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CONSTRUCTION OF FINITELY PRESENTED INFINITE NILL-SEMGROUP

Abstract. The talk is devoted to construction of finitely presented infinite nil-semigroup. In order to do it, we use some geometric methods for the construction.

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The talk is devoted to construction of finitely presented infinite nil-semigroup. This problem was posed by L. N. Shevrin in Sverdlovskaya tetrad, problem 3.61. Our research follows the program, proposed V. N. Latyshev of construction objects via finitely many defining relations. First step of our research was concentrated on construction of finitely presented semigroups with non-integer Gelfand-Kirillov dimension. Generators of the semigroup are interpreted as states of finite automata, and element of semigroup as a set of finite automata, connected in the line. Defining relations corresponds to interaction.

The nil-semigroup problem is an essential part of the existence problem of finitely presented nil-rings of infinite dimension (posed by V. N. Latyshev). In present time there are no examples of such nil-objects which can be finitely presented, thus this construction is an object of special interest. This problem has special interest from the point of view of computer science. Is it possible to construct interaction law of finite automata posed in the line such that if one puts each one in arbitrary state the resulting behavior will be sensefull.

In order to construct finitely presented non-nilpotent nil-semigroup, we use geometric methods. There exists examples of special tiling sets which can be used for aperiodic plane tiling only. There are Penrose and Robinson tilings, and original Berger construction. These constructions use special consideration of tiles as the elements of Turing machine.

Rewriting diagrams allows to use this ideas in semigroups construction. This construction was inspired by Novikov-Adian method. Note that rewriting diagrams in group theory allows to get much better estimations then Van-Kampen diagrams. In semigroup theory our tilings allows to use angle information and more complicated information due to tiling structure. We consider the edges of tiles as the letters in some finite alphabet. Thus, the word in semigroup can be assigned as a path on the tile lattice (mosaic). We can assign relations by equivalences of some paths on this tiling.

In order to construct nil-semigroup we should consider paths on strongly aperiodic tiling (there are no periodic path in such tilings). We assign relations semigroup to obtain the following property:

If the word corresponds to the path that could not situated in the tiling, this word can be reduced to zero word.

In further words, the mosaic can be considered by the special metric space having some special properties.

First, there exist a constant $\lambda > 0$ such that for any two points $A$ and $B$ on distance $D$ there are two geodesic paths $C_1$ and $C_2$ joining them of “widthness” $\lambda \cdot D$. It means that for any $d \leq D/2$ distant from point of $x \in C_1, \rho(x, A) = d$ to the path $C_2$ is not less then $\lambda \cdot D$.

Second, this space have locally finite structure in some sense, i.e. it can be presented by the infinite 2-dimensional complex having vertices of bounded order on each face.

Relations of the semigroup can be assigned by two ways:

(1) Flips of small paths on this tiling (i.e. pairs of edges into pair of edges).

(2) Monomial relations (assuming some words are equal zero). These relations cancel locally non-geodesic paths and locally unextendable paths.

In order to construct this graph we use some DOL-system technique: we assign how to cut squares of finite number of types into some small squares of smaller order. This graph has a special property: each path situated in the some square of high order can be reduced to the path on the boundary of that square or to the zero word. This property used to obtain that any two points can be connected by two geodesic paths (far enough).

Our structure has very specific “fractal” structure, obtained by substitution type construction. In order to use induction, the construction is based on the prototypes such that any path can be transformed to the border of each prototype. In order to achieve first property we must glue some additional fragment. This cause multi-dimensionality.

Consider some arbitrary word in the semigroup. Using flip-relations we can transform this word. We can consider this process as “reconstruction” of the area near the path corresponding to this word. If in some time we can use monomial relation then this path could not situated on the mosaic and this word can be reduced to zero word.

Thus, there are infinite number of geodesic paths on this infinite mosaic. Also, each path that is not situated on the mosaic can be reduced to zero word. Periodic path could not be situated on this mosaic.
REFERENCES

1. P. S. Novikov, S. I. Adian, Burnside problems and identities in words in groups. 
   Moscow, Nauka, 1971.

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