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Fine Structure of the Rydberg Blockade Zone

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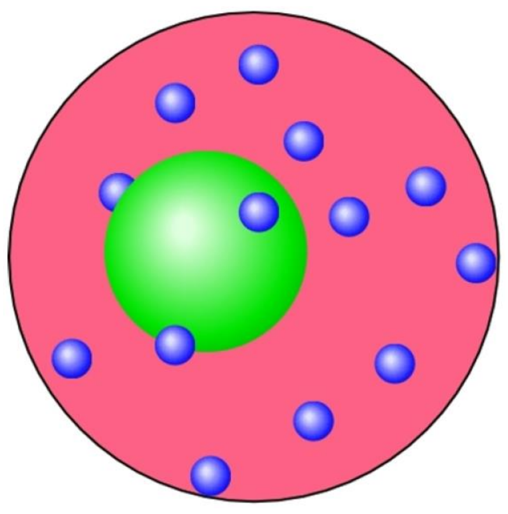
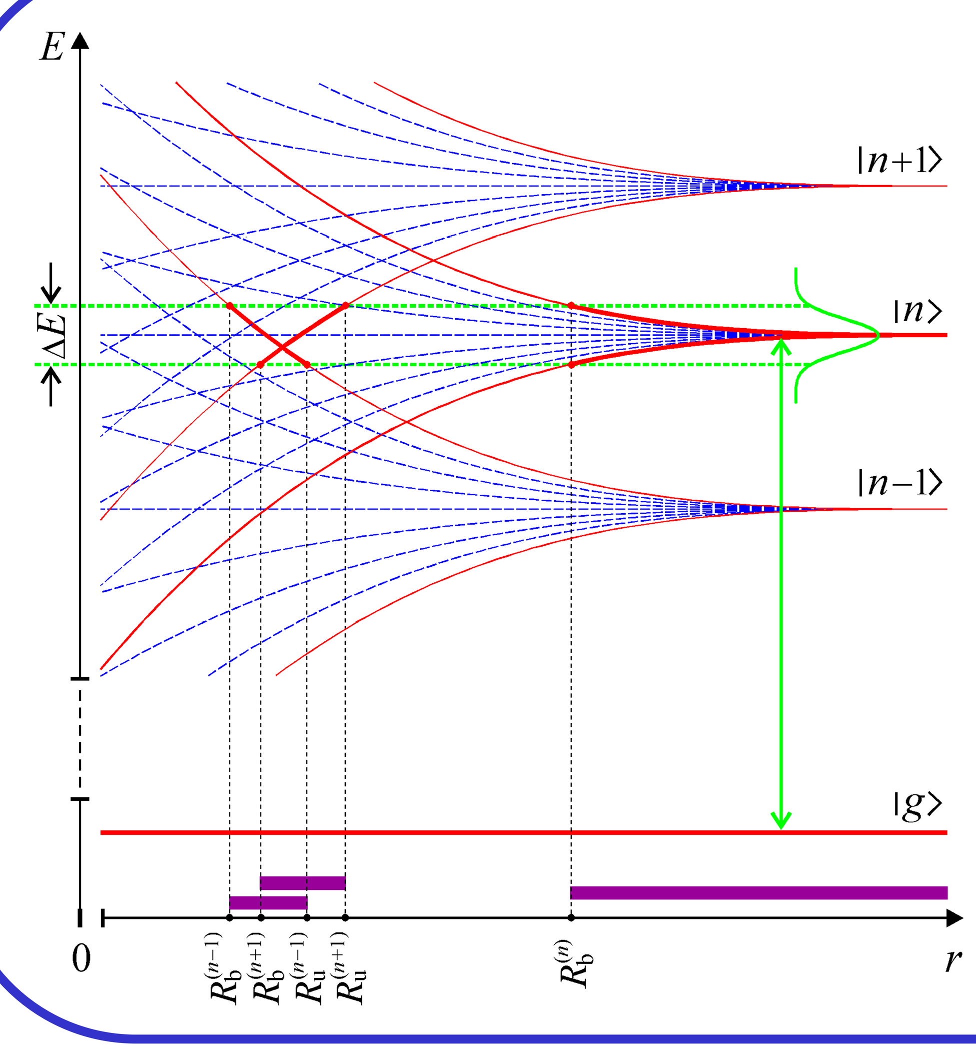
Motivation



- The concept of Rydberg blockade was suggested theoretically in the early 2000's [1], and a few years later this effect was verified experimentally [2-4].
- The idea of Rydberg blockade was used recently in a lot of intriguing phenomena, such as a highly efficient entanglement between the light and atoms [5, 6], observation of the spatially ordered structures in a Rydberg gas [7], realization of strong interaction between photons [8, 9], and, particularly, creation of the photon pairs [10, 11], *etc*.
- However, all these studies have taken into account only blockade of the excitation with the same value of the principal quantum number n as in the original Rydberg atom. In such a case, the blocked zone looks like a solid ball around the already excited atom.
- The aim of the present work is to consider a possibility of excitation to the states with neighboring values of the principal quantum number (*e.g.*, $n-1$ or $n+1$). As a result, it will be shown that Rydberg excitation becomes unblocked again in a certain range of sufficiently small distances from the central atom. **Therefore, the blockade zone should represent a number of co-centric shells rather than a solid ball.**

Basic Idea



- A convenient way to treat the Rydberg blockade is to consider a perturbation of the energy levels of neighboring atoms by the electric field of the central Rydberg atom due to the Stark effect [12].
- In such a case, as is clearly seen in the diagram, the split sublevels from the states with $n-1$ and $n+1$ can pass the energy excitation band ΔE (shown by the green dotted lines) in the intervals of radius $[R_b^{(n-1)}, R_u^{(n-1)}]$ and $[R_b^{(n+1)}, R_u^{(n+1)}]$, respectively. (Here, the subscripts 'b' and 'u' refer to the points where the Rydberg excitation becomes blocked and unblocked.)
- In principle, the same effect can take place also for the states with $n-2$, $n+2$, and so on.

Quantitative Estimates

- The central Rydberg-excited atom produces a dipolar electric field (in atomic units):

$$\tilde{\mathcal{E}}(\tilde{r}) = \frac{C_3 n^2}{\tilde{r}^3} \, ,$$

where $r = a_0 \tilde{r}$, $\mathcal{E} = \mathcal{E}^* \tilde{\mathcal{E}}$, $a_0 = \hbar^2/(me^2)$, $\mathcal{E}^* = m^2 e^5/\hbar^4$, and C_3 is a numerical coefficient on the order of unity.

- The energy levels of the neighboring atoms will be split due to the (first-order) Stark effect [13, 14]:

$$\tilde{E}_{n n_1 n_2} = -\frac{1}{2n^2} + \frac{3}{2}\,(n_1 - n_2)\, n \tilde{\mathcal{E}} \, ,$$

where $E = E^* \tilde{E}$, $E^* = me^4/\hbar^2$; n_1 and n_2 are the parabolic quantum numbers, which satisfy the following relation:

$$n_1 + n_2 + |m| + 1 = n \, .$$

Particularly, at $m = 0$: $n_1 - n_2 = n-1,\ n-3,\ \ldots,\ -n+1$; and so on.

- Blockade of the basic state n develops in the point where

$$\tilde{E}_{n,\pm(n-1)}\big(\tilde{\mathcal{E}}_{\mathrm{b}}^{(n)}\big) \, = \, \tilde{E}_{n,\pm(n-1)}(0) \, \pm \, \frac{1}{2}\, \Delta\tilde{E} \, ,$$

whereas the neighboring state $n-1$ will be unblocked and blocked again under condition:

$$\tilde{E}_{n-1,n-2}\big(\tilde{\mathcal{E}}_{\mathrm{u,b}}^{(n-1)}\big) \, = \, \tilde{E}_{n}(0) \, \mp \, \frac{1}{2}\, \Delta\tilde{E} \, .$$

- Therefore, the boundaries of the additional excitation zone will be given by the formula:

$$\tilde{R}_{\mathrm{u,b}}^{(n-1)} = \left(\frac{3 C_3 n^7}{2} \right)^{1/3} \left\{ 1 \mp \frac{3 C_3 n^7}{2 \big(\tilde{R}_{\mathrm{b}}^{(n)}\big)^3} \right\}^{-1/3} .$$

- In other words, the center of this zone is located at the distance

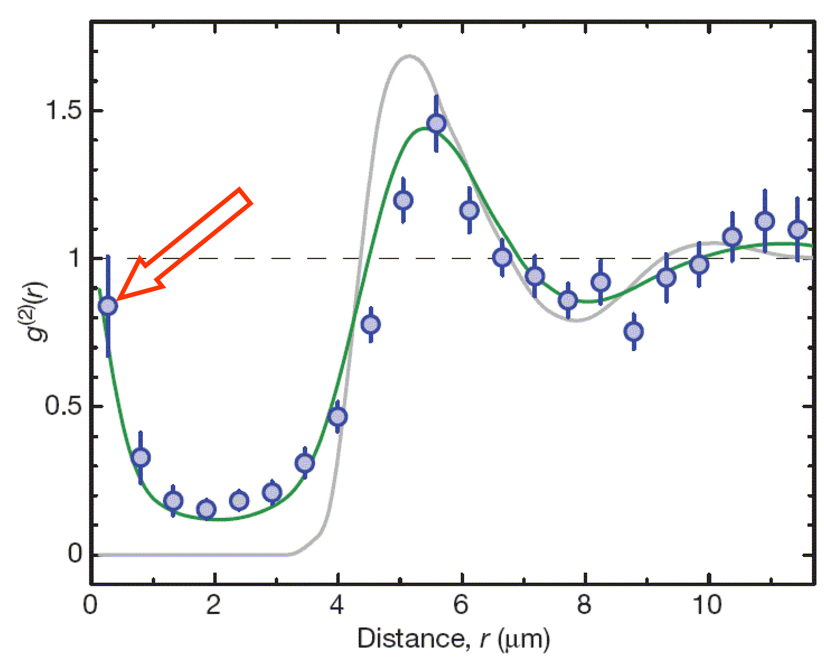
$$\tilde{R}_{\mathrm{c}}^{(n-1)} = \left(\frac{3 C_3 n^7}{2} \right)^{1/3} ,$$

and its characteristic width equals

$$\Delta \tilde{R}^{(n-1)} = \left(\frac{3}{2} \right)^{1/3} \frac{C_3^{4/3}\, n^{28/3}}{\big(\tilde{R}_{\mathrm{b}}^{(n)}\big)^3} \, .$$

Applications

- For the particular parameters of the experiment [7] ($n = 43$, $R_b^{(n)} = 4\,\mu$m or $7.6{\times}10^4$ atomic units), our formula predicts an additional excitation zone at the distance $R_c^{(n-1)} \approx 0.34\,\mu$m or $6.5{\times}10^3$ atomic units. In fact, the pair correlation function $g^{(2)}(r)$ measured in this experiment really exhibits a sharp unexpected maximum at small interparticle separations. This was initially attributed to imperfection of the detection procedures. On the other hand, as follows from our theoretical model, this maximum could have a much deeper physical meaning.



- It might be expected that formation of the additional unblocked zones should facilitate creation of the dense Rydberg gases as well as their subsequent spontaneous avalanche ionization [15].
- Fine structure of the Rydberg blockade zone should be important also in the attempts to produce Rydberg crystals and other kinds of "Rydberg matter". Particularly, it may provide criteria for the optimal interparticle separation in the Rydberg lattices.
- On the other hand, there may be a "negative" effect of the additional unblocked zones in the quantum-information applications, because the possibility of excitation to the neighboring quantum states should reduce a fidelity of the quantum protocols.

Directions of Further Research

- The theoretical estimates presented here are based on the well-known formula for Stark effect in a uniform external field, while the realistic interatomic fields should be substantially nonuniform in the case of small interparticle separation. Such a refined calculation is now in progress.
- Besides, the above formulas refer to the case of "perfect" hydrogen-like atoms, where the first-order Stark effect plays the dominant role. On the other hand, the real experiments are usually performed with alkali atoms (such as rubidium), where the second-order Stark effect can be of crucial importance. This fact should be taken into account in the improved theoretical analysis.
- In general, we expect that the particular expressions for the blocked and unblocked zones will be modified, but the qualitative effect (*i.e.*, emergence of the intermittent shell structure) will survive.

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