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Letter

Dark parametric solitons at the presence of quasi-resonant centers

S V Sazonov

National Research Centre 'Kurchatov Institute', Moscow 123182, Russia

E-mail: sazonov.sergey@gmail.com

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Abstract

The effect of impurity quasi-resonant centers (QRCs) on the second harmonic generation in the mode of formation of the two-color dark temporal solitons at the presence of phase and group mismatchings is studied. It is shown that QRCs, leading to a decrease of the generation efficiency, simultaneously contribute to the formation of such solitons.

Keywords: second harmonics, dark parametric soliton, quasi-resonance

1. Introduction

Optical solitons are objects that can be used in information transmission and processing systems. The studies of various soliton modes of the second harmonic generation have become relevant after the invention of coherent sources of intense light radiation. Thus, the spatial two-color (parametric) solitons were predicted in [1]. Such solitons are continuous two-color laser beams localized in the transverse directions. This localization occurs due to mutual compensation of nonlinear self-focusing and transverse broadening of the beam due to diffraction.

Bright and dark temporal parametric solitons have been studied in detail [2, 3]. The stable localization of such solitons in the direction of propagation is ensured by mutual compensation of nonlinear self-compression of the pulse and its dispersion spreading. Bright temporal solitons are the short laser pulses. In turn, dark temporal solitons are the localized areas in the laser beam, the field intensity inside which reaches zero [4, 5]. Such dark bunches propagate along the laser beam.

The parameters of the medium can be effectively controlled by the impurity quasi-resonant centers (QRCs) [6]. The role of such centers can be fulfilled, for example, by the quantum dots [7, 8]. As is known, the dispersion properties are most effectively manifested in the vicinity of resonant quantum transitions. At the same time, in order to avoid irreversible losses, which are also significant in the vicinity of resonances, the frequency of laser radiation should lie outside from the absorption lines of the impurity centers. Simultaneous satisfaction to the both conditions is usually called the quasi-resonant approximation [6, 9, 10].

In [11] a new two-color spatiotemporal soliton, called as the bright-dark parametric soliton, was predicted. Along the longitudinal coordinate this object looks like a dark temporal soliton. With respect to the transverse directions it has the properties of a bright spatial soliton. Note that in [11] the bright-dark parametric soliton was studied under idealized conditions, when the phase matching (PM) and group matching (GM) conditions were simultaneously satisfied. In addition, the parameter of the dispersion of group velocity (DGV) was considered positive at the fundamental frequency, and equal to zero at the second harmonic frequency. At the same time, in the transparency spectral region of natural crystals the DGV parameter monotonically increases with the increasing of frequency [4, 12]. For the reasons stated above, a crystal with quadratic optical nonlinearity containing doped QRCs may be useful here.

Before the studying of the possibility of forming of a bright-dark parametric spatiotemporal soliton in a crystal containing impurity QRCs in the absence of the both types of matching, it is useful to first consider such possibility for a dark temporal two-color soliton. This study is the subject of the present work.

2. Basic equations

Let the laser pulse propagate along the z - axis, which is perpendicular to the optical axis of the uniaxial crystal. We will also assume that the selected quantum transition of the impurity centers is in quasi-resonance with the laser pulse at the fundamental carrier frequency. Then the equations for the linearly polarized complex slowly varying envelopes (SVEs) ψ_1 and ψ_2 of the electric field of pulses at the fundamental frequency ω and at the second harmonic frequency 2ω have the form

$$i\left(\frac{\partial\psi_1}{\partial z}+\frac{1}{v_1}\frac{\partial\psi_1}{\partial t}\right)=-\frac{\beta_1}{2}\frac{\partial^2\psi_1}{\partial t^2}+\alpha_1\psi_1^*\psi_2\mathrm{e}^{-2i\omega(n_2-n_1)z/c}+\frac{2\pi dn}{cn_1}\omega R\,, \quad (1)$$

$$i\left(\frac{\partial\psi_2}{\partial z}+\frac{1}{v_2}\frac{\partial\psi_2}{\partial t}\right)=-\frac{\beta_2}{2}\frac{\partial^2\psi_2}{\partial t^2}+\alpha_2\psi_1^2\mathrm{e}^{2i\omega(n_2-n_1)z/c}. \quad (2)$$

Here and below the subscripts $j=1$ and $j=2$ refer to the parameters at the fundamental frequency and at the second harmonic frequency, respectively; c is the speed of light in vacuum, n_j is the refractive index, v_j is the group velocity, β_j is the GVD parameter, $\alpha_1=4\pi\omega\chi^{(2)}(2\omega,-\omega)/cn_1$, $\alpha_2=4\pi\omega\chi^{(2)}(\omega,\omega)/cn_2$, $\chi^{(2)}(\omega_1,\omega_2)$ is the frequency-dependent second-order nonlinear susceptibilitiy of the crystal under consideration, n is the concentration of impurity centers, d is the real matrix element of the dipole moment of quasi-resonant quantum transition of the impurity QRCs, R is the complex envelope of the dimensionless non-stationary dipole moment of the impurity center.

Assuming that the temporal duration τ_p of the dark laser soliton is shorter than all times of relaxation, we supplement the system (1) and (2) by the material Bloch equations describing the dynamics of impurity QRCs:

$$\frac{\partial R}{\partial t}=i(\omega_0-\omega)R+i\frac{d}{\hbar}\psi_1 w, \quad (3)$$

$$\frac{\partial w}{\partial t}=2i\frac{d}{\hbar}(\psi_1^*R-\psi_1R^*), \quad (4)$$

where $\hbar$ is the Planck constant, ω_0 is the frequency of the quantum transition of impurity centers involved in the interaction with the laser pulse, and w is the difference between the populations of the excited and ground states of the impurity center.

Let us set the goal of excluding the material variables and of the impurity centers from the system (1)–(4) using the quasi-resonant condition

$$\tau_p^{-1}<<|\omega-\omega_0|<<\omega,\,\omega_0. \quad (5)$$

Taking into account the condition (5), we rewrite the equation (3) in the form

$$R=-\frac{\mathrm{d}\psi_1 w}{\hbar(\omega_0-\omega)}-\frac{i}{\omega_0-\omega}\frac{\partial R}{\partial t}. \quad (6)$$

According to the condition (5), the second term in the right-hand side of equation (6) is a small perturbation. Taking into account this term by the method of the successive approximations, we will have the expansion [13]

$$R=-\frac{\mathrm{d}\psi_1 w}{\hbar(\omega_0-\omega)}+i\frac{d}{\hbar(\omega_0-\omega)^2}\frac{\partial}{\partial t}(\psi_1 w)+\frac{d}{\hbar(\omega_0-\omega)^3}\frac{\partial^2}{\partial t^2}(\psi_1 w)\,.$$

From the physical considerations it is clear that under the condition (5) the excitation of the impurity centers is relatively weak. I.e. the change of populations of the quantum levels of these centers is insignificant. We will assume that in the absence of a laser radiation ($\psi_1=0$) all impurity centers are in the ground state ($w=-1$). Then, assuming approximately in the second and third small terms of the last expansion $w=-1$, we obtain

$$R=-\frac{d}{\hbar(\omega_0-\omega)}\psi_1 w-i\frac{d}{\hbar(\omega_0-\omega)^2}\frac{\partial\psi_1}{\partial t}-\frac{d}{\hbar(\omega_0-\omega)^3}\frac{\partial^2\psi_1}{\partial t^2}. \quad (7)$$

Let us substitute (7), neglecting the last term, into equation (4). Then after integration we will have

$$w=-1+\frac{2d^2|\psi_1|^2}{\hbar^2(\omega_0-\omega)^2}. \quad (8)$$

Using (1), (2), (7), and (8), we arrive to the set of equations

$$i\left(\frac{\partial\Phi_1}{\partial z}+\frac{1}{\tilde v_1}\frac{\partial\Phi_1}{\partial t}\right)=-\frac{\tilde\beta_1}{2}\frac{\partial^2\Phi_1}{\partial t^2}+\alpha_1\Phi_1^*\Phi_2-\sigma|\Phi_1|^2\Phi_1, \quad (9)$$

$$i\left(\frac{\partial\Phi_2}{\partial z}+\frac{1}{v_2}\frac{\partial\Phi_2}{\partial t}\right)=\frac{2\omega}{c}(n_2-\tilde n_1)\Phi_2-\frac{\beta_2}{2}\frac{\partial^2\Phi_2}{\partial t^2}+\alpha_2\Phi_1^2, \quad (10)$$

where

$$\Phi_1=\psi_1\mathrm{e}^{i\omega\Lambda z/c},\quad \Phi_2=\psi_2\mathrm{e}^{-2i\omega(n_2-\tilde n_1)z/c}, \quad (11)$$

$$\sigma=\frac{2d^2}{\hbar^2}\frac{\omega_c\omega_0}{cn_1(\omega_0-\omega)^3}, \quad (12)$$

$$\tilde n_1=n_1+\Lambda. \quad (13)$$

For the addition Λ to the refractive index at the fundamental frequency, caused by impurity centers, we have

$$\Lambda=\frac{\omega_c}{n_1(\omega_0-\omega)}. \quad (14)$$

The group velocity $\tilde v_1$ and the DGV coefficient $\tilde\beta_1$ in the presence of these impurities are determined by the expressions

$$\frac{1}{\tilde v_1}=\frac{1}{v_1}+\frac{\omega_c\omega_0}{cn_1(\omega_0-\omega)^2}, \quad (15)$$

$$\tilde\beta_1=\beta_1+\frac{2\omega_c\omega_0}{cn_1(\omega_0-\omega)^3} \quad (16)$$

$\omega_c=2\pi d^2n/\hbar$ is the collective frequency [14].

The phase transformations (11) lead wave equation (1), (2) to the autonomous set of equations (9) and (10). These transformations generalize the corresponding transformations [15] for the case when QRCs are absence.

From (13)–(16) it follows that QRCs lead, in particular, to changes of the refractive index, group velocity and the DGV parameter at the fundamental frequency, as well as to the appearance of Kerr nonlinearity at this frequency (see last term on the right-hand side of equation (9)).

The parameter $|n_2 - \tilde{n}_1|$ characterizes the magnitude of the phase mismatching between pulses at the fundamental carrier frequency and at the second harmonic frequency. From (13) it is evident that impurity centers also affect on this magnitude.

The group mismatching is characterized by the difference of group velocities $\tilde{v}_1$ and v_2. As it follows from (15), there is a fundamental possibility of controlling by this difference with helping of QRCs.

The frequency ω_0 can be changed using a permanent external electric field, capable of causing a Stark shift of the quantum levels [16].

3. Dark parametric solitons

Let us make some preliminary numerical estimates. To satisfy the conditions (5), we assume $\omega \sim \omega_0 \sim 10^{15}$ s^{-1}, $|\omega_0 - \omega| \sim 10^{13}$ s^{-1}, $\tau_p \sim 10^{-12}$ s. Assuming also that $d \sim 10^{-18}$ SGSE, $n \sim 10^{17}$ cm^{-3}, we find $\omega_c \sim 10^9$ s^{-1}. Then from (12), (14) and (15) we have $\sigma \sim 10^{-7}$ SGSE, $\Lambda \sim 10^{-4}$, and $c\,|1/\tilde{v}_1 - 1/v_2| \sim 10^{-2}$. For the coefficient σ_{cr}, which determines the intrinsic Kerr nonlinearity of the crystal, we have $\sigma_{cr} = 6\pi\omega\chi^{(3)}/cn_1$ [4], where $\chi^{(3)}$ is the third-order optical susceptibility. Assuming that $\chi^{(3)} \sim 10^{-14}$ SGSE for a typical crystal [17], we find $\sigma_{cr} \sim 10^{-9}$ SGSE, that is two orders of magnitude smaller than the parameter σ, which determines the Kerr nonlinearity of the QRCs. Thus, the intrinsic Kerr nonlinearity of the crystal in the absence of QRCs can be neglected.

Assuming that the typical values of the parameter β_1 in the spectral area of optical transparency of the crystals are $|\beta_1| \sim 10^{-27} - 10^{-28}$ s^2 cm^{-1} [4], we find from (16) $\left|\tilde{\beta}_1/\beta_1\right| \sim 10^2 - 10^3$, $\tilde{\beta}_1 \sim 10^{-25}$ s^2 cm^{-1}.

Under conditions of the absence of QRCs the absolute values of DGV parameters at the fundamental frequency and at the second harmonic frequency are usually of the same order: $|\beta_1| \sim |\beta_2|$. Then we have $\left|\tilde{\beta}_1\right| >> |\beta_2|$, as it follows from (16) and from the numerical estimates given above. Therefore, with good accuracy in equation (10) we put $\beta_2 = 0$ [2, 18]. Under this condition the system (9), (10) has an exact solution in the form of dark solitons:

$$\Phi_1 = \Phi_{1m} e^{i(\Omega t - qz)} \tanh\left(\frac{t - z/v_2}{\tau_p}\right), \tag{17}$$

$$\Phi_2 = \Phi_{2m} e^{2i(\Omega t - qz)} \tanh^2\left(\frac{t - z/v_2}{\tau_p}\right), \tag{18}$$

where

$$\Omega = \frac{1}{\tilde{\beta}_1}\left(\frac{1}{v_2} - \frac{1}{\tilde{v}_1}\right), \quad q = \frac{\tilde{\beta}_1}{\tau_p^2} + \frac{1}{2\tilde{\beta}_1}\left(\frac{1}{v_2^2} - \frac{1}{\tilde{v}_1^2}\right), \tag{19}$$

$$\Phi_{1m} = \frac{1}{\tau_p}\sqrt{2\tilde{\beta}_1 \frac{\tilde{\beta}_1 - \mu\tau_p^2}{(\alpha_1\alpha_2 + 2\sigma\mu)\,\tau_p^2 - 2\tilde{\beta}_1\sigma}}, \tag{20}$$

$$\Phi_{2m} = \frac{\tilde{\beta}_1\alpha_2}{(\alpha_1\alpha_2 + 2\sigma\mu)\,\tau_p^2 - 2\tilde{\beta}_1\sigma}, \tag{21}$$

$$\mu = \tilde{\beta}_1\Omega^2 + \frac{\omega}{c}(n_2 - \tilde{n}_1). \tag{22}$$

The duration τ_p of the dark soliton is a free parameter here. The parameter Ω has the meaning of the frequency shift of fundamental soliton component. This parameter is determined by the magnitude of the group mismatching. In turn, the parameter μ is determined by both phase and group mismatchings (see (19) and (22)).

Assuming $\sigma = 0$, we will have from (17)–(22) the solutions, which were obtained in [3] for the case when the QRCs are absent. If, in addition, the conditions of PM and GM are strictly satisfied, we will have the solutions discussed in [2].

In order to match the solution (17)–(22) with the SVE approximation used above, the inequalities $\Omega << \omega$ and $q << \omega/c$ must be satisfied. It is clear from (19) that both inequalities are compatible with each other under the above condition $c\,|1/\tilde{v}_1 - 1/v_2| \sim 10^{-2} << 1$. Indeed, in this case we have

$$\frac{1}{2\tilde{\beta}_1}\left(\frac{1}{v_2^2} - \frac{1}{\tilde{v}_1^2}\right) \approx \frac{1}{v_2\tilde{\beta}_1}\left(\frac{1}{v_2} - \frac{1}{\tilde{v}_1}\right) = \frac{\Omega}{v_2} << \frac{\omega}{c}.$$

Using the above estimate $\tilde{\beta}_1 \sim 10^{-25}$ s^2 cm^{-1}, we have $\Omega \sim 10^{13}$ s^{-1}, which satisfies the inequality $\Omega << \omega$ with a good margin.

In the general case the analysis of the solution (17)–(22) seems to be very painstaking. We will limit ourselves to considering a typical situation inherent in the transparency area of crystals in the near infrared and visible areas [4]. According to the numerical estimates given above, we will assume that the DGV is determined mainly by QRCs.

For crystals such as barium or sodium niobate with the high quadratic nonlinearity we have $\chi^{(2)} \sim 10^{-7}$ SGSE [19]. Then $\alpha_1 \sim \alpha_2 \sim 0.1$ SGSE and $\alpha_1\alpha_2 >> \sigma\mu$. According with the estimates given above, QRCs have virtually no effect on the refractive index n_1. Therefore, we can put with a good accuracy $\tilde{n}_1 \approx n_1$. Assuming in addition to the estimates given above that $|n_2 - n_1| \sim 0.01$, from (22) we find $\mu \approx \omega\,(n_2 - n_1)/c \sim 10^3$. For $\tau_p \sim 10^{-12}$ s we have $\tilde{\beta}_1 << \mu\tau_p^2$. As a result, the expressions (20) and (21) take the form

$$F_{1m} = \frac{1}{\tau_p}\sqrt{\frac{2\tilde{\beta}_1\omega\,(n_1 - n_2)}{c\alpha_1\alpha_2}}, \quad F_{2m} = \frac{\tilde{\beta}_1}{\alpha_1\tau_p^2},. \tag{23}$$

For the intensities $I_{1,2} = c\Phi_{1m,2m}^2/2\pi$ we will have estimates of $I_1 \sim 10^7$ W cm^{-2} and $I_2 \sim 10^3$ W cm^{-2}. Thus, $I_2/I_1 \sim 10^{-4}$. When the conditions of PM and GM are valid, we have

$I_2/I_1 \sim 1$. As can be seen from the first expression (23), the formation of the dark solitons under consideration is mainly influenced by the phase mismatching. From the above estimates it is clear that phase mismatching, as expected, leads to a decrease of the efficiency of second harmonic generation. Taking into account that $\alpha_1\alpha_2 > 0$, from the first expression (23) and (16) under condition $\left|\tilde{\beta}_1\right| >> |\beta_1|$ we will have the necessary condition for the possibility of forming of a dark parametric soliton

$$(n_1 - n_2)(\omega_0 - \omega) > 0. \tag{24}$$

This condition can be satisfied by changing the sign of detuning $\omega_0 - \omega$. As was said above, it is can be done with an external Stark field [16]. Thus, by means of QRCs, satisfying inequality (24), it is possible to effectively formation of the dark two-color solitons.

The temporal solitons are formed at a distance of the order of dispersion length $l_{\text{dis}} = 2\tau_p^2/\left|\tilde{\beta}_1\right|$. Then at the above parameters we have $l_{\text{dis}} \sim 10$ cm. In this case the diffraction broadening should not make itself known. Taking for the aperture D of the input optical pulse $D \sim 1$ mm, we will have $l_{\text{dif}} \sim \omega D^2/c \sim 10^3$ cm, where l_{dif} is the diffraction length. Thus, the condition $l_{\text{dis}} << l_{\text{dif}}$ is satisfied with a good margin.

4. Concluding remarks

The study, which is carried out in this paper, showed that the presence of QRCs in the crystal promotes the formation of temporary dark parametric solitons in the second harmonic generation mode. However, the generation efficiency is relatively low due to the strong influence of phase mismatching.

The next step in the intended direction is to study of the possibility of forming of the spatiotemporal bright-dark solitons in the presence of impurity QRCs, as well as in the presence of phase and group mismatchings. In [11] it was shown that such solitons can be formed under normal DGV. In our case the main contribution into DGV is made by QRCs. In this case the sign of DGV can be effectively controlled by changing of sign of the frequency detuning $\omega_0 - \omega$. Therefore, it should be expected that the presence of QRCs can stimulate the formation of the parametric bright-dark spatiotemporal solitons under real experimental conditions.

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