mathbf{j}_b \cdot \mathbf{k}_G) = \cos l \sin \theta,$$

$$b_{13} = \cos(\mathbf{k}_b, \mathbf{i}_G) = (\mathbf{k}_b \cdot \mathbf{i}_G) = \sin g \sin \theta,$$

$$b_{23} = \cos(\mathbf{k}_b, \mathbf{j}_G) = (\mathbf{k}_b \cdot \mathbf{j}_G) = -\cos g \sin \theta,$$

$$b_{33} = \cos(\mathbf{k}_b, \mathbf{k}_G) = (\mathbf{k}_b \cdot \mathbf{k}_G) = \cos \theta;$$

$$b_{11} = \cos \lambda - \frac{1}{2\Lambda} (\xi \eta \sin \lambda + \eta^2 \cos \lambda),$$

$$b_{21} = \sin \lambda + \frac{1}{2\Lambda} (\xi \eta \cos \lambda - \eta^2 \sin \lambda),$$

$$b_{31} = -\frac{1}{\sqrt{\Lambda}} \cos(\theta/2) \eta,$$

$$b_{12} = -\sin \lambda + \frac{1}{2\Lambda} (\xi^2 \sin \lambda + \xi \eta \cos \lambda),$$

$$b_{22} = \cos \lambda + \frac{1}{2\Lambda} (-\xi^2 \cos \lambda + \xi \eta \sin \lambda),$$

$$b_{32} = \frac{1}{\sqrt{\Lambda}} \cos(\theta/2) \xi,$$

$$b_{13} = \frac{1}{\sqrt{\Lambda}} \cos(\theta/2) (\xi \sin \lambda + \eta \cos \lambda),$$

$$b_{23} = -\frac{1}{\sqrt{\Lambda}} \cos(\theta/2) (\xi \cos \lambda - \eta \sin \lambda),$$

$$b_{33} = \cos \theta.$$

Уравнения вращательного движения в переменных действие-угол

$$\begin{aligned}\frac{d\varphi_1}{dt} &= \frac{\partial F}{\partial I_1}, & \frac{dI_1}{dt} &= -\frac{\partial F}{\partial \varphi_1}, \\ \frac{d\varphi_2}{dt} &= \frac{\partial F}{\partial I_2}, & \frac{dI_2}{dt} &= -\frac{\partial F}{\partial \varphi_2}, \\ \frac{d\varphi_3}{dt} &= \frac{\partial F}{\partial I_3}, & \frac{dI_3}{dt} &= -\frac{\partial F}{\partial \varphi_3},\end{aligned}$$

$$F = F_0(I_1, I_2) - U(I_1, I_2, I_3, \varphi_1, \varphi_2, \varphi_3, t)$$

$$F = \frac{1}{2}L^2\left(\frac{1}{C} - \frac{\sin^2 l}{A} - \frac{\cos^2 l}{B}\right) + \frac{1}{2}G^2\left(\frac{\sin^2 l}{A} + \frac{\cos^2 l}{B}\right) - U(L, G, H, l, g, h, t)$$

$$F_0 = \frac{I_2^2}{2A} \left[1 + \frac{A-C}{C} \frac{\kappa^2}{\lambda^2 + \kappa^2} \right],$$

12.2. Дифференциальные уравнения возмущенного вращательного движения в переменных $G, \rho, \lambda, \varphi, \psi, h$.

$$\dot{G} = \frac{\partial U}{\partial \psi},$$

$$\dot{\lambda} = -\frac{1}{G\mathbf{J}(\lambda)} \frac{\partial U}{\partial \varphi} + \frac{\Lambda(\lambda)}{G\mathbf{J}(\lambda)} \frac{\partial U}{\partial \psi},$$

$$\dot{\rho} = \frac{1}{G} \cot \rho \frac{\partial U}{\partial \psi} - \frac{1}{G} \csc \rho \frac{\partial U}{\partial h},$$

$$\dot{\varphi} = \Omega(G, \lambda) + \frac{1}{G\mathbf{J}(\lambda)} \frac{\partial U}{\partial \lambda},$$

$$\dot{\psi} = \omega(G, \lambda) - \frac{\Lambda(\lambda)}{G\mathbf{J}(\lambda)} \frac{\partial U}{\partial \lambda} - \frac{1}{G} \cot \rho \frac{\partial U}{\partial \rho},$$

$$\dot{h} = \frac{1}{G} \csc \rho \frac{\partial U}{\partial \rho},$$

$$\Omega(G, \lambda) = \frac{\pi G}{2\mathbf{K}(\lambda)} \frac{A-C}{AC} \frac{\kappa}{\sqrt{(1+\kappa^2)(\lambda^2+\kappa^2)}}$$

$$\omega(G, \lambda) = \frac{G}{C} \left(1 - \frac{A-C}{A} \frac{\Pi(\kappa^2, \lambda)}{\mathbf{K}(\lambda)} \right)$$

$$I_1 = G\Lambda(\lambda), \quad I_2 = G, \quad I_3 = G \cos \rho,$$

$$\varphi_1 = \varphi, \quad \varphi_2 = \psi, \quad \varphi_3 = h.$$

$$U = U(\lambda, \rho, \varphi, \psi, h, t)$$

$$\mathbf{J}(\lambda) = \frac{2\kappa\lambda\sqrt{1+\kappa^2}}{\pi(\kappa^2+\lambda^2)^{3/2}} \mathbf{K}(\lambda)$$

$$\Lambda(\lambda) = \frac{2\sqrt{1+\kappa^2}}{\pi\kappa\sqrt{\kappa^2+\lambda^2}} \left\{ (\kappa^2+\lambda^2) \Pi\left(\frac{\pi}{2}, \kappa^2, \lambda\right) - \lambda^2 \mathbf{K}(\lambda) \right\}$$

12.2. Дифференциальные уравнения возмущенного вращательного движения в переменных $G, \rho, \lambda, \varphi, \psi, h$.

$$\dot{G} = \frac{\partial U}{\partial \psi},$$

$$\dot{\lambda} = -\frac{\pi(\kappa^2 + \lambda^2)^{3/2}}{2G\kappa\lambda\sqrt{1+\kappa^2}\mathbf{K}(\lambda)}\frac{\partial U}{\partial \varphi} + \frac{(\kappa^2 + \lambda^2)}{G\lambda\kappa^2\mathbf{K}(\lambda)}\left[(\kappa^2 + \lambda^2)\mathbf{\Pi}\left(\frac{\pi}{2}, \kappa^2, \lambda\right) - \lambda^2\mathbf{K}(\lambda)\right]\frac{\partial U}{\partial \psi},$$

$$\dot{\rho} = \frac{1}{G}\cot\rho\frac{\partial U}{\partial \psi} - \frac{1}{G}\csc\rho\frac{\partial U}{\partial h},$$

$$\dot{\varphi} = \frac{\pi G}{2\mathbf{K}(\lambda)}\frac{A-C}{AC}\frac{\kappa}{\sqrt{(1+\kappa^2)(\lambda^2 + \kappa^2)}} + \frac{\pi(\kappa^2 + \lambda^2)^{3/2}}{2G\kappa\lambda\sqrt{1+\kappa^2}\mathbf{K}(\lambda)}\frac{\partial U}{\partial \lambda},$$

$$\begin{aligned}\dot{\psi} = & \frac{G}{C}\left(1 - \frac{A-C}{A}\frac{\mathbf{\Pi}(\kappa^2, \lambda)}{\mathbf{K}(\lambda)}\right) - \frac{(\kappa^2 + \lambda^2)}{G\lambda\kappa^2\mathbf{K}(\lambda)}\left[(\kappa^2 + \lambda^2)\mathbf{\Pi}\left(\frac{\pi}{2}, \kappa^2, \lambda\right) - \lambda^2\mathbf{K}(\lambda)\right]\frac{\partial U}{\partial \lambda} - \\ & - \frac{1}{G}\cot\rho\frac{\partial U}{\partial \rho},\end{aligned}$$

$$\dot{h} = \frac{1}{G}\csc\rho\frac{\partial U}{\partial \rho},$$

$$U = U(\lambda, \rho, \varphi, \psi, h, t).$$

12.3. К выводу уравнений возмущенного вращательного движения твердого тела в случае непотенциальных сил.

$$\dot{p} = \frac{(B-C)}{A}qr + \frac{M_\xi}{A}, \quad \dot{q} = \frac{(C-A)}{B}rp + \frac{M_\eta}{B}, \quad \dot{r} = \frac{(A-B)}{C}pq + \frac{M_\zeta}{C},$$

$$\dot{G} = \frac{1}{G}(b_{31}M_\xi + b_{32}M_\eta + b_{33}M_\zeta),$$

$$\dot{\lambda} = \frac{\sqrt{\kappa^2 + \lambda^2}}{G} \left(b_{31}M_\xi + b_{32} \frac{1-\lambda^2}{1+\kappa^2} M_\eta - b_{33} \frac{\lambda^2}{\kappa^2} M_\zeta \right),$$

$$\begin{aligned} \dot{\phi}_1 = n_1(\lambda, \kappa) + \frac{\pi(\kappa^2 + \lambda^2)}{2G\mathbf{K}\lambda^2 dnu\sqrt{1+\kappa^2}} & \left\{ \left[b_{32} - \frac{\sqrt{1+\kappa^2}}{\lambda'^2} b_{31}\mathbf{F}(u) \right] M_\xi - \right. \\ & \left. - \left[b_{31} + \frac{1}{\sqrt{1+\kappa^2}} b_{32}\mathbf{F}(u) \right] M_\eta + \left[\frac{\sqrt{1+\kappa^2}}{\lambda'^2 \kappa^2} b_{33}\mathbf{F}(u) \right] M_\zeta \right\}, \end{aligned}$$

$$\dot{g} = \dot{\phi}_2 + g_{\phi_1} \dot{\phi}_1 + g_\lambda \dot{\lambda},$$

$$g_{\phi_1} = -\frac{\mathbf{K}i}{\pi} [Zn(\phi_1 - i\sigma) - Zn(\phi_1 + i\sigma)],$$

$$g_\lambda = \frac{\mathbf{K}i}{\pi} [Zn(\phi_1 - i\sigma) + Zn(\phi_1 + i\sigma)] \sigma_\lambda + \frac{i}{2q} \frac{\partial q}{\partial \lambda} \left[\frac{1}{\theta_4(\phi_1 - i\sigma)} - \frac{1}{\theta_4(\phi_1 + i\sigma)} \right].$$

Непотенциальные силы

$$\dot{G} = M_{\xi} b_{31} + M_{\eta} b_{32} + M_{\zeta} b_{33},$$

$$\dot{\rho} = -\frac{1}{G} (M_{\xi} b_{21} + M_{\eta} b_{22} + M_{\zeta} b_{23}),$$

$$\dot{h} = \frac{1}{G \sin \rho} (M_{\xi} b_{21} + M_{\eta} b_{22} + M_{\zeta} b_{23}),$$

$$\dot{\lambda} = \frac{(\kappa^2 + \lambda^2)}{G\lambda} \left(M_{\xi} b_{31} + \frac{\lambda'^2}{1 + \kappa^2} M_{\eta} b_{32} - \frac{\lambda^2}{\kappa^2} M_{\zeta} b_{33} \right),$$

$$\dot{\phi}_1 = n_1 + \Phi_{11} M_{\xi} + \Phi_{12} M_{\eta} + \Phi_{13} M_{\zeta},$$

$$\dot{\phi}_2 = n_2 + \Phi_{21} M_{\xi} + \Phi_{22} M_{\eta} + \Phi_{23} M_{\zeta},$$

$$\Phi_{11} = \left[\frac{b_{32}}{dnu} - \Gamma \lambda \sqrt{1 + \kappa^2} b_{31} \right],$$

$$\Phi_{12} = - \left[\frac{b_{31}}{dnu} + \Gamma \frac{\lambda \lambda'^2}{\sqrt{1 + \kappa^2}} b_{32} \right],$$

$$\Phi_{13} = \frac{\lambda^3 \sqrt{1 + \kappa^2}}{\kappa^2} b_{33},$$

$$\Gamma = \frac{1}{\lambda \lambda'^2} \left[\frac{\lambda^2 s n u c n u}{dnu} - \mathbf{Z} n u \right],$$

$$\Phi_{21} = \frac{1}{G} \left\{ \cot \rho b_{11} + \cot \theta \cos l + \Phi(u, \lambda) \frac{(\kappa^2 + \lambda^2)}{\lambda} b_{31} \right\},$$

$$\Phi_{22} = \frac{1}{G} \left\{ \cot \rho b_{12} - \cot \theta \sin l + \Phi(u, \lambda) \frac{\lambda'^2 (\kappa^2 + \lambda^2)}{\lambda (1 + \kappa^2)} b_{32} \right\},$$

$$\Phi_{23} = \frac{1}{G} \left\{ \cot \rho b_{13} - \Phi(u, \lambda) \frac{\lambda (\kappa^2 + \lambda^2)}{\kappa^2} b_{33} \right\},$$

$$\begin{aligned} \Phi(u, \lambda) = & \frac{\sqrt{(1 + \kappa^2)(\kappa^2 + \lambda^2)}}{\kappa} \left\{ \left[\frac{2\lambda}{\pi(\kappa^2 + \lambda^2)} \mathbf{\Pi} - \frac{\lambda}{(\kappa^2 + \lambda^2)} \Pi(am u, \kappa^2, \lambda) \right] + \right. \\ & + \frac{2}{\pi} \frac{\lambda}{\lambda'^2 (\lambda^2 - n)} [\mathbf{E} - \lambda'^2 \mathbf{\Pi}] - \frac{1}{dnu(1 + n s n^2 u)} \frac{1}{\mathbf{K}} \frac{\mathbf{E} - \lambda'^2 \mathbf{K}}{\lambda \lambda'^2} u dnu - \\ & \left. - \frac{\lambda}{\lambda'^2 (\lambda^2 - n)} \left[E(am u, \lambda) - \lambda'^2 \Pi(am u, n, \lambda) - \frac{\lambda^2 s n u c n u}{dnu} \right] \right\}. \end{aligned}$$

Непотенциальные силы

$$\dot{G} = M_{\xi} b_{31} + M_{\eta} b_{32} + M_{\zeta} b_{33},$$

$$\dot{\rho} = -\frac{1}{G}(M_{\xi} b_{21} + M_{\eta} b_{22} + M_{\zeta} b_{23}),$$

$$\dot{h} = \frac{1}{G \sin \rho}(M_{\xi} b_{21} + M_{\eta} b_{22} + M_{\zeta} b_{23}),$$

$$\dot{\lambda} = \frac{(\kappa^2 + \lambda^2)}{G\lambda} \left(M_{\xi} b_{31} + \frac{\lambda'^2}{1 + \kappa^2} M_{\eta} b_{32} - \frac{\lambda^2}{\kappa^2} M_{\zeta} b_{33} \right),$$

$$\dot{\phi}_1 = n_1 + \Phi_{11} M_{\xi} + \Phi_{12} M_{\eta} + \Phi_{13} M_{\zeta},$$

$$\dot{\phi}_2 = n_2 + \Phi_{21} M_{\xi} + \Phi_{22} M_{\eta} + \Phi_{23} M_{\zeta},$$

$$\Phi_{11} = -\frac{\pi \sqrt{\kappa^2 + \lambda^2}}{2\mathbf{K}G\lambda\lambda'^2} (snudnu - \mathbf{Z}nucnu),$$

$$\Phi_{21} = \frac{1}{G} \left\{ \cot \rho b_{11} + \cot \theta \cos l + \Phi(u, \lambda) \frac{(\kappa^2 + \lambda^2)}{\lambda} b_{31} \right\},$$

$$\Phi_{12} = -\frac{\pi \sqrt{\kappa^2 + \lambda^2}}{2\mathbf{K}G\lambda\sqrt{1 + \kappa^2}} [cnudnu + \mathbf{Z}nusnu],$$

$$\Phi_{22} = \frac{1}{G} \left\{ \cot \rho b_{12} - \cot \theta \sin l + \Phi(u, \lambda) \frac{\lambda'^2 (\kappa^2 + \lambda^2)}{\lambda(1 + \kappa^2)} b_{32} \right\},$$

$$\Phi_{13} = \frac{\pi \lambda \sqrt{\kappa^2 + \lambda^2}}{2G\kappa\mathbf{K}} dnu,$$

$$\Phi_{23} = \frac{1}{G} \left\{ \cot \rho b_{13} - \Phi(u, \lambda) \frac{\lambda(\kappa^2 + \lambda^2)}{\kappa^2} b_{33} \right\},$$

$$\Phi(u, \lambda) = \frac{\sqrt{(1 + \kappa^2)(\kappa^2 + \lambda^2)}}{\kappa} \left\{ \left[\frac{2\lambda}{\pi(\kappa^2 + \lambda^2)} \Pi\left(\frac{\pi}{2}, \kappa^2, \lambda\right) - \frac{\lambda}{(\kappa^2 + \lambda^2)} \Pi(am u, \kappa^2, \lambda) \right] + \right. \\ \left. + \frac{2}{\pi} \cdot \frac{\partial \Pi\left(\frac{\pi}{2}, \kappa^2, \lambda\right)}{\partial \lambda} - \frac{\partial \Pi(am u, \kappa^2, \lambda)}{\partial (am u)} \cdot \frac{\partial (am u)}{\partial \lambda} - \frac{\partial \Pi(am u, \kappa^2, \lambda)}{\partial \lambda} \right\}.$$

Глава 17. Интегрирование задачи Эйлера-Пуансо методом Гамильтона-Якоби.

17.1. Канонические уравнения задачи Эйлера-Пуансо в переменных Андуайе. Первые интегралы.

17.2. Уравнение Гамильтона-Якоби в задаче Эйлера-Пуансо. Полный интеграл. Общий интеграл.

17.3. Параметры задачи. Области интегрирования.

17.4. Обращение интегралов задачи Эйлера-Пуансо в случае $a > 0$ ($0 \leq \lambda < 1$).

17.4.1. Определение переменной $l = l(t, \beta_1, \alpha_1, \alpha_2)$.

17.4.2. Определение переменной $g(t, \beta_2, \alpha_1, \alpha_2)$.

17.4.3. Определение переменной $L(t, \alpha_1, \alpha_2, \dots)$.

17.4.4. Формулы для проекций угловой скорости вращения тела p, q, r .

17.5. Обращение общего интеграла задачи Эйлера-Пуансо в случае $a > 0$ ($1 < \lambda \leq \infty$).

17.5.1. Определение переменной $l = l(t, \beta_2, \alpha_1, \alpha_2)$.

17.5.2. Определение переменной $g(t, \beta_2, \alpha_1, \alpha_2)$.

17.5.3. Определение переменной $L(t, \alpha_1, \alpha_2)$.

17.6. Канонические уравнения возмущенного движения в переменных Якоби.

$$p = \frac{G}{A} \frac{\lambda}{\sqrt{\kappa^2 + \lambda^2}} \operatorname{cn} u, \quad q = -\frac{G}{B} \frac{\lambda \sqrt{1 + \kappa^2}}{\sqrt{\kappa^2 + \lambda^2}} \operatorname{sn} u, \quad r = \frac{G}{C} \frac{\kappa}{\sqrt{\kappa^2 + \lambda^2}} dm u.$$

$$\cot l = -\sqrt{1 + \kappa^2} \tan z \quad z = am u$$

Глава 20. Переменные действие-угол в задаче Эйлера - Пуансо.

20.1. Переменные действие-угол.

20.2. Некоторые свойства переменных действие-угол.

20.3. Переменные действие-угол в задаче Эйлера-Пуансо (движения в областях I, II)

20.3.1. Переменные действия (области I, II).

20.3.2. Вычисление частных производных $\frac{\partial I_1}{\partial G}, \frac{\partial I_1}{\partial \lambda}, \frac{\partial I_2}{\partial G}, \frac{\partial I_2}{\partial \lambda}.$

20.3.3. Вычисление частных производных $\frac{\partial \lambda}{\partial I_1}, \frac{\partial \lambda}{\partial I_2}, \frac{\partial G}{\partial I_1}, \frac{\partial G}{\partial I_2}.$

20.3.4. Преобразующая функция и формулы канонического преобразования от переменных Андуайе к переменным действие-угол (области движения I, II).

20.3.5. Гамильтониан задачи Эйлера-Пуансо в переменных действие $I_1, I_2.$

20.3.6. Канонические уравнения задачи Эйлера-Пуансо в переменных действие-угол.

20.3.7. Формулы для частот задачи Эйлера-Пуансо.

20.4. Переменные действие-угол (переменные Пуассона) в областях III и IV.

20.4.1. Переменные действия в областях III и IV.

20.4.2. Вычисление производных $\frac{\partial I_1}{\partial G}, \frac{\partial I_1}{\partial \lambda}, \frac{\partial I_2}{\partial G}, \frac{\partial I_2}{\partial \lambda}.$

20.4.3. Формулы преобразования от переменных Андуайе к каноническим переменным действие-угол (области движения III и IV).

20.4.4. Канонические уравнения задачи Эйлера-Пуансо в переменных действие-угол и формулы для частот движения в областях III и IV.

Глава 21. Решение задачи Эйлера-Пуансо в тета-функциях. Формулы для направляющих косинусов $\tilde{B}_{ij}, B_{ij}$ и проекций векторов угловой скорости и углового ускорения на оси твердого тела (движения в областях I и III).

21.1. Формулы связи эллиптических функций Якоби и их производных с тета-функциями θ_l . Значения $\theta_l(0), \theta'_l(0)$ ($l = 1, 2, 3, 4$).

21.2. Выражение эллиптического интеграла третьего рода через функции Якоби и тета-функции.

21.3. Решение задачи Эйлера-Пуансо в тета-функциях (область движений I).

21.3.1. Решение задачи Эйлера-Пуансо в эллиптических функциях и интегралах.

21.3.2. Параметры a, σ в случае движений в области I и выражение переменной Андуайе в тета-функциях.

21.3.3. О значениях тета-функций $\theta_l(z)$ и их производных $\theta'_l(z)$ ($l = 1, 2, 3, 4$) при $z = i\sigma$.

21.3.4. Выражение $\sqrt{1 + \kappa^2 \operatorname{sn}^2 u}$ в тета-функциях.

21.3.5. Формулы решения задачи Эйлера-Пуансо в тета-функциях (область I).

21.3.6. Выражения компонент угловой скорости тела p, q, r и компонент углового ускорения $\dot{p}, \dot{q}, \dot{r}$ в тета-функциях.

21.4. Решение задачи Эйлера-Пуансо в тета-функциях (область движений III).

21.4.1. Решение задачи Эйлера-Пуансо в эллиптических функциях и интегралах.

21.4.2. Параметры a, σ в случае движений в области III и выражение переменной Андуайе в тета-функциях.

21.4.3. О значениях тета-функций $\theta_l(z)$ и их производных $\theta'_l(z)$ ($l = 1, 2, 3, 4$) при $z = i\sigma$.

21.4.4. Выражение $\sqrt{1 + \frac{\kappa^2}{\lambda^2} \operatorname{sn}^2 u}$ в тета-функциях.

21.4.5. Формулы решения задачи Эйлера-Пуансо в тета-функциях (область III).

21.4.6. Выражения компонент угловой скорости тела p, q, r и компонент углового ускорения $\dot{p}, \dot{q}, \dot{r}$ в тета-функциях.

22.1. Направляющие косинусы b_{ij}, B_{ij} и $\tilde{b}_{ij}, \tilde{B}_{ij}$ и соотношения между ними.

$$b_{11} = \cos g \cos l - \sin g \sin l \cos \theta ,$$

$$b_{21} = \sin g \cos l + \cos g \sin l \cos \theta ,$$

$$b_{31} = \sin \theta \sin l ,$$

$$\tilde{b}_{11} = -ib_{11} + b_{21} = e^{ig} (\cos \theta \sin l - i \cos l) ,$$

$$\tilde{b}_{21} = ib_{11} + b_{21} = e^{-ig} (\cos \theta \sin l + i \cos l) ,$$

$$\tilde{b}_{31} = b_{31} = \sin \theta \sin l ,$$

$$b_{12} = -\cos g \sin l - \sin g \cos l \cos \theta ,$$

$$b_{22} = -\sin g \sin l + \cos g \cos l \cos \theta ,$$

$$b_{32} = \sin \theta \cos l ,$$

$$\tilde{b}_{12} = -ib_{12} + b_{22} = e^{ig} (\cos \theta \cos l + i \sin l) ,$$

$$\tilde{b}_{22} = ib_{12} + b_{22} = e^{-ig} (\cos \theta \cos l - i \sin l) ,$$

$$\tilde{b}_{32} = b_{32} = \sin \theta \cos l ,$$

$$b_{13} = \sin g \sin \theta ,$$

$$b_{23} = -\cos g \sin \theta ,$$

$$b_{33} = \cos \theta ,$$

$$\tilde{b}_{13} = -ib_{13} + b_{23} = -\sin \theta e^{ig} ,$$

$$\tilde{b}_{23} = ib_{13} + b_{23} = -\sin \theta e^{-ig} ,$$

$$\tilde{b}_{33} = b_{33} .$$

22.1. Направляющие косинусы b_{ij}, B_{ij} и $\tilde{b}_{ij}, \tilde{B}_{ij}$ и соотношения между ними.

$$\omega = \varphi_2 - g$$

$$\begin{aligned} B_{11} &= \sin \omega \cos \theta \sin l + \cos \omega \cos l, & \tilde{B}_{11} &= B_{21} - iB_{11} = e^{-i\omega} (\cos \theta \sin l - i \cos l), \\ B_{21} &= \cos \omega \cos \theta \sin l - \sin \omega \cos l, & \tilde{B}_{21} &= B_{21} + iB_{11} = e^{i\omega} (\cos \theta \sin l + i \cos l), \\ B_{31} &= \sin \theta \sin l, & \tilde{B}_{31} &= B_{31} = \sin \theta \sin l, \end{aligned}$$

$$\begin{aligned} B_{12} &= \sin \omega \cos \theta \cos l - \cos \omega \sin l, & \tilde{B}_{12} &= B_{22} - iB_{12} = e^{-i\omega} (\cos \theta \cos l + i \sin l), \\ B_{22} &= \cos \omega \cos \theta \cos l + \sin \omega \sin l, & \tilde{B}_{22} &= B_{22} + iB_{12} = e^{i\omega} (\cos \theta \cos l - i \sin l), \\ B_{32} &= \sin \theta \cos l, & \tilde{B}_{32} &= B_{32} = \sin \theta \cos l, \end{aligned}$$

$$\begin{aligned} B_{13} &= -\sin \theta \sin \omega, & \tilde{B}_{13} &= B_{23} - iB_{13} = -\sin \theta e^{-i\omega}, \\ B_{23} &= -\sin \theta \cos \omega, & \tilde{B}_{23} &= B_{23} + iB_{13} = -\sin \theta e^{i\omega}, \\ B_{33} &= \cos \theta, & \tilde{B}_{33} &= B_{33} = \cos \theta. \end{aligned}$$

$$\tilde{B}_{11} = B_{21} - iB_{11} = e^{-i\omega}(\cos\theta \sin l - i \cos l),$$

$$\tilde{B}_{21} = B_{21} + iB_{11} = e^{i\omega}(\cos\theta \sin l + i \cos l),$$

$$\tilde{B}_{31} = B_{31} = \sin\theta \sin l,$$

$$\tilde{B}_{12} = B_{22} - iB_{12} = e^{-i\omega}(\cos\theta \cos l + i \sin l),$$

$$\tilde{B}_{22} = B_{22} + iB_{12} = e^{i\omega}(\cos\theta \cos l - i \sin l),$$

$$\tilde{B}_{32} = B_{32} = \sin\theta \cos l,$$

$$\tilde{B}_{13} = B_{23} - iB_{13} = -\sin\theta e^{-i\omega},$$

$$\tilde{B}_{23} = B_{23} + iB_{13} = -\sin\theta e^{i\omega},$$

$$\tilde{B}_{33} = B_{33} = \cos\theta.$$

$$\tilde{B}_{11} = \frac{(\kappa \operatorname{cn} u \operatorname{dn} u + i\sqrt{1+\kappa^2} \sqrt{\kappa^2 + \lambda^2} \operatorname{sn} u)}{\sqrt{\kappa^2 + \lambda^2} \sqrt{1+\kappa^2 \operatorname{sn}^2 u}} \frac{\sqrt{\theta_4(\varphi_1 - i\sigma)}}{\sqrt{\theta_4(\varphi_1 + i\sigma)}},$$

$$\tilde{B}_{21} = \frac{(\kappa \operatorname{cn} u \operatorname{dn} u - i\sqrt{1+\kappa^2} \sqrt{\kappa^2 + \lambda^2} \operatorname{sn} u)}{\sqrt{\kappa^2 + \lambda^2} \sqrt{1+\kappa^2 \operatorname{sn}^2 u}} \frac{\sqrt{\theta_4(\varphi_1 + i\sigma)}}{\sqrt{\theta_4(\varphi_1 - i\sigma)}},$$

$$\tilde{B}_{31} = \frac{\lambda}{\sqrt{\lambda^2 + \kappa^2}} \operatorname{cn} u,$$

$$\tilde{B}_{12} = \frac{(-\kappa \sqrt{1+\kappa^2} \operatorname{dn} u \operatorname{sn} u + i \operatorname{cn} u \sqrt{\kappa^2 + \lambda^2})}{\sqrt{\kappa^2 + \lambda^2} \sqrt{1+\kappa^2 \operatorname{sn}^2 u}} \frac{\sqrt{\theta_4(\varphi_1 - i\sigma)}}{\sqrt{\theta_4(\varphi_1 + i\sigma)}},$$

$$\tilde{B}_{22} = -\frac{(\kappa \sqrt{1+\kappa^2} \operatorname{dn} u \operatorname{sn} u + i \operatorname{cn} u \sqrt{\kappa^2 + \lambda^2})}{\sqrt{\kappa^2 + \lambda^2} \sqrt{1+\kappa^2 \operatorname{sn}^2 u}} \frac{\sqrt{\theta_4(\varphi_1 + i\sigma)}}{\sqrt{\theta_4(\varphi_1 - i\sigma)}},$$

$$\tilde{B}_{32} = -\frac{\lambda \sqrt{1+\kappa^2}}{\sqrt{\kappa^2 + \lambda^2}} \operatorname{sn} u,$$

$$\tilde{B}_{13} = -\frac{\lambda \sqrt{1+\kappa^2 \operatorname{sn}^2 u}}{\sqrt{\kappa^2 + \lambda^2}} \frac{\sqrt{\theta_4(\varphi_1 - i\sigma)}}{\sqrt{\theta_4(\varphi_1 + i\sigma)}},$$

$$\tilde{B}_{23} = -\frac{\lambda \sqrt{1+\kappa^2 \operatorname{sn}^2 u}}{\sqrt{\kappa^2 + \lambda^2}} \frac{\sqrt{\theta_4(\varphi_1 + i\sigma)}}{\sqrt{\theta_4(\varphi_1 - i\sigma)}},$$

$$\tilde{B}_{33} = \frac{\kappa}{\sqrt{\lambda^2 + \kappa^2}} \operatorname{dn} u.$$

$$\tilde{B}_{11} = \frac{\kappa \sqrt{1 + \kappa^2 \operatorname{sn}^2 u}}{\sqrt{\kappa^2 + \lambda^2}} V_1 \frac{\sqrt{\theta_4(\varphi_1 - i\sigma)}}{\sqrt{\theta_4(\varphi_1 + i\sigma)}},$$

$$\tilde{B}_{21} = -\frac{\kappa \sqrt{1 + \kappa^2 \operatorname{sn}^2 u}}{\sqrt{\kappa^2 + \lambda^2}} V_2 \frac{\sqrt{\theta_4(\varphi_1 + i\sigma)}}{\sqrt{\theta_4(\varphi_1 - i\sigma)}},$$

$$\tilde{B}_{12} = -i\sqrt{1 + \kappa^2 \operatorname{sn}^2 u} V_3 \frac{\sqrt{\theta_4(\varphi_1 - i\sigma)}}{\sqrt{\theta_4(\varphi_1 + i\sigma)}},$$

$$\tilde{B}_{22} = i\sqrt{1 + \kappa^2 \operatorname{sn}^2 u} V_4 \frac{\sqrt{\theta_4(\varphi_1 + i\sigma)}}{\sqrt{\theta_4(\varphi_1 - i\sigma)}},$$

$$\tilde{B}_{13} = -\frac{\lambda \sqrt{1 + \kappa^2 \operatorname{sn}^2 u}}{\sqrt{\kappa^2 + \lambda^2}} \cdot \frac{\sqrt{\theta_4(\varphi_1 - i\sigma)}}{\sqrt{\theta_4(\varphi_1 + i\sigma)}},$$

$$\tilde{B}_{23} = -\frac{\lambda \sqrt{1 + \kappa^2 \operatorname{sn}^2 u}}{\sqrt{\kappa^2 + \lambda^2}} \cdot \frac{\sqrt{\theta_4(\varphi_1 + i\sigma)}}{\sqrt{\theta_4(\varphi_1 - i\sigma)}},$$

$$V_1 = -\frac{\operatorname{sn}(u - a)}{\operatorname{sn} a}, \quad V_2 = -\frac{\operatorname{sn}(u + a)}{\operatorname{sn} a}, \quad V_3 = -\frac{\operatorname{cn}(u - a)}{\operatorname{cn} a}, \quad V_4 = -\frac{\operatorname{cn}(u + a)}{\operatorname{cn} a}$$

$$\sigma = \frac{\pi}{2\mathbf{K}} F\left(\arctan \frac{\kappa}{\lambda}, \lambda'\right), \quad \lambda' = \sqrt{1 - \lambda^2}.$$

$$V_1 = \frac{i\sqrt{(1 + \kappa^2)(\kappa^2 + \lambda^2)} \operatorname{sn} u + \kappa \operatorname{cn} u \operatorname{dn} u}{\kappa(1 + \kappa^2 \operatorname{sn}^2 u)},$$

$$V_2 = \frac{i\sqrt{(1 + \kappa^2)(\kappa^2 + \lambda^2)} \operatorname{sn} u - \kappa \operatorname{cn} u \operatorname{dn} u}{\kappa(1 + \kappa^2 \operatorname{sn}^2 u)},$$

$$V_3 = \frac{-\operatorname{cn} u - \frac{i\sqrt{1 + \kappa^2}}{\sqrt{\kappa^2 + \lambda^2}} \kappa \operatorname{sn} u \operatorname{dn} u}{1 + \kappa^2 \operatorname{sn}^2 u},$$

$$V_4 = \frac{-\operatorname{cn} u + \frac{i\sqrt{1 + \kappa^2}}{\sqrt{\kappa^2 + \lambda^2}} \kappa \operatorname{sn} u \operatorname{dn} u}{1 + \kappa^2 \operatorname{sn}^2 u}.$$

$$u = \frac{2K}{\pi} \varphi_1, \quad a = \frac{2K}{\pi} \sigma i;$$

$$\tilde{B}_{11} = \frac{iT_{11}}{\theta_4(i\sigma)} \cdot \frac{\theta_1(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{21} = -\frac{iT_{11}}{\theta_4(i\sigma)} \cdot \frac{\theta_1(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{31} = T_{31} \cdot \frac{\theta_2(\varphi_1)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{12} = \frac{iT_{12}}{\theta_4(i\sigma)} \cdot \frac{\theta_2(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{22} = -\frac{iT_{12}}{\theta_4(i\sigma)} \cdot \frac{\theta_2(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{32} = -T_{32} \cdot \frac{\theta_1(\varphi_1)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{13} = -\frac{T_{13}}{\theta_4(i\sigma)} \cdot \frac{\theta_4(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{23} = -\frac{T_{13}}{\theta_4(i\sigma)} \cdot \frac{\theta_4(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{33} = T_{33} \cdot \frac{\theta_3(\varphi_1)}{\theta_4(\varphi_1)},$$

$$T_{11} = \frac{\sqrt{2K\lambda\lambda'}}{\sqrt{\pi(\kappa^2 + \lambda^2)}},$$

$$T_{31} = \frac{\sqrt{\lambda\lambda'}}{\sqrt{\kappa^2 + \lambda^2}},$$

$$T_{12} = \frac{\lambda'\sqrt{2K\lambda}}{\sqrt{\pi(\kappa^2 + \lambda^2)}},$$

$$T_{32} = \frac{\sqrt{\lambda}\sqrt{1+\kappa^2}}{\sqrt{\kappa^2 + \lambda^2}},$$

$$T_{13} = \frac{\lambda\sqrt{2K\lambda'}}{\sqrt{\pi(\kappa^2 + \lambda^2)}},$$

$$T_{33} = \frac{\kappa\sqrt{\lambda'}}{\sqrt{\kappa^2 + \lambda^2}}.$$

$$R_l(\varphi_1) = \frac{\theta_l(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)} \quad (l = 1, 2, 4)$$

$$B_{11} = -\frac{i\kappa\theta_4(0)}{2\sqrt{\kappa^2 + \lambda^2}\theta_1(i\sigma)}\left(\frac{\theta_1(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)} + \frac{\theta_1(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)}\right),$$

$$B_{21} = \frac{\kappa\theta_4(0)}{2\sqrt{\kappa^2 + \lambda^2}\theta_1(i\sigma)}\left(\frac{\theta_1(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)} - \frac{\theta_1(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)}\right),$$

$$B_{31} = \frac{\sqrt{\lambda\lambda'}}{\sqrt{\kappa^2 + \lambda^2}} \cdot \frac{\theta_2(\varphi_1)}{\theta_4(\varphi_1)},$$

$$B_{12} = -\frac{\theta_4(0)}{2\theta_2(i\sigma)}\left(\frac{\theta_2(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)} + \frac{\theta_2(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)}\right),$$

$$B_{22} = \frac{i\theta_4(0)}{2\theta_2(i\sigma)}\left(\frac{\theta_2(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)} - \frac{\theta_2(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)}\right),$$

$$B_{32} = -\frac{\sqrt{\lambda}\sqrt{1+\kappa^2}}{\sqrt{\kappa^2 + \lambda^2}} \cdot \frac{\theta_1(\varphi_1)}{\theta_4(\varphi_1)},$$

$$B_{13} = \frac{i\lambda\theta_4(0)}{2\sqrt{\kappa^2 + \lambda^2}\theta_4(i\sigma)}\left(\frac{\theta_4(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)} - \frac{\theta_4(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)}\right),$$

$$B_{23} = \frac{-\lambda\theta_4(0)}{2\sqrt{\kappa^2 + \lambda^2}\theta_4(i\sigma)}\left(\frac{\theta_4(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)} + \frac{\theta_4(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)}\right),$$

$$B_{33} = \frac{\kappa\lambda'}{\sqrt{\kappa^2 + \lambda^2}} \cdot \frac{\theta_3(\varphi_1)}{\theta_4(\varphi_1)}.$$

$$R_l(\varphi_1) = \frac{\theta_l(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)} \quad (l = 1, 2, 4)$$

$$R_l(\varphi_1) = \frac{\theta_l(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)} \quad (l = 1, 2, 4)$$

$$R_l(\varphi_1) = \frac{\theta_l(\varphi_1)}{\theta_4(\varphi_1)} \quad (l = 1, 2, 3)$$

$$\begin{aligned}
\frac{\theta_1(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)} &= -it_1\theta_4(i\sigma) \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2} e^{-\sigma}}{1 - e^{-2\sigma} q^{2m+1}} e^{i(2m+1)\varphi_1}, \\
\frac{\theta_1(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)} &= -it_1\theta_4(i\sigma) \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2} e^{\sigma}}{1 - e^{2\sigma} q^{2m+1}} e^{i(2m+1)\varphi_1}, \\
\frac{\theta_1(\varphi_1)}{\theta_4(\varphi_1)} &= -it_1^0 \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1 - q^{2m+1}} e^{i(2m+1)\varphi_1}, \\
\frac{\theta_2(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)} &= t_2\theta_4(i\sigma) \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2} e^{-\sigma}}{1 + e^{-2\sigma} q^{2m+1}} e^{i(2m+1)\varphi_1}, \\
\frac{\theta_2(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)} &= t_2\theta_4(i\sigma) \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2} e^{\sigma}}{1 + e^{2\sigma} q^{2m+1}} e^{i(2m+1)\varphi_1}, \\
\frac{\theta_2(\varphi_1)}{\theta_4(\varphi_1)} &= t_2^0 \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1 + q^{2m+1}} e^{i(2m+1)\varphi_1}, \\
\frac{\theta_3(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)} &= t_3\theta_4(i\sigma) \sum_{m=-\infty}^{\infty} \frac{q^m e^{-\sigma}}{1 + q^{2m} e^{-2\sigma}} e^{2im\varphi_1}, \\
\frac{\theta_3(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)} &= t_3\theta_4(i\sigma) \sum_{m=-\infty}^{\infty} \frac{q^m e^{\sigma}}{1 + q^{2m} e^{2\sigma}} e^{2im\varphi_1}, \\
\frac{\theta_3(\varphi_1)}{\theta_4(\varphi_1)} &= t_3^0 \sum_{m=-\infty}^{\infty} \frac{q^m}{1 + q^{2m}} e^{2im\varphi_1}, \\
\frac{\theta_4(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)} &= t_4\theta_4(i\sigma) \sum_{m=-\infty}^{\infty} \frac{q^m e^{-\sigma}}{1 - q^{2m} e^{-2\sigma}} e^{2im\varphi_1}, \\
\frac{\theta_4(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)} &= -t_4\theta_4(i\sigma) \sum_{m=-\infty}^{\infty} \frac{q^m e^{\sigma}}{1 - q^{2m} e^{2\sigma}} e^{2im\varphi_1};
\end{aligned}$$

23.3. Полный набор тригонометрических

разложений функций $\frac{\theta_l(\varphi_1 \pm i\sigma)}{\theta_4(\varphi_1)}, \frac{\theta_l(\varphi_1)}{\theta_4(\varphi_1)}$

($l = 1, 2, 3, 4$).

$$\begin{aligned}
t_1 &= \frac{2}{\theta_2(0)\theta_3(0)\theta_4(0)} = \frac{\pi\sqrt{\pi}}{\mathbf{K}\sqrt{2\mathbf{K}\lambda\lambda'}}, \\
t_2 &= \frac{2\theta_3(i\sigma)}{\theta_2(0)\theta_3(0)\theta_4(i\sigma)} = \frac{\pi\sqrt{\pi}\sqrt{1+\kappa^2}}{\mathbf{K}\lambda'\sqrt{2\mathbf{K}\lambda}}, \\
t_1^0 &= t_1\theta_4(0) = \frac{\pi}{\mathbf{K}\sqrt{\lambda}}, \\
t_2^0 &= \frac{2}{\theta_2(0)\theta_4(0)} = \frac{\pi}{\mathbf{K}\sqrt{\lambda\lambda'}}, \\
t_2 &= \frac{2\theta_3(i\sigma)}{\theta_2(0)\theta_3(0)\theta_4(0)\theta_4(i\sigma)} = \frac{\pi\sqrt{\pi}\sqrt{1+\kappa^2}}{\mathbf{K}\lambda'\sqrt{2\mathbf{K}\lambda}}, \\
t_3 &= \frac{2\theta_2(i\sigma)}{\theta_2(0)\theta_3(0)\theta_4(0)\theta_4(i\sigma)} = \frac{\pi\sqrt{\pi(\lambda^2 + \kappa^2)}}{\mathbf{K}\lambda\lambda'\sqrt{2\mathbf{K}}}, \\
t_4 &= \frac{-2i\theta_1(i\sigma)}{\theta_2(0)\theta_3(0)\theta_4(0)\theta_4(i\sigma)} = \frac{\pi\sqrt{\pi\kappa}}{\mathbf{K}\lambda\sqrt{2\mathbf{K}\lambda'}}, \\
t_3^0 &= \frac{2}{\theta_3(0)\theta_4(0)} = \frac{\pi}{\mathbf{K}\sqrt{\lambda'}}.
\end{aligned}$$

$$q = e^{-2d}, d = \frac{\pi\mathbf{K}'}{2\mathbf{K}}; \quad \sigma = \frac{\pi}{2\mathbf{K}} F(\arctan \frac{\kappa}{\lambda}, \lambda')$$

23.4. Ряды Фурье для направляющих косинусов $\tilde{B}_{ij}$ в случае движений в области I.

$$\begin{aligned}
 \tilde{B}_{11} &= \frac{\pi e^\sigma}{K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1 - e^{2\sigma} q^{2m+1}} e^{i(2m+1)\varphi_1}, & B_{11} &= \frac{\tilde{B}_{21} - \tilde{B}_{11}}{2i} = \frac{i\pi}{2K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} q^{m+\frac{1}{2}} \left[\frac{e^{-\sigma}}{1 - e^{-2\sigma} q^{2m+1}} + \frac{e^\sigma}{1 - e^{2\sigma} q^{2m+1}} \right] e^{i(2m+1)\varphi_1}, \\
 \tilde{B}_{21} &= -\frac{\pi e^{-\sigma}}{K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1 - e^{-2\sigma} q^{2m+1}} e^{i(2m+1)\varphi_1}, & B_{21} &= \frac{\tilde{B}_{21} + \tilde{B}_{11}}{2} = \frac{\pi}{2K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} q^{m+\frac{1}{2}} \left[\frac{e^\sigma}{1 - e^{2\sigma} q^{2m+1}} - \frac{e^{-\sigma}}{1 - e^{-2\sigma} q^{2m+1}} \right] e^{i(2m+1)\varphi_1}, \\
 \tilde{B}_{31} &= \frac{\pi}{K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1 + q^{2m+1}} e^{i(2m+1)\varphi_1}, & B_{31} &= \tilde{B}_{31} = \frac{\pi}{K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{m+\frac{1}{2}}}{1 + q^{2m+1}} e^{i(2m+1)\varphi_1}, \\
 \\
 \tilde{B}_{12} &= \frac{i\pi\sqrt{1+\kappa^2}e^\sigma}{K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1 + e^{2\sigma} q^{2m+1}} e^{i(2m+1)\varphi_1}, & B_{12} &= \frac{\tilde{B}_{22} - \tilde{B}_{12}}{2i} = -\frac{\pi\sqrt{1+\kappa^2}}{2K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} q^{m+\frac{1}{2}} \left[\frac{e^{-\sigma}}{1 + e^{-2\sigma} q^{2m+1}} + \frac{e^\sigma}{1 + e^{2\sigma} q^{2m+1}} \right] e^{i(2m+1)\varphi_1}, \\
 \tilde{B}_{22} &= -\frac{i\pi\sqrt{1+\kappa^2}e^{-\sigma}}{K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1 + e^{-2\sigma} q^{2m+1}} e^{i(2m+1)\varphi_1}, & B_{22} &= \frac{\tilde{B}_{22} + \tilde{B}_{12}}{2} = \frac{i\pi\sqrt{1+\kappa^2}}{2K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} q^{m+\frac{1}{2}} \left[\frac{e^\sigma}{1 + e^{2\sigma} q^{2m+1}} - \frac{e^{-\sigma}}{1 - e^{-2\sigma} q^{2m+1}} \right] e^{i(2m+1)\varphi_1}, \\
 \tilde{B}_{32} &= \frac{i\pi\sqrt{1+\kappa^2}}{K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1 - q^{2m+1}} e^{i(2m+1)\varphi_1}, & B_{32} &= \tilde{B}_{32} = \frac{i\pi\sqrt{1+\kappa^2}}{K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{m+\frac{1}{2}}}{1 - q^{2m+1}} e^{i(2m+1)\varphi_1}, \\
 \\
 \tilde{B}_{13} &= \frac{\pi\kappa e^\sigma}{K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^m}{1 - q^{2m} e^{-2\sigma}} e^{i2m\varphi_1}, & B_{13} &= \frac{\tilde{B}_{23} - \tilde{B}_{13}}{2i} = \frac{i\pi\kappa}{2K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} q^m \left[\frac{e^{-\sigma}}{1 - q^{2m} e^{-2\sigma}} + \frac{e^\sigma}{1 - q^{2m} e^{2\sigma}} \right] e^{i2m\varphi_1}, \\
 \tilde{B}_{23} &= -\frac{\pi\kappa e^{-\sigma}}{K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^m}{1 - q^{2m} e^{-2\sigma}} e^{i2m\varphi_1}, & B_{23} &= \frac{\tilde{B}_{23} + \tilde{B}_{13}}{2} = \frac{\pi\kappa}{2K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} q^m \left[\frac{e^\sigma}{1 - q^{2m} e^{2\sigma}} - \frac{e^{-\sigma}}{1 - q^{2m} e^{-2\sigma}} \right] e^{i2m\varphi_1}, \\
 \tilde{B}_{33} &= \frac{\pi\kappa}{K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^m}{1 + q^{2m}} e^{i2m\varphi_1}. & B_{33} &= \tilde{B}_{33} = \frac{\pi\kappa}{K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^m}{1 + q^{2m}} e^{i2m\varphi_1}.
 \end{aligned}$$

$$b_{11} = -\frac{\pi}{\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{e^{\sigma} q^{m+\frac{1}{2}}}{1-q^{2m+1}e^{2\sigma}} \sin[(2m+1)\varphi_1 + \varphi_2] + \frac{e^{-\sigma} q^{m+\frac{1}{2}}}{1-q^{2m+1}e^{-2\sigma}} \sin[(2m+1)\varphi_1 - \varphi_2] \right\},$$

$$b_{12} = -\frac{\pi}{\mathbf{K}} \sqrt{\frac{1+\kappa^2}{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{e^{\sigma} q^{m+\frac{1}{2}}}{1+q^{2m+1}e^{2\sigma}} \cos[(2m+1)\varphi_1 + \varphi_2] + \frac{e^{-\sigma} q^{m+\frac{1}{2}}}{1+q^{2m+1}e^{-2\sigma}} \cos[(2m+1)\varphi_1 - \varphi_2] \right\},$$

$$b_{13} = \frac{\pi\kappa}{\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{e^{\sigma} q^m}{1-q^{2m}e^{2\sigma}} \sin[2m\varphi_1 + \varphi_2] + \frac{e^{-\sigma} q^m}{1-q^{2m}e^{-2\sigma}} \sin[2m\varphi_1 - \varphi_2] \right\} (1 + \delta_{m0})^{-1},$$

$$b_{21} = \frac{\pi}{\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{e^{\sigma} q^{m+\frac{1}{2}}}{1-q^{2m+1}e^{2\sigma}} \cos[(2m+1)\varphi_1 + \varphi_2] - \frac{e^{-\sigma} q^{m+\frac{1}{2}}}{1-q^{2m+1}e^{-2\sigma}} \cos[(2m+1)\varphi_1 - \varphi_2] \right\},$$

$$b_{22} = -\frac{\pi}{\mathbf{K}} \sqrt{\frac{1+\kappa^2}{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{e^{\sigma} q^{m+\frac{1}{2}}}{1+q^{2m+1}e^{2\sigma}} \sin[(2m+1)\varphi_1 + \varphi_2] - \frac{e^{-\sigma} q^{m+\frac{1}{2}}}{1+q^{2m+1}e^{-2\sigma}} \sin[(2m+1)\varphi_1 - \varphi_2] \right\},$$

$$b_{23} = -\frac{\pi\kappa}{\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{e^{\sigma} q^m}{1-q^{2m}e^{2\sigma}} \cos[2m\varphi_1 + \varphi_2] - \frac{e^{-\sigma} q^m}{1-q^{2m}e^{-2\sigma}} \cos[2m\varphi_1 - \varphi_2] \right\} (1 + \delta_{m0})^{-1},$$

$$b_{31} = \frac{2\pi}{\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{q^{m+\frac{1}{2}}}{1+q^{2m+1}} \cos[(2m+1)\varphi_1] \right\},$$

$$b_{32} = -\frac{2\pi}{\mathbf{K}} \sqrt{\frac{\kappa^2 + 1}{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{q^{m+\frac{1}{2}}}{1-q^{2m+1}} \sin[(2m+1)\varphi_1] \right\},$$

$$b_{33} = \frac{2\pi}{\mathbf{K}} \frac{\kappa}{\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{q^m}{1+q^{2m}} \cos[2m\varphi_1] \right\} (1 + \delta_{m0})^{-1},$$

24.1.1. Ряды Фурье для направляющих косинусов b_{ij} .

$$q = e^{-2d}, \quad d = \frac{\pi\mathbf{K}(\lambda')}{2\mathbf{K}(\lambda)} \geq \sigma$$

$$\sigma = \frac{\pi}{2\mathbf{K}} F\left(\arctan \frac{\kappa}{\lambda}, \lambda'\right)$$

23.9. Ряды Фурье для проекций векторов угловой скорости и углового ускорения (p, q, r и $\dot{p}, \dot{q}, \dot{r}$) в случае движений в области I.

$$\begin{aligned} p &= \frac{G}{A} \sin \theta \sin l = \frac{G}{A} b_{31} = \frac{G}{A} \tilde{B}_{31}, \\ q &= \frac{G}{B} \sin \theta \cos l = \frac{G}{B} b_{32} = \frac{G}{B} \tilde{B}_{32}, \\ r &= \frac{G}{C} \cos \theta = \frac{G}{C} b_{33} = \frac{G}{C} \tilde{B}_{33}, \end{aligned}$$

$$\varphi_1 = n_1 t + \varphi_1^{(0)}, \quad n_1 = \frac{A - C}{AC} \frac{\pi G}{2 \mathbf{K}(\lambda)} \frac{\kappa}{\sqrt{(1 + \kappa^2)(\kappa^2 + \lambda^2)}}.$$

$$\dot{p} = \frac{i\pi^2 G^2 \kappa}{2A^2 \mathbf{K}^2(\kappa^2 + \lambda^2) \sqrt{1 + \kappa^2}} \frac{(A - C)}{C} \sum_{m=-\infty}^{\infty} \frac{(2m+1) q^{m+\frac{1}{2}}}{1 + q^{2m+1}} e^{i(2m+1)\varphi_1},$$

$$\dot{q} = -\frac{\pi^2 G^2 \kappa}{2AB \mathbf{K}^2(\kappa^2 + \lambda^2)} \frac{(A - C)}{C} \sum_{m=-\infty}^{\infty} \frac{(2m+1) q^{m+\frac{1}{2}}}{1 - q^{2m+1}} e^{i(2m+1)\varphi_1},$$

$$\dot{r} = \frac{iG^2 \pi^2 \kappa^2}{AC \mathbf{K}^2(\kappa^2 + \lambda^2) \sqrt{1 + \kappa^2}} \frac{(A - C)}{C} \sum_{m=-\infty}^{\infty} \frac{mq^m}{1 + q^{2m}} e^{2im\varphi_1}.$$

$$b_{11} = -\frac{\pi}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{\sin[(2m+1)\varphi_1 + \varphi_2]}{\sinh[(2m+1)d - \sigma]} + \frac{\sin[(2m+1)\varphi_1 - \varphi_2]}{\sinh[(2m+1)d + \sigma]} \right\},$$

$$b_{21} = \frac{\pi}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{\cos[(2m+1)\varphi_1 + \varphi_2]}{\sinh[(2m+1)d - \sigma]} - \frac{\cos[(2m+1)\varphi_1 - \varphi_2]}{\sinh[(2m+1)d + \sigma]} \right\},$$

$$b_{31} = \frac{\pi}{\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{\cos[(2m+1)\varphi_1]}{\cosh[(2m+1)d]} \right\},$$

$$b_{12} = -\frac{\pi}{2\mathbf{K}} \sqrt{\frac{1+\kappa^2}{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{\cos[(2m+1)\varphi_1 + \varphi_2]}{\cosh[(2m+1)d - \sigma]} + \frac{\cos[(2m+1)\varphi_1 - \varphi_2]}{\cosh[(2m+1)d + \sigma]} \right\},$$

$$b_{22} = -\frac{\pi}{2\mathbf{K}} \sqrt{\frac{1+\kappa^2}{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{\sin[(2m+1)\varphi_1 + \varphi_2]}{\cosh[(2m+1)d - \sigma]} - \frac{\sin[(2m+1)\varphi_1 - \varphi_2]}{\cosh[(2m+1)d + \sigma]} \right\},$$

$$b_{32} = -\frac{\pi}{\mathbf{K}} \sqrt{\frac{1+\kappa^2}{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{\sin[(2m+1)\varphi_1]}{\sinh[(2m+1)d]} \right\},$$

$$b_{13} = \frac{\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{\sin[2m\varphi_1 + \varphi_2]}{\sinh[2md - \sigma]} + \frac{\sin[2m\varphi_1 - \varphi_2]}{\sinh[2md + \sigma]} \right\} (1 + \delta_{m0})^{-1},$$

$$b_{23} = -\frac{\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{\cos[2m\varphi_1 + \varphi_2]}{\sinh[2md - \sigma]} - \frac{\cos[2m\varphi_1 - \varphi_2]}{\sinh[2md + \sigma]} \right\} (1 + \delta_{m0})^{-1},$$

$$b_{33} = \frac{\pi\kappa}{\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{\cos[2m\varphi_1]}{\cosh[2md]} \right\} (1 + \delta_{m0})^{-1};$$

$$d = \frac{\pi\mathbf{K}(\lambda')}{2\mathbf{K}(\lambda)} \geq \sigma$$

$$\sigma = \frac{\pi}{2\mathbf{K}} F\left(\arctan \frac{\kappa}{\lambda}, \lambda'\right)$$

$$\dot{b}_{11} = -\frac{\pi}{2K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{(2m+1)n_1 + n_2}{sh[(2m+1)d - \sigma]} \cos[(2m+1)\varphi_1 + \varphi_2] + \frac{(2m+1)n_1 - n_2}{sh[(2m+1)d + \sigma]} \cos[(2m+1)\varphi_1 - \varphi_2] \right\},$$

$$\dot{b}_{21} = -\frac{\pi}{2K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{(2m+1)n_1 + n_2}{sh[(2m+1)d - \sigma]} \sin[(2m+1)\varphi_1 + \varphi_2] + \frac{(2m+1)n_1 - n_2}{sh[(2m+1)d + \sigma]} \sin[(2m+1)\varphi_1 - \varphi_2] \right\},$$

$$\dot{b}_{31} = -\frac{\pi}{K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{(2m+1)n_1}{ch[(2m+1)d]} \sin[(2m+1)\varphi_1] \right\},$$

24.1.1. Ряды Фурье для производных направляющих косинусов $\dot{b}_{ij}$.

$$\dot{b}_{12} = \frac{\pi}{2K} \sqrt{\frac{1+\kappa^2}{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{(2m+1)n_1 + n_2}{ch[(2m+1)d - \sigma]} \sin[(2m+1)\varphi_1 + \varphi_2] + \frac{(2m+1)n_1 - n_2}{ch[(2m+1)d + \sigma]} \sin[(2m+1)\varphi_1 - \varphi_2] \right\},$$

$$\dot{b}_{22} = -\frac{\pi}{2K} \sqrt{\frac{1+\kappa^2}{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{(2m+1)n_1 + n_2}{ch[(2m+1)d - \sigma]} \cos[(2m+1)\varphi_1 + \varphi_2] - \frac{(2m+1)n_1 - n_2}{ch[(2m+1)d + \sigma]} \cos[(2m+1)\varphi_1 - \varphi_2] \right\}$$

$$\dot{b}_{32} = -\frac{\pi}{K} \sqrt{\frac{1+\kappa^2}{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{(2m+1)n_1}{sh[(2m+1)d]} \cos[(2m+1)\varphi_1] \right\}, \quad \sigma = \frac{\pi}{2\mathbf{K}} F(\arctan \frac{\kappa}{\lambda}, \lambda'), \quad d = \frac{\pi \mathbf{K}'}{2\mathbf{K}}$$

$$\dot{b}_{13} = \frac{\pi\kappa}{2K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{2mn_1 + n_2}{sh[2md - \sigma]} \cos[2m\varphi_1 + \varphi_2] + \frac{2mn_1 - n_2}{sh[2md + \sigma]} \cos[2m\varphi_1 - \varphi_2] \right\} (1 + \delta_{m0})^{-1},$$

$$\dot{b}_{23} = \frac{\pi\kappa}{2K\sqrt{\kappa^2 + \lambda^2}} \sum_{m=0}^{\infty} \left\{ \frac{2mn_1 + n_2}{sh[2md - \sigma]} \sin[2m\varphi_1 + \varphi_2] - \frac{2mn_1 - n_2}{sh[2md + \sigma]} \sin[2m\varphi_1 - \varphi_2] \right\} (1 + \delta_{m0})^{-1},$$

24.2.2. Ряды Фурье для направляющих косинусов b_{ij} .

Область движений III.

$$b_{11} = \sum_{m=0}^{\infty} \left\{ b_{2m+1,1}^{(11)} \sin[(2m+1)\varphi_1 + \varphi_2] + b_{2m+1,-1}^{(11)} \sin[(2m+1)\varphi_1 - \varphi_2] \right\},$$

$$b_{21} = \sum_{m=0}^{\infty} \left\{ b_{2m+1,1}^{(2,1)} \cos[(2m+1)\varphi_1 + \varphi_2] + b_{2m+1,-1}^{(2,1)} \cos[(2m+1)\varphi_1 - \varphi_2] \right\},$$

$$b_{31} = \sum_{m=0}^{\infty} \left\{ b_{2m,0}^{(3,1)} \cos[2m\varphi_1] \right\},$$

$$b_{12} = \sum_{m=0}^{\infty} \left\{ b_{2m,1}^{(1,2)} \cos[2m\varphi_1 + \varphi_2] + b_{2m,-1}^{(1,2)} \cos[2m\varphi_1 - \varphi_2] \right\},$$

$$b_{22} = \sum_{m=0}^{\infty} \left\{ b_{2m,1}^{(2,2)} \sin[2m\varphi_1 + \varphi_2] + b_{2m,-1}^{(2,2)} \sin[2m\varphi_1 - \varphi_2] \right\},$$

$$b_{32} = \sum_{m=0}^{\infty} \left\{ b_{2m+1,0}^{(3,2)} \sin[(2m+1)\varphi_1] \right\},$$

$$b_{13} = \sum_{m=0}^{\infty} \left\{ b_{2m,1}^{(1,3)} \sin[2m\varphi_1 + \varphi_2] + b_{2m,-1}^{(1,3)} \sin[2m\varphi_1 - \varphi_2] \right\},$$

$$b_{23} = \sum_{m=0}^{\infty} \left\{ b_{2m,1}^{(2,3)} \cos[2m\varphi_1 + \varphi_2] + b_{2m,-1}^{(2,3)} \cos[2m\varphi_1 - \varphi_2] \right\},$$

$$b_{33} = \sum_{m=0}^{\infty} \left\{ b_{2m+1,0}^{(3,3)} \cos[(2m+1)\varphi_1] \right\},$$

$$D = \frac{\pi \mathbf{K}'(\lambda')}{2\mathbf{K}\left(\frac{1}{\lambda}\right)}, \quad \sigma = \frac{\pi F(\arctan \kappa, \lambda')}{2\mathbf{K}\left(\frac{1}{\lambda}\right)}, \quad \lambda' = \sqrt{1 - \frac{1}{\lambda^2}}.$$

$$b_{2m+1,1}^{(11)} = \frac{-\pi\lambda}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{sh}[(2m+1)D + \sigma]},$$

$$b_{2m+1,-1}^{(11)} = \frac{-\pi\lambda}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{sh}[(2m+1)D - \sigma]},$$

$$b_{2m+1,1}^{(21)} = \frac{\pi\lambda}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{sh}[(2m+1)D + \sigma]},$$

$$b_{2m+1,-1}^{(21)} = -\frac{\pi\lambda}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{sh}[(2m+1)D - \sigma]},$$

$$b_{2m,0}^{(31)} = \frac{\pi\lambda}{\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{ch}[2md]} (1 + \delta_{m0})^{-1},$$

$$b_{2m,1}^{(12)} = -\frac{\pi\lambda\sqrt{1+\kappa^2}}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{ch}[2mD + \sigma]} (1 + \delta_{m0})^{-1},$$

$$b_{2m,-1}^{(12)} = -\frac{\pi\lambda\sqrt{1+\kappa^2}}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{ch}[2mD - \sigma]} (1 + \delta_{m0})^{-1},$$

$$b_{2m,1}^{(22)} = -\frac{\pi\lambda\sqrt{1+\kappa^2}}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{ch}[2mD + \sigma]} (1 + \delta_{m0})^{-1},$$

$$b_{2m,-1}^{(22)} = \frac{\pi\lambda\sqrt{1+\kappa^2}}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{ch}[2mD - \sigma]} (1 + \delta_{m0})^{-1},$$

$$b_{2m+1,0}^{(32)} = \frac{\pi\lambda\sqrt{1+\kappa^2}}{\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{sh}[(2m+1)D]},$$

$$b_{2m,1}^{(13)} = \frac{\pi\kappa\lambda}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{sh}[2mD + \sigma]} (1 + \delta_{m0})^{-1},$$

$$b_{2m,-1}^{(13)} = \frac{\pi\kappa\lambda}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{sh}[2mD - \sigma]} (1 + \delta_{m0})^{-1},$$

$$b_{2m,1}^{(23)} = \frac{-\pi\kappa\lambda}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{sh}[2mD + \sigma]} (1 + \delta_{m0})^{-1},$$

$$b_{2m,-1}^{(23)} = \frac{\pi\kappa\lambda}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{sh}[2mD - \sigma]} (1 + \delta_{m0})^{-1},$$

$$b_{2m+1,0}^{(33)} = \frac{\pi\kappa\lambda}{\mathbf{K}\sqrt{\kappa^2 + \lambda^2} \operatorname{ch}[(2m+1)D]} (1 + \delta_{m0})^{-1}.$$

$$p = \frac{G}{A} \frac{\lambda \sqrt{\lambda'}}{\sqrt{\kappa^2 + \lambda^2}} \frac{\theta_3(\varphi_1)}{\theta_4(\varphi_1)},$$

$$q = -\frac{G}{B} \frac{\sqrt{\lambda} \sqrt{1 + \kappa^2}}{\sqrt{\kappa^2 + \lambda^2}} \frac{\theta_1(\varphi_1)}{\theta_4(\varphi_1)},$$

$$r = \frac{G}{C} \frac{\kappa \sqrt{\lambda \lambda'}}{\sqrt{\kappa^2 + \lambda^2}} \frac{\theta_2(\varphi_1)}{\theta_4(\varphi_1)},$$

$$\dot{p} = G^2 \frac{(C - B)}{ABC} \frac{\lambda \sqrt{\lambda'} \sqrt{1 + \kappa^2}}{\kappa^2 + \lambda^2} \cdot \frac{\theta_1(\varphi_1) \theta_2(\varphi_1)}{\theta_4^2(\varphi_1)},$$

$$\dot{q} = G^2 \frac{(C - A)}{ABC} \frac{\lambda \lambda' \sqrt{\lambda}}{\kappa^2 + \lambda^2} \cdot \frac{\theta_3(\varphi_1) \theta_2(\varphi_1)}{\theta_4^2(\varphi_1)},$$

$$\dot{r} = G^2 \frac{(B - A)}{ABC} \frac{\lambda \sqrt{\lambda \lambda'} \sqrt{1 + \kappa^2}}{\kappa^2 + \lambda^2} \cdot \frac{\theta_1(\varphi_1) \theta_3(\varphi_1)}{\theta_4^2(\varphi_1)}.$$

25. Ряды Фурье для произведений направляющих косинусов и их квадратов (движения в областях I и III).

	b_{11}	b_{12}	b_{13}	b_{21}	b_{22}	b_{23}	b_{31}	b_{32}	b_{33}
b_{11}	P_1								
b_{12}	P_2	P_3							
b_{13}	P_4	P_5	P_6						
b_{21}	P_7	P_8	P_9	P_{10}					
b_{22}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}				
b_{23}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}			
b_{31}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}		
b_{32}	P_{29}	P_{30}	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	P_{36}	
b_{33}	P_{37}	P_{38}	P_{39}	P_{40}	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}

Рис. 25.1. Диаграмма взаимных произведений направляющих косинусов b_{ij} .

25.1. Выражения произведений и квадратов направляющих косинусов $b_{ij}(P_k)$ через произведения и квадраты направляющих косинусов $\tilde{B}_{ij}(P_k)$.

$$\begin{aligned}
1. \quad P_1 = b_{11}^2 &= -\frac{1}{4}(\tilde{B}_{21}^2 e^{-2i\varphi_2} + \tilde{B}_{11}^2 e^{2i\varphi_2}) + \frac{1}{2}(1 - B_{31}^2) \\
2. \quad P_2 = b_{12}b_{11} &= -\frac{1}{4}(\tilde{B}_{21}\tilde{B}_{22}e^{-2i\varphi_2} + \tilde{B}_{11}\tilde{B}_{12}e^{2i\varphi_2}) - \frac{1}{2}B_{31}B_{32} \\
3. \quad P_3 = b_{12}^2 &= -\frac{1}{4}(\tilde{B}_{22}^2 e^{-2i\varphi_2} + \tilde{B}_{12}^2 e^{2i\varphi_2}) + \frac{1}{2}(1 - B_{32}^2) \\
4. \quad P_4 = b_{13}b_{11} &= -\frac{1}{4}(\tilde{B}_{21}\tilde{B}_{23}e^{-2i\varphi_2} + \tilde{B}_{11}\tilde{B}_{13}e^{2i\varphi_2}) - \frac{1}{2}B_{31}B_{33} \\
5. \quad P_5 = b_{13}b_{12} &= -\frac{1}{4}(\tilde{B}_{22}\tilde{B}_{23}e^{-2i\varphi_2} + \tilde{B}_{12}\tilde{B}_{13}e^{2i\varphi_2}) - \frac{1}{2}B_{32}B_{33} \\
6. \quad P_6 = b_{13}^2 &= -\frac{1}{4}(\tilde{B}_{23}^2 e^{-2i\varphi_2} + \tilde{B}_{13}^2 e^{2i\varphi_2}) + \frac{1}{2}(1 - B_{33}^2)
\end{aligned}$$

.....

$$\begin{aligned}
42. \quad P_{42} = b_{33}b_{23} &= \frac{1}{2}(B_{33}\tilde{B}_{23}e^{-i\varphi_2} + B_{33}\tilde{B}_{13}e^{i\varphi_2}) \\
43. \quad P_{43} = b_{33}b_{31} &= \tilde{B}_{31}B_{33} \\
44. \quad P_{44} = b_{33}b_{32} &= \tilde{B}_{32}B_{33} \\
45. \quad P_{45} = b_{33}^2 &= B_{33}^2
\end{aligned}$$

$$\begin{aligned}
b_{11} &= \frac{1}{2i}(-\tilde{B}_{11}e^{i\varphi_2} + \tilde{B}_{21}e^{-i\varphi_2}), \\
b_{21} &= \frac{1}{2}(\tilde{B}_{21}e^{-i\varphi_2} + \tilde{B}_{11}e^{i\varphi_2}), \\
b_{31} &= \tilde{B}_{31}, \\
b_{12} &= \frac{1}{2i}(-\tilde{B}_{12}e^{i\varphi_2} + \tilde{B}_{22}e^{-i\varphi_2}), \\
b_{22} &= \frac{1}{2}(\tilde{B}_{12}e^{i\varphi_2} + \tilde{B}_{22}e^{-i\varphi_2}), \\
b_{32} &= \tilde{B}_{32},
\end{aligned}$$

$$\begin{aligned}
b_{13} &= \frac{1}{2i}(-\tilde{B}_{13}e^{i\varphi_2} + \tilde{B}_{23}e^{-i\varphi_2}), \\
b_{23} &= \frac{1}{2}(\tilde{B}_{13}e^{i\varphi_2} + \tilde{B}_{23}e^{-i\varphi_2}), \\
b_{33} &= \tilde{B}_{33}.
\end{aligned}$$

$$\begin{aligned}
1. \quad \tilde{B}_{21}^2 &= -\frac{T_{11}^2}{\theta_4^2(i\sigma)} \cdot \frac{\theta_1^2(\varphi_1 + i\sigma)}{\theta_4^2(\varphi_1)} \\
2. \quad \tilde{B}_{11}^2 &= -\frac{T_{11}^2}{\theta_4^2(i\sigma)} \cdot \frac{\theta_1^2(\varphi_1 - i\sigma)}{\theta_4^2(\varphi_1)} \\
3. \quad \tilde{B}_{21}\tilde{B}_{22} &= -\frac{T_{11}T_{12}}{\theta_4^2(i\sigma)} \cdot \frac{\theta_1(\varphi_1 + i\sigma)\theta_2(\varphi_1 + i\sigma)}{\theta_4^2(\varphi_1)} \\
4. \quad \tilde{B}_{11}\tilde{B}_{12} &= -\frac{T_{11}T_{12}}{\theta_4^2(i\sigma)} \cdot \frac{\theta_1(\varphi_1 - i\sigma)\theta_2(\varphi_1 - i\sigma)}{\theta_4^2(\varphi_1)} \\
5. \quad \tilde{B}_{22}^2 &= -\frac{T_{12}^2}{\theta_4^2(i\sigma)} \cdot \frac{\theta_2^2(\varphi_1 + i\sigma)}{\theta_4^2(\varphi_1)} \\
6. \quad \tilde{B}_{22}^2 &= -\frac{T_{12}^2}{\theta_4^2(i\sigma)} \cdot \frac{\theta_2^2(\varphi_1 - i\sigma)}{\theta_4^2(\varphi_1)}
\end{aligned}$$

.....

$$\begin{aligned}
34. \quad \tilde{B}_{31}B_{33} &= T_{31}T_{33} \cdot \frac{\theta_2(\varphi_1)\theta_3(\varphi_1)}{\theta_4^2(\varphi_1)} \\
35. \quad \tilde{B}_{32}B_{33} &= -T_{32}T_{33} \cdot \frac{\theta_1(\varphi_1)\theta_3(\varphi_1)}{\theta_4^2(\varphi_1)} \\
36. \quad B_{33}^2 &= T_{33}^2 \cdot \frac{\theta_3^2(\varphi_1)}{\theta_4^2(\varphi_1)}
\end{aligned}$$

25.3. Ряды Фурье для произведений и квадратов направляющих косинусов $\tilde{B}_{ij}$.

$$\tilde{B}_{ij}) \quad R_l = \frac{\theta_l(\varphi_1 + i\sigma_l)}{\theta_4(\varphi_1)} \quad (l = 1, 2, 3, 4), \quad sn^2\left(i \frac{2K}{\pi} \sigma\right) = -\frac{n}{k^2},$$

где σ_l принимает значения $\sigma, -\sigma, 0$ при $l = 1, 2, 3$ и $\sigma = \sigma, -\sigma$ при $l = 4$.

$$\tilde{B}_{ij} \tilde{B}_{nk}) \quad R_{ij} = \frac{\theta_i(\varphi_1 + i\sigma_i) \theta_j(\varphi_1 + i\sigma_j)}{\theta_4^2(\varphi_1)}, \quad (i, j = 1, 2, 3, 4; i \leq j) \quad \sigma_i, \sigma_j \Rightarrow (\sigma, -\sigma, 0)$$

$$R_{ij} = \sum_{m=-\infty}^{\infty} a_m^{(i,j)} e^{im\varphi_1}$$

$$a_m^{(i,j)} = \frac{1}{2\pi} \int_0^{2\pi} R_{ij}(\varphi_1) e^{-im\varphi_1} d\varphi_1$$

$$(i, j = 1, 2, 3, 4; \quad i \leq j)$$

$$R_{ij} = R_{ij;k} \Rightarrow$$

$$k = 1, \quad \sigma_i = \sigma, \quad \sigma_j = \sigma;$$

$$k = 2, \quad \sigma_i = -\sigma, \quad \sigma_j = -\sigma;$$

$$k = 3, \quad \sigma_i = \sigma, \quad \sigma_j = -\sigma;$$

$$k = 4, \quad \sigma_i = -\sigma, \quad \sigma_j = \sigma;$$

$$k = 5, \quad \sigma_i = \sigma, \quad \sigma_j = 0;$$

$$k = 6, \quad \sigma_i = -\sigma, \quad \sigma_j = 0;$$

$$k = 7, \quad \sigma_i = 0, \quad \sigma_j = \sigma;$$

$$k = 8, \quad \sigma_i = 0, \quad \sigma_j = -\sigma;$$

$$k = 9, \quad \sigma_i = 0, \quad \sigma_j = 0.$$

25.3.2. Свойства функций R_{ij} . Выражение контурного интеграла через вычеты.

Формула для вычетов.

$$a_m^{(i,j)} = \frac{-\Delta_i \Delta_j q^{-\frac{m}{2}} e^{\sigma_i + \sigma_j} \theta_{5-i}(i\sigma_i) \theta_{5-j}(i\sigma_j)}{\phi^2 \left[1 - \delta_i \delta_j (-1)^{i+j} e^{2(\sigma_i + \sigma_j)} q^{-m} \right]} \left(m + im_{5-i}^{(1)} + im_{5-j}^{(1)} \right) \left(1 + (-1)^m \delta_i \delta_j \right)$$

где, дополнительно, $\Delta_j = i$ при $j = 1, 4$ и $\Delta_j = 1$ при $j = 2, 3$,

$$m_j^{(1)} = \frac{\theta'_j(i\sigma_j)}{\theta_j(i\sigma_j)}, \quad \phi = \theta_2(0)\theta_3(0)\theta_4(0).$$

$$m_1^{(1)} = \frac{\theta'_1(i\sigma)}{\theta_1(i\sigma)} = -i \frac{2\Pi}{\pi\kappa} \sqrt{(1+\kappa^2)(\kappa^2 + \lambda^2)},$$

$$m_2^{(1)} = \frac{\theta'_2(i\sigma)}{\theta_2(i\sigma)} = i \frac{2}{\pi\kappa} \sqrt{\frac{1+\kappa^2}{\kappa^2 + \lambda^2}} \left[\mathbf{K}\lambda^2 - \Pi(\kappa^2 + \lambda^2) \right],$$

$$m_3^{(1)} = \frac{\theta'_3(i\sigma)}{\theta_3(i\sigma)} = i \frac{2\sqrt{\kappa^2 + \lambda^2}}{\pi\kappa\sqrt{1+\kappa^2}} \left[\mathbf{K} - \Pi(1+\kappa^2) \right],$$

$$m_4^{(1)} = \frac{\theta'_4(i\sigma)}{\theta_4(i\sigma)} = i \frac{2(\mathbf{K} - \Pi)}{\mathbf{K}\kappa} \sqrt{(1+\kappa^2)(\kappa^2 + \lambda^2)}.$$

25.3.6.1. Общие формулы для рядов Фурье. Выражения коэффициентов $t_{ij;k}, P_{ij;k}$.

$$R_{ij;k} = \sum_{m=-\infty}^{\infty} a_m^{(i,j;k)} e^{-im\varphi_1}$$

$$, j = 1, 2, 3, 4; \quad i \leq j, \quad k \in (1, \dots, 9) \text{.}$$

$$a_m^{(i,j)} = \frac{-\Delta_i \Delta_j q^{-\frac{m}{2}} e^{\sigma_i + \sigma_j} \theta_{5-i}(i\sigma_i) \theta_{5-j}(i\sigma_j)}{\phi^2 \left[1 - \delta_i \delta_j (-1)^{i+j} e^{2(\sigma_i + \sigma_j)} q^{-m} \right]} \left(m + im_{5-i}^{(1)} + im_{5-j}^{(1)} \right) (1 + (-1)^m \delta_i \delta_j),$$

где, дополнительно, $\Delta_j = i$ при $j = 1, 4$ и $\Delta_j = 1$ при $j = 2, 3$,

$$m_j^{(1)} = \frac{\theta'_j(i\sigma_j)}{\theta_j(i\sigma_j)}, \quad \phi = \theta_2(0)\theta_3(0)\theta_4(0).$$

$$\begin{aligned}
1. \quad & \frac{\theta_1^2(\varphi_1 + i\sigma)}{\theta_4^2(\varphi_1)} = t_{11;1} \sum_{m=-\infty}^{\infty} (m + im_4^{(1)}) \frac{q^{-m} e^{2\sigma}}{1 - e^{4\sigma} q^{-2m}} e^{2im\varphi_1} \\
2. \quad & \frac{\theta_1^2(\varphi_1 - i\sigma)}{\theta_4^2(\varphi_1)} = t_{11;1} \sum_{m=-\infty}^{\infty} (m - im_4^{(1)}) \frac{q^{-m} e^{2\sigma}}{1 - e^{4\sigma} q^{-2m}} e^{2im\varphi_1} \\
3. \quad & \frac{\theta_1(\varphi_1 + i\sigma)\theta_1(\varphi_1 - i\sigma)}{\theta_4^2(\varphi_1)} = t_{11;1} \sum_{m=-\infty}^{\infty} \frac{mq^{-m}}{1 - q^{-2m}} e^{2im\varphi_1} + t_{11;1}^0 \\
4. \quad & \frac{\theta_1(\varphi_1 + i\sigma)\theta_1(\varphi_1)}{\theta_4^2(\varphi_1)} = t_{11;5} \sum_{m=-\infty}^{\infty} \left(m + \frac{i}{2} m_4^{(1)}\right) \frac{q^{-m} e^{\sigma}}{1 - e^{4\sigma} q^{-2m}} e^{2im\varphi_1} \\
5. \quad & \frac{\theta_1(\varphi_1 - i\sigma)\theta_1(\varphi_1)}{\theta_4^2(\varphi_1)} = t_{11;5} \sum_{m=-\infty}^{\infty} \left(m - \frac{i}{2} m_4^{(1)}\right) \frac{q^{-m} e^{-\sigma}}{1 - e^{4\sigma} q^{-2m}} e^{2im\varphi_1}
\end{aligned}$$

$$\begin{aligned}
(1., 2., 3.) \quad & p_{1,1}^{(1,1)} = \frac{\pi^3}{2\mathbf{K}^3 k k'} \\
(4., 5.) \quad & p_{1,1}^{(1,0)} = \frac{\pi^2 \sqrt{\pi}}{\sqrt{2\mathbf{K} k'} \mathbf{K}^2 k} \\
(6.) \quad & p_{1,1}^{(0,0)} = \frac{\pi^2}{\mathbf{K}^2 k} \\
(7., 8., 9., 10.) \quad & p_{1,2}^{(1,1)} = \frac{\pi^3 d n a}{2\mathbf{K}^3 k k' \sqrt{k'}}
\end{aligned}$$

.....

$$\begin{aligned}
64. \quad & \frac{\theta_4^2(\varphi_1 + i\sigma)}{\theta_4^2(\varphi_1)} = t_{44;1} \sum_{m=-\infty}^{\infty} [m + im_1^{(1)}] \frac{q^{-m} e^{2\sigma}}{1 - e^{4\sigma} q^{-2m}} e^{2im\varphi_1} \\
65. \quad & \frac{\theta_4^2(\varphi_1 - i\sigma)}{\theta_4^2(\varphi_1)} = t_{44;1} \sum_{m=-\infty}^{\infty} [m - im_1^{(1)}] \frac{q^{-m} e^{-2\sigma}}{1 - e^{-4\sigma} q^{-2m}} e^{2im\varphi_1} \\
66. \quad & \frac{\theta_4(\varphi_1 + i\sigma)\theta_4(\varphi_1 - i\sigma)}{\theta_4^2(\varphi_1)} = t_{44;1} \sum_{m=-\infty}^{\infty} \frac{mq^{-m}}{1 - q^{-2m}} e^{2im\varphi_1} + t_{44;1}^0
\end{aligned}$$

$$\begin{aligned}
(1., 2., 3.) \quad & t_{11;1} = \frac{4}{\phi^2} \theta_4^2(i\sigma) = p_{11;1} \theta_4^2(i\sigma) \\
(4., 5.) \quad & t_{11;5} = \frac{4\theta_4(0)}{\phi^2} \theta_4(i\sigma) = p_{11;5} \theta_4(i\sigma) \\
(6.) \quad & t_{11;9} = \frac{4\theta_4^2(0)}{\phi^2} = p_{11;9} \\
(7., 8., 9., 10.) \quad & t_{12;1} = -\frac{4i\theta_3(i\sigma)\theta_4(i\sigma)}{\phi^2} = -i p_{12;1} \theta_4^2(i\sigma)
\end{aligned}$$

$$m_1^{(1)}(i\sigma) = \frac{\theta_1'(i\sigma)}{\theta_1(i\sigma)} = \frac{2\Pi}{\pi} \frac{cnadna}{sna},$$

$$m_2^{(1)}(i\sigma) = \frac{\theta_2'(i\sigma)}{\theta_2(i\sigma)} = \frac{2dna}{\pi snacna} (\Pi cn^2 a - \mathbf{K}),$$

$$m_3^{(1)}(i\sigma) = \frac{\theta_3'(i\sigma)}{\theta_3(i\sigma)} = \frac{2cna}{\pi snadna} (\Pi dn^2 a - \mathbf{K}),$$

$$m_4^{(1)}(i\sigma) = \frac{\theta_4'(i\sigma)}{\theta_4(i\sigma)} = \frac{2(\Pi - \mathbf{K})}{\pi} \frac{cna}{sna} \frac{dna}{sna}.$$

$$m_1^{(1)} = \frac{\theta_1'(i\sigma)}{\theta_1(i\sigma)} = -i \frac{2\Pi}{\pi\kappa} \sqrt{(1+\kappa^2)(\kappa^2 + \lambda^2)},$$

$$m_2^{(1)} = \frac{\theta_2'(i\sigma)}{\theta_2(i\sigma)} = i \frac{2}{\pi\kappa} \sqrt{\frac{1+\kappa^2}{\kappa^2 + \lambda^2}} \left[\mathbf{K}\lambda^2 - \Pi(\kappa^2 + \lambda^2) \right],$$

$$m_3^{(1)} = \frac{\theta_3'(i\sigma)}{\theta_3(i\sigma)} = i \frac{2\sqrt{\kappa^2 + \lambda^2}}{\pi\kappa\sqrt{1+\kappa^2}} \left[\mathbf{K} - \Pi(1+\kappa^2) \right],$$

$$m_4^{(1)} = \frac{\theta_4'(i\sigma)}{\theta_4(i\sigma)} = i \frac{2(\mathbf{K} - \Pi)}{\mathbf{K}\kappa} \sqrt{(1+\kappa^2)(\kappa^2 + \lambda^2)}.$$

Приложение 3. Контроль правильности рядов Фурье для произведений и квадратов направляющих косинусов $b_{ij}b_{nk}$.

$$[1] \quad b_{11}^2 + b_{12}^2 + b_{13}^2 = 1.$$

$$[1] \quad b_{11}^2 + b_{12}^2 + b_{13}^2 = 1,$$

$$[2] \quad b_{21}^2 + b_{22}^2 + b_{23}^2 = 1,$$

$$[3] \quad b_{31}^2 + b_{32}^2 + b_{33}^2 = 1,$$

$$b_{11}^2 = b_{0,0}^{(11,11)} + b_{0,2}^{(11,11)} \cos 2\varphi_2 + \sum_{m=1}^{\infty} \left\{ b_{2m,0}^{(11,11)} \cos 2(m\varphi_1) + b_{2m,2}^{(11,11)} \cos 2(m\varphi_1 + \varphi_2) + b_{2m,-2}^{(11,11)} \cos 2(m\varphi_1 - \varphi_2) \right\}$$

$$[4] \quad b_{11}b_{21} + b_{12}b_{22} + b_{13}b_{23} = 0,$$

$$[5] \quad b_{11}b_{31} + b_{12}b_{32} + b_{13}b_{33} = 0,$$

$$[6] \quad b_{21}b_{31} + b_{22}b_{32} + b_{23}b_{33} = 0,$$

$$b_{12}^2 = b_{0,0}^{(12,12)} + b_{0,2}^{(12,12)} \cos 2\varphi_2 + \sum_{m=1}^{\infty} \left\{ b_{2m,0}^{(12,12)} \cos 2(m\varphi_1) + b_{2m,2}^{(12,12)} \cos 2(m\varphi_1 + \varphi_2) + b_{2m,-2}^{(12,12)} \cos 2(m\varphi_1 - \varphi_2) \right\}$$

$$b_{13}^2 = b_{0,0}^{(13,13)} + b_{0,2}^{(13,13)} \cos 2\varphi_2 + \sum_{m=1}^{\infty} \left\{ b_{2m,0}^{(13,13)} \cos 2(m\varphi_1) + b_{2m,2}^{(13,13)} \cos 2(m\varphi_1 + \varphi_2) + b_{2m,-2}^{(13,13)} \cos 2(m\varphi_1 - \varphi_2) \right\}$$

$$[7] \quad b_{11}^2 + b_{21}^2 + b_{31}^2 = 1,$$

$$[8] \quad b_{12}^2 + b_{22}^2 + b_{32}^2 = 1,$$

$$[9] \quad b_{13}^2 + b_{23}^2 + b_{33}^2 = 1,$$

$$b_{0,0}^{(11,11)} = \frac{\mathbf{K}(1+\kappa^2) - \mathbf{E}}{2\mathbf{K}(\kappa^2 + \lambda^2)},$$

$$b_{0,2}^{(11,11)} = \frac{-\pi^2}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{\mathbf{M}_4}{\sinh(2\sigma)},$$

$$b_{2m,0}^{(11,11)} = \frac{-\pi^2}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m}{\sinh(2md)},$$

$$b_{2m,2}^{(11,11)} = \frac{-\pi^2}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m - 2\mathbf{M}_4}{\sinh[2(md - \sigma)]},$$

$$b_{2m,-2}^{(11,11)} = \frac{-\pi^2}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m + 2\mathbf{M}_4}{\sinh[2(md + \sigma)]},$$

$$b_{0,0}^{(12,12)} = \frac{1}{2} \left[1 + \frac{(1+\kappa^2)(\mathbf{E} - \mathbf{K})}{\mathbf{K}(\kappa^2 + \lambda^2)} \right],$$

$$b_{0,2}^{(12,12)} = \frac{\pi^2(1+\kappa^2)}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{\mathbf{M}_3}{\sinh(2\sigma)},$$

$$b_{2m,0}^{(12,12)} = \frac{\pi^2(1+\kappa^2)}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m}{\sinh(2md)},$$

$$b_{2m,2}^{(12,12)} = \frac{\pi^2(1+\kappa^2)}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m - 2\mathbf{M}_3}{\sinh[2(md - \sigma)]},$$

$$b_{2m,-2}^{(12,12)} = \frac{\pi^2(1+\kappa^2)}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m + 2\mathbf{M}_3}{\sinh[2(md + \sigma)]},$$

$$b_{0,0}^{(13,13)} = \frac{1}{2} \left[1 - \frac{\kappa^2 \mathbf{E}}{\mathbf{K}(\kappa^2 + \lambda^2)} \right],$$

$$b_{0,2}^{(13,13)} = -\frac{\pi^2 \kappa^2}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{\mathbf{M}_1}{\sinh(2\sigma)},$$

$$b_{2m,0}^{(13,13)} = -\frac{\pi^2 \kappa^2}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m}{\sinh(2md)},$$

$$b_{2m,2}^{(13,13)} = -\frac{\pi^2 \kappa^2}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m - 2\mathbf{M}_1}{\sinh[2(md - \sigma)]},$$

$$b_{2m,-2}^{(13,13)} = -\frac{\pi^2 \kappa^2}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m + 2\mathbf{M}_1}{\sinh[2(md + \sigma)]}.$$

$$[10] \quad b_{11}b_{12} + b_{21}b_{22} + b_{31}b_{32} = 0,$$

$$[11] \quad b_{11}b_{13} + b_{21}b_{23} + b_{31}b_{33} = 0,$$

$$[12] \quad b_{12}b_{13} + b_{22}b_{23} + b_{32}b_{33} = 0.$$

$$[13] \quad b_{33} = b_{11}b_{22} - b_{21}b_{12},$$

$$[14] \quad b_{32} = b_{21}b_{13} - b_{11}b_{23},$$

$$[15] \quad b_{31} = b_{23}b_{12} - b_{22}b_{13},$$

$$\text{Проверка [1]: } b_{11}^2 + b_{12}^2 + b_{13}^2 = 1.$$

$$[16] \quad b_{23} = b_{31}b_{12} - b_{11}b_{32},$$

$$[17] \quad b_{22} = b_{11}b_{33} - b_{31}b_{13},$$

$$[18] \quad b_{21} = b_{32}b_{13} - b_{12}b_{33},$$

$$[19] \quad b_{13} = b_{21}b_{32} - b_{31}b_{22},$$

$$[20] \quad b_{12} = b_{31}b_{23} - b_{21}b_{33},$$

$$[21] \quad b_{11} = b_{22}b_{33} - b_{32}b_{23}.$$

$$\sum_{i=1}^3 b_{0,0}^{(1i,1i)} = \frac{1}{2K(\kappa^2 + \lambda^2)} \left\{ \mathbf{K}[1 + \kappa^2 + \lambda^2 - 1 + \kappa^2 + \lambda^2] + \mathbf{E}[-1 + 1 + \kappa^2 - \kappa^2] \right\} \equiv 1$$

$$\sum_{i=1}^3 b_{0,2}^{(1i,1i)} = -\frac{\pi^2}{2K^2(\kappa^2 + \lambda^2) \sinh(2\sigma)} [\mathbf{M}_4 - (1 + \kappa^2)\mathbf{M}_3 + \kappa^2\mathbf{M}_1] \equiv 0$$

$$\sum_{i=1}^3 b_{2m,0}^{(1i,1i)} = \frac{\pi^2 m}{2K^2(\kappa^2 + \lambda^2) \sinh(2\sigma)} [-1 + 1 + \kappa^2 - \kappa^2] \equiv 0$$

$$\sum_{i=1}^3 b_{2m,2\varepsilon}^{(1i,1i)} = \frac{\pi^2}{2K^2(\kappa^2 + \lambda^2) \sinh[2(md - \varepsilon\sigma)]} [m(-1 + 1 + \kappa^2 - \kappa^2) + 2\varepsilon[\mathbf{M}_4 - (1 + \kappa^2)\mathbf{M}_3 + \kappa^2\mathbf{M}_1]] \equiv 0$$

Приложение 3. Контроль правильности рядов Фурье для произведений и квадратов направляющих косинусов $b_{ij}b_{nk}$. [1] $b_{11}^2 + b_{12}^2 + b_{13}^2 = 1$.

$$b_{11}^2 = b_{0,0}^{(11,11)} + b_{0,2}^{(11,11)} \cos 2\varphi_2 + \sum_{m=1}^{\infty} \left\{ b_{2m,0}^{(11,11)} \cos 2(m\varphi_1) + b_{2m,2}^{(11,11)} \cos 2(m\varphi_1 + \varphi_2) + b_{2m,-2}^{(11,11)} \cos 2(m\varphi_1 - \varphi_2) \right\}$$

$$b_{12}^2 = b_{0,0}^{(12,12)} + b_{0,2}^{(12,12)} \cos 2\varphi_2 + \sum_{m=1}^{\infty} \left\{ b_{2m,0}^{(12,12)} \cos 2(m\varphi_1) + b_{2m,2}^{(12,12)} \cos 2(m\varphi_1 + \varphi_2) + b_{2m,-2}^{(12,12)} \cos 2(m\varphi_1 - \varphi_2) \right\}$$

$$b_{13}^2 = b_{0,0}^{(13,13)} + b_{0,2}^{(13,13)} \cos 2\varphi_2 + \sum_{m=1}^{\infty} \left\{ b_{2m,0}^{(13,13)} \cos 2(m\varphi_1) + b_{2m,2}^{(13,13)} \cos 2(m\varphi_1 + \varphi_2) + b_{2m,-2}^{(13,13)} \cos 2(m\varphi_1 - \varphi_2) \right\}$$

$$\begin{aligned} b_{0,0}^{(11,11)} &= \frac{\mathbf{K}(1+\kappa^2) - \mathbf{E}}{2\mathbf{K}(\kappa^2 + \lambda^2)}, & b_{0,0}^{(12,12)} &= \frac{1}{2} \left[1 + \frac{(1+\kappa^2)(\mathbf{E} - \mathbf{K})}{\mathbf{K}(\kappa^2 + \lambda^2)} \right], & b_{0,0}^{(13,13)} &= \frac{1}{2} \left[1 - \frac{\kappa^2 \mathbf{E}}{\mathbf{K}(\kappa^2 + \lambda^2)} \right], \\ b_{0,2}^{(11,11)} &= \frac{-\pi^2}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{\mathbf{M}_4}{\sinh(2\sigma)}, & b_{0,2}^{(12,12)} &= \frac{\pi^2(1+\kappa^2)}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{\mathbf{M}_3}{\sinh(2\sigma)}, & b_{0,2}^{(13,13)} &= -\frac{\pi^2 \kappa^2}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{\mathbf{M}_1}{\sinh(2\sigma)}, \\ b_{2m,0}^{(11,11)} &= \frac{-\pi^2}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m}{\sinh(2md)}, & b_{2m,0}^{(12,12)} &= \frac{\pi^2(1+\kappa^2)}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m}{\sinh(2md)}, & b_{2m,0}^{(13,13)} &= -\frac{\pi^2 \kappa^2}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m}{\sinh(2md)}, \\ b_{2m,2}^{(11,11)} &= \frac{-\pi^2}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m - 2\mathbf{M}_4}{\sinh[2(md - \sigma)]}, & b_{2m,2}^{(12,12)} &= \frac{\pi^2(1+\kappa^2)}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m - 2\mathbf{M}_3}{\sinh[2(md - \sigma)]}, & b_{2m,2}^{(13,13)} &= -\frac{\pi^2 \kappa^2}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m - 2\mathbf{M}_1}{\sinh[2(md - \sigma)]}, \\ b_{2m,-2}^{(11,11)} &= \frac{-\pi^2}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m + 2\mathbf{M}_4}{\sinh[2(md + \sigma)]}, & b_{2m,-2}^{(12,12)} &= \frac{\pi^2(1+\kappa^2)}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m + 2\mathbf{M}_3}{\sinh[2(md + \sigma)]}, & b_{2m,-2}^{(13,13)} &= -\frac{\pi^2 \kappa^2}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m + 2\mathbf{M}_1}{\sinh[2(md + \sigma)]}. \end{aligned}$$

Проверка [1]: $b_{11}^2 + b_{12}^2 + b_{13}^2 = 1$.

$$\sum_{i=1}^3 b_{0,0}^{(1i,1i)} = \frac{1}{2K(\kappa^2 + \lambda^2)} \left\{ \mathbf{K}[1 + \kappa^2 + \lambda^2 - 1 + \kappa^2 + \lambda^2] + \mathbf{E}[-1 + 1 + \kappa^2 - \kappa^2] \right\} \equiv 1$$

$$\sum_{i=1}^3 b_{0,2}^{(1i,1i)} = -\frac{\pi^2}{2K^2(\kappa^2 + \lambda^2) \sinh(2\sigma)} [\mathbf{M}_4 - (1 + \kappa^2)\mathbf{M}_3 + \kappa^2 \mathbf{M}_1] \equiv 0$$

$$\sum_{i=1}^3 b_{2m,0}^{(1i,1i)} = \frac{\pi^2 m}{2K^2(\kappa^2 + \lambda^2) \sinh(2\sigma)} [-1 + 1 + \kappa^2 - \kappa^2] \equiv 0$$

$$\sum_{i=1}^3 b_{2m,2\varepsilon}^{(1i,1i)} = \frac{\pi^2}{2K^2(\kappa^2 + \lambda^2) \sinh[2(md - \varepsilon\sigma)]} [m(-1 + 1 + \kappa^2 - \kappa^2) + 2\varepsilon[\mathbf{M}_4 - (1 + \kappa^2)\mathbf{M}_3 + \kappa^2 \mathbf{M}_1]] \equiv 0$$

$$[10] \quad b_{11}b_{12} + b_{21}b_{22} + b_{31}b_{32} = 0.$$

$$b_{11}b_{12} = b_{0,2}^{(12,11)} \sin 2\varphi_2 + \sum_{m=1}^{\infty} \left\{ b_{2m,0}^{(12,11)} \sin 2(m\varphi_1) + b_{2m,2}^{(12,11)} \sin 2(m\varphi_1 + \varphi_2) + b_{2m,-2}^{(12,11)} \sin 2(m\varphi_1 - \varphi_2) \right\},$$

$$b_{21}b_{22} = b_{0,2}^{(22,21)} \sin 2\varphi_2 + \sum_{m=1}^{\infty} \left\{ b_{2m,0}^{(22,21)} \sin 2(m\varphi_1) + b_{2m,2}^{(22,21)} \sin 2(m\varphi_1 + \varphi_2) + b_{2m,-2}^{(22,21)} \sin 2(m\varphi_1 - \varphi_2) \right\},$$

$$b_{31}b_{32} = \sum_{m=1}^{\infty} \left\{ b_{2m,0}^{(32,31)} \sin 2(m\varphi_1) \right\}.$$

$$b_{0,2}^{(12,11)} = -\frac{\pi^2 \sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{\mathbf{M}_3 + \mathbf{M}_4}{\cosh(2\sigma)},$$

$$b_{2m,0}^{(12,11)} = \frac{\pi^2 \sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m}{\cosh(2md)},$$

$$b_{2m,2}^{(12,11)} = \frac{\pi^2 \sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m - \mathbf{M}_3 - \mathbf{M}_4}{\cosh[2(md - \sigma)]},$$

$$b_{2m,-2}^{(12,11)} = \frac{\pi^2 \sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m + \mathbf{M}_3 + \mathbf{M}_4}{\cosh[2(md + \sigma)]}.$$

$$b_{2m,0}^{(32,31)} = -\frac{\pi^2 \sqrt{1+\kappa^2}}{\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m}{\cosh(2md)},$$

$$b_{0,2}^{(22,21)} = \frac{\pi^2 \sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{\mathbf{M}_3 + \mathbf{M}_4}{\cosh(2\sigma)},$$

$$b_{2m,0}^{(22,21)} = \frac{\pi^2 \sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m}{\cosh(2md)},$$

$$b_{2m,2}^{(22,21)} = -\frac{\pi^2 \sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m - \mathbf{M}_3 - \mathbf{M}_4}{\cosh[2(md - \sigma)]},$$

$$b_{2m,-2}^{(22,21)} = -\frac{\pi^2 \sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m + \mathbf{M}_3 + \mathbf{M}_4}{\cosh[2(md + \sigma)]}.$$

Проверка [10].

$$\sum_{i=1}^3 b_{0,2}^{(i1,i2)} = \frac{\pi^2 \sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2 + \lambda^2) \cosh(2\sigma)} [-\mathbf{M}_3 - \mathbf{M}_4 + \mathbf{M}_3 + \mathbf{M}_4] \equiv 0,$$

$$\sum_{i=1}^3 b_{2m,0}^{(i1,i2)} = \frac{\pi^2 \sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2 + \lambda^2) \cosh(2md)} [m + m - 2m] \equiv 0,$$

$$\sum_{i=1}^3 b_{2m,2\varepsilon}^{(i1,i2)} = \frac{\pi^2 \sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2 + \lambda^2) \cosh[2(md - \varepsilon\sigma)]} [m - m - \varepsilon(\mathbf{M}_3 + \mathbf{M}_4) + \varepsilon(\mathbf{M}_3 + \mathbf{M}_4)] \equiv 0.$$

$$[16] \quad b_{23} = b_{31}b_{12} - b_{11}b_{32}.$$

$$b_{31}b_{12} = \sum_{m=1}^{\infty} \left\{ b_{0,1}^{(31,12)} \cos(\varphi_2) + b_{2m,1}^{(31,12)} \cos(2m\varphi_1 + \varphi_2) + b_{2m,-1}^{(31,12)} \cos(2m\varphi_1 - \varphi_2) \right\},$$

$$b_{11}b_{32} = \sum_{m=1}^{\infty} \left\{ b_{0,1}^{(11,32)} \cos(\varphi_2) + b_{2m,1}^{(11,32)} \cos(2m\varphi_1 + \varphi_2) + b_{2m,-1}^{(11,32)} \cos(2m\varphi_1 - \varphi_2) \right\},$$

$$b_{23} = b_{0,1}^{(23)} \cos(\varphi_2) + \sum_{m=0}^{\infty} \left\{ b_{2m,1}^{(23)} \cos(2m\varphi_1 + \varphi_2) + b_{2m,-1}^{(23)} \cos(2m\varphi_1 - \varphi_2) \right\},$$

$$\begin{aligned} b_{0,1}^{(31,12)} &= -\frac{\pi^2 \sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{\mathbf{M}_3}{\sinh(\sigma)}, & b_{0,1}^{(11,32)} &= -\frac{\pi^2 \sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{\mathbf{M}_4}{\sinh(\sigma)}, & b_{0,1}^{(23)} &= \frac{\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \cdot \frac{1}{\sinh(\sigma)}, \\ b_{2m,1}^{(31,12)} &= -\frac{\pi^2 \sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m - \mathbf{M}_3}{\sinh(2md - \sigma)}, & b_{2m,1}^{(11,32)} &= -\frac{\pi^2 \sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m - \mathbf{M}_4}{\sinh(2md - \sigma)}, & b_{2m,1}^{(23)} &= -\frac{\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \cdot \frac{1}{\sinh(2md - \sigma)}, \\ b_{2m,-1}^{(31,12)} &= -\frac{\pi^2 \sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m + \mathbf{M}_3}{\sinh(2md + \sigma)}, & b_{2m,-1}^{(11,32)} &= -\frac{\pi^2 \sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m + \mathbf{M}_4}{\sinh(2md + \sigma)}, & b_{2m,-1}^{(23)} &= \frac{\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \cdot \frac{1}{\sinh(2md + \sigma)}. \end{aligned}$$

Проверка [16].

$$\mathbf{M}_1 = \frac{\Pi}{\pi\kappa} \sqrt{(1+\kappa^2)(\kappa^2 + \lambda^2)},$$

$$\mathbf{M}_2 = \frac{\sqrt{1+\kappa^2}}{\pi\kappa\sqrt{\kappa^2 + \lambda^2}} \left[\Pi(\kappa^2 + \lambda^2) - \mathbf{K}\lambda^2 \right],$$

$$\mathbf{M}_3 = \frac{\sqrt{\kappa^2 + \lambda^2}}{\pi\kappa\sqrt{1+\kappa^2}} \left[\Pi(1+\kappa^2) - \mathbf{K} \right],$$

$$\mathbf{M}_4 = \frac{\Pi - \mathbf{K}}{\pi\kappa} \sqrt{(1+\kappa^2)(\kappa^2 + \lambda^2)},$$

$$\begin{aligned} b_{0,1}^{(31,12)} - b_{0,1}^{(11,32)} &= \frac{\pi^2 \sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2 + \lambda^2) \sinh[\sigma]} [\mathbf{M}_4 - \mathbf{M}_3] = \\ &= \frac{\pi^2 \sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2 + \lambda^2) \sinh[\sigma]} \left[-\frac{\kappa\mathbf{K}\sqrt{\kappa^2 + \lambda^2}}{\pi\sqrt{1+\kappa^2}} \right] = -\frac{\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2 + \lambda^2}} \cdot \frac{1}{\sinh[\sigma]} = -b_{0,1}^{(23)}, \end{aligned}$$

$$b_{2m,\varepsilon}^{(31,12)} - b_{2m,\varepsilon}^{(11,32)} = -\frac{\pi^2 \kappa \sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2 + \lambda^2) \sinh[2md - \varepsilon\sigma]} [\varepsilon m - \varepsilon m + \mathbf{M}_4 - \mathbf{M}_3] = -b_{2m+1,\varepsilon}^{(23)}.$$

26.2. Ряды Фурье для проекций вектора угловой скорости и вектора углового ускорения на оси промежуточной системы координат, связанной с вектором кинетического момента.

$$\omega_{G_1} = \omega_{G_1}^{(0)} \sum_{m=1}^{\infty} \left\{ \frac{\sin(\varphi_2)}{\cosh(\sigma)} + \frac{\sin(2m\varphi_1 + \varphi_2)}{\cosh(2md - \sigma)} - \frac{\sin(2m\varphi_1 - \varphi_2)}{\cosh(2md + \sigma)} \right\},$$

$$\omega_{G_2} = -\omega_{G_2}^{(0)} \sum_{m=1}^{\infty} \left\{ \frac{\cos(\varphi_2)}{\cosh(\sigma)} + \frac{\cos(2m\varphi_1 + \varphi_2)}{\cosh(2md - \sigma)} + \frac{\cos(2m\varphi_1 - \varphi_2)}{\cosh(2md + \sigma)} \right\},$$

$$\omega_{G_3} = \omega_{G_3}^{(0)} = G \frac{A-B}{AB} \frac{1+\kappa^2}{\kappa^2 + \lambda^2}, \quad \omega_{G_1}^{(0)} = G \frac{\pi(A-B)}{2ABK} \frac{\sqrt{1+\kappa^2}}{\kappa\sqrt{\kappa^2 + \lambda^2}}.$$

$$\varepsilon_{G_1} = \omega_{G_1}^{(0)} \sum_{m=1}^{\infty} \left\{ \frac{n_2}{\cosh(\sigma)} \cos(\varphi_2) + \frac{2mn_1 + n_2}{\cosh(2md - \sigma)} \cos(2m\varphi_1 + \varphi_2) - \frac{2mn_1 - n_2}{\cosh(2md + \sigma)} \cos(2m\varphi_1 - \varphi_2) \right\},$$

$$\varepsilon_{G_2} = \omega_{G_2}^{(0)} \sum_{m=1}^{\infty} \left\{ \frac{n_2}{\cosh(\sigma)} \sin(\varphi_2) + \frac{2mn_1 + n_2}{\cosh(2md - \sigma)} \sin(2m\varphi_1 + \varphi_2) + \frac{2mn_1 - n_2}{\cosh(2md + \sigma)} \sin(2m\varphi_1 - \varphi_2) \right\},$$

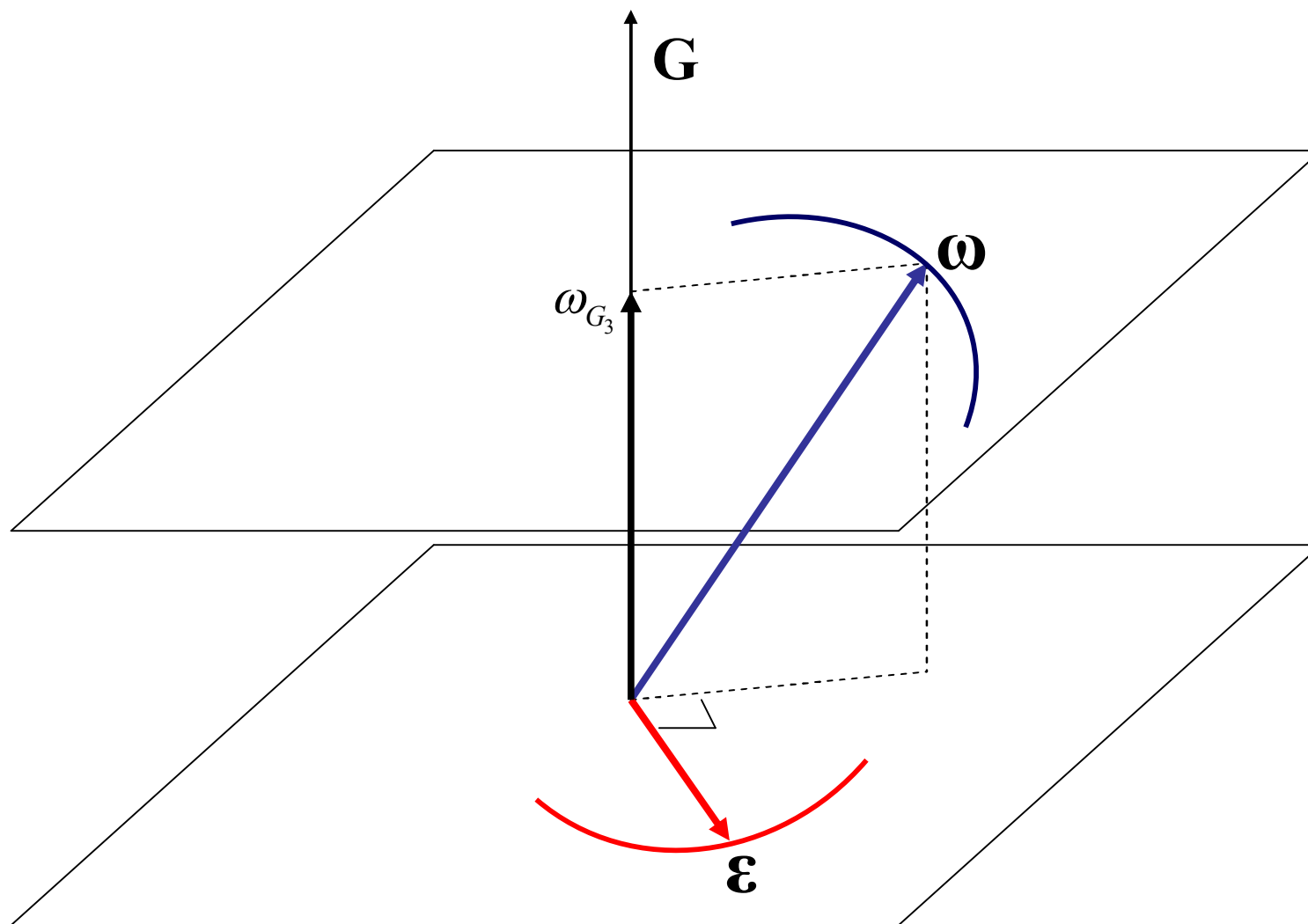
$$\varepsilon_{G_3} = 0,$$

где

$$\omega_{G_3} = \omega_{G_3}^{(0)} = G \frac{A-B}{AB} \frac{1+\kappa^2}{\kappa^2 + \lambda^2}, \quad \omega_{G_1}^{(0)} = G \frac{\pi(A-B)}{2ABK} \frac{\sqrt{1+\kappa^2}}{\kappa\sqrt{\kappa^2 + \lambda^2}},$$

$$\kappa^2 = \frac{C(A-B)}{A(B-C)}, \quad n_1 = \frac{I_2(A-C)}{2AC} \frac{\pi\kappa}{\sqrt{(1+\kappa^2)(\kappa^2 + \lambda^2)}K(\lambda)}.$$

1. Ряды Фурье функций : $e^{-2i\omega}$, $e^{2i\omega}$, $\cos 2\omega$, $\sin 2\omega$, $\cos^2 \omega$, $\sin^2 \omega$, $\cos 2g$, $\sin 2g$, $\cos^2 g$, $\sin^2 g$.
2. Ряды Фурье функций: $e^{-4i\omega}$, $e^{4i\omega}$, $\cos 4\omega$, $\sin 4\omega$, $\cos^2 2\omega$, $\sin^2 2\omega$, $\cos 4g$, $\sin 4g$, $\cos^2 2g$, $\sin^2 2g$.
3. Ряды Фурье функций: $\cos \theta$, $\sin \theta$, $\cos^2 \frac{\theta}{2}$, $\sin^2 \frac{\theta}{2}$.
4. О разложениях скоростей $\dot{l}$, $\dot{L}$, $\dot{g}$, $\dot{\theta}$ и правых частей уравнений Андуайе для задачи Эйлера-Пуансо.
5. Ряды Фурье функций: $\cos^2 l$, $\sin^2 l$, $\cos 2l$, $\sin 2l$.
6. Ряды Фурье функций: $\cos^2 \theta$, $\sin^2 \theta$, $\cos 2\theta$, $\sin 2\theta$.
7. Ряды Фурье функций: $\cos \theta \cos^2 l$, $\cos \theta \sin^2 l$, $\cos \theta \cos 2l$, $\cos \theta \sin 2l$.
8. Ряды Фурье функций для переменных: g , $\dot{g}$, $\ddot{g}$.
9. Ряды Фурье функций для переменных: l , $\dot{l}$, $\ddot{l}$.
10. Ряды Фурье функции $\sin \theta \sin l \cos l$ и переменных θ , $\dot{\theta}$, $\ddot{\theta}$.
11. Решение задачи Эйлера-Пуансо в рядах Фурье для переменных Андуайе.



Гессиан задачи:

$$\Delta = \begin{vmatrix} \frac{\partial^2 F_0}{\partial I_1^2} & \frac{\partial^2 F_0}{\partial I_1 \partial I_2} \\ \frac{\partial^2 F_0}{\partial I_2 \partial I_1} & \frac{\partial^2 F_0}{\partial I_2^2} \end{vmatrix} = \begin{vmatrix} \frac{\partial n_1}{\partial I_1} & \frac{\partial n_1}{\partial I_2} \\ \frac{\partial n_2}{\partial I_1} & \frac{\partial n_2}{\partial I_2} \end{vmatrix}$$

$$F_0 = \frac{I_2^2}{2A} \left[1 + \frac{A-C}{C} \frac{\kappa^2}{\lambda^2 + \kappa^2} \right],$$

$$n_1 = -\frac{1}{G\mathbf{J}(\lambda)} \frac{\partial F_0}{\partial \lambda}, \quad n_2 = \frac{\partial F_0}{\partial G} + \frac{\Lambda(\lambda)}{G\mathbf{J}(\lambda)} \frac{\partial F_0}{\partial \lambda}.$$

$$\Delta = \frac{\pi^2 (A-C)}{4AC^2 (1+\kappa^2) \lambda^2 \lambda'^2 \mathbf{K}^3} \left\{ \lambda^2 \left\{ \kappa^2 \mathbf{K} + [1 + (C-A)/A] \mathbf{E} \right\} + \kappa^2 (\mathbf{E} - \mathbf{K}) \right\} > 0$$

16.8. Соотношения между углами θ, γ и θ_ω в задаче Эйлера-Пуансо для осесимметричного тела. Ряды невозмущенного движения сфероида. $C > A$

$$\tan \theta_\omega = \sqrt{\frac{1+e}{1-e}} \tan \theta \quad 0 < e = \frac{C^2 - A^2}{A^2 + C^2} \quad \sigma = \frac{\sqrt{1+e} - \sqrt{1-e}}{\sqrt{1+e} + \sqrt{1-e}} = \frac{C-A}{C+A}$$

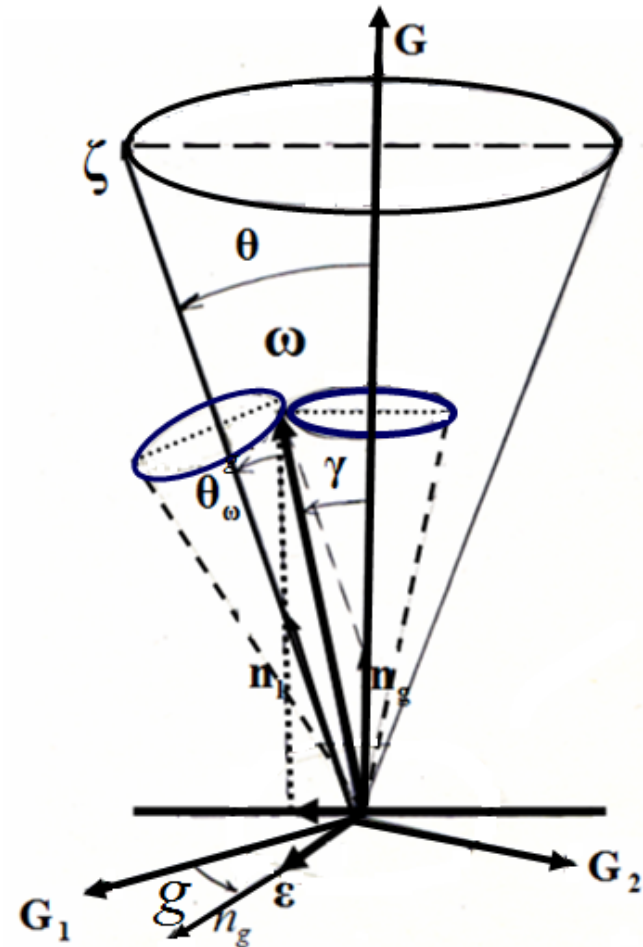
$$\theta_\omega - \theta = 2 \sum_{m=1}^{\infty} \frac{\sigma^m \sin m\theta}{m},$$

$$\theta - \theta_\omega = 2 \sum_{m=1}^{\infty} \frac{(-1)^m \sigma^m \sin m\theta}{m},$$

$$\cos \theta_\omega = -\sigma + (1 - \sigma^2) \sum_{m=1}^{\infty} \sigma^{m-1} \cos m\theta,$$

$$\sin \theta_\omega = (1 - \sigma^2) \sum_{m=1}^{\infty} \sigma^{m-1} \sin m\theta,$$

$$\cos \theta = \sigma + (1 - \sigma^2) \sum_{m=1}^{\infty} (-1)^{m-1} \sigma^{m-1} \cos m\theta_\omega.$$



30.1. Уравнения возмущенного вращательного движения деформируемого тела.

$$H = H_0 + H_1$$

$$\begin{aligned}\frac{d\varphi_1}{dt} &= \frac{\partial H}{\partial I_1}, & \frac{dI_1}{dt} &= -\frac{\partial H}{\partial \varphi_1}, \\ \frac{d\varphi_2}{dt} &= \frac{\partial H}{\partial I_2}, & \frac{dI_2}{dt} &= -\frac{\partial H}{\partial \varphi_2}, \\ \frac{d\varphi_3}{dt} &= \frac{\partial H}{\partial I_3}, & \frac{dI_3}{dt} &= -\frac{\partial H}{\partial \varphi_3},\end{aligned}$$

$$H_0 = \frac{I_2^2}{2A} \left[1 + \frac{A-C}{C} \frac{\kappa^2}{\lambda^2 + \kappa^2} \right] = H_0(I_1, I_2)$$

$$H_1 = H - H_0 = H_1(I_i, \varphi_i, t)$$

Уравнения возмущенного вращательного движения в переменных:

$$G = I_2, \quad \lambda = \lambda(I_1 / I_2), \quad \rho = \rho(I_3 / I_2), \quad \varphi = \varphi_1, \quad \psi = \varphi_2, \quad h = \varphi_3.$$

$$\dot{G} = \frac{\partial W}{\partial \psi},$$

$$\dot{\lambda} = -\frac{1}{G\mathbf{J}(\lambda)} \frac{\partial W}{\partial \varphi} + \frac{\Lambda(\lambda)}{G\mathbf{J}(\lambda)} \frac{\partial W}{\partial \psi},$$

$$\dot{\rho} = \frac{1}{G} \operatorname{ctg} \rho \frac{\partial W}{\partial \psi} - \frac{1}{G} \operatorname{cosec} \rho \frac{\partial W}{\partial h},$$

$$\dot{\varphi} = \Omega(G, \lambda) + \frac{1}{G\mathbf{J}(\lambda)} \frac{\partial W}{\partial \lambda},$$

$$\dot{\psi} = \omega(G, \lambda) - \frac{\Lambda(\lambda)}{G\mathbf{J}(\lambda)} \frac{\partial W}{\partial \lambda} - \frac{1}{G} \operatorname{ctg} \rho \frac{\partial W}{\partial \rho},$$

$$\dot{h} = \frac{1}{G} \operatorname{cosec} \rho \frac{\partial W}{\partial \rho},$$

$$W = -H_1 = H_0 - H = W(\lambda, \rho, \varphi, \psi, h, t)$$

$$\Omega(G, \lambda) = \frac{\pi G}{2\mathbf{K}(\lambda)} \frac{A-C}{AC} \frac{\kappa}{\sqrt{(1+\kappa^2)(\lambda^2 + \kappa^2)}} \quad \omega(G, \lambda) = \frac{G}{C} \left(1 - \frac{A-C}{A} \frac{\Pi(\kappa^2, \lambda)}{\mathbf{K}(\lambda)} \right)$$

$$\Lambda(\lambda) = \frac{2\sqrt{1+\kappa^2}}{\pi\kappa\sqrt{\kappa^2 + \lambda^2}} \left\{ (\kappa^2 + \lambda^2) \Pi\left(\frac{\pi}{2}, \kappa^2, \lambda\right) - \lambda^2 \mathbf{K}(\lambda) \right\} \quad \mathbf{J}(\lambda) = \frac{2\kappa\lambda\sqrt{1+\kappa^2}}{\pi(\kappa^2 + \lambda^2)^{3/2}} \mathbf{K}(\lambda)$$

Уравнения возмущенного вращательного движения в неинерциальной системе отсчета

$$\dot{G} = \frac{\partial U}{\partial \psi},$$

$$\dot{\lambda} = -\frac{1}{G\mathbf{J}(\lambda)} \frac{\partial U}{\partial \varphi} + \frac{\Lambda(\lambda)}{G\mathbf{J}(\lambda)} \frac{\partial U}{\partial \psi},$$

$$\dot{\rho} = \sin \pi_1 \sin(h - \Pi_1) \frac{d\Pi_1}{dt} + \cos(h - \Pi_1) \frac{d\pi_1}{dt} + \frac{1}{G} \operatorname{ctg} \rho \frac{\partial U}{\partial \psi} - \frac{1}{G} \operatorname{cosec} \rho \frac{\partial U}{\partial h},$$

$$\dot{\phi} = \Omega(G, \lambda) + \frac{1}{G\mathbf{J}(\lambda)} \frac{\partial U}{\partial \lambda},$$

$$\dot{\psi} = \omega(G, \lambda) + \operatorname{cosec} \rho \sin \pi_1 \cos(h - \Pi_1) \frac{d\Pi_1}{dt} - \sin(h - \Pi_1) \frac{d\pi_1}{dt} - \frac{\Lambda(\lambda)}{G\mathbf{J}(\lambda)} \frac{\partial U}{\partial \lambda} - \frac{1}{G} \operatorname{ctg} \rho \frac{\partial U}{\partial \rho},$$

$$\dot{h} = (1 - \cos \pi_1) \frac{d\Pi_1}{dt} - \cot \rho \left[\sin \pi_1 \cos(h - \Pi_1) \frac{d\Pi_1}{dt} - \frac{d\pi_1}{dt} \sin(h - \Pi_1) \right] + \frac{1}{G} \operatorname{csc} \rho \frac{\partial U}{\partial \rho},$$

$$U = U(\lambda, \rho, \varphi, \psi, h, t)$$

30.2. Возмущения первого порядка во вращательном движении слабodeформируемой планеты в переменных действие-угол.

Невозмущенное эйлеровское (чандлеровско-эйлеровское) движение.

$$H_1 = 0, R_E = 0 \text{ и } U = 0$$

$$G = G_0, \rho = \rho_0, \lambda = \lambda_0,$$

$$\varphi = \Omega(G_0, \lambda_0)t + \varphi_0, \psi = \omega(G_0, \lambda_0)t + \psi_0, h = h_0$$

$G_0, \rho_0, \lambda_0, \varphi_0, \psi_0, h_0$ - начальные условия задачи

К вычислению возмущений первого порядка

$$G_1 = \int \frac{\partial U}{\partial \psi} dt,$$

$$\rho_1 = \frac{1}{G} \cot \rho \int \frac{\partial U}{\partial \psi} dt - \frac{1}{G} \csc \rho \int \frac{\partial U}{\partial h} dt,$$

$$\lambda_1 = -\frac{1}{G\mathbf{J}(\lambda)} \int \frac{\partial U}{\partial \varphi} dt + \frac{\Lambda(\lambda)}{G\mathbf{J}(\lambda)} \int \frac{\partial U}{\partial \psi} dt,$$

$$h_1 = \frac{1}{G} \csc \rho \int \frac{\partial U}{\partial \rho} dt,$$

$$\varphi_1 = \frac{\partial \Omega(G, \lambda)}{\partial G} \int G_1 dt + \frac{\partial \Omega(G, \lambda)}{\partial \lambda} \int \lambda_1 dt + \frac{1}{G\mathbf{J}(\lambda)} \int \frac{\partial U}{\partial \lambda} dt,$$

$$\psi_1 = \frac{\partial \omega(G, \lambda)}{\partial G} \int G_1 dt + \frac{\partial \omega(G, \lambda)}{\partial \lambda} \int \lambda_1 dt - \frac{\Lambda(\lambda)}{G\mathbf{J}(\lambda)} \int \frac{\partial U}{\partial \lambda} dt - \frac{1}{G} \operatorname{ctg} \rho \int \frac{\partial U}{\partial \rho} dt$$

$$\Omega(G, \lambda) = \frac{\pi G}{2\mathbf{K}(\lambda)} \frac{A-C}{AC} \frac{\kappa}{\sqrt{(1+\kappa^2)(\lambda^2 + \kappa^2)}}, \quad \omega(G, \lambda) = \frac{G}{C} \left(1 - \frac{A-C}{A} \frac{\mathbf{\Pi}(\kappa^2, \lambda)}{\mathbf{K}(\lambda)} \right).$$

$$U = \sum_{\mathbf{v}; k_1, k_2} U_{\mathbf{v}; k_1, k_2}(\lambda, \rho) \cos(\Theta_{\mathbf{v}} + k_1 \varphi_1 + k_2 \varphi_2)$$

30.2.1. Возмущения модуля углового момента G .

$$\begin{aligned} \delta G = & \sum_{\mathbf{v}} \sum_{\mu=\pm 1} G_{\mathbf{v};0,\mu} \cos(\Theta_{\mathbf{v}} + \mu\varphi_2) + \sum_{\mathbf{v}} \sum_{\mu=\pm 1} G_{\mathbf{v};0,2\mu} \cos(\Theta_{\mathbf{v}} + 2\mu\varphi_2) + \\ & + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} G_{\mathbf{v};2\varepsilon m,\mu} \cos(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + \mu\varphi_2) + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} G_{\mathbf{v};2\varepsilon m,2\mu} \cos(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + 2\mu\varphi_2) \end{aligned}$$

$$G_{\mathbf{v};0,\mu} = P \frac{\mu f_{0,1}(\lambda) R_{\mathbf{v};\mu}(\rho)}{\Omega_{\mathbf{v}} + \mu\omega},$$

$$G_{\mathbf{v};0,2\mu} = 2P \frac{\mu b_{0,2}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\mu\omega},$$

$$G_{\mathbf{v};2\varepsilon m,\mu} = P \frac{\mu f_{2m,\varepsilon\mu}(\lambda) R_{\mathbf{v};\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega},$$

$$G_{\mathbf{v};2\varepsilon m,2\mu} = 2P \frac{\mu b_{2m,2\varepsilon\mu}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega}.$$

30.2.2. Возмущения эксцентриситета фазовой траектории λ .

$$\begin{aligned} \delta\lambda = & \sum_{\mathbf{v}} \sum_{\mu=\pm 1} \lambda_{\mathbf{v};0,\mu} \cos(\Theta_{\mathbf{v}} + \mu\varphi_2) + \sum_{\mathbf{v}} \sum_{\mu=\pm 1} \lambda_{\mathbf{v};0,2\mu} \cos(\Theta_{\mathbf{v}} + 2\mu\varphi_2) + \\ & + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} \lambda_{\mathbf{v};2\varepsilon m,\mu} \cos(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + \mu\varphi_2) + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} \lambda_{\mathbf{v};2\varepsilon m,2\mu} \cos(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + 2\mu\varphi_2) \end{aligned}$$

$$\lambda_{\mathbf{v};0,\mu} = \frac{P\Lambda(\lambda)}{G\mathbf{J}(\lambda)} \cdot \frac{\mu f_{0,1}(\lambda) R_{\mathbf{v};\mu}(\rho)}{\Omega_{\mathbf{v}} + \mu\omega},$$

$$\lambda_{\mathbf{v};0,2\mu} = 2 \frac{P\Lambda(\lambda)}{G\mathbf{J}(\lambda)} \cdot \frac{\mu b_{0,2}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\mu\omega},$$

$$\lambda_{\mathbf{v};2\varepsilon m,\mu} = \frac{P}{G\mathbf{J}(\lambda)} \cdot \frac{\mu [\Lambda(\lambda) - 2\varepsilon m] f_{2m,\varepsilon\mu}(\lambda) R_{\mathbf{v};\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega},$$

$$\lambda_{\mathbf{v};2\varepsilon m,2\mu} = \frac{2P}{G\mathbf{J}(\lambda)} \cdot \frac{\mu [\Lambda(\lambda) - \varepsilon m] b_{2m,2\varepsilon\mu}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega}.$$

30.2.3. Возмущения наклонности вектора кинетического момента ρ .

$$\begin{aligned} \delta\rho = & \sum_{\mathbf{v}} \rho_{\mathbf{v};0,0} \cos(\Theta_{\mathbf{v}}) + \sum_{\mathbf{v}} \sum_{\mu=\pm 1} \rho_{\mathbf{v};0,\mu} \cos(\Theta_{\mathbf{v}} + \mu\varphi_2) + \sum_{\mathbf{v}} \sum_{\mu=\pm 1} \rho_{\mathbf{v};0,2\mu} \cos(\Theta_{\mathbf{v}} + 2\mu\varphi_2) + \\ & + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} \rho_{\mathbf{v};2\varepsilon m,\mu} \cos(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + \mu\varphi_2) + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} \rho_{\mathbf{v};2\varepsilon m,2\mu} \cos(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + 2\mu\varphi_2) \end{aligned}$$

$$\rho_{\mathbf{v};0,0} = 2 \frac{P}{G} \csc \rho \frac{v_5 (b_{0,0} R_{\mathbf{v}}^{(22)} + c_{0,0} R_{\mathbf{v}}^{(33)})}{\Omega_{\mathbf{v}}},$$

$$\rho_{\mathbf{v};0,\mu} = \frac{P}{G} \csc \rho \frac{(\mu \cos \rho + v_5) f_{0,1}(\lambda) R_{\mathbf{v};\mu}(\rho)}{\Omega_{\mathbf{v}} + \mu\omega},$$

$$\rho_{\mathbf{v};0,2\mu} = \frac{P}{G} \csc \rho \frac{(2\mu \cos \rho + v_5) b_{0,2}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\mu\omega},$$

$$\rho_{\mathbf{v};2\varepsilon m,\mu} = \frac{P}{G} \csc \rho \frac{(\mu \cos \rho + v_5) f_{2m,\varepsilon\mu}(\lambda) R_{\mathbf{v};\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega}$$

$$\rho_{\mathbf{v};2\varepsilon m,2\mu} = \frac{P}{G} \csc \rho \frac{(2\mu \cos \rho + v_5) b_{2m,2\varepsilon\mu}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega}.$$

30.2.4. Возмущения долготы восходящего узла плоскости Андуайе ортогональной вектору кинетического момента

$$\begin{aligned} \delta h = & \sum_{\mathbf{v}} h_{\mathbf{v};0,0} \sin(\Theta_{\mathbf{v}}) + \sum_{\mathbf{v}} \sum_{\mu=\pm 1} h_{\mathbf{v};0,\mu} \sin(\Theta_{\mathbf{v}} + \mu\varphi_2) + \sum_{\mathbf{v}} \sum_{\mu=\pm 1} h_{\mathbf{v};0,2\mu} \sin(\Theta_{\mathbf{v}} + 2\mu\varphi_2) + \\ & + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} h_{\mathbf{v};2\varepsilon m,\mu} \sin(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + \mu\varphi_2) + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} h_{\mathbf{v};2\varepsilon m,2\mu} \sin(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + 2\mu\varphi_2) \end{aligned}$$

$$h_{\mathbf{v};0,0} = \frac{P}{G} \csc \rho \frac{(1 + \delta - 3c_{0,0})}{\Omega_{\mathbf{v}}} \cdot \frac{\partial R_{\mathbf{v}}^{(22)}}{\partial \rho},$$

$$h_{\mathbf{v};0,\mu} = \frac{P}{G} \csc \rho \frac{f_{0,1}(\lambda)}{\Omega_{\mathbf{v}} + \mu\omega} \cdot \frac{\partial R_{\mathbf{v};\mu}(\rho)}{\partial \rho},$$

$$h_{\mathbf{v};0,2\mu} = \frac{P}{G} \csc \rho \frac{b_{0,2}(\lambda)}{\Omega_{\mathbf{v}} + \mu\omega} \cdot \frac{\partial R_{\mathbf{v};2\mu}(\rho)}{\partial \rho},$$

$$h_{\mathbf{v};2\varepsilon m,\mu} = \frac{P}{G} \csc \rho \frac{f_{2m,\varepsilon\mu}(\lambda)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega} \cdot \frac{\partial R_{\mathbf{v};\mu}(\rho)}{\partial \rho},$$

$$h_{\mathbf{v};2\varepsilon m,2\mu} = \frac{P}{G} \csc \rho \frac{b_{2m,2\varepsilon\mu}(\lambda)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega} \cdot \frac{\partial R_{\mathbf{v};2\mu}(\rho)}{\partial \rho}.$$

30.2.5. Возмущения переменной φ_1 .

$$\begin{aligned} \delta\varphi_1 = & \sum_{\mathbf{v}} \varphi_{\mathbf{v};0,0} \sin(\Theta_{\mathbf{v}}) + \sum_{\mathbf{v}} \sum_{\mu=\pm 1} \varphi_{\mathbf{v};0,\mu} \sin(\Theta_{\mathbf{v}} + \mu\varphi_2) + \sum_{\mathbf{v}} \sum_{\mu=\pm 1} \varphi_{\mathbf{v};0,2\mu} \sin(\Theta_{\mathbf{v}} + 2\mu\varphi_2) + \\ & + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} \varphi_{\mathbf{v};2\varepsilon m,\mu} \sin(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + \mu\varphi_2) + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} \varphi_{\mathbf{v};2\varepsilon m,2\mu} \sin(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + 2\mu\varphi_2) \end{aligned}$$

$$\varphi_{\mathbf{v};0,0} = \frac{2P}{G\mathbf{J}(\lambda)} \cdot \frac{\partial b_{0,0}}{\partial \lambda} \cdot \frac{R_{\mathbf{v};0}(\rho)}{\Omega_{\mathbf{v}}},$$

$$\varphi_{\mathbf{v};0,\mu} = \frac{P}{G\mathbf{J}(\lambda)} \left\{ (\Omega_{\mathbf{v}} + \mu\omega) \frac{\partial f_{0,1}(\lambda)}{\partial \lambda} + \mu [\Omega \mathbf{J}(\lambda) + \Omega_{\lambda} \Lambda(\lambda)] f_{0,1}(\lambda) \right\} \times \frac{R_{\mathbf{v};\mu}(\rho)}{(\Omega_{\mathbf{v}} + \mu\omega)^2},$$

$$\varphi_{\mathbf{v};0,2\mu} = \frac{P}{G\mathbf{J}(\lambda)} \left\{ (\Omega_{\mathbf{v}} + 2\mu\omega) \frac{\partial b_{0,2}(\lambda)}{\partial \lambda} + 2\mu [\Omega \mathbf{J}(\lambda) + \Omega_{\lambda} \Lambda(\lambda)] b_{0,2}(\lambda) \right\} \times \frac{R_{\mathbf{v};2\mu}(\rho)}{(\Omega_{\mathbf{v}} + 2\mu\omega)^2},$$

$$\varphi_{\mathbf{v};2\varepsilon m,\mu} = \frac{P}{G\mathbf{J}(\lambda)} \left\{ (\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega) \frac{\partial f_{2m,\varepsilon\mu}(\lambda)}{\partial \lambda} + \mu (\Omega \mathbf{J}(\lambda) + \Omega_{\lambda} [\Lambda(\lambda) - 2\varepsilon m]) f_{2m,\varepsilon\mu}(\lambda) \right\} \times \frac{R_{\mathbf{v};\mu}(\rho)}{(\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega)^2},$$

$$\varphi_{\mathbf{v};2\varepsilon m,2\mu} = \frac{P}{G\mathbf{J}(\lambda)} \left\{ (\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega) \frac{\partial b_{2m,2\varepsilon\mu}(\lambda)}{\partial \lambda} + 2\mu (\Omega \mathbf{J}(\lambda) + \Omega_{\lambda} [\Lambda(\lambda) - \varepsilon m]) b_{2m,2\varepsilon\mu}(\lambda) \right\} \times \frac{R_{\mathbf{v};2\mu}(\rho)}{(\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega)^2}.$$

30.2.6. Возмущения переменной φ_2 .

$$\begin{aligned} \delta\varphi_2 = & \sum_{\mathbf{v}} \psi_{\mathbf{v};0,0} \sin(\Theta_{\mathbf{v}}) + \sum_{\mathbf{v}} \sum_{\mu=\pm 1} \psi_{\mathbf{v};0,\mu} \sin(\Theta_{\mathbf{v}} + \mu\varphi_2) + \sum_{\mathbf{v}} \sum_{\mu=\pm 1} \psi_{\mathbf{v};0,2\mu} \sin(\Theta_{\mathbf{v}} + 2\mu\varphi_2) + \\ & + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} \psi_{\mathbf{v};2\varepsilon m,\mu} \sin(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + \mu\varphi_2) + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} \psi_{\mathbf{v};2\varepsilon m,2\mu} \sin(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + 2\mu\varphi_2) \end{aligned}$$

$$\begin{aligned} \psi_{\mathbf{v};0,0} = & -\frac{2P}{G\mathbf{J}(\lambda)} \left\{ \Lambda(\lambda) \frac{\partial b_{0,0}}{\partial \lambda} \cdot \frac{R_{\mathbf{v};0}(\rho)}{\Omega_{\mathbf{v}}} + \cot \rho \frac{\mathbf{J}(\lambda)(1+\delta-3c_{0,0})}{2\Omega_{\mathbf{v}}} \frac{\partial R_{\mathbf{v}}^{(22)}}{\partial \rho} \right\} \\ \psi_{\mathbf{v};0,\mu} = & \frac{P}{G\mathbf{J}(\lambda)} \left[\left\{ -\Lambda(\lambda)(\Omega_{\mathbf{v}} + \mu\omega) \frac{\partial f_{0,1}(\lambda)}{\partial \lambda} + \mu [\omega \mathbf{J}(\lambda) + \omega_{\lambda} \Lambda(\lambda)] f_{0,1}(\lambda) \right\} \times \frac{R_{\mathbf{v};\mu}(\rho)}{(\Omega_{\mathbf{v}} + \mu\omega)^2} - \cot \rho \frac{\mathbf{J}(\lambda) f_{0,1}(\lambda)}{\Omega_{\mathbf{v}} + \mu\omega} \cdot \frac{\partial R_{\mathbf{v};\mu}(\rho)}{\partial \rho} \right] \\ \psi_{\mathbf{v};0,2\mu} = & \frac{P}{G\mathbf{J}(\lambda)} \left[\left\{ -\Lambda(\lambda)(\Omega_{\mathbf{v}} + 2\mu\omega) \frac{\partial b_{0,2}(\lambda)}{\partial \lambda} + 2\mu [\omega \mathbf{J}(\lambda) + \omega_{\lambda} \Lambda(\lambda)] b_{0,2}(\lambda) \right\} \times \frac{R_{\mathbf{v};2\mu}(\rho)}{(\Omega_{\mathbf{v}} + 2\mu\omega)^2} - \cot \rho \frac{\mathbf{J}(\lambda) b_{0,2}(\lambda)}{\Omega_{\mathbf{v}} + \mu\omega} \cdot \frac{\partial R_{\mathbf{v};2\mu}(\rho)}{\partial \rho} \right] \\ \psi_{\mathbf{v};2\varepsilon m,\mu} = & \frac{P}{G\mathbf{J}(\lambda)} \left[\left\{ -\Lambda(\lambda)(\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega) \frac{\partial f_{2m,\varepsilon\mu}(\lambda)}{\partial \lambda} + \mu (\omega \mathbf{J}(\lambda) + \omega_{\lambda} [\Lambda(\lambda) - 2\varepsilon m]) f_{2m,\varepsilon\mu}(\lambda) \right\} \times \right. \\ & \times \frac{R_{\mathbf{v};\mu}(\rho)}{(\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega)^2} - \cot \rho \frac{\mathbf{J}(\lambda) f_{2m,\varepsilon\mu}(\lambda)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega} \cdot \frac{\partial R_{\mathbf{v};\mu}(\rho)}{\partial \rho} \left. \right] \\ \psi_{\mathbf{v};2\varepsilon m,2\mu} = & \frac{P}{G\mathbf{J}(\lambda)} \times \left[\left\{ -\Lambda(\lambda)(\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega) \frac{\partial b_{2m,2\varepsilon\mu}(\lambda)}{\partial \lambda} + 2\mu (\omega \mathbf{J}(\lambda) + \omega_{\lambda} [\Lambda(\lambda) - \varepsilon m]) b_{2m,2\varepsilon\mu}(\lambda) \right\} \times \right. \\ & \times \frac{R_{\mathbf{v};2\mu}(\rho)}{(\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega)^2} - \cot \rho \frac{\mathbf{J}(\lambda) b_{2m,2\varepsilon\mu}(\lambda)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega} \cdot \frac{\partial R_{\mathbf{v};2\mu}(\rho)}{\partial \rho} \left. \right] \end{aligned}$$

$$\sigma = \frac{\pi}{2\mathbf{K}} F(\arctan \frac{\kappa}{\lambda}, \lambda'), \quad d = \frac{\pi \mathbf{K}'}{2\mathbf{K}}$$

Вспомогательные
формулы

$$\frac{\partial \sigma}{\partial \lambda} = -\frac{\pi^2}{4\lambda\lambda'^2\mathbf{K}^2} \Lambda_0(n, \lambda), \quad n = \arctan \frac{\kappa}{\lambda},$$

$$\Lambda_0(n, \lambda) = \frac{1}{\pi} \{ F(n, \lambda) [\mathbf{E}(\lambda) - \mathbf{K}(\lambda)] + \mathbf{K}(\lambda) E(n, \lambda) \},$$

$$\frac{\partial d}{\partial \lambda} = -\frac{\pi}{2\lambda\lambda'^2\mathbf{K}^2} [\mathbf{E}'\mathbf{K} + \mathbf{E}\mathbf{K}' - \mathbf{K}\mathbf{K}'] = -\frac{\pi}{2\lambda\lambda'\mathbf{K}^2}.$$

$$\Omega_\lambda = \frac{C-A}{AC} \frac{\kappa}{\sqrt{1+\kappa^2}} \frac{(\kappa^2 + \lambda^2)\mathbf{E} - \lambda'^2\kappa^2\mathbf{K}}{\mathbf{K}^2\lambda\lambda'^2(\kappa^2 + \lambda^2)^{3/2}},$$

$$\omega_\lambda = \frac{C-A}{AC} \frac{[\lambda^2\mathbf{K}\mathbf{E} - (\kappa^2 + \lambda^2)\mathbf{\Pi}\mathbf{E} + \lambda'^2\kappa^2\mathbf{\Pi}\mathbf{K}]}{\mathbf{K}^2\lambda\lambda'^2(\kappa^2 + \lambda^2)}$$

$$R_{v;2\mu}(\rho) = R_v^{(22)} + \mu R_v^{(12)} = -A_v^{(0)} \sin^2 \rho + 2 \sin \rho (\cos \rho - \mu) A_v^{(1)} - \frac{1}{2} A_v^{(2)} (1 - \mu \cos \rho)^2,$$

$$R_{v;\mu}(\rho) = R_v^{(23)} + \mu R_v^{(13)} = -A_v^{(0)} \sin 2\rho + 2 A_v^{(1)} (\cos 2\rho - \mu \cos \rho) + A_v^{(2)} \sin \rho (\cos \rho - \mu),$$

$$R_{v;0}(\rho) = R_v^{(22)} - 2R_v^{(33)} = -A_v^{(0)} (1 - 3 \cos^2 \rho) + 3 \sin 2\rho A_v^{(1)} + \frac{3}{2} A_v^{(2)} \sin^2 \rho.$$

$$\begin{aligned}
R_{\mathbf{v};2\mu}(\rho) &= R_{\mathbf{v}}^{(22)} + \mu R_{\mathbf{v}}^{(12)} = -A_{\mathbf{v}}^{(0)} \sin^2 \rho + 2 \sin \rho (\cos \rho - \mu) A_{\mathbf{v}}^{(1)} - \frac{1}{2} A_{\mathbf{v}}^{(2)} (1 - \mu \cos \rho)^2, \\
R_{\mathbf{v};\mu}(\rho) &= R_{\mathbf{v}}^{(23)} + \mu R_{\mathbf{v}}^{(13)} = -A_{\mathbf{v}}^{(0)} \sin 2\rho + 2 A_{\mathbf{v}}^{(1)} (\cos 2\rho - \mu \cos \rho) + A_{\mathbf{v}}^{(2)} \sin \rho (\cos \rho - \mu), \\
R_{\mathbf{v};0}(\rho) &= R_{\mathbf{v}}^{(22)} - 2 R_{\mathbf{v}}^{(33)} = -A_{\mathbf{v}}^{(0)} (1 - 3 \cos^2 \rho) + 3 \sin 2\rho A_{\mathbf{v}}^{(1)} + \frac{3}{2} A_{\mathbf{v}}^{(2)} \sin^2 \rho.
\end{aligned}$$

$$\begin{aligned}
\frac{\partial R_{\mathbf{v};2\mu}(\rho)}{\partial \rho} &= -A_{\mathbf{v}}^{(0)} \sin 2\rho + 2(\cos 2\rho - \mu \cos \rho) A_{\mathbf{v}}^{(1)} - A_{\mathbf{v}}^{(2)} (1 - \mu \cos \rho) \mu \sin \rho, \\
\frac{\partial R_{\mathbf{v};\mu}(\rho)}{\partial \rho} &= -2 A_{\mathbf{v}}^{(0)} \cos 2\rho + 2 A_{\mathbf{v}}^{(1)} (\mu \sin \rho - \sin 2\rho) + A_{\mathbf{v}}^{(2)} (\cos 2\rho - \mu \cos \rho), \\
\frac{\partial R_{\mathbf{v};0}(\rho)}{\partial \rho} &= 6 \cos 2\rho A_{\mathbf{v}}^{(1)} + \left[-3 A_{\mathbf{v}}^{(0)} + \frac{3}{2} A_{\mathbf{v}}^{(2)} \right] \sin 2\rho, \\
\frac{\partial R_{\mathbf{v}}^{(22)}}{\partial \rho} &= - \left(A_{\mathbf{v}}^{(0)} - \frac{1}{2} A_{\mathbf{v}}^{(2)} \right) \sin 2\rho + 2 A_{\mathbf{v}}^{(1)} \cos 2\rho.
\end{aligned}$$

$$b_{0,0} = \frac{1}{2}(1 + \delta) - \frac{\delta(1 + \kappa^2)}{2(\kappa^2 + \lambda^2)} + \frac{D\mathbf{E}}{2\mathbf{K}(\kappa^2 + \lambda^2)},$$

$$c_{0,0} = \frac{\delta(1 + \kappa^2)}{\kappa^2 + \lambda^2} - \frac{D\mathbf{E}}{\mathbf{K}(\kappa^2 + \lambda^2)}.$$

$$\frac{\partial b_{0,0}}{\partial \lambda} = \frac{b_1 \mathbf{K}^2 + b_2 \mathbf{E}\mathbf{K} + b_3 \mathbf{E}^2}{2(\kappa^2 + \lambda^2)^2 \lambda \lambda'^2 \mathbf{K}^2},$$

$$\frac{\partial c_{0,0}}{\partial \lambda} = -2 \frac{\partial b_{0,0}}{\partial \lambda} = -\frac{b_1 \mathbf{K}^2 + b_2 \mathbf{E}\mathbf{K} + b_3 \mathbf{E}^2}{(\kappa^2 + \lambda^2)^2 \lambda \lambda'^2 \mathbf{K}^2}.$$

$$b_1 = \lambda'^2 \left[2\delta(1 + \kappa^2)\lambda^2 - D(\kappa^2 + \lambda^2) \right], \quad b_2 = 2D\kappa^2 \lambda'^2,$$

$$b_3 = -D(\kappa^2 + \lambda^2), \quad D = \delta(1 + \kappa^2) - \kappa^2,$$

$$b_{0,1} = -\frac{\pi^2 \mathbf{M}}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{1}{\cosh(\sigma)},$$

$$b_{0,2} = \frac{\pi^2 \mathbf{M}}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{1}{\sinh(2\sigma)},$$

$$b_{2m,0} = \frac{\pi^2 D}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{m}{\sinh(2md)},$$

$$b_{2m,\varepsilon\mu} = -\varepsilon\mu \frac{\pi^2(mD + \varepsilon\mu\mathbf{M})}{2\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{1}{\cosh(2md - \varepsilon\mu\sigma)},$$

$$b_{2m,2\varepsilon\mu} = -\frac{\pi^2(mD + 2\varepsilon\mu\mathbf{M})}{4\mathbf{K}^2(\kappa^2 + \lambda^2)} \cdot \frac{1}{\sinh[2(md - \varepsilon\mu\sigma)]};$$

$$\sigma = \frac{\pi}{2\mathbf{K}} F\left(\arctan \frac{\kappa}{\lambda}, \lambda'\right), \quad d = \frac{\pi\mathbf{K}'}{2\mathbf{K}}$$

$$\frac{\partial f_{0,1}}{\partial \lambda} = \frac{A_{0,1}}{\cosh(\sigma)} + B_{0,1} \frac{\sinh(\sigma)}{\cosh^2(\sigma)},$$

$$\frac{\partial b_{0,2}}{\partial \lambda} = \frac{A_{0,2}}{\sinh(2\sigma)} + B_{0,2} \frac{\cosh(2\sigma)}{\sinh^2(2\sigma)},$$

$$\frac{\partial b_{2m,0}}{\partial \lambda} = \frac{A_{2m,0}}{\cosh(2md)} + B_{2m,0} \frac{\sinh(2md)}{\cosh^2(2md)},$$

$$\frac{\partial f_{2m,\varepsilon\mu}}{\partial \lambda} = \frac{A_{2m,\varepsilon\mu}}{\cosh(2md - \varepsilon\mu\sigma)} + B_{2m,\varepsilon\mu} \frac{\sinh(2md - \varepsilon\mu\sigma)}{\cosh^2(2md - \varepsilon\mu\sigma)},$$

$$\frac{\partial b_{2m,2\varepsilon\mu}}{\partial \lambda} = \frac{A_{2m,2\varepsilon\mu}}{\sinh[2(md - \varepsilon\mu\sigma)]} + B_{2m,2\varepsilon\mu} \frac{\cosh[2(md - \varepsilon\mu\sigma)]}{\sinh^2[2(md - \varepsilon\mu\sigma)]}.$$

$$A_{0,1} = -\frac{\pi\kappa\sqrt{1+\kappa^2}\left[\delta(\lambda^2-\kappa^2)\lambda'^2\mathbf{K}+(1-\delta)(\kappa^2+\lambda^2)\mathbf{E}+2D\lambda^2\lambda'^2\Pi\right]}{2\lambda\lambda'^2(\lambda^2-\kappa^2)\mathbf{K}^2(\kappa^2+\lambda^2)^{3/2}} + \frac{\pi^2\mathbf{M}\left[\kappa^2\lambda'^2\mathbf{K}+(\kappa^2+\lambda^2)\mathbf{E}\right]}{\lambda\lambda'^2(\kappa^2+\lambda^2)^2\mathbf{K}^3},$$

$$A_{0,2} = \frac{\pi\kappa\sqrt{1+\kappa^2}\left[\delta(\lambda^2-\kappa^2)\lambda'^2\mathbf{K}+(1-\delta)(\kappa^2+\lambda^2)\mathbf{E}+2D\lambda^2\lambda'^2\Pi\right]}{2\lambda\lambda'^2(\lambda^2-\kappa^2)\mathbf{K}^2(\kappa^2+\lambda^2)^{3/2}} - \frac{\pi^2\mathbf{M}\left[\kappa^2\lambda'^2\mathbf{K}+(\kappa^2+\lambda^2)\mathbf{E}\right]}{\lambda\lambda'^2(\kappa^2+\lambda^2)^2\mathbf{K}^3},$$

$$A_{2m,\varepsilon\mu} = -\frac{\pi\varepsilon\mu\kappa\sqrt{1+\kappa^2}\left[\delta(\lambda^2-\kappa^2)\lambda'^2\mathbf{K}+(1-\delta)(\kappa^2+\lambda^2)\mathbf{E}+2D\lambda^2\lambda'^2\Pi\right]}{2\lambda\lambda'^2(\lambda^2-\kappa^2)(\kappa^2+\lambda^2)^{3/2}\mathbf{K}^2} + \frac{\pi^2(\varepsilon\mu mD+\mathbf{M})\left[-\kappa^2\lambda'^2\mathbf{K}+(\kappa^2+\lambda^2)\mathbf{E}\right]}{\lambda\lambda'^2(\kappa^2+\lambda^2)^2\mathbf{K}^3},$$

$$A_{2m,2\varepsilon\mu} = -\frac{\pi\varepsilon\mu\kappa\sqrt{1+\kappa^2}\left[\delta(\lambda^2-\kappa^2)\lambda'^2\mathbf{K}+(1-\delta)(\kappa^2+\lambda^2)\mathbf{E}+2D\lambda^2\lambda'^2\Pi\right]}{2\lambda\lambda'^2(\lambda^2-\kappa^2)(\kappa^2+\lambda^2)^{3/2}\mathbf{K}^2} + \frac{\pi^2(mD+2\varepsilon\mu\mathbf{M})\left[-\kappa^2\lambda'^2\mathbf{K}+(\kappa^2+\lambda^2)\mathbf{E}\right]}{\lambda\lambda'^2(\kappa^2+\lambda^2)^2\mathbf{K}^3},$$

$$B_{0,1} = -\frac{\pi^4\mathbf{M}\Lambda_0(\kappa^2,\lambda)}{8\lambda\lambda'^2(\kappa^2+\lambda^2)\mathbf{K}^4}, \quad B_{0,2} = \frac{\pi^4\mathbf{M}\Lambda_0(\kappa^2,\lambda)}{4\lambda\lambda'^2(\kappa^2+\lambda^2)\mathbf{K}^4}, \quad B_{2m,0} = \frac{\pi^3 Dm^2}{2\lambda\lambda'^2(\kappa^2+\lambda^2)\mathbf{K}^4},$$

$$B_{2m,\varepsilon\mu} = \frac{\pi^3(\varepsilon\mu mD+\mathbf{M})\left[-4m+\varepsilon\mu\pi\Lambda_0(n,\lambda)\right]}{8\lambda\lambda'^2(\kappa^2+\lambda^2)\mathbf{K}^4}, \quad B_{2m,2\varepsilon\mu} = \frac{\pi^3(mD+2\varepsilon\mu\mathbf{M})\left[-2m+\varepsilon\mu\pi\Lambda_0(n,\lambda)\right]}{4\lambda\lambda'^2(\kappa^2+\lambda^2)\mathbf{K}^4},$$

30.3. Возмущенное вращательное движение небесного тела в случае малых значений параметра λ .

$$U = U_0 + \lambda U_1 + \lambda^2 U_2$$

$$U_0 = \frac{3}{4} n^2 (A - C) \sum_{\mathbf{v}} \left\{ R_{\mathbf{v}}(\rho, \delta, \kappa) \cos \Theta_{\mathbf{v}} + \right. \\ \left. + B_{2,2}^{(0)} \left[R_{\mathbf{v},2}(\rho) \cos(\Theta_{\mathbf{v}} + 2\varphi_1 + 2\varphi_2) + R_{\mathbf{v},-2}(\rho) \cos(\Theta_{\mathbf{v}} - 2\varphi_1 - 2\varphi_2) \right] \right\},$$

$$\delta G = P B_{2,2}^{(0)} \sum_{\mathbf{v}} \left[\frac{R_{\mathbf{v},2}(\rho)}{\Omega_{\mathbf{v}} + 2\Omega + 2\omega} \cos(\Theta_{\mathbf{v}} + 2\varphi_1 + 2\varphi_2) - \frac{R_{\mathbf{v},-2}(\rho)}{\Omega_{\mathbf{v}} - 2\Omega - 2\omega} \cos(\Theta_{\mathbf{v}} - 2\varphi_1 - 2\varphi_2) \right],$$

$$P = \frac{3}{2} n^2 (A - C), \quad B_{2,2}^{(0)} = \frac{1}{2} \delta \sqrt{1 + \kappa^2}.$$

$$\frac{(\delta G)_{CH}}{(\delta G)_{EU}} = \frac{\sqrt{1 + \kappa_{CH}^2}}{\sqrt{1 + \kappa_{EU}^2}} = 1.001348$$

30.3.2. Возмущения наклонности вектора кинетического момента ρ .

$$\dot{\rho} = \frac{1}{G} \operatorname{ctg} \rho \frac{\partial U_0}{\partial \varphi_2} - \frac{1}{G} \operatorname{cosec} \rho \frac{\partial U_0}{\partial h} \qquad \dot{\rho} = \frac{\lambda}{G} \operatorname{csc} \rho \left(\cos \rho \frac{\partial U_1}{\partial \varphi_2} - \frac{\partial U_1}{\partial h} \right)$$

$$\begin{aligned} \delta \rho = & 2 \frac{P}{G} \operatorname{csc} \rho \sum_{\mathbf{v}} \frac{\nu_5 (b_{0,0} R_{\mathbf{v}}^{(22)} + c_{0,0} R_{\mathbf{v}}^{(33)})}{\Omega_{\mathbf{v}}} \cos(\Theta_{\mathbf{v}}) + \\ & + \frac{P}{G} \operatorname{csc} \rho \sum_{\mathbf{v}} \sum_{\mu=\pm 1} \frac{(\mu \cos \rho + \nu_5) f_{0,1}(\lambda) R_{\mathbf{v};\mu}(\rho)}{\Omega_{\mathbf{v}} + \mu \omega} \cos(\Theta_{\mathbf{v}} + \mu \varphi_2) + \\ & + \frac{P}{G} \operatorname{csc} \rho \sum_{\mathbf{v}} \sum_{\mu=\pm 1} \frac{(2\mu \cos \rho + \nu_5) b_{0,2}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\mu \omega} \cos(\Theta_{\mathbf{v}} + 2\mu \varphi_2) + \\ & + \frac{P}{G} \operatorname{csc} \rho \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} \frac{(\mu \cos \rho + \nu_5) f_{2m,\varepsilon\mu}(\lambda) R_{\mathbf{v};\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m \Omega + \mu \omega} \cos(\Theta_{\mathbf{v}} + 2\varepsilon m \varphi_1 + \mu \varphi_2) + \\ & + \frac{P}{G} \operatorname{csc} \rho \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1; \\ \varepsilon=\pm 1}} \frac{(2\mu \cos \rho + \nu_5) b_{2m,2\varepsilon\mu}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m \Omega + 2\mu \omega} \cos(\Theta_{\mathbf{v}} + 2\varepsilon m \varphi_1 + 2\mu \varphi_2). \end{aligned}$$

30.3.3. Возмущения переменной λ .

$$\dot{\lambda} = \frac{1}{G} \left[\frac{\Lambda(\lambda)}{\mathbf{J}(\lambda)} - \frac{1}{\mathbf{J}(\lambda)} \right] \frac{\partial U_0}{\partial \varphi_1} = \frac{\lambda}{2} \left[1 + \frac{\kappa^2 - 6}{2\sqrt{1 + \kappa^2}} \right] \frac{\partial U_0}{\partial \varphi_1}$$

$$\delta\lambda = \frac{1}{2} \lambda PL_{2,2}(\delta, \kappa^2) \sum_{\mathbf{v}} \left[\frac{R_{\mathbf{v},2}(\rho)}{\Omega_{\mathbf{v}} + 2\Omega + 2\omega} \cos(\Theta_{\mathbf{v}} + 2\varphi_1 + 2\varphi_2) - \frac{R_{\mathbf{v},-2}(\rho)}{\Omega_{\mathbf{v}} - 2\Omega - 2\omega} \cos(\Theta_{\mathbf{v}} - 2\varphi_1 - 2\varphi_2) \right],$$

$$L_{2,2}(\delta, \kappa^2) = \frac{1}{2} \delta \sqrt{1 + \kappa^2} \left[1 + \frac{\kappa^2 - 6}{2\sqrt{1 + \kappa^2}} \right].$$

Возмущения от дополнительной части возмущающей функции порядка λ

$$U_1 = \frac{1}{2} P \sum_{\mathbf{v}} \left\{ B_{2,1}^{(1)} \left[R_{\mathbf{v},1}(\rho) \cos(\Theta_{\mathbf{v}} + 2\varphi_1 + \varphi_2) + R_{\mathbf{v},-1}(\rho) \cos(\Theta_{\mathbf{v}} - 2\varphi_1 - \varphi_2) \right] + \right. \\ \left. + B_{0,1}^{(1)} \left[R_{\mathbf{v},1}(\rho) \cos(\Theta_{\mathbf{v}} + \varphi_2) + R_{\mathbf{v},-1}(\rho) \cos(\Theta_{\mathbf{v}} - \varphi_2) \right] \right\},$$

$$P = \frac{3}{2} n^2 (A - C), \quad B_{0,1}^{(1)} = \frac{2\mathbf{M}^{(0)}}{\kappa^3}, \quad B_{2,1}^{(1)} = \frac{A - B}{4\kappa A} \left[-1 + \frac{A}{C - A} \sqrt{1 + \kappa^2} \right];$$

$$\delta\lambda = -\frac{1}{2} PL_{2,1}(\delta, \kappa) \sum_{\mathbf{v}} \left[\frac{R_{\mathbf{v},1}(\rho)}{\Omega_{\mathbf{v}} + 2\Omega + \omega} \cos(\Theta_{\mathbf{v}} + 2\varphi_1 + \varphi_2) - \frac{R_{\mathbf{v},-1}(\rho)}{\Omega_{\mathbf{v}} - 2\Omega - \omega} \cos(\Theta_{\mathbf{v}} - 2\varphi_1 - \varphi_2) \right] \\ - \frac{1}{2} PL_{0,1}(\delta, \kappa) \sum_{\mathbf{v}} \left\{ \left[\frac{R_{\mathbf{v},1}(\rho)}{\Omega_{\mathbf{v}} + \omega} \cos(\Theta_{\mathbf{v}} + \varphi_2) - R_{\mathbf{v},-1}(\rho) \frac{R_{\mathbf{v},-1}(\rho)}{\Omega_{\mathbf{v}} - \omega} \cos(\Theta_{\mathbf{v}} - \varphi_2) \right] \right\},$$

$$L_{2,1}(\delta, \kappa) = -\frac{\kappa}{4(1 + \kappa^2)} \left[\delta(1 + \kappa^2 + \sqrt{1 + \kappa^2}) - \kappa^2 \right], \quad L_{0,1}(\delta, \kappa) = \frac{1}{\kappa(1 + \kappa^2)} \left[\delta(1 + \kappa^2 - \sqrt{1 + \kappa^2}) - \kappa^2 \right].$$

30.3.4. Возмущения угловой переменной φ .

$$\dot{\varphi} = \frac{3\kappa^2}{4\lambda\sqrt{1+\kappa^2}} n^2 (A-C) \sum_{\mathbf{v}} \left\{ B_{2,1}^{(1)} \left[R_{\mathbf{v},1}(\rho) \cos(\Theta_{\mathbf{v}} + 2\varphi_1 + \varphi_2) + R_{\mathbf{v},-1}(\rho) \cos(\Theta_{\mathbf{v}} - 2\varphi_1 - \varphi_2) \right] + \right. \\ \left. + \frac{3\kappa^2}{4\lambda\sqrt{1+\kappa^2}} n^2 (A-C) B_{0,1}^{(1)} \sum_{\mathbf{v}} \left[R_{\mathbf{v},1}(\rho) \cos(\Theta_{\mathbf{v}} + \varphi_2) + R_{\mathbf{v},-1}(\rho) \cos(\Theta_{\mathbf{v}} - \varphi_2) \right] \right\},$$

$$\delta\varphi = P\Phi_{2,1}(\delta, \kappa) \sum_{\mathbf{v}} \left[\frac{R_{\mathbf{v},1}(\rho)}{\Omega_{\mathbf{v}} + 2\Omega + \omega} \sin(\Theta_{\mathbf{v}} + 2\varphi_1 + \varphi_2) + \frac{R_{\mathbf{v},-1}(\rho)}{\Omega_{\mathbf{v}} - 2\Omega - \omega} \sin(\Theta_{\mathbf{v}} - 2\varphi_1 - \varphi_2) \right] \\ + P\Phi_{0,1}(\delta, \kappa) \sum_{\mathbf{v}} \left\{ \left[\frac{R_{\mathbf{v},1}(\rho)}{\Omega_{\mathbf{v}} + \omega} \sin(\Theta_{\mathbf{v}} + \varphi_2) + R_{\mathbf{v},-1}(\rho) \frac{R_{\mathbf{v},-1}(\rho)}{\Omega_{\mathbf{v}} - \omega} \sin(\Theta_{\mathbf{v}} - \varphi_2) \right] \right\},$$

$$\Phi_{2,1}(\delta, \kappa) = \frac{\kappa^2}{2\lambda\sqrt{1+\kappa^2}} B_{2,1}^{(1)}, \quad P = \frac{3}{2} n^2 (A-C), \quad \Phi_{0,1}(\delta, \kappa) = \frac{\kappa^2}{2\lambda\sqrt{1+\kappa^2}} B_{0,1}^{(1)};$$

30.3.5. Возмущения угловой переменной ψ .

$$\delta\psi = -\frac{\kappa^2}{\lambda\sqrt{1+\kappa^2}}\int U_1 dt - \frac{1}{G}\operatorname{ctg}\rho\int\frac{\partial U_0}{\partial\rho}dt$$

30.3.6. Возмущения долготы восходящего узла плоскости Андуайе h .

$$\delta h = \frac{1}{G} \csc \rho \int \frac{\partial U_0}{\partial \rho} dt + \frac{\lambda}{G} \csc \rho \int \frac{\partial U_1}{\partial \rho} dt$$

$$\begin{aligned} \delta h = & \frac{1}{2} \lambda \frac{P}{G} \csc \rho B_{2,1}^{(1)} \sum_{\mathbf{v}} \left[\frac{R'_{\mathbf{v},1}(\rho)}{\Omega_{\mathbf{v}} + 2\Omega + \omega} \sin(\Theta_{\mathbf{v}} + 2\varphi_1 + \varphi_2) + \frac{R'_{\mathbf{v},-1}(\rho)}{\Omega_{\mathbf{v}} - 2\Omega - \omega} \sin(\Theta_{\mathbf{v}} - 2\varphi_1 - \varphi_2) \right] + \\ & + \frac{1}{2} \lambda \frac{P}{G} \csc \rho B_{0,1}^{(1)} \sum_{\mathbf{v}} \left[\frac{R'_{\mathbf{v},1}(\rho)}{\Omega_{\mathbf{v}} + \omega} \sin(\Theta_{\mathbf{v}} + \varphi_2) + \frac{R'_{\mathbf{v},-1}(\rho)}{\Omega_{\mathbf{v}} - \omega} \sin(\Theta_{\mathbf{v}} - \varphi_2) \right], \end{aligned}$$

$$B_{2,1}^{(1)} = -\frac{1}{4\kappa} \left[\delta(1 + \kappa^2 + \sqrt{1 + \kappa^2}) - \kappa^2 \right], \quad B_{0,1}^{(1)} = \frac{1}{\kappa^3} \left[\delta(1 + \kappa^2 - \sqrt{1 + \kappa^2}) - \kappa^2 \right].$$

30.4. Вековые возмущения Чандлеровского-эйлеровского движения небесных тел (Земли).

$$\dot{G} = 0,$$

$$\dot{\lambda} = 0,$$

$$\dot{\rho} = \sin \pi_1 \sin(h - \Pi_1) \frac{d\Pi_1}{dt} + \cos(h - \Pi_1) \frac{d\pi_1}{dt},$$

$$\dot{\phi} = \Omega(G, \lambda) + \frac{1}{G\mathbf{J}(\lambda)} \frac{\partial \langle U \rangle}{\partial \lambda},$$

$$\dot{\psi} = \omega(G, \lambda) + \operatorname{cosec} \rho \sin \pi_1 \cos(h - \Pi_1) \frac{d\Pi_1}{dt} - \sin(h - \Pi_1) \frac{d\pi_1}{dt} - \frac{\Lambda(\lambda)}{G\mathbf{J}(\lambda)} \frac{\partial \langle U \rangle}{\partial \lambda} - \frac{1}{G} \operatorname{ctg} \rho \frac{\partial \langle U \rangle}{\partial \rho},$$

$$\dot{h} = (1 - \cos \pi_1) \frac{d\Pi_1}{dt} - \cot \rho \left[\sin \pi_1 \cos(h - \Pi_1) \frac{d\Pi_1}{dt} - \frac{d\pi_1}{dt} \sin(h - \Pi_1) \right] + \frac{1}{G} \operatorname{csc} \rho \frac{\partial \langle U \rangle}{\partial \rho},$$

$$\langle U \rangle = U = P \left[b_{0,0}(\lambda) R_{0,0,0,0,0}^{(22)}(\rho) + c_{0,0}(\lambda) R_{0,0,0,0,0}^{(33)}(\rho) \right], \quad P = \frac{3}{2} n^2 (A - C);$$

$$b_{0,0} = \frac{[\delta(1 + \kappa^2) - \kappa^2] \mathbf{E} + [\delta(\lambda^2 - 1) + \kappa^2 + \lambda^2] \mathbf{K}}{2\mathbf{K}(\kappa^2 + \lambda^2)}, \quad c_{0,0} = \frac{-\mathbf{E}D + \delta(1 + \kappa^2) \mathbf{K}}{\mathbf{K}(\kappa^2 + \lambda^2)},$$

$$R_{0,0,0,0,0}^{(22)} = -A_{0,0,0,0,0}^{(0)} \sin^2 \rho - \frac{1}{2} (1 + \cos^2 \rho) A_{0,0,0,0,0}^{(2)} + A_{0,0,0,0,0}^{(1)} \sin 2\rho,$$

$$R_{0,0,0,0,0}^{(33)} = -A_{0,0,0,0,0}^{(0)} \cos^2 \rho - \frac{1}{2} (1 + \sin^2 \rho) A_{0,0,0,0,0}^{(2)} - A_{0,0,0,0,0}^{(1)} \sin 2\rho.$$

$$\langle \dot{h} \rangle = \frac{1}{G} \csc \rho \frac{\partial \langle U \rangle}{\partial \rho},$$

$$\langle \dot{\phi} \rangle = \frac{1}{G\mathbf{J}(\lambda)} \frac{\partial \langle U \rangle}{\partial \lambda},$$

$$\langle \dot{\psi} \rangle = -\frac{\Lambda(\lambda)}{G\mathbf{J}(\lambda)} \frac{\partial \langle U \rangle}{\partial \lambda} - \frac{1}{G} \cot \rho \frac{\partial \langle U \rangle}{\partial \rho},$$

$$\langle U \rangle = P \left[b_{0,0}(\lambda) R_{0.0.0.0.0}^{(22)}(\rho) + c_{0,0}(\lambda) R_{0.0.0.0.0}^{(33)}(\rho) \right],$$

$$b_{0,0} = \frac{[\delta(1+\kappa^2) - \kappa^2]\mathbf{E} + [\delta(\lambda^2 - 1) + \kappa^2 + \lambda^2]\mathbf{K}}{2\mathbf{K}(\kappa^2 + \lambda^2)},$$

$$c_{0,0} = \frac{-\mathbf{E}D + \delta(1+\kappa^2)\mathbf{K}}{\mathbf{K}(\kappa^2 + \lambda^2)}, \quad D = \delta(1+\kappa^2) - \kappa^2.$$

30.4.1. Гравитационная составляющая чандлеровского движения полюса.

$$\dot{\phi} = \frac{3\pi n^2 (A - C) (b_1 \mathbf{K}^2 + b_2 \mathbf{E} \mathbf{K} + b_3 \mathbf{E}^2)}{8G\kappa\lambda^2\lambda'^2\sqrt{(1+\kappa^2)(\kappa^2+\lambda^2)}\mathbf{K}^3} \left[-A_0^{(0)}(1-3\cos^2 \rho) + 3\sin 2\rho A_0^{(1)} + \frac{3}{2} A_0^{(2)} \sin^2 \rho \right]$$

$$b_1 = \lambda'^2 \left[2\delta(1+\kappa^2)\lambda^2 - D(\kappa^2 + \lambda^2) \right], \quad b_2 = 2D\kappa^2\lambda'^2, \\ b_3 = -D(\kappa^2 + \lambda^2), \quad D = [\delta(1+\kappa^2) - \kappa^2].$$

$$\dot{\phi} = \frac{3n^2 (A - C)(2 - D)}{4\omega C\sqrt{1+\kappa^2}} \left[-A_0^{(0)}(1-3\cos^2 \rho) + 3\sin 2\rho A_0^{(1)} + \frac{3}{2} A_0^{(2)} \sin^2 \rho \right]$$

30.4.2. Постоянная прецессии.

$$\dot{h} = \frac{3n^2(A-C)}{4\omega C \sin \rho} \left[1 + \delta - \frac{3\delta(1+\kappa^2)}{\kappa^2 + \lambda^2} + \frac{3DE}{\mathbf{K}(\kappa^2 + \lambda^2)} \right] \left\{ - \left(A_0^{(0)} - \frac{1}{2} A_0^{(2)} \right) \sin 2\rho + 2A_0^{(1)} \cos 2\rho \right\}$$

$$\dot{h} = \frac{3n^2(A-C)(\delta-2)}{4\omega C \sin \rho} \left\{ - \left(A_0^{(0)} - \frac{1}{2} A_0^{(2)} \right) \sin 2\rho + 2A_0^{(1)} \cos 2\rho \right\},$$

$$\Delta \dot{h} = \frac{9\lambda^2}{8\kappa^2} \cdot \frac{n^2(A-C)}{\omega C \sin \rho} \left[2 - \delta(1+\kappa^2) + \kappa^2 \right] \left\{ - \left(A_0^{(0)} - \frac{1}{2} A_0^{(2)} \right) \sin 2\rho + 2A_0^{(1)} \cos 2\rho \right\}.$$

Вклад чандлеровского движения полюса в постоянную прецессии.

$$p = p_0(1 - 0.222921 \times 10^{-11})$$



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