Physical and Mathematical Modelling of Heat Flow in the Snow Samples

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Abstract. The study of snow thermal conductivity was performed using laboratory experiments and mathematical modeling. Laboratory experiments on thermometry of snow were considered. Thermometry was carried out in a cold room under the influence of a cooling plate from below. Thermometry of snow samples was carried out. As a result, the cold wave propagation velocity in the sample was measured and the heat transfer coefficient was measured. Mathematical modeling of the sample cooling process was also performed.

Keywords: snow cover · heat transfer coefficient · cold chamber · thermometry · physical experiment · mathematical modeling · finite differences

1 Introduction

The study of snow thermo physical properties plays a key role in understanding climatic processes, especially in regions with seasonal snow cover. This paper reviews laboratory experiments on snow thermometry aimed at determining its effective thermal conductivity and the peculiarities of heat and mass transfer in the snow column.

Among all relatively stable components of the Earth's cryosphere, the snow cover is distinguished by its seasonal existence, extreme variability of properties and structure in time and space. It is a natural barrier to cold penetration from the atmosphere into the soil, which is due to its porous structure and low density. These qualities are most pronounced in loose layers of freshly fallen snow deposited in low wind or calm conditions. Precisely because of its structure, freshly fallen snow has a very low thermal conductivity, which is comparable to the thermal conductivity of air (Voytkovsky, 1977), while ice crusts have a thermal conductivity comparable to that of ice, which determines the variability of thermal insulation properties of snow cover in a large range (Osokin et al., 2013).

The structure of the snow column changes in time in the process of natural compaction and sublimation rounding. In natural conditions of snow cover deposition, these processes last from several hours to several days depending on the intensity of recrystallization processes, which is ultimately determined by snow temperature and mass (Chernov, 2016). Snow cover compaction and spatial transformations of its structure lead to variability of its thermal properties. The influence of snow density and temperature on its thermal conductivity was repeatedly noted earlier (Osokin et al., 2000).

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In addition, it has been established that snow structure is related to heat transfer processes, but numerical estimates of this relationship have been obtained relatively recently (Osokin et al., 2014; Schneebeli and Sokratov, 2004).

Snow thermal conductivity is an important thermophysical characteristic that largely determines the heat exchange in the system atmosphere - snow cover - soil. It affects the stability of frozen rocks, including the development of slope processes in the mountains. The temperature distribution in the snow cover depends on snow thermal conductivity, which determines snow metamorphism and changes in its structure, and the latter, in turn, has an inverse effect on thermal conductivity. Such cyclic dependence is supplemented by many other factors of snow column development (Osokin and Sosnovskiy, 2014). Each snow component (ice crystals, water vapor, air) affects its effective thermal conductivity both independently and by interacting with each other. The contribution of these components to the effective thermal conductivity of snow depends on many parameters - temperature, temperature gradient, and structural features. The effective thermal conductivity of the entire snow column also depends on the stratigraphy of the snow cover, daily air temperature fluctuations, and base temperature.

The effective thermal conductivity coefficient is a complex parameter characterizing all thermophysical processes in the snow cover related to heat transfer: conductive heat conduction, water vapor diffusion, sublimation and condensation, and heat exchange between ice crystals and air. The first dependences of snow thermal conductivity on density were obtained at the end of the 19th century (Sturm et al., 1997). Since then, many formulas for calculating the thermal conductivity of snow of different structures have appeared (Kotlyakov et al., 2018). The thermal conductivity of snow is determined by different methods: using needle probes, heat flux meters, computational techniques, including microtomography. Despite the great interest in this issue, the problem of determining snow thermal conductivity and its variability depending on different factors remains open. One of the reasons for this is the large variation in the values of the snow thermal conductivity coefficient (Calonne et al., 2011). That is why the measurement of this indicator and the analysis of the obtained results are still relevant.

2 Experimental Setup

The experiments were conducted in a specially equipped cold room at the Faculty of Geography of Moscow State University. The key element of the installation was an innovative system that creates a vertical temperature gradient in the snow column (Fig. 1).

Main Components of the Installation:

- Polyurethane foam boxes without top and bottom for placing snow samples
- Metal panel with adjustable temperature (thermostat)
- A system of thermocouples spaced 1.5 cm vertically
- Refrigeration chamber for ambient temperature control

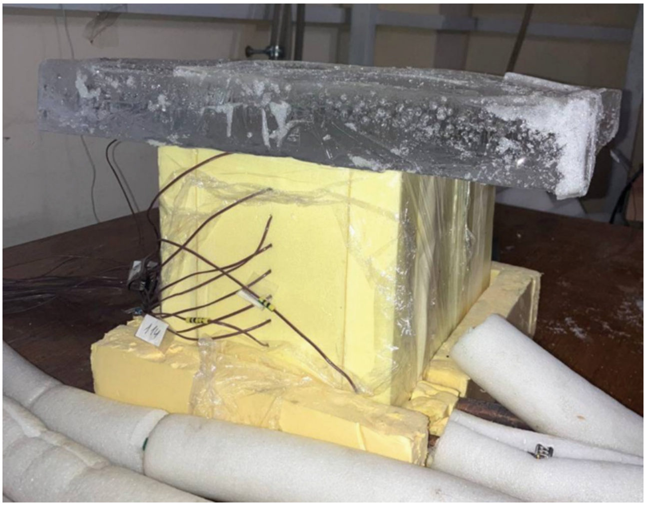

Fig. 1. Exterior view of the experimental setup in the cold room

3 Methodology

The experiment began by setting the temperature at the lower (using a thermostat) and upper (cold room temperature) boundaries of the snow sample. The thickness of the sample was 18 cm.

The key steps of the experiment were:

1. Placing the snow in a polyurethane foam box
2. Placement of thermocouples at different levels of the sample
3. Setting the boundary temperatures
4. Regularly measuring the temperature inside the sample

4 Materials and Methods

Thus, experiments to determine the coefficient of effective thermal conductivity of the snow column, as well as the peculiarities of heat and mass transfer under conditions of application of different in value and direction temperature gradient were conducted in the cold room of the Faculty of Geography. An installation was assembled to create a vertical temperature gradient in the snow column where snow was placed in polyurethane foam boxes without top and bottom. At the bottom there was a metal panel on which the temperature was set by means of a thermostat, and on the upper side the temperature was equal to the temperature in the refrigerating chamber. Thermocouples were inserted into the side of the snow shell with a vertical spacing of 1.5 cm. Special attention was paid to the study of cold wave propagation in the sample at different temperature gradients and initial snow temperatures.

The experiment started with setting the temperature on the thermostat and in the cold chamber (at the lower and upper boundaries of the snow sample). Then the temperature inside the sample was measured regularly on thermocouples to determine the spread of the temperature set at the boundaries into the depth of the sample. From the measured temperature data at different levels in the sample and their changes over time, the effective thermal conductivity coefficient of the snow was determined.

As a result of the experiment, it turned out that at different direction of the temperature gradient and at different snow temperature, the coefficient of effective thermal conductivity for snow is different.

In general, the study of the effective thermal conductivity of snow has been carried out for more than 100 years. The collected results of previous studies are presented in Fig. 2.

In general, the measurement of the effective thermal conductivity coefficient of snow can be performed by two methods: the method of stationary heat flux and the method of heat wave. In our case, we used a method approximated to the method of stationary heat flux.

Mathematical modeling of the sample cooling process was also performed. The initial data of the calculation were the results of temperature measurement closer to the surface. The calculations were performed by means of the difference scheme obtained on the basis of the Fourier heat conduction equation.

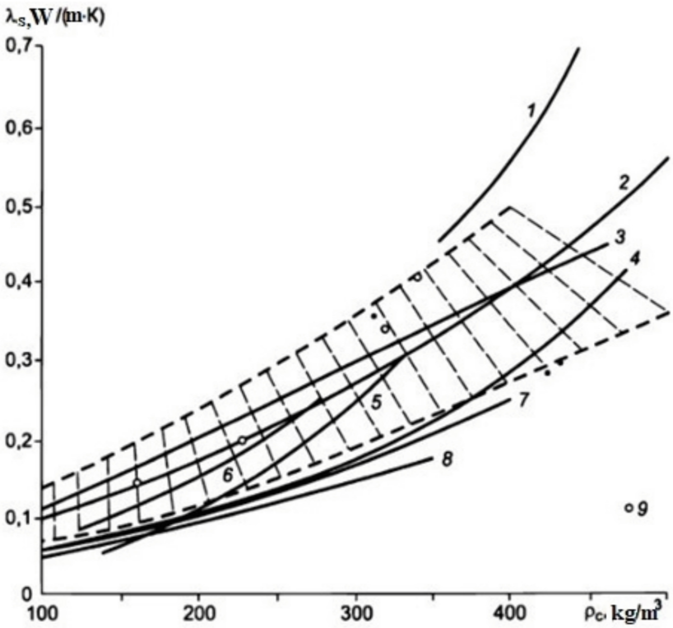

Fig. 2. Comparison of the values of the snow cover thermal conductivity coefficient λc calculated depending on the density ρc according to the formulas of different authors: 1 - A.S. Kondratieva; 2 - M. Janson; 3 - B.V. Proskuryakova; 4 - T.V. Dyachkova, N.V. Serova, G.F. Abels; 6 - V.V. Piotrovich; 7 - Ts. Iosida; 8 - G.K. Sulakvelidze; 9 - calculated data of G.V. Porhaev; the range of possible changes in λc is shaded (Pavlov, 2008).

5 Mathematical Modeling

The Fourier heat conduction equation was used to describe the temperature field in snow (Lykov (1967), Basics of Permafrost Forecasting (1974), Samarskiy and Gulin (1989), Tikhonov and Samarskiy (1999)):

$$\partial \mathrm{T}/\partial \mathrm{t} = (\lambda_s/\mathrm{Cs}) * (\partial^2 \mathrm{T}/\partial \mathrm{x}^2)$$

where:

- λ_s - conductive heat transfer coefficient of snow
- C_s - heat capacity of snow
- T - temperature (°C)
- t - time
- x - spatial varia

Thus, the temperature field in snow is described by the Fourier heat conduction equation:

$$C_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_s \frac{\partial T}{\partial x}\right) \tag{1}$$

The snow sample thickness of 18 cm in depth was divided into i = 1, n (= 200) parts. For the numerical solution, the difference equation was written down and the difference scheme based on the finite difference method was applied:

$$C_s \frac{T_i^{j+1} - T_i^j}{\Delta t} = \lambda_s \frac{T_{i+1}^j - 2T_i^j + T_{i-1}^j}{\Delta x^2}, \tag{2}$$

where j – time step value, i – space step value.

Equation (2) gives the new value of snow temperature at the new time layer:

$$T_i^{j+1} = T_i^j + \Delta t/C_s \cdot K_s \frac{T_{i+1}^j - 2T_i^j + T_{i-1}^j}{\Delta x^2} \tag{3}$$

A time step j of 1 min was broken into intervals of 0.1 s and calculated at a new step using boundary conditions:

$$\lambda \left.\frac{T_0 - T_1}{\Delta x}\right| = 0,\ T_{n+1} = Tsurf \tag{4}$$

The difference scheme for performing a step on j (in time) was written as:

$$T_i^{j+1} = T_i^j + \Delta t/C_s \cdot K_s/\Delta x^2 (T_{i+1}^j - 2T_i^j + T_{i-1}^j) \tag{5}$$

for i = 1, n, and the boundary conditions were applied:

$$T_0 = T_1, T_{n+1} = T_{surf} \tag{6}$$

The calculations were performed in MATLAB programming environment. The results of numerical modeling of the cold wave propagation during one-sided cooling of the snow sample in laboratory conditions and the temperature change during cooling of the snow sample agree with the thermometry data. This method of studying the thermal conductivity of snow cover, both by laboratory experiments and by means of mathematical modeling gives improved results.

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References

Lykov, A.V.: Theory of Heat Conduction, p. 599. Higher School, Moscow (1967)

Basics of Permafrost Forecasting in Engineering and Geological Research. In: Kudryavtsev, V.A. (ed.) Moscow State University Publishing House, Moscow, p. 431 (1974)

Samarskiy, A.A., Gulin, A.V.: Numerical Methods, p. 432. Nauka, Moscow (1989)

Tikhonov, A.N., Samarskiy, A.A.: Equations of Mathematical Physics, 6th edition, corrected and supplemented, p. 800. Moscow State University (1999)

Pavlov, A.V.: Cryolithozone Monitoring, p. 230. GEO, Novosibirsk (2008)

Voytkovsky, K.F.: Mechanical properties of snow, p. 126. Izd-vo Nauka, Moscow (1977)

Osokin, N.I., Samoilov, R.S., Sosnovskiy, A.V., Sokratov, S.A.: About the role of some natural factors in soil freezing. MGI. **88**, 41–45 (2000)

Osokin, N.I., Sosnovskiy, A.V.: Experimental studies of the effective heat conductivity coefficient of the snow cover in West Spitsbergen. Ice & Snow **54**(3), 50–58 (2014). https://doi.org/10.15356/2076-6734-2014-3-50-58

Osokin, N.I., Sosnovsky, A.V., Chernov, R.A.: Influence of snow cover stratigraphy on its thermal resistance. Ice and snow **3**(123), 63–70 (2013)

Kotlyakov, V.M., Sosnovsky, A.V., Osokin, N.I.: Estimation of the heat conductivity coefficient of snow by its density and hardness in Western Svalbard. Ice and Snow. **58**(3), 343–352 (2018). https://doi.org/10.15356/2076-6734-2018-3-343352

Chernov, R.A.: Metamorphism and thermal properties of freshly fallen snow (based on studies in the Moscow region). Ice and Snow **56**(2), 199–206 (2016). https://doi.org/10.15356/2076-6734-2016-2-199-206

Calonne, N., Flin, F., Morin, S., Lesaffre, B., du Roscoat, S.R., Geindreau, C.: Numerical and experimental investigations of the effective thermal conductivity of snow. Geophysical Research Letters. **38**, L23501 (2011). https://doi.org/10.1029/2011GL049234

Schneebeli, M., Sokratov, S.A.: Tomography of temperature gradient metamorphism of snow and associated changes in heat conductivity. Hydrological Processes. **18**, 3655–3665 (2004)

Sturm, M., Holmgren, J., Konig, M., Morris, K.: The thermal conductivity of seasonal snow. Journal of Glaciology **43**(143), 26–41 (1997)