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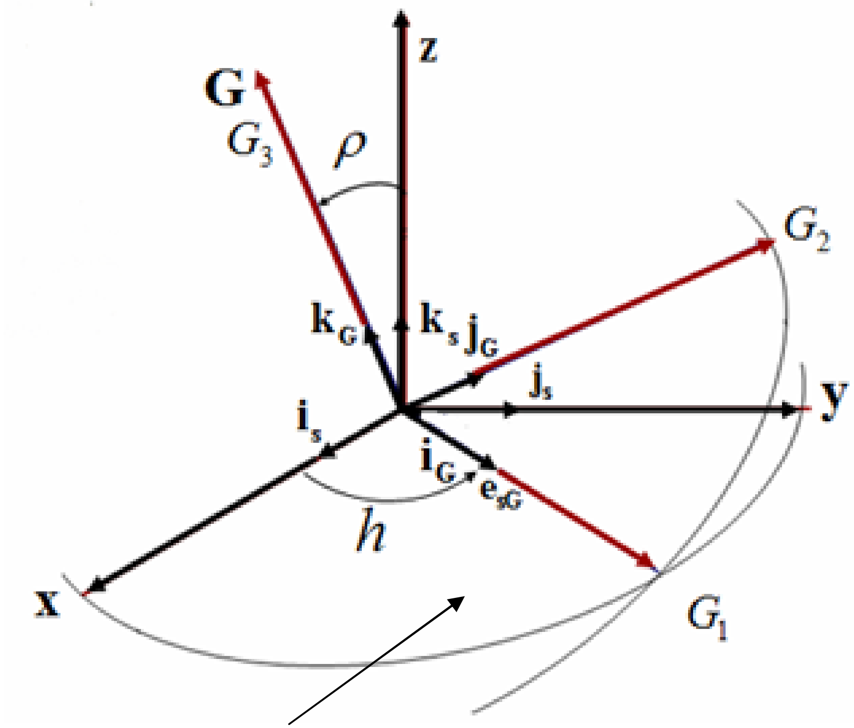
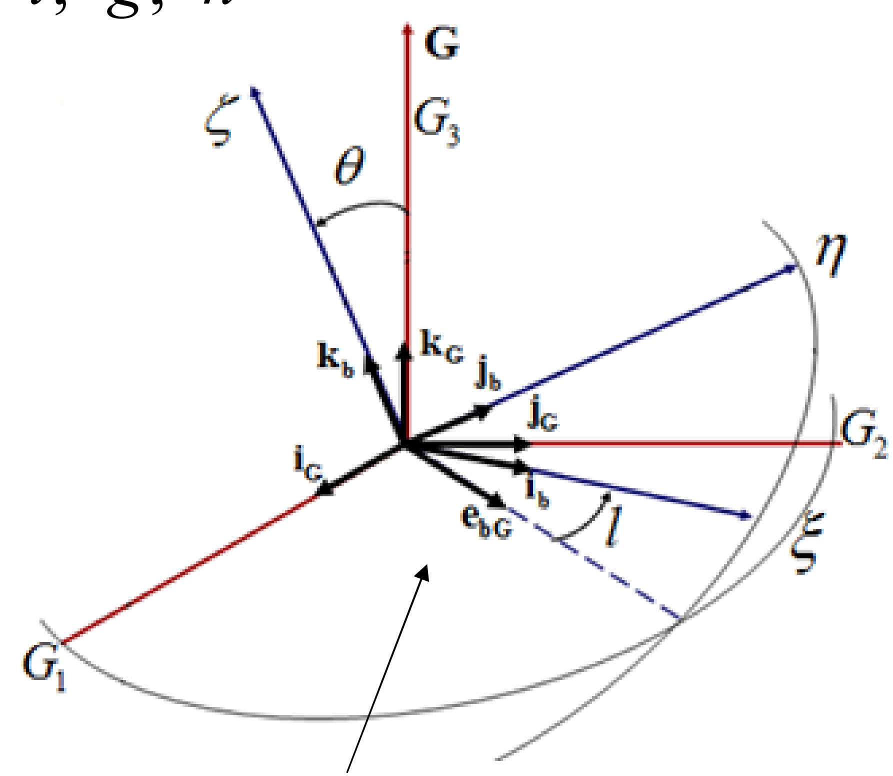
Динамика возмущенного вращательного движения небесных тел

Переменные Андуайе

$$\mathbf{G}: \; G,\ \theta,\ \rho,\ l,\ g,\ h$$



Основная плоскость



Промежуточная плоскость Андуайе

Канонические переменные Андуайе

$$\mathbf{G}: \; L,\ G,\ H,\ l,\ g,\ h$$

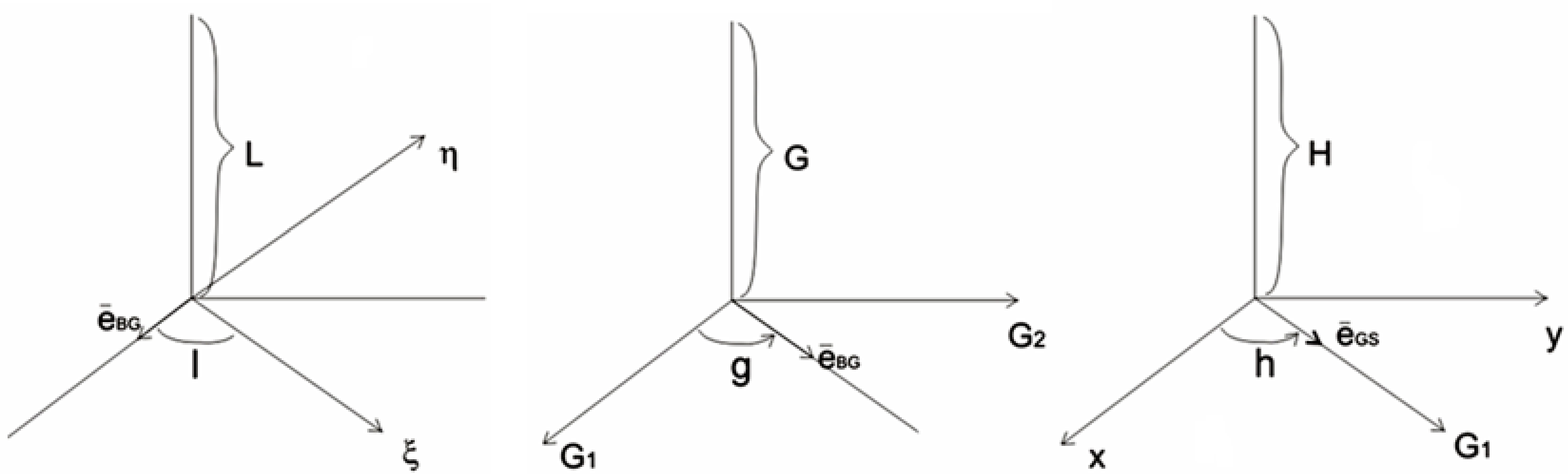
$$L = \mathbf{G}\cdot\mathbf{k_b} = G\left(\mathbf{k_G}\cdot\mathbf{k_b}\right) = Gb_{33} = G\cos\theta$$

$$H = \mathbf{G}\cdot\mathbf{k_s} = G\left(\mathbf{k_G}\cdot\mathbf{k_s}\right) = Gg_{33} = G\cos\rho$$

Глава 11. Дифференциальные уравнения вращательного движения твердого тела в переменных Андуайе и Пуанкаре.

$$G\ ,\quad \theta\ ,\quad \rho\ ,\quad l\ ,\quad g\ ,\quad h\ ,$$

$$L = \mathbf{G}\cdot\mathbf{k_b} = G\left(\mathbf{k_G}\cdot\mathbf{k_b}\right) = Gb_{33} = G\cos\theta\ ,\quad H = \mathbf{G}\cdot\mathbf{k_s} = G\left(\mathbf{k_G}\cdot\mathbf{k_s}\right) = Gg_{33} = G\cos\rho\ ;$$



$$L = -Ep - Dq + Cr\ ,$$

$$G = \sqrt{\left(A^2+F^2+E^2\right)p^2 + \left(F^2+B^2+D^2\right)r^2 + +2\left(-AF-FB+ED\right)pq + 2\left(-AE+FD-EC\right)pr + 2\left(FE-BD-DC\right)qr}\ ,$$

$$H = p\left(A\sin\Theta\sin\Phi - F\sin\Theta\cos\Phi - E\cos\Theta\right) + q\left(-F\sin\Theta\sin\Phi + B\sin\Theta\cos\Phi - D\cos\Theta\right) +$$

$$+ r\left(-E\sin\Theta\sin\Phi - D\sin\Theta\cos\Phi + C\cos\Theta\right).$$

$$L = p_\Phi ,$$

$$G = \sqrt{p_\Theta^2 + p_\Phi^2 + \operatorname{cosec}^2\Theta \left(p_\Psi - p_\Phi \cos\Theta\right)^2} ,$$

$$H = p_\Psi .$$

$$p = G\left[\sin\theta\left(a\sin l - f\cos l\right) - e\cos\theta\right],$$

$$q = G\left[\sin\theta\left(-f\sin l + b\cos l\right) - d\cos\theta\right],$$

$$r = G\left[\sin\theta\left(-e\sin l - d\cos l\right) + c\cos\theta\right].$$

$$p = \frac{G}{A}\sin\theta\sin l ,$$

$$q = \frac{G}{B}\sin\theta\cos l ,$$

$$r = \frac{G}{C}\cos\theta .$$

$$\omega = \sqrt{p^2 + q^2 + r^2} =$$

$$= G\sqrt{\begin{aligned}&\sin^2\theta\sin^2 l\left(a^2 + f^2 + e^2\right) + \sin^2\theta\cos^2 l\left(f^2 + b^2 + d^2\right) + \cos^2\theta\left(e^2 + d^2 + c^2\right) + \\ &+ \sin^2\theta\sin 2l\left(e - af - bf\right) + \sin 2\theta\sin l\left(df - ea - ce\right) + \sin 2\theta\cos l\left(ef - db - cd\right)\end{aligned}} ,$$

$$\omega = G\sqrt{\sin^2\theta\left(\frac{\sin^2 l}{A^2} + \frac{\cos^2 l}{B^2}\right) + \cos^2\theta\frac{1}{C^2}}$$

11.4. Доказательство каноничности преобразования от переменных Эйлера $\Psi, \Theta, \Phi, p_\Psi, p_\Theta, p_\Phi$ к переменным Андуайе L, G, H, l, g, h.

$$\mathbf{G}d\mathbf{\Omega} = p_\Psi d\Psi + p_\Theta d\Theta + p_\Phi d\Phi \qquad Ldl + Gdg + Hdh = \mathbf{G}d\mathbf{\Omega}$$

$$L = p_\Phi,$$

$$G = \sqrt{p_\Theta^2 + p_\Phi^2 + \operatorname{cosec}^2\Theta\left(p_\Psi - p_\Phi\cos\Theta\right)^2},$$

$$H = p_\Psi,$$

$$l = \Phi - \arccos\left\{\frac{p_\Psi - p_\Phi\cos\Theta}{\sqrt{p_\Theta^2 + \operatorname{cosec}^2\Theta\left(p_\Psi - p_\Phi\cos\Theta\right)^2}\sin\Theta}\right\},$$

$$g = \arccos\left\{\frac{p_\Psi p_\Phi - \left[p_\Theta^2 + p_\Phi^2 + \operatorname{cosec}^2\Theta\left(p_\Psi - p_\Phi\cos\Theta\right)^2\right]\cos\Theta}{\sqrt{p_\Theta^2 + \operatorname{cosec}^2\Theta\left(p_\Psi - p_\Phi\cos\Theta\right)^2}\sqrt{p_\Theta^2 + p_\Phi^2\operatorname{cosec}^2\Theta - c\tan^2\Theta p_\Psi^2 - 2\operatorname{cosec}\Theta c\tan\Theta p_\Psi p_\Phi}}\right\},$$

$$h = \Psi - \arccos\left\{\frac{p_\Phi - p_\Psi\cos\Theta}{\sqrt{p_\Theta^2 + p_\Phi^2\operatorname{cosec}^2\Theta - c\tan^2\Theta p_\Psi^2 - 2\operatorname{cosec}\Theta c\tan\Theta p_\Psi p_\Phi}}\right\}.$$

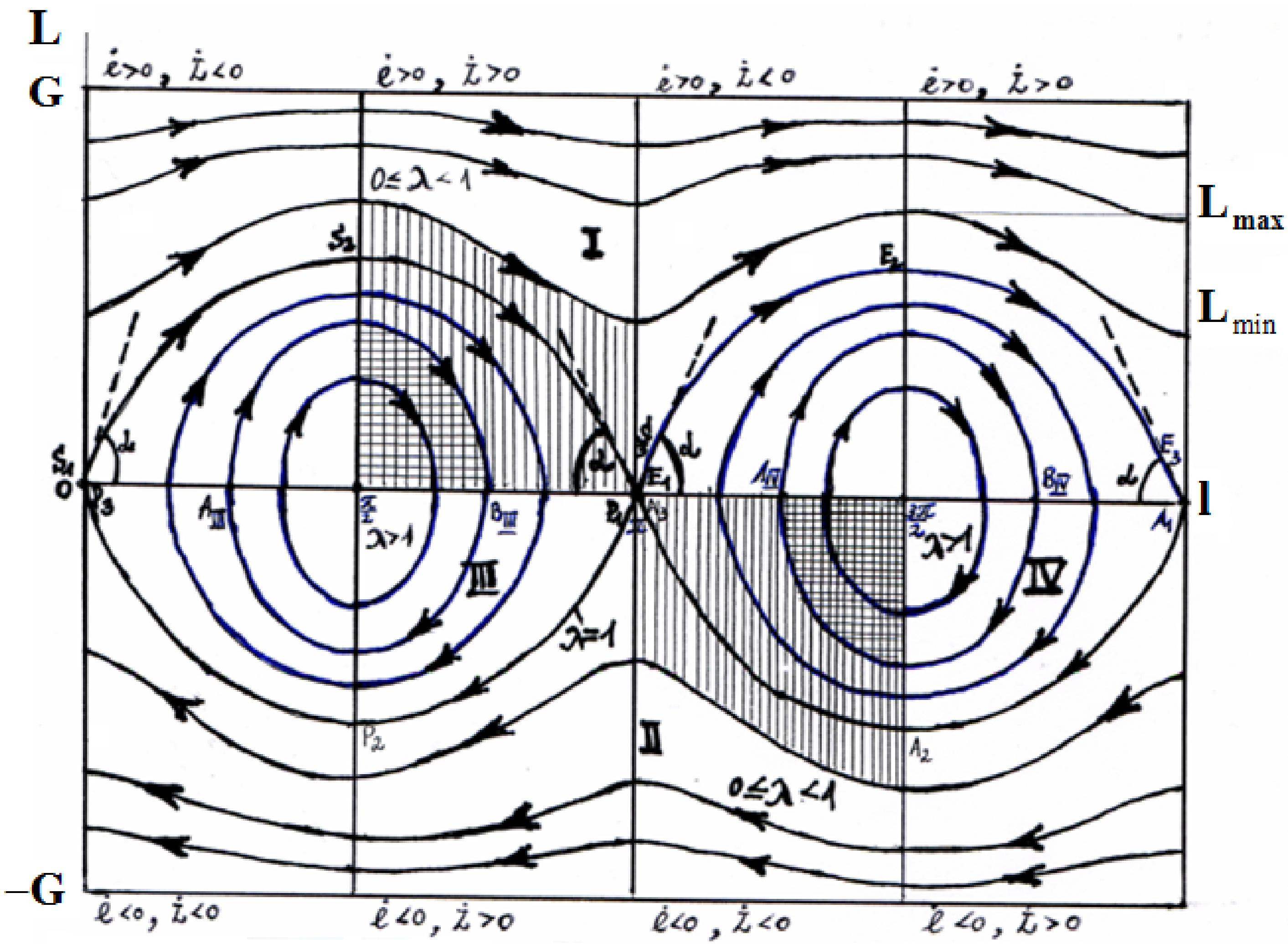
Фазовая плоскость. Области движения I и III.

$$L = L(l, \lambda, \kappa, G) = \frac{G\kappa}{\sqrt{\kappa^2 + \lambda^2}} \sqrt{\frac{1 - \lambda^2 + (\kappa^2 + \lambda^2)\sin^2 l}{1 + \kappa^2 \sin^2 l}}$$

$$\kappa^2 = \frac{C(B-A)}{A(C-B)} > 0,$$

$$\lambda^2 = \frac{A^2}{C^2} \kappa^2 \frac{p_0^2}{r_0^2} > 0$$

$$\lambda = \sqrt{1 - \frac{L_{\min}^2}{L_{\max}^2}}$$



11.5. Канонические уравнения вращательного движения твердого тела в переменных Андуайе.

$$\frac{dL}{dt} = -\frac{\partial F}{\partial l}, \qquad \frac{dl}{dt} = \frac{\partial F}{\partial L},$$

$$\frac{dG}{dt} = -\frac{\partial F}{\partial g}, \qquad \frac{dg}{dt} = \frac{\partial F}{\partial G},$$

$$\frac{dH}{dt} = -\frac{\partial F}{\partial h}, \qquad \frac{dh}{dt} = \frac{\partial F}{\partial H},$$

$$F = T - U$$

$$F = \frac{1}{2}\left(G^2 - L^2\right)\left(a\sin^2 l + b\cos^2 l - f\sin 2l\right) + \frac{1}{2}cL^2 -$$

$$-L\sqrt{G^2 - L^2}\left(d\cos l + e\sin l\right) - U\left(L, G, H, l, g, h, t\right).$$

Уравнения движения в переменных Андуайе:

$$G,\ \theta,\ \rho,\ l,\ g,\ h$$

$$\frac{dG}{dt} = \frac{\partial U}{\partial g}$$

$$\frac{d\theta}{dt} = \frac{1}{2} G \sin\theta \left(\frac{1}{A} - \frac{1}{B} \right) \sin 2l + \frac{1}{G} \cot\theta \frac{\partial U}{\partial g} - \frac{1}{G} \csc\theta \frac{\partial U}{\partial l}$$

$$\frac{d\rho}{dt} = \frac{1}{G} \cot\rho \frac{\partial U}{\partial g} - \frac{1}{G} \csc\rho \frac{\partial U}{\partial h}$$

$$\frac{dl}{dt} = -G \cos\theta \left(\frac{1}{C} - \frac{\sin^2 l}{A} - \frac{\cos^2 l}{B} \right) + \frac{1}{G} \csc\theta \frac{\partial U}{\partial \theta}$$

$$\frac{dg}{dt} = G \left(\frac{\sin^2 l}{A} + \frac{\cos^2 l}{B} \right) - \frac{1}{G} \cot\theta \frac{\partial U}{\partial \theta} - \frac{1}{G} \cot\rho \frac{\partial U}{\partial \rho}$$

$$\frac{dh}{dt} = -\frac{1}{G} \csc\rho \frac{\partial U}{\partial \rho}$$

$$U = U(\theta, \rho, l, g, h, t)$$

11.7.1. Канонические уравнения в переменных Пуанкаре.

$$\frac{d\Lambda}{dt} = -\frac{\partial F}{\partial \lambda}, \qquad \frac{d\lambda}{dt} = \frac{\partial F}{\partial \Lambda},$$

$$\frac{d\Gamma}{dt} = -\frac{\partial F}{\partial \gamma}, \qquad \frac{d\gamma}{dt} = \frac{\partial F}{\partial \Gamma}, \qquad F = F_0 - U$$

$$\frac{dZ}{dt} = -\frac{\partial F}{\partial z}, \qquad \frac{dz}{dt} = \frac{\partial F}{\partial Z},$$

$$F_0 = \frac{1}{2}\left(\frac{\sin^2\gamma}{A} + \frac{\cos^2\gamma}{B}\right)\Lambda^2 + \frac{1}{2}\left(\frac{1}{C} - \frac{\sin^2\gamma}{A} - \frac{\cos^2\gamma}{B}\right)\left(\Lambda^2 - 2\Lambda\Gamma + \Gamma^2\right)$$

$$\Lambda = G, \qquad \lambda = l + g + h,$$

$$\Gamma = 2G\sin^2\frac{\theta}{2} = G(1-\cos\theta), \quad \gamma = -l,$$

$$Z = 2G\sin^2\frac{\rho}{2} = G(1-\cos\rho), \quad z = -h.$$

$$\Lambda = \Lambda\,, \qquad \lambda = \lambda\,,$$
$$\xi = \sqrt{2\Gamma}\cos\gamma\,, \quad \eta = \sqrt{2\Gamma}\sin\gamma\,,$$
$$p = \sqrt{2Z}\cos z\,, \quad q = \sqrt{2Z}\sin z\,.$$

$$\frac{d\Lambda}{dt} = -\frac{\partial F}{\partial \lambda}, \qquad \frac{d\lambda}{dt} = \frac{\partial F}{\partial \Lambda},$$
$$\frac{d\eta}{dt} = -\frac{\partial F}{\partial \xi}, \qquad \frac{d\xi}{dt} = \frac{\partial F}{\partial \eta},$$
$$\frac{dp}{dt} = -\frac{\partial F}{\partial q}, \qquad \frac{dq}{dt} = \frac{\partial F}{\partial p},$$
$$F = F(\Lambda, \xi, \eta, p, q, \lambda, t)\,.$$

$$\Lambda = G\,, \qquad \lambda = l + g + h\,,$$
$$\xi = 2\sqrt{G}\sin\frac{\theta}{2}\cos l\,, \quad \eta = -2\sqrt{G}\sin\frac{\theta}{2}\sin l\,,$$
$$p = 2\sqrt{G}\sin\frac{\rho}{2}\cos h\,, \quad q = -2\sqrt{G}\sin\frac{\rho}{2}\sin h\,.$$

$$\Lambda = \Lambda\,, \qquad \lambda = \lambda\,,$$
$$\xi = \sqrt{2\Gamma}\cos\gamma\,, \quad \eta = \sqrt{2\Gamma}\sin\gamma\,,$$
$$p = \sqrt{2Z}\cos z\,, \quad q = \sqrt{2Z}\sin z\,.$$

$$\frac{d\Lambda}{dt} = -\frac{\partial F}{\partial \lambda}, \qquad \frac{d\lambda}{dt} = \frac{\partial F}{\partial \Lambda},$$
$$\frac{d\eta}{dt} = -\frac{\partial F}{\partial \xi}, \qquad \frac{d\xi}{dt} = \frac{\partial F}{\partial \eta},$$
$$\frac{dp}{dt} = -\frac{\partial F}{\partial q}, \qquad \frac{dq}{dt} = \frac{\partial F}{\partial p},$$
$$F = F(\Lambda, \xi, \eta, p, q, \lambda, t)\,.$$

$$\Lambda = G\,, \qquad \lambda = l + g + h\,,$$
$$\xi = 2\sqrt{G}\sin\frac{\theta}{2}\cos l\,, \quad \eta = -2\sqrt{G}\sin\frac{\theta}{2}\sin l\,,$$
$$p = 2\sqrt{G}\sin\frac{\rho}{2}\cos h\,, \quad q = -2\sqrt{G}\sin\frac{\rho}{2}\sin h\,.$$

$$U = U\left(\theta, \rho, l, g, h, t\right) = U\left(b_{ij}, \rho, h, t\right)$$

$$b_{11} = \cos\left(\mathbf{i_b}, \mathbf{i_G}\right) = \left(\mathbf{i_b} \cdot \mathbf{i_G}\right) = \cos l \cos g - \sin l \sin g \cos\theta,$$
$$b_{21} = \cos\left(\mathbf{i_b}, \mathbf{j_G}\right) = \left(\mathbf{i_b} \cdot \mathbf{j_G}\right) = \cos l \sin g + \sin l \cos g \cos\theta,$$
$$b_{31} = \cos\left(\mathbf{i_b}, \mathbf{k_G}\right) = \left(\mathbf{i_b} \cdot \mathbf{k_G}\right) = \sin\theta \sin l,$$
$$b_{12} = \cos\left(\mathbf{j_b}, \mathbf{i_G}\right) = \left(\mathbf{j_b} \cdot \mathbf{i_G}\right) = -\sin l \cos g - \cos l \sin g \cos\theta,$$
$$b_{22} = \cos\left(\mathbf{j_b}, \mathbf{j_G}\right) = \left(\mathbf{j_b} \cdot \mathbf{j_G}\right) = -\sin l \sin g + \cos l \cos g \cos\theta,$$
$$b_{32} = \cos\left(\mathbf{j_b}, \mathbf{k_G}\right) = \left(\mathbf{j_b} \cdot \mathbf{k_G}\right) = \cos l \sin\theta,$$
$$b_{13} = \cos\left(\mathbf{k_b}, \mathbf{i_G}\right) = \left(\mathbf{k_b} \cdot \mathbf{i_G}\right) = \sin g \sin\theta,$$
$$b_{23} = \cos\left(\mathbf{k_b}, \mathbf{j_G}\right) = \left(\mathbf{k_b} \cdot \mathbf{j_G}\right) = -\cos g \sin\theta,$$
$$b_{33} = \cos\left(\mathbf{k_b}, \mathbf{k_G}\right) = \left(\mathbf{k_b} \cdot \mathbf{k_G}\right) = \cos\theta;$$

$$b_{11} = \cos\lambda - \frac{1}{2\Lambda}\left(\xi\eta \sin\lambda + \eta^2 \cos\lambda\right),$$
$$b_{21} = \sin\lambda + \frac{1}{2\Lambda}\left(\xi\eta \cos\lambda - \eta^2 \sin\lambda\right),$$
$$b_{31} = -\frac{1}{\sqrt{\Lambda}}\cos\left(\theta/2\right)\eta,$$
$$b_{12} = -\sin\lambda + \frac{1}{2\Lambda}\left(\xi^2 \sin\lambda + \xi\eta \cos\lambda\right),$$
$$b_{22} = \cos\lambda + \frac{1}{2\Lambda}\left(-\xi^2 \cos\lambda + \xi\eta \sin\lambda\right),$$
$$b_{32} = \frac{1}{\sqrt{\Lambda}}\cos\left(\theta/2\right)\xi,$$
$$b_{13} = \frac{1}{\sqrt{\Lambda}}\cos\left(\theta/2\right)\left(\xi \sin\lambda + \eta \cos\lambda\right),$$
$$b_{23} = -\frac{1}{\sqrt{\Lambda}}\cos\left(\theta/2\right)\left(\xi \cos\lambda - \eta \sin\lambda\right),$$
$$b_{33} = \cos\theta.$$

Уравнения вращательного движения в переменных действие-угол

$$\frac{d\varphi_1}{dt}=\frac{\partial F}{\partial I_1},\qquad \frac{dI_1}{dt}=-\frac{\partial F}{\partial \varphi_1},$$

$$\frac{d\varphi_2}{dt}=\frac{\partial F}{\partial I_2},\qquad \frac{dI_2}{dt}=-\frac{\partial F}{\partial \varphi_2},$$

$$\frac{d\varphi_3}{dt}=\frac{\partial F}{\partial I_3},\qquad \frac{dI_3}{dt}=-\frac{\partial F}{\partial \varphi_3},$$

$$F=F_0\left(I_1,I_2\right)-U\left(I_1,I_2,I_3,\varphi_1,\varphi_2,\varphi_3,t\right)$$

$$F=\frac{1}{2}L^2\left(\frac{1}{C}-\frac{\sin^2 l}{A}-\frac{\cos^2 l}{B}\right)+\frac{1}{2}G^2\left(\frac{\sin^2 l}{A}+\frac{\cos^2 l}{B}\right)-U(L,G,H,l,g,h,t)$$

$$F_0=\frac{I_2^2}{2A}\left[1+\frac{A-C}{C}\frac{\kappa^2}{\lambda^2+\kappa^2}\right],$$

12.2. Дифференциальные уравнения возмущенного вращательного движения в переменных $G,\rho,\lambda,\varphi,\psi,h$.

$$I_1 = G\mathbf{\Lambda}(\lambda),\quad I_2 = G,\quad I_3 = G\cos\rho,$$

$$\varphi_1 = \varphi,\quad \varphi_2 = \psi,\quad \varphi_3 = h.$$

$$U = U(\lambda,\rho,\varphi,\psi,h,t)$$

$$\dot{G} = \frac{\partial U}{\partial \psi},$$

$$\dot{\lambda} = -\frac{1}{G\mathbf{J}(\lambda)}\frac{\partial U}{\partial \varphi} + \frac{\mathbf{\Lambda}(\lambda)}{G\mathbf{J}(\lambda)}\frac{\partial U}{\partial \psi},$$

$$\dot{\rho} = \frac{1}{G}\cot\rho\,\frac{\partial U}{\partial \psi} - \frac{1}{G}\csc\rho\,\frac{\partial U}{\partial h},$$

$$\dot{\varphi} = \mathbf{\Omega}(G,\lambda) + \frac{1}{G\mathbf{J}(\lambda)}\frac{\partial U}{\partial \lambda},$$

$$\dot{\psi} = \boldsymbol{\omega}(G,\lambda) - \frac{\mathbf{\Lambda}(\lambda)}{G\mathbf{J}(\lambda)}\frac{\partial U}{\partial \lambda} - \frac{1}{G}\cot\rho\,\frac{\partial U}{\partial \rho},$$

$$\dot{h} = \frac{1}{G}\csc\rho\,\frac{\partial U}{\partial \rho},$$

$$\mathbf{\Omega}(G,\lambda) = \frac{\pi G}{2\mathbf{K}(\lambda)}\frac{A-C}{AC}\frac{\kappa}{\sqrt{(1+\kappa^2)(\lambda^2+\kappa^2)}}\qquad \mathbf{J}(\lambda) = \frac{2\kappa\lambda\sqrt{1+\kappa^2}}{\pi(\kappa^2+\lambda^2)^{3/2}}\mathbf{K}(\lambda)$$

$$\boldsymbol{\omega}(G,\lambda) = \frac{G}{C}\left(1 - \frac{A-C}{A}\frac{\mathbf{\Pi}(\kappa^2,\lambda)}{\mathbf{K}(\lambda)}\right)\quad \mathbf{\Lambda}(\lambda) = \frac{2\sqrt{1+\kappa^2}}{\pi\kappa\sqrt{\kappa^2+\lambda^2}}\left\{(\kappa^2+\lambda^2)\mathbf{\Pi}\left(\frac{\pi}{2},\kappa^2,\lambda\right) - \lambda^2\mathbf{K}(\lambda)\right\}$$

12.2. Дифференциальные уравнения возмущенного вращательного движения в переменных $G,\rho,\lambda,\varphi,\psi,h$.

$$\dot{G}=\frac{\partial U}{\partial \psi},$$

$$\dot{\lambda}=-\frac{\pi\left(\kappa^2+\lambda^2\right)^{3/2}}{2G\kappa\lambda\sqrt{1+\kappa^2}\mathbf{K}(\lambda)}\frac{\partial U}{\partial \varphi}+\frac{\left(\kappa^2+\lambda^2\right)}{G\lambda\kappa^2\mathbf{K}(\lambda)}\left[\left(\kappa^2+\lambda^2\right)\mathbf{\Pi}\left(\frac{\pi}{2},\kappa^2,\lambda\right)-\lambda^2\mathbf{K}(\lambda)\right]\frac{\partial U}{\partial \psi},$$

$$\dot{\rho}=\frac{1}{G}\cot\rho\,\frac{\partial U}{\partial \psi}-\frac{1}{G}\csc\rho\,\frac{\partial U}{\partial h},$$

$$\dot{\varphi}=\frac{\pi G}{2\mathbf{K}(\lambda)}\frac{A-C}{AC}\frac{\kappa}{\sqrt{\left(1+\kappa^2\right)\left(\lambda^2+\kappa^2\right)}}+\frac{\pi\left(\kappa^2+\lambda^2\right)^{3/2}}{2G\kappa\lambda\sqrt{1+\kappa^2}\mathbf{K}(\lambda)}\frac{\partial U}{\partial \lambda},$$

$$\dot{\psi}=\frac{G}{C}\left(1-\frac{A-C}{A}\frac{\mathbf{\Pi}\left(\kappa^2,\lambda\right)}{\mathbf{K}(\lambda)}\right)-\frac{\left(\kappa^2+\lambda^2\right)}{G\lambda\kappa^2\mathbf{K}(\lambda)}\left[\left(\kappa^2+\lambda^2\right)\mathbf{\Pi}\left(\frac{\pi}{2},\kappa^2,\lambda\right)-\lambda^2\mathbf{K}(\lambda)\right]\frac{\partial U}{\partial \lambda}-$$

$$-\frac{1}{G}\cot\rho\,\frac{\partial U}{\partial \rho},$$

$$\dot{h}=\frac{1}{G}\csc\rho\,\frac{\partial U}{\partial \rho},$$

$$U=U\left(\lambda,\rho,\varphi,\psi,h,t\right).$$

12.3. К выводу уравнений возмущенного вращательного движения твердого тела в случае непотенциальных сил.

$$\dot p=\frac{(B-C)}{A}qr+\frac{M_\xi}{A},\quad \dot q=\frac{(C-A)}{B}rp+\frac{M_\eta}{B},\quad \dot r=\frac{(A-B)}{C}pq+\frac{M_\zeta}{C},$$

$$\dot G=\frac{1}{G}(b_{31}M_\xi+b_{32}M_\eta+b_{33}M_\zeta),$$

$$\dot\lambda=\frac{\sqrt{\kappa^2+\lambda^2}}{G}\left(b_{31}M_\xi+b_{32}\frac{1-\lambda^2}{1+\kappa^2}M_\eta-b_{33}\frac{\lambda^2}{\kappa^2}M_\zeta\right),$$

$$\dot\varphi_1=n_1(\lambda,\kappa)+\frac{\pi(\kappa^2+\lambda^2)}{2G\mathbf{K}\lambda^2 dnu\sqrt{1+\kappa^2}}\left\{\left[b_{32}-\frac{\sqrt{1+\kappa^2}}{\lambda'^2}b_{31}\mathbf{F}(u)\right]M_\xi-\right.$$

$$\left.-\left[b_{31}+\frac{1}{\sqrt{1+\kappa^2}}b_{32}\mathbf{F}(u)\right]M_\eta+\left[\frac{\sqrt{1+\kappa^2}}{\lambda'^2\kappa^2}b_{33}\mathbf{F}(u)\right]M_\zeta\right\},$$

$$\dot g=\dot\varphi_2+g_{\varphi_1}\dot\varphi_1+g_\lambda\dot\lambda,$$

$$g_{\varphi_1}=-\frac{\mathbf{K}i}{\pi}\left[Zn(\varphi_1-i\sigma)-Zn(\varphi_1+i\sigma)\right],$$

$$g_\lambda=\frac{\mathbf{K}i}{\pi}\left[Zn(\varphi_1-i\sigma)+Zn(\varphi_1+i\sigma)\right]\sigma_\lambda+\frac{i}{2q}\frac{\partial q}{\partial\lambda}\left[\frac{1}{\theta_4(\varphi_1-i\sigma)}-\frac{1}{\theta_4(\varphi_1+i\sigma)}\right].$$

Непотенциальные силы

$$\dot{G} = M_\xi b_{31} + M_\eta b_{32} + M_\zeta b_{33},$$

$$\dot{\rho} = -\frac{1}{G}(M_\xi b_{21} + M_\eta b_{22} + M_\zeta b_{23}),$$

$$\dot{h} = \frac{1}{G\sin\rho}(M_\xi b_{21} + M_\eta b_{22} + M_\zeta b_{23}),$$

$$\dot{\lambda} = \frac{(\kappa^2+\lambda^2)}{G\lambda}\left(M_\xi b_{31} + \frac{\lambda'^2}{1+\kappa^2}M_\eta b_{32} - \frac{\lambda^2}{\kappa^2}M_\zeta b_{33}\right),$$

$$\dot{\varphi}_1 = n_1 + \Phi_{11}M_\xi + \Phi_{12}M_\eta + \Phi_{13}M_\zeta,$$

$$\dot{\varphi}_2 = n_2 + \Phi_{21}M_\xi + \Phi_{22}M_\eta + \Phi_{23}M_\zeta,$$

$$\Phi_{11} = \left[\frac{b_{32}}{dnu} - \Gamma\lambda\sqrt{1+\kappa^2}b_{31}\right],$$

$$\Phi_{12} = -\left[\frac{b_{31}}{dnu} + \Gamma\frac{\lambda\lambda'^2}{\sqrt{1+\kappa^2}}b_{32}\right],$$

$$\Phi_{13} = \frac{\lambda^3\sqrt{1+\kappa^2}}{\kappa^2}b_{33},$$

$$\Gamma = \frac{1}{\lambda\lambda'^2}\left[\frac{\lambda^2 snucnu}{dnu} - \mathbf{Zn}u\right],$$

$$\Phi_{21} = \frac{1}{G}\left\{\cot\rho b_{11} + \cot\theta\cos l + \Phi(u,\lambda)\frac{(\kappa^2+\lambda^2)}{\lambda}b_{31}\right\},$$

$$\Phi_{22} = \frac{1}{G}\left\{\cot\rho b_{12} - \cot\theta\sin l + \Phi(u,\lambda)\frac{\lambda'^2(\kappa^2+\lambda^2)}{\lambda(1+\kappa^2)}b_{32}\right\},$$

$$\Phi_{23} = \frac{1}{G}\left\{\cot\rho b_{13} - \Phi(u,\lambda)\frac{\lambda(\kappa^2+\lambda^2)}{\kappa^2}b_{33}\right\},$$

$$\Phi(u,\lambda) = \frac{\sqrt{\left(1+\kappa^2\right)\left(\kappa^2+\lambda^2\right)}}{\kappa}\left\{\left[\frac{2\lambda}{\pi(\kappa^2+\lambda^2)}\mathbf{\Pi} - \frac{\lambda}{(\kappa^2+\lambda^2)}\Pi\left(amu,\kappa^2,\lambda\right)\right] + \right.$$

$$+\frac{2}{\pi}\frac{\lambda}{\lambda'^2(\lambda^2-n)}[\mathbf{E} - \lambda'^2\mathbf{\Pi}] - \frac{1}{dnu(1+nsn^2u)}\frac{1}{\mathbf{K}}\frac{\mathbf{E}-\lambda'^2\mathbf{K}}{\lambda\lambda'^2}udnu -$$

$$\left. -\frac{\lambda}{\lambda'^2(\lambda^2-n)}\left[E(amu,\lambda) - \lambda'^2\Pi\left(amu,n,\lambda\right) - \frac{\lambda^2 snucnu}{dnu}\right]\right\}.$$

Непотенциальные силы

$$\dot{G} = M_\xi b_{31} + M_\eta b_{32} + M_\zeta b_{33},$$

$$\dot{\rho} = -\frac{1}{G}(M_\xi b_{21} + M_\eta b_{22} + M_\zeta b_{23}),$$

$$\dot{h} = \frac{1}{G\sin\rho}(M_\xi b_{21} + M_\eta b_{22} + M_\zeta b_{23}),$$

$$\dot{\lambda} = \frac{(\kappa^2+\lambda^2)}{G\lambda}\left(M_\xi b_{31} + \frac{\lambda'^2}{1+\kappa^2}M_\eta b_{32} - \frac{\lambda^2}{\kappa^2}M_\zeta b_{33}\right),$$

$$\dot{\varphi}_1 = n_1 + \Phi_{11}M_\xi + \Phi_{12}M_\eta + \Phi_{13}M_\zeta,$$

$$\dot{\varphi}_2 = n_2 + \Phi_{21}M_\xi + \Phi_{22}M_\eta + \Phi_{23}M_\zeta,$$

$$\Phi_{11} = -\frac{\pi\sqrt{\kappa^2+\lambda^2}}{2\mathbf{K}G\lambda\lambda'^2}(snudnu - \mathbf{Zn}ucnu),$$

$$\Phi_{12} = -\frac{\pi\sqrt{\kappa^2+\lambda^2}}{2\mathbf{K}G\lambda\sqrt{1+\kappa^2}}[cnudnu + \mathbf{Zn}usnu],$$

$$\Phi_{13} = \frac{\pi\lambda\sqrt{\kappa^2+\lambda^2}}{2G\kappa\mathbf{K}}dnu,$$

$$\Phi_{21} = \frac{1}{G}\left\{\cot\rho b_{11} + \cot\theta\cos l + \Phi(u,\lambda)\frac{(\kappa^2+\lambda^2)}{\lambda}b_{31}\right\},$$

$$\Phi_{22} = \frac{1}{G}\left\{\cot\rho b_{12} - \cot\theta\sin l + \Phi(u,\lambda)\frac{\lambda'^2(\kappa^2+\lambda^2)}{\lambda(1+\kappa^2)}b_{32}\right\},$$

$$\Phi_{23} = \frac{1}{G}\left\{\cot\rho b_{13} - \Phi(u,\lambda)\frac{\lambda(\kappa^2+\lambda^2)}{\kappa^2}b_{33}\right\},$$

$$\Phi(u,\lambda) = \frac{\sqrt{\left(1+\kappa^2\right)\left(\kappa^2+\lambda^2\right)}}{\kappa}\left\{\left[\frac{2\lambda}{\pi(\kappa^2+\lambda^2)}\mathbf{\Pi}\left(\frac{\pi}{2},\kappa^2,\lambda\right) - \frac{\lambda}{(\kappa^2+\lambda^2)}\Pi\left(amu,\kappa^2,\lambda\right)\right] + \right.$$

$$\left. + \frac{2}{\pi}\cdot\frac{\partial\mathbf{\Pi}\left(\frac{\pi}{2},\kappa^2,\lambda\right)}{\partial\lambda} - \frac{\partial\Pi\left(amu,\kappa^2,\lambda\right)}{\partial(amu)}\cdot\frac{\partial(amu)}{\partial\lambda} - \frac{\partial\Pi\left(amu,\kappa^2,\lambda\right)}{\partial\lambda}\right\}.$$

Глава 17. Интегрирование задачи Эйлера-Пуансо методом Гамильтона-Якоби.

17.1. Канонические уравнения задачи Эйлера-Пуансо в переменных Андуайе. Первые интегралы.
17.2. Уравнение Гамильтона-Якоби в задаче Эйлера-Пуансо. Полный интеграл. Общий интеграл.
17.3. Параметры задачи. Области интегрирования.
17.4. Обращение интегралов задачи Эйлера-Пуансо в случае $a>0$ ($0\le\lambda<1$).
17.4.1. Определение переменной $l=l\left(t,\beta_1,\alpha_1,\alpha_2\right)$.
17.4.2. Определение переменной $g\left(t,\beta_2,\alpha_1,\alpha_2\right)$.
17.4.3. Определение переменной $L\left(t,\alpha_1,\alpha_2,\ldots\right)$.
17.4.4. Формулы для проекций угловой скорости вращения тела p,q,r.
17.5. Обращение общего интеграла задачи Эйлера-Пуансо в случае $a>0$ ($1<\lambda\le\infty$).
17.5.1. Определение переменной $l=l\left(t,\beta_2,\alpha_1,\alpha_2\right)$.
17.5.2. Определение переменной $g\left(t,\beta_2,\alpha_1,\alpha_2\right)$.
17.5.3. Определение переменной $L\left(t,\alpha_1,\alpha_2\right)$.
17.6. Канонические уравнения возмущенного движения в переменных Якоби.

$$p=\frac{G}{A}\frac{\lambda}{\sqrt{\kappa^2+\lambda^2}}cn\,u,\qquad q=-\frac{G}{B}\frac{\lambda\sqrt{1+\kappa^2}}{\sqrt{\kappa^2+\lambda^2}}sn\,u,\qquad r=\frac{G}{C}\frac{\kappa}{\sqrt{\kappa^2+\lambda^2}}dn u\,.$$

$$\cot l=-\sqrt{1+\kappa^2}\,\tan z \qquad\qquad z=am\,u$$

Глава 20. Переменные действие-угол в задаче Эйлера - Пуансо.

20.1. Переменные действие-угол.
20.2. Некоторые свойства переменных действие-угол.
20.3. Переменные действие-угол в задаче Эйлера-Пуансо (движения в областях I, II)
20.3.1. Переменные действия (области I, II).
20.3.2. Вычисление частных производных $\dfrac{\partial I_1}{\partial G}, \dfrac{\partial I_1}{\partial \lambda}, \dfrac{\partial I_2}{\partial G}, \dfrac{\partial I_2}{\partial \lambda}$.
20.3.3. Вычисление частных производных $\dfrac{\partial \lambda}{\partial I_1}, \dfrac{\partial \lambda}{\partial I_2}, \dfrac{\partial G}{\partial I_1}, \dfrac{\partial G}{\partial I_2}$.
20.3.4. Преобразующая функция и формулы канонического преобразования от переменных Андуайе к переменным действие-угол (области движения I, II).
20.3.5. Гамильтониан задачи Эйлера-Пуансо в переменных действие I_1, I_2.
20.3.6. Канонические уравнения задачи Эйлера-Пуансо в переменных действие-угол.
20.3.7. Формулы для частот задачи Эйлера-Пуансо.
20.4. Переменные действие-угол (переменные Пуассона) в областях III и IV.
20.4.1. Переменные действия в областях III и IV.
20.4.2. Вычисление производных $\dfrac{\partial I_1}{\partial G}, \dfrac{\partial I_1}{\partial \lambda}, \dfrac{\partial I_2}{\partial G}, \dfrac{\partial I_2}{\partial \lambda}$.
20.4.3. Формулы преобразования от переменных Андуайе к каноническим переменным действие-угол (области движения III и IV).
20.4.4. Канонические уравнения задачи Эйлера-Пуансо в переменных действие-угол и формулы для частот движения в областях III и IV.

Глава 21. Решение задачи Эйлера-Пуансо в тета-функциях. Формулы для направляющих косинусов $\widetilde{B}_{ij}, B_{ij}$ и проекций векторов угловой скорости и углового ускорения на оси твердого тела (движения в областях I и III).

21.1. Формулы связи эллиптических функций Якоби и их производных с тета-функциями θ_l. Значения $\theta_l(0), \theta_l'(0)\ (l = 1,2,3,4)$.

21.2. Выражение эллиптического интеграла третьего рода через функции Якоби и тета- функции.

21.3. Решение задачи Эйлера-Пуансо в тета-функциях (область движений I).

21.3.1. Решение задачи Эйлера-Пуансо в эллиптических функциях и интегралах.

21.3.2. Параметры a, σ в случае движений в области I и выражение переменной Андуайе в тета-функциях.

21.3.3. О значениях тета-функций $\theta_l(z)$ и их производных $\theta_l'(z)\ (l = 1,2,3,4)$ при $z = i\sigma$.

21.3.4. Выражение $\sqrt{1+\kappa^2 sn^2 u}$ в тета-функциях.

21.3.5. Формулы решения задачи Эйлера-Пуансо в тета-функциях (область I).

21.3.6. Выражения компонент угловой скорости тела p, q, r и компонент углового ускорения $\dot{p}, \dot{q}, \dot{r}$ в тета-функциях.

21.4. Решение задачи Эйлера-Пуансо в тета-функциях (область движений III).

21.4.1. Решение задачи Эйлера-Пуансо в эллиптических функция и интегралах.

21.4.2. Параметры a, σ в случае движений в области III и выражение переменной Андуайе в тета-функциях.

21.4.3. О значениях тета-функций $\theta_l(z)$ и их производных $\theta_l'(z)\ (l = 1,2,3,4)$ при $z = i\sigma$.

21.4.4. Выражение $\sqrt{1+\dfrac{\kappa^2}{\lambda^2} sn^2 u}$ в тета-функциях.

21.4.5. Формулы решения задачи Эйлера-Пуансо в тета-функциях (область III).

21.4.6. Выражения компонент угловой скорости тела p, q, r и компонент углового ускорения $\dot{p}, \dot{q}, \dot{r}$ в тета-функциях.

22.1. Направляющие косинусы b_{ij}, B_{ij} и $\widetilde{b}_{ij}, \widetilde{B}_{ij}$ и соотношения между ними.

$$b_{11} = \cos g \cos l - \sin g \sin l \cos\theta \,,$$
$$b_{21} = \sin g \cos l + \cos g \sin l \cos\theta \,,$$
$$b_{31} = \sin\theta \sin l \,,$$

$$b_{12} = -\cos g \sin l - \sin g \cos l \cos\theta \,,$$
$$b_{22} = -\sin g \sin l + \cos g \cos l \cos\theta \,,$$
$$b_{32} = \sin\theta \cos l \,,$$

$$b_{13} = \sin g \sin\theta \,,$$
$$b_{23} = -\cos g \sin\theta \,,$$
$$b_{33} = \cos\theta \,,$$

$$\widetilde{b}_{11} = -ib_{11} + b_{21} = e^{ig}\left(\cos\theta \sin l - i\cos l\right),$$
$$\widetilde{b}_{21} = \quad ib_{11} + b_{21} = e^{-ig}\left(\cos\theta \sin l + i\cos l\right),$$
$$\widetilde{b}_{31} = \quad b_{31} \quad = \sin\theta \sin l \,,$$

$$\widetilde{b}_{12} = -ib_{12} + b_{22} = e^{ig}\left(\cos\theta \cos l + i\sin l\right),$$
$$\widetilde{b}_{22} = \quad ib_{12} + b_{22} = e^{-ig}\left(\cos\theta \cos l - i\sin l\right),$$
$$\widetilde{b}_{32} = \quad b_{32} \quad = \sin\theta \cos l \,,$$

$$\widetilde{b}_{13} = -ib_{13} + b_{23} = -\sin\theta \, e^{ig} \,,$$
$$\widetilde{b}_{23} = \quad ib_{13} + b_{23} = -\sin\theta \, e^{-ig} \,,$$
$$\widetilde{b}_{33} = b_{33} \,.$$

22.1. Направляющие косинусы b_{ij}, B_{ij} и $\widetilde{b}_{ij}, \widetilde{B}_{ij}$ и соотношения между ними.

$$\omega = \varphi_2 - g$$

$$B_{11} = \sin\omega\cos\theta\sin l + \cos\omega\cos l\,,$$
$$B_{21} = \cos\omega\cos\theta\sin l - \sin\omega\cos l\,,$$
$$B_{31} = \sin\theta\sin l\,,$$

$$B_{12} = \sin\omega\cos\theta\cos l - \cos\omega\sin l\,,$$
$$B_{22} = \cos\omega\cos\theta\cos l + \sin\omega\sin l\,,$$
$$B_{32} = \sin\theta\cos l\,,$$

$$B_{13} = -\sin\theta\sin\omega\,,$$
$$B_{23} = -\sin\theta\cos\omega\,,$$
$$B_{33} = \cos\theta\,.$$

$$\widetilde{B}_{11} = B_{21} - iB_{11} = e^{-i\omega}\left(\cos\theta\sin l - i\cos l\right),$$
$$\widetilde{B}_{21} = B_{21} + iB_{11} = e^{i\omega}\left(\cos\theta\sin l + i\cos l\right),$$
$$\widetilde{B}_{31} = B_{31} = \sin\theta\sin l\,,$$

$$\widetilde{B}_{12} = B_{22} - iB_{12} = e^{-i\omega}\left(\cos\theta\cos l + i\sin l\right),$$
$$\widetilde{B}_{22} = B_{22} + iB_{12} = e^{i\omega}\left(\cos\theta\cos l - i\sin l\right),$$
$$\widetilde{B}_{32} = B_{32} = \sin\theta\cos l\,,$$

$$\widetilde{B}_{13} = B_{23} - iB_{13} = -\sin\theta\, e^{-i\omega}\,,$$
$$\widetilde{B}_{23} = B_{23} + iB_{13} = -\sin\theta\, e^{i\omega}\,,$$
$$\widetilde{B}_{33} = B_{33} = \cos\theta\,.$$

$$\widetilde{B}_{11} = B_{21} - iB_{11} = e^{-i\omega}\left(\cos\theta \sin l - i\cos l\right),$$
$$\widetilde{B}_{21} = B_{21} + iB_{11} = e^{i\omega}\left(\cos\theta \sin l + i\cos l\right),$$
$$\widetilde{B}_{31} = B_{31} = \sin\theta \sin l\,,$$

$$\widetilde{B}_{12} = B_{22} - iB_{12} = e^{-i\omega}\left(\cos\theta \cos l + i\sin l\right),$$
$$\widetilde{B}_{22} = B_{22} + iB_{12} = e^{i\omega}\left(\cos\theta \cos l - i\sin l\right),$$
$$\widetilde{B}_{32} = B_{32} = \sin\theta \cos l\,,$$

$$\widetilde{B}_{13} = B_{23} - iB_{13} = -\sin\theta\, e^{-i\omega}\,,$$
$$\widetilde{B}_{23} = B_{23} + iB_{13} = -\sin\theta\, e^{i\omega}\,,$$
$$\widetilde{B}_{33} = B_{33} = \cos\theta\,.$$

$$\widetilde{B}_{11} = \frac{\left(\kappa\, cn u\, dn u + i\sqrt{1+\kappa^2}\sqrt{\kappa^2+\lambda^2}\, sn u\right)}{\sqrt{\kappa^2+\lambda^2}\sqrt{1+\kappa^2 sn^2 u}} \frac{\sqrt{\theta_4(\varphi_1 - i\sigma)}}{\sqrt{\theta_4(\varphi_1 + i\sigma)}}\,,$$
$$\widetilde{B}_{21} = \frac{\left(\kappa\, cn u\, dn u - i\sqrt{1+\kappa^2}\sqrt{\kappa^2+\lambda^2}\, sn u\right)}{\sqrt{\kappa^2+\lambda^2}\sqrt{1+\kappa^2 sn^2 u}} \frac{\sqrt{\theta_4(\varphi_1 + i\sigma)}}{\sqrt{\theta_4(\varphi_1 - i\sigma)}}\,,$$
$$\tilde{B}_{31} = \frac{\lambda}{\sqrt{\lambda^2+\kappa^2}}\, cnu\,,$$

$$\widetilde{B}_{12} = \frac{\left(-\kappa\sqrt{1+\kappa^2}\, dn u\, sn u + i\, cn u\sqrt{\kappa^2+\lambda^2}\right)}{\sqrt{\kappa^2+\lambda^2}\sqrt{1+\kappa^2 sn^2 u}} \frac{\sqrt{\theta_4(\varphi_1 - i\sigma)}}{\sqrt{\theta_4(\varphi_1 + i\sigma)}}\,,$$
$$\widetilde{B}_{22} = -\frac{\left(\kappa\sqrt{1+\kappa^2}\, dn u\, sn u + i\, cn u\sqrt{\kappa^2+\lambda^2}\right)}{\sqrt{\kappa^2+\lambda^2}\sqrt{1+\kappa^2 sn^2 u}} \frac{\sqrt{\theta_4(\varphi_1 + i\sigma)}}{\sqrt{\theta_4(\varphi_1 - i\sigma)}}\,,$$
$$\tilde{B}_{32} = -\frac{\lambda\sqrt{1+\kappa^2}}{\sqrt{\kappa^2+\lambda^2}}\, snu\,,$$

$$\widetilde{B}_{13} = -\frac{\lambda\sqrt{1+\kappa^2 sn^2 u}}{\sqrt{\kappa^2+\lambda^2}} \frac{\sqrt{\theta_4(\varphi_1 - i\sigma)}}{\sqrt{\theta_4(\varphi_1 + i\sigma)}}\,,$$
$$\widetilde{B}_{23} = -\frac{\lambda\sqrt{1+\kappa^2 sn^2 u}}{\sqrt{\kappa^2+\lambda^2}} \frac{\sqrt{\theta_4(\varphi_1 + i\sigma)}}{\sqrt{\theta_4(\varphi_1 - i\sigma)}}\,,$$
$$\tilde{B}_{33} = \frac{\kappa}{\sqrt{\lambda^2+\kappa^2}}\, dnu\,.$$

$$\widetilde{B}_{11} = \frac{\kappa\sqrt{1+\kappa^2 sn^2 u}}{\sqrt{\kappa^2+\lambda^2}} V_1 \frac{\sqrt{\theta_4(\varphi_1 - i\sigma)}}{\sqrt{\theta_4(\varphi_1 + i\sigma)}} , \qquad \sigma = \frac{\pi}{2\mathbf{K}} F\left(\arctan\frac{\kappa}{\lambda}, \lambda'\right), \ \lambda' = \sqrt{1-\lambda^2} .$$

$$\widetilde{B}_{21} = -\frac{\kappa\sqrt{1+\kappa^2 sn^2 u}}{\sqrt{\kappa^2+\lambda^2}} V_2 \frac{\sqrt{\theta_4(\varphi_1 + i\sigma)}}{\sqrt{\theta_4(\varphi_1 - i\sigma)}} , \qquad V_1 = \frac{i\sqrt{(1+\kappa^2)(\kappa^2+\lambda^2)}\, sn\, u + \kappa\, cn\, u\, dn\, u}{\kappa(1+\kappa^2 sn^2 u)} ,$$

$$\widetilde{B}_{12} = -i\sqrt{1+\kappa^2 sn^2 u}\, V_3 \frac{\sqrt{\theta_4(\varphi_1 - i\sigma)}}{\sqrt{\theta_4(\varphi_1 + i\sigma)}} , \qquad V_2 = \frac{i\sqrt{(1+\kappa^2)(\kappa^2+\lambda^2)}\, sn\, u - \kappa\, cn\, u\, dn\, u}{\kappa(1+\kappa^2 sn^2 u)} ,$$

$$\widetilde{B}_{22} = i\sqrt{1+\kappa^2 sn^2 u}\, V_4 \frac{\sqrt{\theta_4(\varphi_1 + i\sigma)}}{\sqrt{\theta_4(\varphi_1 - i\sigma)}} , \qquad V_3 = \frac{-cn\, u - \dfrac{i\sqrt{1+\kappa^2}}{\sqrt{\kappa^2+\lambda^2}} \kappa\, sn\, u\, dn\, u}{1+\kappa^2 sn^2 u} ,$$

$$\widetilde{B}_{13} = -\frac{\lambda\sqrt{1+\kappa^2 sn^2 u}}{\sqrt{\kappa^2+\lambda^2}} \cdot \frac{\sqrt{\theta_4(\varphi_1 - i\sigma)}}{\sqrt{\theta_4(\varphi_1 + i\sigma)}} , \qquad V_4 = \frac{-cn\, u + \dfrac{i\sqrt{1+\kappa^2}}{\sqrt{\kappa^2+\lambda^2}} \kappa\, sn\, u\, dn\, u}{1+\kappa^2 sn^2 u} .$$

$$\widetilde{B}_{23} = -\frac{\lambda\sqrt{1+\kappa^2 sn^2 u}}{\sqrt{\kappa^2+\lambda^2}} \cdot \frac{\sqrt{\theta_4(\varphi_1 + i\sigma)}}{\sqrt{\theta_4(\varphi_1 - i\sigma)}} , \qquad u = \frac{2K}{\pi}\varphi_1, \quad a = \frac{2K}{\pi}\sigma i;$$

$$V_1 = -\frac{sn(u-a)}{sn\, a}, \quad V_2 = -\frac{sn(u+a)}{sn\, a}, \quad V_3 = -\frac{cn(u-a)}{cn\, a}, \quad V_4 = -\frac{cn(u+a)}{cn\, a}$$

$$\tilde{B}_{11} = \frac{iT_{11}}{\theta_4(i\sigma)} \cdot \frac{\theta_1(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{21} = -\frac{iT_{11}}{\theta_4(i\sigma)} \cdot \frac{\theta_1(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{31} = T_{31} \cdot \frac{\theta_2(\varphi_1)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{12} = \frac{iT_{12}}{\theta_4(i\sigma)} \cdot \frac{\theta_2(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{22} = -\frac{iT_{12}}{\theta_4(i\sigma)} \cdot \frac{\theta_2(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{32} = -T_{32} \cdot \frac{\theta_1(\varphi_1)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{13} = -\frac{T_{13}}{\theta_4(i\sigma)} \cdot \frac{\theta_4(\varphi_1 - i\sigma)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{23} = -\frac{T_{13}}{\theta_4(i\sigma)} \cdot \frac{\theta_4(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)},$$

$$\tilde{B}_{33} = T_{33} \cdot \frac{\theta_3(\varphi_1)}{\theta_4(\varphi_1)},$$

$$T_{11} = \frac{\sqrt{2K\lambda\lambda'}}{\sqrt{\pi\left(\kappa^2 + \lambda^2\right)}},$$

$$T_{31} = \frac{\sqrt{\lambda\lambda'}}{\sqrt{\kappa^2 + \lambda^2}},$$

$$T_{12} = \frac{\lambda'\sqrt{2K\lambda}}{\sqrt{\pi\left(\kappa^2 + \lambda^2\right)}},$$

$$T_{32} = \frac{\sqrt{\lambda}\sqrt{1+\kappa^2}}{\sqrt{\kappa^2 + \lambda^2}},$$

$$T_{13} = \frac{\lambda\sqrt{2K\lambda'}}{\sqrt{\pi\left(\kappa^2 + \lambda^2\right)}},$$

$$T_{33} = \frac{\kappa\sqrt{\lambda'}}{\sqrt{\kappa^2 + \lambda^2}}.$$

$$R_l(\varphi_1) = \frac{\theta_l(\varphi_1 + i\sigma)}{\theta_4(\varphi_1)} \qquad (l = 1, 2, 4)$$

$$B_{11}=-\frac{i\kappa\theta_4(0)}{2\sqrt{\kappa^2+\lambda^2}\,\theta_1(i\sigma)}\left(\frac{\theta_1(\varphi_1+i\sigma)}{\theta_4(\varphi_1)}+\frac{\theta_1(\varphi_1-i\sigma)}{\theta_4(\varphi_1)}\right),$$

$$B_{21}=\frac{\kappa\theta_4(0)}{2\sqrt{\kappa^2+\lambda^2}\,\theta_1(i\sigma)}\left(\frac{\theta_1(\varphi_1+i\sigma)}{\theta_4(\varphi_1)}-\frac{\theta_1(\varphi_1-i\sigma)}{\theta_4(\varphi_1)}\right),$$

$$B_{31}=\frac{\sqrt{\lambda\lambda'}}{\sqrt{\kappa^2+\lambda^2}}\cdot\frac{\theta_2(\varphi_1)}{\theta_4(\varphi_1)},$$

$$B_{12}=-\frac{\theta_4(0)}{2\theta_2(i\sigma)}\left(\frac{\theta_2(\varphi_1+i\sigma)}{\theta_4(\varphi_1)}+\frac{\theta_2(\varphi_1-i\sigma)}{\theta_4(\varphi_1)}\right),$$

$$B_{22}=\frac{i\theta_4(0)}{2\theta_2(i\sigma)}\left(\frac{\theta_2(\varphi_1-i\sigma)}{\theta_4(\varphi_1)}-\frac{\theta_2(\varphi_1+i\sigma)}{\theta_4(\varphi_1)}\right),$$

$$B_{32}=-\frac{\sqrt{\lambda}\sqrt{1+\kappa^2}}{\sqrt{\kappa^2+\lambda^2}}\cdot\frac{\theta_1(\varphi_1)}{\theta_4(\varphi_1)},$$

$$B_{13}=\frac{i\lambda\theta_4(0)}{2\sqrt{\kappa^2+\lambda^2}\,\theta_4(i\sigma)}\left(\frac{\theta_4(\varphi_1+i\sigma)}{\theta_4(\varphi_1)}-\frac{\theta_4(\varphi_1-i\sigma)}{\theta_4(\varphi_1)}\right),$$

$$B_{23}=\frac{-\lambda\theta_4(0)}{2\sqrt{\kappa^2+\lambda^2}\,\theta_4(i\sigma)}\left(\frac{\theta_4(\varphi_1+i\sigma)}{\theta_4(\varphi_1)}+\frac{\theta_4(\varphi_1-i\sigma)}{\theta_4(\varphi_1)}\right),$$

$$B_{33}=\frac{\kappa\lambda'}{\sqrt{\kappa^2+\lambda^2}}\cdot\frac{\theta_3(\varphi_1)}{\theta_4(\varphi_1)}.$$

$$R_l(\varphi_1)=\frac{\theta_l(\varphi_1+i\sigma)}{\theta_4(\varphi_1)}\qquad(l=1,2,4)$$

$$R_l(\varphi_1)=\frac{\theta_l(\varphi_1-i\sigma)}{\theta_4(\varphi_1)}\qquad(l=1,2,4)$$

$$R_l(\varphi_1)=\frac{\theta_l(\varphi_1)}{\theta_4(\varphi_1)}\qquad(l=1,2,3)$$

$$\frac{\theta_1(\varphi_1+i\sigma)}{\theta_4(\varphi_1)}=-it_1\theta_4(i\sigma)\sum_{m=-\infty}^{\infty}\frac{q^{(2m+1)/2}e^{-\sigma}}{1-e^{-2\sigma}q^{2m+1}}e^{i(2m+1)\varphi_1},$$

$$\frac{\theta_1(\varphi_1-i\sigma)}{\theta_4(\varphi_1)}=-it_1\theta_4(i\sigma)\sum_{m=-\infty}^{\infty}\frac{q^{(2m+1)/2}e^{\sigma}}{1-e^{2\sigma}q^{2m+1}}e^{i(2m+1)\varphi_1},$$

$$\frac{\theta_1(\varphi_1)}{\theta_4(\varphi_1)}=-it_1^0\sum_{m=-\infty}^{\infty}\frac{q^{(2m+1)/2}}{1-q^{2m+1}}e^{i(2m+1)\varphi_1},$$

$$\frac{\theta_2(\varphi_1+i\sigma)}{\theta_4(\varphi_1)}=t_2\theta_4(i\sigma)\sum_{m=-\infty}^{\infty}\frac{q^{(2m+1)/2}e^{-\sigma}}{1+e^{-2\sigma}q^{2m+1}}e^{i(2m+1)\varphi_1},$$

$$\frac{\theta_2(\varphi_1-i\sigma)}{\theta_4(\varphi_1)}=t_2\theta_4(i\sigma)\sum_{m=-\infty}^{\infty}\frac{q^{(2m+1)/2}e^{\sigma}}{1+e^{2\sigma}q^{2m+1}}e^{i(2m+1)\varphi_1},$$

$$\frac{\theta_2(\varphi_1)}{\theta_4(\varphi_1)}=t_2^0\sum_{m=-\infty}^{\infty}\frac{q^{(2m+1)/2}}{1+q^{2m+1}}e^{i(2m+1)\varphi_1},$$

$$\frac{\theta_3(\varphi_1+i\sigma)}{\theta_4(\varphi_1)}=t_3\theta_4(i\sigma)\sum_{m=-\infty}^{\infty}\frac{q^{m}e^{-\sigma}}{1+q^{2m}e^{-2\sigma}}e^{2im\varphi_1},$$

$$\frac{\theta_3(\varphi_1-i\sigma)}{\theta_4(\varphi_1)}=t_3\theta_4(i\sigma)\sum_{m=-\infty}^{\infty}\frac{q^{m}e^{\sigma}}{1+q^{2m}e^{2\sigma}}e^{2im\varphi_1},$$

$$\frac{\theta_3(\varphi_1)}{\theta_4(\varphi_1)}=t_3^0\sum_{m=-\infty}^{\infty}\frac{q^{m}}{1+q^{2m}}e^{2im\varphi_1},$$

$$\frac{\theta_4(\varphi_1+i\sigma)}{\theta_4(\varphi_1)}=t_4\theta_4(i\sigma)\sum_{m=-\infty}^{\infty}\frac{q^{m}e^{-\sigma}}{1-q^{2m}e^{-2\sigma}}e^{2im\varphi_1},$$

$$\frac{\theta_4(\varphi_1-i\sigma)}{\theta_4(\varphi_1)}=-t_4\theta_4(i\sigma)\sum_{m=-\infty}^{\infty}\frac{q^{m}e^{\sigma}}{1-q^{2m}e^{-2\sigma}}e^{2im\varphi_1};$$

23.3. Полный набор тригонометрических разложений функций $\frac{\theta_l(\varphi_1\pm i\sigma)}{\theta_4(\varphi_1)},\frac{\theta_l(\varphi_1)}{\theta_4(\varphi_1)}$ $(l=1,2,3,4)$.

$$t_1=\frac{2}{\theta_2(0)\theta_3(0)\theta_4(0)}=\frac{\pi\sqrt{\pi}}{\mathbf{K}\sqrt{2\mathbf{K}\lambda\lambda'}},$$

$$t_2=\frac{2\theta_3(i\sigma)}{\theta_2(0)\theta_3(0)\theta_4(i\sigma)}=\frac{\pi\sqrt{\pi}\sqrt{1+\kappa^2}}{\mathbf{K}\lambda'\sqrt{2\mathbf{K}\lambda}},$$

$$t_1^0=t_1\theta_4(0)=\frac{\pi}{\mathbf{K}\sqrt{\lambda}},$$

$$t_2^0=\frac{2}{\theta_2(0)\theta_4(0)}=\frac{\pi}{\mathbf{K}\sqrt{\lambda\lambda'}},$$

$$t_2=\frac{2\theta_3(i\sigma)}{\theta_2(0)\theta_3(0)\theta_4(0)\theta_4(i\sigma)}=\frac{\pi\sqrt{\pi}\sqrt{1+\kappa^2}}{\mathbf{K}\lambda'\sqrt{2\mathbf{K}\lambda}},$$

$$t_3=\frac{2\theta_2(i\sigma)}{\theta_2(0)\theta_3(0)\theta_4(0)\theta_4(i\sigma)}=\frac{\pi\sqrt{\pi\left(\lambda^2+\kappa^2\right)}}{\mathbf{K}\lambda\lambda'\sqrt{2\mathbf{K}}},$$

$$t_4=\frac{-2i\theta_1(i\sigma)}{\theta_2(0)\theta_3(0)\theta_4(0)\theta_4(i\sigma)}=\frac{\pi\sqrt{\pi}\kappa}{\mathbf{K}\lambda\sqrt{2K\lambda'}},$$

$$t_3^0=\frac{2}{\theta_3(0)\theta_4(0)}=\frac{\pi}{\mathbf{K}\sqrt{\lambda'}}.$$

$$q=e^{-2d},\ d=\frac{\pi\mathbf{K}'}{2\mathbf{K}};\quad \sigma=\frac{\pi}{2\mathbf{K}}F(\arctan\frac{\kappa}{\lambda},\lambda')$$

23.4. Ряды Фурье для направляющих косинусов $\widetilde{B}_{ij}$ в случае движений в области I.

$$\tilde{B}_{11} = \frac{\pi e^{\sigma}}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1-e^{2\sigma}q^{2m+1}} e^{i(2m+1)\varphi_1},$$

$$\tilde{B}_{21} = -\frac{\pi e^{-\sigma}}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1-e^{-2\sigma}q^{2m+1}} e^{i(2m+1)\varphi_1},$$

$$\tilde{B}_{31} = \frac{\pi}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1+q^{2m+1}} e^{i(2m+1)\varphi_1},$$

$$B_{11} = \frac{\tilde{B}_{21}-\tilde{B}_{11}}{2i} = \frac{i\pi}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} q^{m+\frac{1}{2}} \left[\frac{e^{-\sigma}}{1-e^{-2\sigma}q^{2m+1}} + \frac{e^{\sigma}}{1-e^{2\sigma}q^{2m+1}}\right] e^{i(2m+1)\varphi_1},$$

$$B_{21} = \frac{\tilde{B}_{21}+\tilde{B}_{11}}{2} = \frac{\pi}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} q^{m+\frac{1}{2}} \left[\frac{e^{\sigma}}{1-e^{2\sigma}q^{2m+1}} - \frac{e^{-\sigma}}{1-e^{-2\sigma}q^{2m+1}}\right] e^{i(2m+1)\varphi_1},$$

$$B_{31} = \tilde{B}_{31} = \frac{\pi}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{m+\frac{1}{2}}}{1+q^{2m+1}} e^{i(2m+1)\varphi_1},$$

$$\tilde{B}_{12} = \frac{i\pi\sqrt{1+\kappa^2}\,e^{\sigma}}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1+e^{2\sigma}q^{2m+1}} e^{i(2m+1)\varphi_1},$$

$$\tilde{B}_{22} = -\frac{i\pi\sqrt{1+\kappa^2}\,e^{-\sigma}}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1+e^{-2\sigma}q^{2m+1}} e^{i(2m+1)\varphi_1},$$

$$\tilde{B}_{32} = \frac{i\pi\sqrt{1+\kappa^2}}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{(2m+1)/2}}{1-q^{2m+1}} e^{i(2m+1)\varphi_1},$$

$$B_{12} = \frac{\tilde{B}_{22}-\tilde{B}_{12}}{2i} = -\frac{\pi\sqrt{1+\kappa^2}}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} q^{m+\frac{1}{2}} \left[\frac{e^{-\sigma}}{1+e^{-2\sigma}q^{2m+1}} + \frac{e^{\sigma}}{1+e^{2\sigma}q^{2m+1}}\right] e^{i(2m+1)\varphi_1},$$

$$B_{22} = \frac{\tilde{B}_{22}+\tilde{B}_{12}}{2} = \frac{i\pi\sqrt{1+\kappa^2}}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} q^{m+\frac{1}{2}} \left[\frac{e^{\sigma}}{1+e^{2\sigma}q^{2m+1}} - \frac{e^{-\sigma}}{1-e^{-2\sigma}q^{2m+1}}\right] e^{i(2m+1)\varphi_1},$$

$$B_{32} = \tilde{B}_{32} = \frac{i\pi\sqrt{1+\kappa^2}}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{m+\frac{1}{2}}}{1-q^{2m+1}} e^{i(2m+1)\varphi_1},$$

$$\tilde{B}_{13} = \frac{\pi\kappa e^{\sigma}}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{m}}{1-q^{2m}e^{-2\sigma}} e^{i2m\varphi_1},$$

$$\tilde{B}_{23} = -\frac{\pi\kappa e^{-\sigma}}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{m}}{1-q^{2m}e^{-2\sigma}} e^{i2m\varphi_1},$$

$$\tilde{B}_{33} = \frac{\pi\kappa}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{m}}{1+q^{2m}} e^{i2m\varphi_1}.$$

$$B_{13} = \frac{\tilde{B}_{23}-\tilde{B}_{13}}{2i} = \frac{i\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} q^{m} \left[\frac{e^{-\sigma}}{1-q^{2m}e^{-2\sigma}} + \frac{e^{\sigma}}{1-q^{2m}e^{2\sigma}}\right] e^{i2m\varphi_1},$$

$$B_{23} = \frac{\tilde{B}_{23}+\tilde{B}_{13}}{2} = \frac{\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} q^{m} \left[\frac{e^{\sigma}}{1-q^{2m}e^{2\sigma}} - \frac{e^{-\sigma}}{1-q^{2m}e^{-2\sigma}}\right] e^{i2m\varphi_1},$$

$$B_{33} = \tilde{B}_{33} = \frac{\pi\kappa}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \sum_{m=-\infty}^{\infty} \frac{q^{m}}{1+q^{2m}} e^{i2m\varphi_1}.$$

$$b_{11}=-\frac{\pi}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{e^{\sigma}q^{m+\frac{1}{2}}}{1-q^{2m+1}e^{2\sigma}}\sin\left[(2m+1)\varphi_1+\varphi_2\right]+\frac{e^{-\sigma}q^{m+\frac{1}{2}}}{1-q^{2m+1}e^{-2\sigma}}\sin\left[(2m+1)\varphi_1-\varphi_2\right]\right\},$$

$$b_{12}=-\frac{\pi}{\mathbf{K}}\sqrt{\frac{1+\kappa^2}{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{e^{\sigma}q^{m+\frac{1}{2}}}{1+q^{2m+1}e^{2\sigma}}\cos\left[(2m+1)\varphi_1+\varphi_2\right]+\frac{e^{-\sigma}q^{m+\frac{1}{2}}}{1+q^{2m+1}e^{-2\sigma}}\cos\left[(2m+1)\varphi_1-\varphi_2\right]\right\},$$

$$b_{13}=\frac{\pi\kappa}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{e^{\sigma}q^{m}}{1-q^{2m}e^{2\sigma}}\sin\left[2m\varphi_1+\varphi_2\right]+\frac{e^{-\sigma}q^{m}}{1-q^{2m}e^{-2\sigma}}\sin\left[2m\varphi_1-\varphi_2\right]\right\}(1+\delta_{m0})^{-1},$$

$$b_{21}=\frac{\pi}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{e^{\sigma}q^{m+\frac{1}{2}}}{1-q^{2m+1}e^{2\sigma}}\cos\left[(2m+1)\varphi_1+\varphi_2\right]-\frac{e^{-\sigma}q^{m+\frac{1}{2}}}{1-q^{2m+1}e^{-2\sigma}}\cos\left[(2m+1)\varphi_1-\varphi_2\right]\right\},$$

$$b_{22}=-\frac{\pi}{\mathbf{K}}\sqrt{\frac{1+\kappa^2}{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{e^{\sigma}q^{m+\frac{1}{2}}}{1+q^{2m+1}e^{2\sigma}}\sin\left[(2m+1)\varphi_1+\varphi_2\right]-\frac{e^{-\sigma}q^{m+\frac{1}{2}}}{1+q^{2m+1}e^{-2\sigma}}\sin\left[(2m+1)\varphi_1-\varphi_2\right]\right\},$$

$$b_{23}=-\frac{\pi\kappa}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{e^{\sigma}q^{m}}{1-q^{2m}e^{2\sigma}}\cos\left[2m\varphi_1+\varphi_2\right]-\frac{e^{-\sigma}q^{m}}{1-q^{2m}e^{-2\sigma}}\cos\left[2m\varphi_1-\varphi_2\right]\right\}(1+\delta_{m0})^{-1},$$

$$b_{31}=\frac{2\pi}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{q^{m+\frac{1}{2}}}{1+q^{2m+1}}\cos\left[(2m+1)\varphi_1\right]\right\},$$

$$b_{32}=-\frac{2\pi}{\mathbf{K}}\sqrt{\frac{\kappa^2+1}{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{q^{m+\frac{1}{2}}}{1-q^{2m+1}}\sin\left[(2m+1)\varphi_1\right]\right\},$$

$$b_{33}=\frac{2\pi}{\mathbf{K}}\frac{\kappa}{\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{q^{m}}{1+q^{2m}}\cos\left[2m\varphi_1\right]\right\}(1+\delta_{m0})^{-1},$$

24.1.1. Ряды Фурье для направляющих косинусов b_{ij}.

$$q=e^{-2d},\quad d=\frac{\pi\mathbf{K}(\lambda')}{2\mathbf{K}(\lambda)}\geq\sigma$$

$$\sigma=\frac{\pi}{2\mathbf{K}}F(\arctan\frac{\kappa}{\lambda},\lambda')$$

23.9. Ряды Фурье для проекций векторов угловой скорости и углового ускорения (p, q, r и $\dot{p}, \dot{q}, \dot{r}$) в случае движений в области I.

$$p = \frac{G}{A}\sin\theta\sin l = \frac{G}{A} b_{31} = \frac{G}{A}\widetilde{B}_{31},$$
$$q = \frac{G}{B}\sin\theta\cos l = \frac{G}{B} b_{32} = \frac{G}{B}\widetilde{B}_{32},$$
$$r = \frac{G}{C}\cos\theta = \frac{G}{C} b_{33} = \frac{G}{C}\widetilde{B}_{33},$$

$$\varphi_1 = n_1 t + \varphi_1^{(0)}, \qquad n_1 = \frac{A - C}{AC}\frac{\pi G}{2\mathbf{K}(\lambda)}\frac{\kappa}{\sqrt{\left(1+\kappa^2\right)\left(\kappa^2+\lambda^2\right)}}.$$

$$\dot{p} = \frac{i\pi^2 G^2 \kappa}{2A^2\mathbf{K}^2\left(\kappa^2+\lambda^2\right)\sqrt{1+\kappa^2}}\frac{(A-C)}{C}\sum_{m=-\infty}^{\infty}\frac{(2m+1)q^{m+\frac{1}{2}}}{1+q^{2m+1}}e^{i(2m+1)\varphi_1},$$

$$\dot{q} = -\frac{\pi^2 G^2 \kappa}{2AB\mathbf{K}^2\left(\kappa^2+\lambda^2\right)}\frac{(A-C)}{C}\sum_{m=-\infty}^{\infty}\frac{(2m+1)q^{m+\frac{1}{2}}}{1-q^{2m+1}}e^{i(2m+1)\varphi_1},$$

$$\dot{r} = \frac{iG^2\pi^2\kappa^2}{AC\mathbf{K}^2\left(\kappa^2+\lambda^2\right)\sqrt{1+\kappa^2}}\frac{(A-C)}{C}\sum_{m=-\infty}^{\infty}\frac{mq^m}{1+q^{2m}}e^{2im\varphi_1}.$$

$$b_{11} = -\frac{\pi}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{\sin\left[\left(2m+1\right)\varphi_1+\varphi_2\right]}{\sinh\left[\left(2m+1\right)d-\sigma\right]}+\frac{\sin\left[\left(2m+1\right)\varphi_1-\varphi_2\right]}{\sinh\left[\left(2m+1\right)d+\sigma\right]}\right\},$$

$$b_{21} = \frac{\pi}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{\cos\left[\left(2m+1\right)\varphi_1+\varphi_2\right]}{\sinh\left[\left(2m+1\right)d-\sigma\right]}-\frac{\cos\left[\left(2m+1\right)\varphi_1-\varphi_2\right]}{\sinh\left[\left(2m+1\right)d+\sigma\right]}\right\},$$

$$b_{31} = \frac{\pi}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{\cos\left[\left(2m+1\right)\varphi_1\right]}{\cosh\left[\left(2m+1\right)d\right]}\right\},$$

$$b_{12} = -\frac{\pi}{2\mathbf{K}}\sqrt{\frac{1+\kappa^2}{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{\cos\left[\left(2m+1\right)\varphi_1+\varphi_2\right]}{\cosh\left[\left(2m+1\right)d-\sigma\right]}+\frac{\cos\left[\left(2m+1\right)\varphi_1-\varphi_2\right]}{\cosh\left[\left(2m+1\right)d+\sigma\right]}\right\},$$

$$b_{22} = -\frac{\pi}{2\mathbf{K}}\sqrt{\frac{1+\kappa^2}{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{\sin\left[\left(2m+1\right)\varphi_1+\varphi_2\right]}{\cosh\left[\left(2m+1\right)d-\sigma\right]}-\frac{\sin\left[\left(2m+1\right)\varphi_1-\varphi_2\right]}{\cosh\left[\left(2m+1\right)d+\sigma\right]}\right\},$$

$$b_{32} = -\frac{\pi}{\mathbf{K}}\sqrt{\frac{1+\kappa^2}{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{\sin\left[\left(2m+1\right)\varphi_1\right]}{\sinh\left[\left(2m+1\right)d\right]}\right\},$$

$$b_{13} = \frac{\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{\sin\left[2m\varphi_1+\varphi_2\right]}{\sinh\left[2md-\sigma\right]}+\frac{\sin\left[2m\varphi_1-\varphi_2\right]}{\sinh\left[2md+\sigma\right]}\right\}\left(1+\delta_{m0}\right)^{-1},$$

$$b_{23} = -\frac{\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{\cos\left[2m\varphi_1+\varphi_2\right]}{\sinh\left[2md-\sigma\right]}-\frac{\cos\left[2m\varphi_1-\varphi_2\right]}{\sinh\left[2md+\sigma\right]}\right\}\left(1+\delta_{m0}\right)^{-1},$$

$$d = \frac{\pi\mathbf{K}\left(\lambda'\right)}{2\mathbf{K}\left(\lambda\right)} \geq \sigma$$

$$b_{33} = \frac{\pi\kappa}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{\cos\left[2m\varphi_1\right]}{\cosh\left[2md\right]}\right\}\left(1+\delta_{m0}\right)^{-1};$$

$$\sigma = \frac{\pi}{2\mathbf{K}}F(\arctan\frac{\kappa}{\lambda},\lambda')$$

24.1.1. Ряды Фурье для производных направляющих косинусов $\dot{b}_{ij}$.

$$\dot{b}_{11}=-\frac{\pi}{2K\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{(2m+1)n_1+n_2}{sh\left[(2m+1)d-\sigma\right]}\cos\left[(2m+1)\varphi_1+\varphi_2\right]+\frac{(2m+1)n_1-n_2}{sh\left[(2m+1)d+\sigma\right]}\cos\left[(2m+1)\varphi_1-\varphi_2\right]\right\},$$

$$\dot{b}_{21}=-\frac{\pi}{2K\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{(2m+1)n_1+n_2}{sh\left[(2m+1)d-\sigma\right]}\sin\left[(2m+1)\varphi_1+\varphi_2\right]+\frac{(2m+1)n_1-n_2}{sh\left[(2m+1)d+\sigma\right]}\sin\left[(2m+1)\varphi_1-\varphi_2\right]\right\},$$

$$\dot{b}_{31}=-\frac{\pi}{K\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{(2m+1)n_1}{ch\left[(2m+1)d\right]}\sin\left[(2m+1)\varphi_1\right]\right\},$$

$$\dot{b}_{12}=\frac{\pi}{2K}\sqrt{\frac{1+\kappa^2}{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{(2m+1)n_1+n_2}{ch\left[(2m+1)d-\sigma\right]}\sin\left[(2m+1)\varphi_1+\varphi_2\right]+\frac{(2m+1)n_1-n_2}{ch\left[(2m+1)d+\sigma\right]}\sin\left[(2m+1)\varphi_1-\varphi_2\right]\right\},$$

$$\dot{b}_{22}=-\frac{\pi}{2K}\sqrt{\frac{1+\kappa^2}{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{(2m+1)n_1+n_2}{ch\left[(2m+1)d-\sigma\right]}\cos\left[(2m+1)\varphi_1+\varphi_2\right]-\frac{(2m+1)n_1-n_2}{ch\left[(2m+1)d+\sigma\right]}\cos\left[(2m+1)\varphi_1-\varphi_2\right]\right\}$$

$$\dot{b}_{32}=-\frac{\pi}{K}\sqrt{\frac{1+\kappa^2}{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{(2m+1)n_1}{sh\left[(2m+1)d\right]}\cos\left[(2m+1)\varphi_1\right]\right\},$$

$$\sigma=\frac{\pi}{2\mathbf{K}}F(\arctan\frac{\kappa}{\lambda},\lambda'),\quad d=\frac{\pi\mathbf{K}'}{2\mathbf{K}}$$

$$\dot{b}_{13}=\frac{\pi\kappa}{2K\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{2mn_1+n_2}{sh\left[2md-\sigma\right]}\cos\left[2m\varphi_1+\varphi_2\right]+\frac{2mn_1-n_2}{sh\left[2md+\sigma\right]}\cos\left[2m\varphi_1-\varphi_2\right]\right\}(1+\delta_{m0})^{-1},$$

$$\dot{b}_{23}=\frac{\pi\kappa}{2K\sqrt{\kappa^2+\lambda^2}}\sum_{m=0}^{\infty}\left\{\frac{2mn_1+n_2}{sh\left[2md-\sigma\right]}\sin\left[2m\varphi_1+\varphi_2\right]-\frac{2mn_1-n_2}{sh\left[2md+\sigma\right]}\sin\left[2m\varphi_1-\varphi_2\right]\right\}(1+\delta_{m0})^{-1},$$

24.2.2. Ряды Фурье для направляющих косинусов b_{ij}. Область движений III.

$$b_{11}=\sum_{m=0}^{\infty}\left\{b_{2m+1,1}^{(11)}\sin\left[(2m+1)\varphi_1+\varphi_2\right]+b_{2m+1,-1}^{(11)}\sin\left[(2m+1)\varphi_1-\varphi_2\right]\right\},$$

$$b_{21}=\sum_{m=0}^{\infty}\left\{b_{2m+1,1}^{(2.1)}\cos\left[(2m+1)\varphi_1+\varphi_2\right]+b_{2m+1,-1}^{(2.1)}\cos\left[(2m+1)\varphi_1-\varphi_2\right]\right\},$$

$$b_{31}=\sum_{m=0}^{\infty}\left\{b_{2m,0}^{(3.1)}\cos\left[2m\varphi_1\right]\right\},$$

$$b_{12}=\sum_{m=0}^{\infty}\left\{b_{2m,1}^{(1.2)}\cos\left[2m\varphi_1+\varphi_2\right]+b_{2m,-1}^{(1.2)}\cos\left[2m\varphi_1-\varphi_2\right]\right\},$$

$$b_{22}=\sum_{m=0}^{\infty}\left\{b_{2m,1}^{(2.2)}\sin\left[2m\varphi_1+\varphi_2\right]+b_{2m,-1}^{(2.2)}\sin\left[2m\varphi_1-\varphi_2\right]\right\},$$

$$b_{32}=\sum_{m=0}^{\infty}\left\{b_{2m+1,0}^{(3.2)}\sin\left[(2m+1)\varphi_1\right]\right\},$$

$$b_{13}=\sum_{m=0}^{\infty}\left\{b_{2m,1}^{(1.3)}\sin\left[2m\varphi_1+\varphi_2\right]+b_{2m,-1}^{(1.3)}\sin\left[2m\varphi_1-\varphi_2\right]\right\},$$

$$b_{23}=\sum_{m=0}^{\infty}\left\{b_{2m,1}^{(2.3)}\cos\left[2m\varphi_1+\varphi_2\right]+b_{2m,-1}^{(2.3)}\cos\left[2m\varphi_1-\varphi_2\right]\right\},$$

$$b_{33}=\sum_{m=0}^{\infty}\left\{b_{2m+1,0}^{(3.3)}\cos\left[(2m+1)\varphi_1\right]\right\},$$

$$D=\frac{\pi\mathbf{K}'(\lambda')}{2\mathbf{K}\left(\frac{1}{\lambda}\right)},\qquad \sigma=\frac{\pi F(\arctan\kappa,\lambda')}{2\mathbf{K}\left(\frac{1}{\lambda}\right)},\qquad \lambda'=\sqrt{1-\frac{1}{\lambda^2}}.$$

$$b_{2m+1,1}^{(11)}=\frac{-\pi\lambda}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,sh\left[(2m+1)D+\sigma\right]},$$

$$b_{2m+1,-1}^{(11)}=\frac{-\pi\lambda}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,sh\left[(2m+1)D-\sigma\right]},$$

$$b_{2m+1,1}^{(21)}=\frac{\pi\lambda}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,sh\left[(2m+1)D+\sigma\right]},$$

$$b_{2m+1,-1}^{(21)}=-\frac{\pi\lambda}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,sh\left[(2m+1)D-\sigma\right]},$$

$$b_{2m,0}^{(31)}=\frac{\pi\lambda}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,ch\left[2md\right]}\left(1+\delta_{m0}\right)^{-1},$$

$$b_{2m,1}^{(12)}=-\frac{\pi\lambda\sqrt{1+\kappa^2}}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,ch\left[2mD+\sigma\right]}\left(1+\delta_{m0}\right)^{-1},$$

$$b_{2m,-1}^{(12)}=-\frac{\pi\lambda\sqrt{1+\kappa^2}}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,ch\left[2mD-\sigma\right]}\left(1+\delta_{m0}\right)^{-1}$$

$$b_{2m,1}^{(22)}=-\frac{\pi\lambda\sqrt{1+\kappa^2}}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,ch\left[2mD+\sigma\right]}\left(1+\delta_{m0}\right)^{-1},$$

$$b_{2m,-1}^{(22)}=\frac{\pi\lambda\sqrt{1+\kappa^2}}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,ch\left[2mD-\sigma\right]}\left(1+\delta_{m0}\right)^{-1},$$

$$b_{2m+1,0}^{(32)}=\frac{\pi\lambda\sqrt{1+\kappa^2}}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,sh\left[(2m+1)D\right]},$$

$$b_{2m,1}^{(13)}=\frac{\pi\kappa\lambda}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,sh\left[2mD+\sigma\right]}\left(1+\delta_{m0}\right)^{-1},$$

$$b_{2m,-1}^{(13)}=\frac{\pi\kappa\lambda}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,sh\left[2mD-\sigma\right]}\left(1+\delta_{m0}\right)^{-1},$$

$$b_{2m,1}^{(23)}=\frac{-\pi\kappa\lambda}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,sh\left[2mD+\sigma\right]}\left(1+\delta_{m0}\right)^{-1},$$

$$b_{2m,-1}^{(23)}=\frac{\pi\kappa\lambda}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,sh\left[2mD-\sigma\right]}\left(1+\delta_{m0}\right)^{-1},$$

$$b_{2m+1,0}^{(33)}=\frac{\pi\kappa\lambda}{\mathbf{K}\sqrt{\kappa^2+\lambda^2}\,ch\left[(2m+1)D\right]}\left(1+\delta_{m0}\right)^{-1}.$$

$$p = \frac{G}{A}\frac{\lambda\sqrt{\lambda'}}{\sqrt{\kappa^2+\lambda^2}}\frac{\theta_3(\varphi_1)}{\theta_4(\varphi_1)},$$

$$q = -\frac{G}{B}\frac{\sqrt{\lambda}\sqrt{1+\kappa^2}}{\sqrt{\kappa^2+\lambda^2}}\frac{\theta_1(\varphi_1)}{\theta_4(\varphi_1)},$$

$$r = \frac{G}{C}\frac{\kappa\sqrt{\lambda\lambda'}}{\sqrt{\kappa^2+\lambda^2}}\frac{\theta_2(\varphi_1)}{\theta_4(\varphi_1)},$$

$$\dot{p} = G^2\frac{(C-B)}{ABC}\frac{\lambda\sqrt{\lambda'}\sqrt{1+\kappa^2}}{\kappa^2+\lambda^2}\cdot\frac{\theta_1(\varphi_1)\theta_2(\varphi_1)}{\theta_4^2(\varphi_1)},$$

$$\dot{q} = G^2\frac{(C-A)}{ABC}\frac{\lambda\lambda'\sqrt{\lambda}}{\kappa^2+\lambda^2}\cdot\frac{\theta_3(\varphi_1)\theta_2(\varphi_1)}{\theta_4^2(\varphi_1)},$$

$$\dot{r} = G^2\frac{(B-A)}{ABC}\frac{\lambda\sqrt{\lambda\lambda'}\sqrt{1+\kappa^2}}{\kappa^2+\lambda^2}\cdot\frac{\theta_1(\varphi_1)\theta_3(\varphi_1)}{\theta_4^2(\varphi_1)}.$$

25. Ряды Фурье для произведений направляющих косинусов и их квадратов (движения в областях I и III).

	b_{11}	b_{12}	b_{13}	b_{21}	b_{22}	b_{23}	b_{31}	b_{32}	b_{33}
b_{11}	P_1								
b_{12}	P_2	P_3							
b_{13}	P_4	P_5	P_6						
b_{21}	P_7	P_8	P_9	P_{10}					
b_{22}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}				
b_{23}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}			
b_{31}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}		
b_{32}	P_{29}	P_{30}	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	P_{36}	
b_{33}	P_{37}	P_{38}	P_{39}	P_{40}	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}

Рис. 25.1. Диаграмма взаимных произведений направляющих косинусов b_{ij} **.**

25.1. Выражения произведений и квадратов направляющих косинусов $b_{ij}\left(P_k\right)$ через произведения и квадраты направляющих косинусов $\widetilde{B}_{ij}\left(P_k\right)$.

1. $P_1 = b_{11}^2 = -\frac{1}{4}\left(\widetilde{B}_{21}^2 e^{-2i\varphi_2} + \widetilde{B}_{11}^2 e^{2i\varphi_2}\right) + \frac{1}{2}\left(1 - B_{31}^2\right)$

2. $P_2 = b_{12} b_{11} = -\frac{1}{4}\left(\widetilde{B}_{21}\widetilde{B}_{22} e^{-2i\varphi_2} + \widetilde{B}_{11}\widetilde{B}_{12} e^{2i\varphi_2}\right) - \frac{1}{2} B_{31} B_{32}$

3. $P_3 = b_{12}^2 = -\frac{1}{4}\left(\widetilde{B}_{22}^2 e^{-2i\varphi_2} + \widetilde{B}_{12}^2 e^{2i\varphi_2}\right) + \frac{1}{2}\left(1 - B_{32}^2\right)$

4. $P_4 = b_{13} b_{11} = -\frac{1}{4}\left(\widetilde{B}_{21}\widetilde{B}_{23} e^{-2i\varphi_2} + \widetilde{B}_{11}\widetilde{B}_{13} e^{2i\varphi_2}\right) - \frac{1}{2} B_{31} B_{33}$

5. $P_5 = b_{13} b_{12} = -\frac{1}{4}\left(\widetilde{B}_{22}\widetilde{B}_{23} e^{-2i\varphi_2} + \widetilde{B}_{12}\widetilde{B}_{13} e^{2i\varphi_2}\right) - \frac{1}{2} B_{32} B_{33}$

6. $P_6 = b_{13}^2 = -\frac{1}{4}\left(\widetilde{B}_{23}^2 e^{-2i\varphi_2} + \widetilde{B}_{13}^2 e^{2i\varphi_2}\right) + \frac{1}{2}\left(1 - B_{33}^2\right)$

....

42. $P_{42} = b_{33} b_{23} = \frac{1}{2}\left(B_{33}\widetilde{B}_{23} e^{-i\varphi_2} + B_{33}\widetilde{B}_{13} e^{i\varphi_2}\right)$

43. $P_{43} = b_{33} b_{31} = \widetilde{B}_{31} B_{33}$

44. $P_{44} = b_{33} b_{32} = \widetilde{B}_{32} B_{33}$

45. $P_{45} = b_{33}^2 = B_{33}^2$

$$b_{11} = \frac{1}{2i}\left(-\widetilde{B}_{11} e^{i\varphi_2} + \widetilde{B}_{21} e^{-i\varphi_2}\right),$$

$$b_{21} = \frac{1}{2}\left(\widetilde{B}_{21} e^{-i\varphi_2} + \widetilde{B}_{11} e^{i\varphi_2}\right),$$

$$b_{31} = \widetilde{B}_{31},$$

$$b_{12} = \frac{1}{2i}\left(-\widetilde{B}_{12} e^{i\varphi_2} + \widetilde{B}_{22} e^{-i\varphi_2}\right),$$

$$b_{22} = \frac{1}{2}\left(\widetilde{B}_{12} e^{i\varphi_2} + \widetilde{B}_{22} e^{-i\varphi_2}\right),$$

$$b_{32} = \widetilde{B}_{32},$$

$$b_{13} = \frac{1}{2i}\left(-\widetilde{B}_{13} e^{i\varphi_2} + \widetilde{B}_{23} e^{-i\varphi_2}\right),$$

$$b_{23} = \frac{1}{2}\left(\widetilde{B}_{13} e^{i\varphi_2} + \widetilde{B}_{23} e^{-i\varphi_2}\right),$$

$$b_{33} = \widetilde{B}_{33}.$$

1. $$\widetilde{B}_{21}^{2} = -\frac{T_{11}^{2}}{\theta_4^2(i\sigma)} \cdot \frac{\theta_1^2(\varphi_1 + i\sigma)}{\theta_4^2(\varphi_1)}$$

2. $$\widetilde{B}_{11}^{2} = -\frac{T_{11}^{2}}{\theta_4^2(i\sigma)} \cdot \frac{\theta_1^2(\varphi_1 - i\sigma)}{\theta_4^2(\varphi_1)}$$

3. $$\widetilde{B}_{21}\widetilde{B}_{22} = -\frac{T_{11}T_{12}}{\theta_4^2(i\sigma)} \cdot \frac{\theta_1(\varphi_1 + i\sigma)\theta_2(\varphi_1 + i\sigma)}{\theta_4^2(\varphi_1)}$$

4. $$\widetilde{B}_{11}\widetilde{B}_{12} = -\frac{T_{11}T_{12}}{\theta_4^2(i\sigma)} \cdot \frac{\theta_1(\varphi_1 - i\sigma)\theta_2(\varphi_1 - i\sigma)}{\theta_4^2(\varphi_1)}$$

5. $$\widetilde{B}_{22}^{2} = -\frac{T_{12}^{2}}{\theta_4^2(i\sigma)} \cdot \frac{\theta_2^2(\varphi_1 + i\sigma)}{\theta_4^2(\varphi_1)}$$

6. $$\widetilde{B}_{22}^{2} = -\frac{T_{12}^{2}}{\theta_4^2(i\sigma)} \cdot \frac{\theta_2^2(\varphi_1 - i\sigma)}{\theta_4^2(\varphi_1)}$$

....

34. $$\widetilde{B}_{31}B_{33} = T_{31}T_{33} \cdot \frac{\theta_2(\varphi_1)\theta_3(\varphi_1)}{\theta_4^2(\varphi_1)}$$

35. $$\widetilde{B}_{32}B_{33} = -T_{32}T_{33} \cdot \frac{\theta_1(\varphi_1)\theta_3(\varphi_1)}{\theta_4^2(\varphi_1)}$$

36. $$B_{33}^{2} = T_{33}^{2} \cdot \frac{\theta_3^2(\varphi_1)}{\theta_4^2(\varphi_1)}$$

25.3. Ряды Фурье для произведений и квадратов направляющих косинусов $\widetilde{B}_{ij}$.

$\tilde{B}_{ij})$ $$R_l = \frac{\theta_l\left(\varphi_1 + i\sigma_l\right)}{\theta_4\left(\varphi_1\right)} \qquad \left(l = 1,2,3,4\right), \qquad sn^2\left(i\frac{2\mathbf{K}}{\pi}\sigma\right) = -\frac{n}{k^2},$$

где σ_l принимает значения $\sigma, -\sigma, 0$ при $l = 1,2,3$ и $\sigma = \sigma, -\sigma$ при $l = 4$.

$\tilde{B}_{ij}\tilde{B}_{nk})$ $$R_{ij} = \frac{\theta_i\left(\varphi_1 + i\sigma_i\right)\theta_j\left(\varphi_1 + i\sigma_j\right)}{\theta_4^2\left(\varphi_1\right)}, \quad \left(i, j = 1,2,3,4; i \le j\right) \quad \sigma_i, \sigma_j \Rightarrow \left(\sigma, -\sigma, 0\right)$$

$$R_{ij} = \sum_{m=-\infty}^{\infty} a_m^{(i,j)} e^{im\varphi_1}$$

$$a_m^{(i,j)} = \frac{1}{2\pi}\int_0^{2\pi} R_{ij}\left(\varphi_1\right)e^{-im\varphi_1}\,d\varphi_1 \qquad R_{ij} = R_{ij;k} \quad \Rightarrow$$

$\left(i, j = 1,2,3,4; \quad i \le j\right)$

$k = 1, \quad \sigma_i = \sigma, \quad \sigma_j = \sigma;$

$k = 2, \quad \sigma_i = -\sigma, \quad \sigma_j = -\sigma;$

$k = 3, \quad \sigma_i = \sigma, \quad \sigma_j = -\sigma;$

$k = 4, \quad \sigma_i = -\sigma, \quad \sigma_j = \sigma;$

$k = 5, \quad \sigma_i = \sigma, \quad \sigma_j = 0;$

$k = 6, \quad \sigma_i = -\sigma, \quad \sigma_j = 0;$

$k = 7, \quad \sigma_i = 0, \quad \sigma_j = \sigma;$

$k = 8, \quad \sigma_i = 0, \quad \sigma_j = -\sigma;$

$k = 9, \quad \sigma_i = 0, \quad \sigma_j = 0.$

25.3.2. Свойства функций R_{ij}. Выражение контурного интеграла через вычеты. Формула для вычетов.

$$a_m^{(i,j)}=\frac{-\Delta_i\Delta_j q^{-\frac{m}{2}}e^{\sigma_i+\sigma_j}\theta_{5-i}\left(i\sigma_i\right)\theta_{5-j}\left(i\sigma_j\right)}{\phi^2\left[1-\delta_i\delta_j(-1)^{i+j}e^{2\left(\sigma_i+\sigma_j\right)}q^{-m}\right]}\left(m+im_{5-i}^{(1)}+im_{5-j}^{(1)}\right)\left(1+(-1)^m\delta_i\delta_j\right)$$

где, дополнительно, $\Delta_j=i$ при $j=1,4$ и $\Delta_j=1$ при $j=2,3$,

$$m_j^{(1)}=\frac{\theta_j'\left(i\sigma_j\right)}{\theta_j\left(i\sigma_j\right)},\qquad \phi=\theta_2(0)\theta_3(0)\theta_4(0).$$

$$m_1^{(1)}=\frac{\theta_1'(i\sigma)}{\theta_1(i\sigma)}=-i\frac{2\mathbf{\Pi}}{\pi\kappa}\sqrt{\left(1+\kappa^2\right)\left(\kappa^2+\lambda^2\right)},$$

$$m_2^{(1)}=\frac{\theta_2'(i\sigma)}{\theta_2(i\sigma)}=i\frac{2}{\pi\kappa}\sqrt{\frac{1+\kappa^2}{\kappa^2+\lambda^2}}\left[\mathbf{K}\lambda^2-\mathbf{\Pi}\left(\kappa^2+\lambda^2\right)\right],$$

$$m_3^{(1)}=\frac{\theta_3'(i\sigma)}{\theta_3(i\sigma)}=i\frac{2\sqrt{\kappa^2+\lambda^2}}{\pi\kappa\sqrt{1+\kappa^2}}\left[\mathbf{K}-\mathbf{\Pi}\left(1+\kappa^2\right)\right],$$

$$m_4^{(1)}=\frac{\theta_4'(i\sigma)}{\theta_4(i\sigma)}=i\frac{2\left(\mathbf{K}-\mathbf{\Pi}\right)}{\mathbf{K}\kappa}\sqrt{\left(1+\kappa^2\right)\left(\kappa^2+\lambda^2\right)}.$$

25.3.6.1. Общие формулы для рядов Фурье. Выражения коэффициентов $t_{ij;k}, P_{ij;k}$.

$$R_{ij;k} = \sum_{m=-\infty}^{\infty} a_m^{(i,j;k)} e^{-im\varphi_1}$$

$$, j = 1,2,3,4; \quad i \le j, \quad k \in \left(1,\ldots,9\right)\right).$$

$$a_m^{(i,j)} = \frac{-\Delta_i \Delta_j q^{-\frac{m}{2}} e^{\sigma_i+\sigma_j} \theta_{5-i}\left(i\sigma_i\right)\theta_{5-j}\left(i\sigma_j\right)}{\phi^2\left[1-\delta_i\delta_j(-1)^{i+j} e^{2\left(\sigma_i+\sigma_j\right)} q^{-m}\right]}\left(m + i m_{5-i}^{(1)} + i m_{5-j}^{(1)}\right)\left(1+(-1)^m \delta_i \delta_j\right),$$

где, дополнительно, $\Delta_j = i$ при $j = 1,4$ и $\Delta_j = 1$ при $j = 2,3$,

$$m_j^{(1)} = \frac{\theta_j'\left(i\sigma_j\right)}{\theta_j\left(i\sigma_j\right)}, \qquad \phi = \theta_2(0)\theta_3(0)\theta_4(0).$$

1. $$\frac{\theta_1^2\left(\varphi_1+i\sigma\right)}{\theta_4^2\left(\varphi_1\right)}=t_{11;1}\sum_{m=-\infty}^{\infty}\left(m+im_4^{(1)}\right)\frac{q^{-m}e^{2\sigma}}{1-e^{4\sigma}q^{-2m}}e^{2im\varphi_1}$$

2. $$\frac{\theta_1^2\left(\varphi_1-i\sigma\right)}{\theta_4^2\left(\varphi_1\right)}=t_{11;1}\sum_{m=-\infty}^{\infty}\left(m-im_4^{(1)}\right)\frac{q^{-m}e^{2\sigma}}{1-e^{4\sigma}q^{-2m}}e^{2im\varphi_1}$$

3. $$\frac{\theta_1\left(\varphi_1+i\sigma\right)\theta_1\left(\varphi_1-i\sigma\right)}{\theta_4^2\left(\varphi_1\right)}=t_{11;1}\sum_{m=-\infty}^{\infty}{}'\,\frac{mq^{-m}}{1-q^{-2m}}e^{2im\varphi_1}+t_{11;1}^{0}$$

4. $$\frac{\theta_1\left(\varphi_1+i\sigma\right)\theta_1\left(\varphi_1\right)}{\theta_4^2\left(\varphi_1\right)}=t_{11;5}\sum_{m=-\infty}^{\infty}\left(m+\frac{i}{2}m_4^{(1)}\right)\frac{q^{-m}e^{\sigma}}{1-e^{4\sigma}q^{-2m}}e^{2im\varphi_1}$$

5. $$\frac{\theta_1\left(\varphi_1-i\sigma\right)\theta_1\left(\varphi_1\right)}{\theta_4^2\left(\varphi_1\right)}=t_{11;5}\sum_{m=-\infty}^{\infty}\left(m-\frac{i}{2}m_4^{(1)}\right)\frac{q^{-m}e^{-\sigma}}{1-e^{4\sigma}q^{-2m}}e^{2im\varphi_1}$$

(1., 2., 3.) $$p_{1,1}^{(1,1)}=\frac{\pi^3}{2\mathbf{K}^3kk'}$$

(4., 5.) $$p_{1,1}^{(1,0)}=\frac{\pi^2\sqrt{\pi}}{\sqrt{2\mathbf{K}k'}\mathbf{K}^2k}$$

(6.) $$p_{1,1}^{(0,0)}=\frac{\pi^2}{\mathbf{K}^2k}$$

(7., 8., 9., 10.) $$p_{1,2}^{(1,1)}=\frac{\pi^3dna}{2\mathbf{K}^3kk'\sqrt{k'}}$$

. . . .

64. $$\frac{\theta_4^2\left(\varphi_1+i\sigma\right)}{\theta_4^2\left(\varphi_1\right)}=t_{44;1}\sum_{m=-\infty}^{\infty}\left[m+im_1^{(1)}\right]\frac{q^{-m}e^{2\sigma}}{1-e^{4\sigma}q^{-2m}}e^{2im\varphi_1}$$

65. $$\frac{\theta_4^2\left(\varphi_1-i\sigma\right)}{\theta_4^2\left(\varphi_1\right)}=t_{44;1}\sum_{m=-\infty}^{\infty}\left[m-im_1^{(1)}\right]\frac{q^{-m}e^{-2\sigma}}{1-e^{-4\sigma}q^{-2m}}e^{2im\varphi_1}$$

66. $$\frac{\theta_4\left(\varphi_1+i\sigma\right)\theta_4\left(\varphi_1-i\sigma\right)}{\theta_4^2\left(\varphi_1\right)}=t_{44;1}\sum_{m=-\infty}^{\infty}{}'\,\frac{mq^{-m}}{1-q^{-2m}}e^{2im\varphi_1}+t_{44;1}^{0}$$

(1., 2., 3.) $$t_{11;1}=\frac{4}{\phi^2}\theta_4^2\left(i\sigma\right)=p_{11;1}\,\theta_4^2\left(i\sigma\right)$$

(4., 5.) $$t_{11;5}=\frac{4\theta_4\left(0\right)}{\phi^2}\theta_4\left(i\sigma\right)=p_{11;5}\,\theta_4\left(i\sigma\right)$$

(6.) $$t_{11;9}=\frac{4\theta_4^2\left(0\right)}{\phi^2}=p_{11;9}$$

(7., 8., 9., 10.) $$t_{12;1}=-\frac{4i\theta_3\left(i\sigma\right)\theta_4\left(i\sigma\right)}{\phi^2}=-i\,p_{12;1}\,\theta_4^2\left(i\sigma\right)$$

$$m_1^{(1)}\left(i\sigma\right)=\frac{\theta_1'\left(i\sigma\right)}{\theta_1\left(i\sigma\right)}=\frac{2\mathbf{\Pi}}{\pi}\frac{cnadna}{sna},$$

$$m_2^{(1)}\left(i\sigma\right)=\frac{\theta_2'\left(i\sigma\right)}{\theta_2\left(i\sigma\right)}=\frac{2dna}{\pi snacna}\left(\mathbf{\Pi}cn^2a-\mathbf{K}\right),$$

$$m_3^{(1)}\left(i\sigma\right)=\frac{\theta_3'\left(i\sigma\right)}{\theta_3\left(i\sigma\right)}=\frac{2cna}{\pi snadna}\left(\mathbf{\Pi}dn^2a-\mathbf{K}\right),$$

$$m_4^{(1)}\left(i\sigma\right)=\frac{\theta_4'\left(i\sigma\right)}{\theta_4\left(i\sigma\right)}=\frac{2\left(\mathbf{\Pi}-\mathbf{K}\right)}{\pi}\frac{cn\,a\;dn\,a}{sn\,a}.$$

$$m_1^{(1)}=\frac{\theta_1'\left(i\sigma\right)}{\theta_1\left(i\sigma\right)}=-i\frac{2\mathbf{\Pi}}{\pi\kappa}\sqrt{\left(1+\kappa^2\right)\left(\kappa^2+\lambda^2\right)},$$

$$m_2^{(1)}=\frac{\theta_2'\left(i\sigma\right)}{\theta_2\left(i\sigma\right)}=i\frac{2}{\pi\kappa}\sqrt{\frac{1+\kappa^2}{\kappa^2+\lambda^2}}\left[\mathbf{K}\lambda^2-\mathbf{\Pi}\left(\kappa^2+\lambda^2\right)\right],$$

$$m_3^{(1)}=\frac{\theta_3'\left(i\sigma\right)}{\theta_3\left(i\sigma\right)}=i\frac{2\sqrt{\kappa^2+\lambda^2}}{\pi\kappa\sqrt{1+\kappa^2}}\left[\mathbf{K}-\mathbf{\Pi}\left(1+\kappa^2\right)\right],$$

$$m_4^{(1)}=\frac{\theta_4'\left(i\sigma\right)}{\theta_4\left(i\sigma\right)}=i\frac{2\left(\mathbf{K}-\mathbf{\Pi}\right)}{\mathbf{K}\kappa}\sqrt{\left(1+\kappa^2\right)\left(\kappa^2+\lambda^2\right)}.$$

Приложение 3. Контроль правильности рядов Фурье для произведений и квадратов направляющих косинусов $b_{ij}b_{nk}$.

[1] $b_{11}^2+b_{12}^2+b_{13}^2=1$,

[2] $b_{21}^2+b_{22}^2+b_{23}^2=1$,

[3] $b_{31}^2+b_{32}^2+b_{33}^2=1$,

[4] $b_{11}b_{21}+b_{12}b_{22}+b_{13}b_{23}=0$,

[5] $b_{11}b_{31}+b_{12}b_{32}+b_{13}b_{33}=0$,

[6] $b_{21}b_{31}+b_{22}b_{32}+b_{23}b_{33}=0$,

[7] $b_{11}^2+b_{21}^2+b_{31}^2=1$,

[8] $b_{12}^2+b_{22}^2+b_{32}^2=1$,

[9] $b_{13}^2+b_{23}^2+b_{33}^2=1$,

[10] $b_{11}b_{12}+b_{21}b_{22}+b_{31}b_{32}=0$,

[11] $b_{11}b_{13}+b_{21}b_{23}+b_{31}b_{33}=0$,

[12] $b_{12}b_{13}+b_{22}b_{23}+b_{32}b_{33}=0$.

[13] $b_{33}=b_{11}b_{22}-b_{21}b_{12}$.

[14] $b_{32}=b_{21}b_{13}-b_{11}b_{23}$,

[15] $b_{31}=b_{23}b_{12}-b_{22}b_{13}$,

[16] $b_{23}=b_{31}b_{12}-b_{11}b_{32}$,

[17] $b_{22}=b_{11}b_{33}-b_{31}b_{13}$,

[18] $b_{21}=b_{32}b_{13}-b_{12}b_{33}$,

[19] $b_{13}=b_{21}b_{32}-b_{31}b_{22}$,

[20] $b_{12}=b_{31}b_{23}-b_{21}b_{33}$,

[21] $b_{11}=b_{22}b_{33}-b_{32}b_{23}$.

[1] $b_{11}^2+b_{12}^2+b_{13}^2=1$.

$$b_{11}^2=b_{0,0}^{(11,11)}+b_{0,2}^{(11,11)}\cos 2\varphi_2+\sum_{m=1}^{\infty}\left\{b_{2m,0}^{(11,11)}\cos 2(m\varphi_1)+b_{2m,2}^{(11,11)}\cos 2(m\varphi_1+\varphi_2)+b_{2m,-2}^{(11,11)}\cos 2(m\varphi_1-\varphi_2)\right\}$$

$$b_{12}^2=b_{0,0}^{(12,12)}+b_{0,2}^{(12,12)}\cos 2\varphi_2+\sum_{m=1}^{\infty}\left\{b_{2m,0}^{(12,12)}\cos 2(m\varphi_1)+b_{2m,2}^{(12,12)}\cos 2(m\varphi_1+\varphi_2)+b_{2m,-2}^{(12,12)}\cos 2(m\varphi_1-\varphi_2)\right\}$$

$$b_{13}^2=b_{0,0}^{(13,13)}+b_{0,2}^{(13,13)}\cos 2\varphi_2+\sum_{m=1}^{\infty}\left\{b_{2m,0}^{(13,13)}\cos 2(m\varphi_1)+b_{2m,2}^{(13,13)}\cos 2(m\varphi_1+\varphi_2)+b_{2m,-2}^{(13,13)}\cos 2(m\varphi_1-\varphi_2)\right\}$$

$$b_{0,0}^{(11,11)}=\frac{\mathbf{K}(1+\kappa^2)-\mathbf{E}}{2\mathbf{K}(\kappa^2+\lambda^2)},$$

$$b_{0,2}^{(11,11)}=\frac{-\pi^2}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{\mathbf{M}_4}{\sinh(2\sigma)},$$

$$b_{2m,0}^{(11,11)}=\frac{-\pi^2}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m}{\sinh(2md)},$$

$$b_{2m,2}^{(11,11)}=\frac{-\pi^2}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m-2\mathbf{M}_4}{\sinh\left[2(md-\sigma)\right]},$$

$$b_{2m,-2}^{(11,11)}=\frac{-\pi^2}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m+2\mathbf{M}_4}{\sinh\left[2(md+\sigma)\right]},$$

$$b_{0,0}^{(12,12)}=\frac{1}{2}\left[1+\frac{(1+\kappa^2)(\mathbf{E}-\mathbf{K})}{\mathbf{K}(\kappa^2+\lambda^2)}\right],$$

$$b_{0,2}^{(12,12)}=\frac{\pi^2(1+\kappa^2)}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{\mathbf{M}_3}{\sinh(2\sigma)},$$

$$b_{2m,0}^{(12,12)}=\frac{\pi^2(1+\kappa^2)}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m}{\sinh(2md)},$$

$$b_{2m,2}^{(12,12)}=\frac{\pi^2(1+\kappa^2)}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m-2\mathbf{M}_3}{\sinh\left[2(md-\sigma)\right]},$$

$$b_{2m,-2}^{(12,12)}=\frac{\pi^2(1+\kappa^2)}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m+2\mathbf{M}_3}{\sinh\left[2(md+\sigma)\right]}.$$

$$b_{0,0}^{(13,13)}=\frac{1}{2}\left[1-\frac{\kappa^2\mathbf{E}}{\mathbf{K}(\kappa^2+\lambda^2)}\right],$$

$$b_{0,2}^{(12,12)}=-\frac{\pi^2\kappa^2}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{\mathbf{M}_1}{\sinh(2\sigma)},$$

$$b_{2m,0}^{(13,13)}=-\frac{\pi^2\kappa^2}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m}{\sinh(2md)},$$

$$b_{2m,2}^{(13,13)}=-\frac{\pi^2\kappa^2}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m-2\mathbf{M}_1}{\sinh\left[2(md-\sigma)\right]},$$

$$b_{2m,-2}^{(13,13)}=-\frac{\pi^2\kappa^2}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m+2\mathbf{M}_1}{\sinh\left[2(md+\sigma)\right]}.$$

Проверка [1]: $b_{11}^2+b_{12}^2+b_{13}^2=1$.

$$\sum_{i=1}^{3}b_{0,0}^{(1i,1i)}=\frac{1}{2K(\kappa^2+\lambda^2)}\left\{\mathbf{K}[1+\kappa^2+\lambda^2-1+\kappa^2+\lambda^2]+\mathbf{E}[-1+1+\kappa^2-\kappa^2]\right\}\equiv 1$$

$$\sum_{i=1}^{3}b_{0,2}^{(1i,1i)}=-\frac{\pi^2}{2K^2(\kappa^2+\lambda^2)\sinh(2\sigma)}[\mathbf{M_4}-(1+\kappa^2)\mathbf{M_3}+\kappa^2\mathbf{M_1}]\equiv 0$$

$$\sum_{i=1}^{3}b_{2m,0}^{(1i,1i)}=\frac{\pi^2 m}{2K^2(\kappa^2+\lambda^2)\sinh(2\sigma)}[-1+1+\kappa^2-\kappa^2]\equiv 0$$

$$\sum_{i=1}^{3}b_{2m,2\varepsilon}^{(1i,1i)}=\frac{\pi^2}{2K^2(\kappa^2+\lambda^2)\sinh\left[2(md-\varepsilon\sigma)\right]}[m(-1+1+\kappa^2-\kappa^2)+2\varepsilon[\mathbf{M_4}-(1+\kappa^2)\mathbf{M_3}+\kappa^2\mathbf{M_1}]\equiv 0$$

Приложение 3. Контроль правильности рядов Фурье для произведений и квадратов направляющих косинусов $b_{ij}b_{nk}$. [1] $b_{11}^2+b_{12}^2+b_{13}^2=1$.

$$b_{11}^2=b_{0,0}^{(11,11)}+b_{0,2}^{(11,11)}\cos 2\varphi_2+\sum_{m=1}^{\infty}\left\{b_{2m,0}^{(11,11)}\cos 2(m\varphi_1)+b_{2m,2}^{(11,11)}\cos 2(m\varphi_1+\varphi_2)+b_{2m,-2}^{(11,11)}\cos 2(m\varphi_1-\varphi_2)\right\}$$

$$b_{12}^2=b_{0,0}^{(12,12)}+b_{0,2}^{(12,12)}\cos 2\varphi_2+\sum_{m=1}^{\infty}\left\{b_{2m,0}^{(12,12)}\cos 2(m\varphi_1)+b_{2m,2}^{(12,12)}\cos 2(m\varphi_1+\varphi_2)+b_{2m,-2}^{(12,12)}\cos 2(m\varphi_1-\varphi_2)\right\}$$

$$b_{13}^2=b_{0,0}^{(13,13)}+b_{0,2}^{(13,13)}\cos 2\varphi_2+\sum_{m=1}^{\infty}\left\{b_{2m,0}^{(13,13)}\cos 2(m\varphi_1)+b_{2m,2}^{(13,13)}\cos 2(m\varphi_1+\varphi_2)+b_{2m,-2}^{(13,13)}\cos 2(m\varphi_1-\varphi_2)\right\}$$

$$b_{0,0}^{(11,11)}=\frac{\mathbf{K}(1+\kappa^2)-\mathbf{E}}{2\mathbf{K}(\kappa^2+\lambda^2)},$$

$$b_{0,2}^{(11,11)}=\frac{-\pi^2}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{\mathbf{M_4}}{\sinh(2\sigma)},$$

$$b_{2m,0}^{(11,11)}=\frac{-\pi^2}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m}{\sinh(2md)},$$

$$b_{2m,2}^{(11,11)}=\frac{-\pi^2}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m-2\mathbf{M_4}}{\sinh\left[2(md-\sigma)\right]},$$

$$b_{2m,-2}^{(11,11)}=\frac{-\pi^2}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m+2\mathbf{M_4}}{\sinh\left[2(md+\sigma)\right]},$$

$$b_{0,0}^{(12,12)}=\frac{1}{2}\left[1+\frac{(1+\kappa^2)(\mathbf{E}-\mathbf{K})}{\mathbf{K}(\kappa^2+\lambda^2)}\right],$$

$$b_{0,2}^{(12,12)}=\frac{\pi^2(1+\kappa^2)}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{\mathbf{M}_3}{\sinh(2\sigma)},$$

$$b_{2m,0}^{(12,12)}=\frac{\pi^2(1+\kappa^2)}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m}{\sinh(2md)},$$

$$b_{2m,2}^{(12,12)}=\frac{\pi^2(1+\kappa^2)}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m-2\mathbf{M}_3}{\sinh\left[2(md-\sigma)\right]},$$

$$b_{2m,-2}^{(12,12)}=\frac{\pi^2(1+\kappa^2)}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m+2\mathbf{M}_3}{\sinh\left[2(md+\sigma)\right]}.$$

$$b_{0,0}^{(13,13)}=\frac{1}{2}\left[1-\frac{\kappa^2\mathbf{E}}{\mathbf{K}(\kappa^2+\lambda^2)}\right],$$

$$b_{0,2}^{(12,12)}=-\frac{\pi^2\kappa^2}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{\mathbf{M}_1}{\sinh(2\sigma)},$$

$$b_{2m,0}^{(13,13)}=-\frac{\pi^2\kappa^2}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m}{\sinh(2md)},$$

$$b_{2m,2}^{(13,13)}=-\frac{\pi^2\kappa^2}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m-2\mathbf{M}_1}{\sinh\left[2(md-\sigma)\right]},$$

$$b_{2m,-2}^{(13,13)}=-\frac{\pi^2\kappa^2}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m+2\mathbf{M}_1}{\sinh\left[2(md+\sigma)\right]}.$$

Проверка [1]: $b_{11}^2+b_{12}^2+b_{13}^2=1$.

$$\sum_{i=1}^{3}b_{0,0}^{(1i,1i)}=\frac{1}{2K(\kappa^2+\lambda^2)}\left\{\mathbf{K}[1+\kappa^2+\lambda^2-1+\kappa^2+\lambda^2]+\mathbf{E}[-1+1+\kappa^2-\kappa^2]\right\}\equiv 1$$

$$\sum_{i=1}^{3}b_{0,2}^{(1i,1i)}=-\frac{\pi^2}{2K^2(\kappa^2+\lambda^2)\sinh(2\sigma)}[\mathbf{M_4}-(1+\kappa^2)\mathbf{M_3}+\kappa^2\mathbf{M_1}]\equiv 0$$

$$\sum_{i=1}^{3}b_{2m,0}^{(1i,1i)}=\frac{\pi^2 m}{2K^2(\kappa^2+\lambda^2)\sinh(2\sigma)}[-1+1+\kappa^2-\kappa^2]\equiv 0$$

$$\sum_{i=1}^{3}b_{2m,2\varepsilon}^{(1i,1i)}=\frac{\pi^2}{2K^2(\kappa^2+\lambda^2)\sinh\left[2(md-\varepsilon\sigma)\right]}[m(-1+1+\kappa^2-\kappa^2)+2\varepsilon[\mathbf{M_4}-(1+\kappa^2)\mathbf{M_3}+\kappa^2\mathbf{M_1}]\equiv 0$$

$$\textbf{[10]}\ b_{11}b_{12}+b_{21}b_{22}+b_{31}b_{32}=0\,.$$

$$b_{11}b_{12}=b_{0,2}^{(12,11)}\sin 2\varphi_2+\sum_{m=1}^{\infty}\left\{b_{2m,0}^{(12,11)}\sin 2(m\varphi_1)+b_{2m,2}^{(12,11)}\sin 2(m\varphi_1+\varphi_2)+b_{2m,-2}^{(12,11)}\sin 2(m\varphi_1-\varphi_2)\right\},$$

$$b_{21}b_{22}=b_{0,2}^{(22,21)}\sin 2\varphi_2+\sum_{m=1}^{\infty}\left\{b_{2m,0}^{(22,21)}\sin 2(m\varphi_1)+b_{2m,2}^{(22,21)}\sin 2(m\varphi_1+\varphi_2)+b_{2m,-2}^{(22,21)}\sin 2(m\varphi_1-\varphi_2)\right\},$$

$$b_{31}b_{32}=\sum_{m=1}^{\infty}\left\{b_{2m,0}^{(32,31)}\sin 2(m\varphi_1)\right\}.$$

$$b_{2m,0}^{(32,31)}=-\frac{\pi^2\sqrt{1+\kappa^2}}{\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m}{\cosh(2md)},$$

$$b_{0,2}^{(12,11)}=-\frac{\pi^2\sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{\mathbf{M}_3+\mathbf{M}_4}{\cosh(2\sigma)},$$

$$b_{2m,0}^{(12,11)}=\frac{\pi^2\sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m}{\cosh(2md)},$$

$$b_{2m,2}^{(12,11)}=\frac{\pi^2\sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m-\mathbf{M}_3-\mathbf{M}_4}{\cosh\left[2(md-\sigma)\right]},$$

$$b_{2m,-2}^{(12,11)}=\frac{\pi^2\sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m+\mathbf{M}_3+\mathbf{M}_4}{\cosh\left[2(md+\sigma)\right]}.$$

$$b_{0,2}^{(22,21)}=\frac{\pi^2\sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{\mathbf{M}_3+\mathbf{M}_4}{\cosh(2\sigma)},$$

$$b_{2m,0}^{(22,21)}=\frac{\pi^2\sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m}{\cosh(2md)},$$

$$b_{2m,2}^{(22,21)}=-\frac{\pi^2\sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m-\mathbf{M}_3-\mathbf{M}_4}{\cosh\left[2(md-\sigma)\right]},$$

$$b_{2m,-2}^{(22,21)}=-\frac{\pi^2\sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2+\lambda^2)}\cdot\frac{m+\mathbf{M}_3+\mathbf{M}_4}{\cosh\left[2(md+\sigma)\right]}.$$

Проверка [10].

$$\sum_{i=1}^{3}b_{0,2}^{(i1,i2)}=\frac{\pi^2\sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2+\lambda^2)\cosh(2\sigma)}[-\mathbf{M_3}-\mathbf{M_4}+\mathbf{M_3}+\mathbf{M_4}]\equiv 0\,,$$

$$\sum_{i=1}^{3}b_{2m,0}^{(i1,i2)}=\frac{\pi^2\sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2+\lambda^2)\cosh(2md)}[m+m-2m]\equiv 0\,,$$

$$\sum_{i=1}^{3}b_{2m,2\varepsilon}^{(i1,i2)}=\frac{\pi^2\sqrt{1+\kappa^2}}{4\mathbf{K}^2(\kappa^2+\lambda^2)\cosh\left[2(md-\varepsilon\sigma)\right]}[m-m-\varepsilon(\mathbf{M_3}+\mathbf{M_4})+\varepsilon(\mathbf{M_3}+\mathbf{M_4})]\equiv 0\,.$$

$$\textbf{[16]}\ \ b_{23} = b_{31} b_{12} - b_{11} b_{32}\,.$$

$$b_{31} b_{12} = \sum_{m=1}^{\infty} \left\{ b_{0,1}^{(31,12)} \cos(\varphi_2) + b_{2m,1}^{(31,12)} \cos(2m\varphi_1 + \varphi_2) + b_{2m,-1}^{(31,12)} \cos(2m\varphi_1 - \varphi_2) \right\},$$

$$b_{11} b_{32} = \sum_{m=1}^{\infty} \left\{ b_{0,1}^{(11,32)} \cos(\varphi_2) + b_{2m,1}^{(11,32)} \cos(2m\varphi_1 + \varphi_2) + b_{2m,-1}^{(11,32)} \cos(2m\varphi_1 - \varphi_2) \right\},$$

$$b_{23} = b_{0,1}^{(23)} \cos(\varphi_2) + \sum_{m=0}^{\infty} \left\{ b_{2m,1}^{(23)} \cos(2m\varphi_1 + \varphi_2) + b_{2m,-1}^{(23)} \cos(2m\varphi_1 - \varphi_2) \right\},$$

$$b_{0,1}^{(31,12)} = -\frac{\pi^2\sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2+\lambda^2)} \cdot \frac{\mathbf{M}_3}{\sinh(\sigma)}, \qquad b_{0,1}^{(11,32)} = -\frac{\pi^2\sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2+\lambda^2)} \cdot \frac{\mathbf{M}_4}{\sinh(\sigma)}, \qquad b_{0,1}^{(23)} = \frac{\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \cdot \frac{1}{\sinh(\sigma)},$$

$$b_{2m,1}^{(31,12)} = -\frac{\pi^2\sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2+\lambda^2)} \cdot \frac{m-\mathbf{M}_3}{\sinh(2md-\sigma)}, \quad b_{2m,1}^{(11,32)} = -\frac{\pi^2\sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2+\lambda^2)} \cdot \frac{m-\mathbf{M}_4}{\sinh(2md-\sigma)}, \quad b_{2m,1}^{(23)} = -\frac{\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \cdot \frac{1}{\sinh(2md-\sigma)},$$

$$b_{2m,-1}^{(31,12)} = -\frac{\pi^2\sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2+\lambda^2)} \cdot \frac{m+\mathbf{M}_3}{\sinh(2md+\sigma)}. \quad b_{2m,-1}^{(11,32)} = -\frac{\pi^2\sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2+\lambda^2)} \cdot \frac{m+\mathbf{M}_4}{\sinh(2md+\sigma)}. \quad b_{2m,-1}^{(23)} = \frac{\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \cdot \frac{1}{\sinh(2md+\sigma)}.$$

$$\mathbf{M}_1 = \frac{\mathbf{\Pi}}{\pi\kappa}\sqrt{(1+\kappa^2)(\kappa^2+\lambda^2)},$$

$$\mathbf{M}_2 = \frac{\sqrt{1+\kappa^2}}{\pi\kappa\sqrt{\kappa^2+\lambda^2}}\left[\mathbf{\Pi}(\kappa^2+\lambda^2) - \mathbf{K}\lambda^2\right],$$

$$\mathbf{M}_3 = \frac{\sqrt{\kappa^2+\lambda^2}}{\pi\kappa\sqrt{1+\kappa^2}}\left[\mathbf{\Pi}(1+\kappa^2) - \mathbf{K}\right],$$

$$\mathbf{M}_4 = \frac{\mathbf{\Pi}-\mathbf{K}}{\pi\kappa}\sqrt{(1+\kappa^2)(\kappa^2+\lambda^2)},$$

Проверка [16].

$$b_{0,1}^{(31,12)} - b_{0,1}^{(11,32)} = \frac{\pi^2\sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2+\lambda^2)\sinh[\sigma]}[\mathbf{M}_4 - \mathbf{M}_3] =$$

$$= \frac{\pi^2\sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2+\lambda^2)\sinh[\sigma]}\left[-\frac{\kappa\mathbf{K}\sqrt{\kappa^2+\lambda^2}}{\pi\sqrt{1+\kappa^2}}\right] = -\frac{\pi\kappa}{2\mathbf{K}\sqrt{\kappa^2+\lambda^2}} \cdot \frac{1}{\sinh[\sigma]} = -b_{0,1}^{(23)},$$

$$b_{2m,\varepsilon}^{(31,12)} - b_{2m,\varepsilon}^{(11,32)} = -\frac{\pi^2\kappa\sqrt{1+\kappa^2}}{2\mathbf{K}^2(\kappa^2+\lambda^2)\sinh[2md-\varepsilon\sigma]}[\varepsilon m - \varepsilon m + \mathbf{M}_4 - \mathbf{M}_3] = -b_{2m+1,\varepsilon}^{(23)}.$$

26.2. Ряды Фурье для проекций вектора угловой скорости и вектора углового ускорения на оси промежуточной системы координат, связанной с вектором кинетического момента.

$$\omega_{G_1} = \omega_{G_1}^{(0)} \sum_{m=1}^{\infty} \left\{ \frac{\sin(\varphi_2)}{\cosh(\sigma)} + \frac{\sin(2m\varphi_1 + \varphi_2)}{\cosh(2md - \sigma)} - \frac{\sin(2m\varphi_1 - \varphi_2}{\cosh(2md + \sigma)}) \right\},$$

$$\omega_{G_2} = -\omega_{G_2}^{(0)} \sum_{m=1}^{\infty} \left\{ \frac{\cos(\varphi_2)}{\cosh(\sigma)} + \frac{\cos(2m\varphi_1 + \varphi_2)}{\cosh(2md - \sigma)} + \frac{\cos(2m\varphi_1 - \varphi_2}{\cosh(2md + \sigma)}) \right\},$$

$$\omega_{G_3} = \omega_{G_3}^{(0)} = G \frac{A-B}{AB} \frac{1+\kappa^2}{\kappa^2+\lambda^2}, \quad \omega_{G_1}^{(0)} = G \frac{\pi(A-B)}{2AB\mathbf{K}} \frac{\sqrt{1+\kappa^2}}{\kappa\sqrt{\kappa^2+\lambda^2}}.$$

$$\varepsilon_{G_1} = \omega_{G_1}^{(0)} \sum_{m=1}^{\infty} \left\{ \frac{n_2}{\cosh(\sigma)} \cos(\varphi_2) + \frac{2mn_1 + n_2}{\cosh(2md - \sigma)} \cos(2m\varphi_1 + \varphi_2) - \frac{2mn_1 - n_2}{\cosh(2md + \sigma)} \cos(2m\varphi_1 - \varphi_2) \right\},$$

$$\varepsilon_{G_2} = \omega_{G_2}^{(0)} \sum_{m=1}^{\infty} \left\{ \frac{n_2}{\cosh(\sigma)} \sin(\varphi_2) + \frac{2mn_1 + n_2}{\cosh(2md - \sigma)} \sin(2m\varphi_1 + \varphi_2) + \frac{2mn_1 - n_2}{\cosh(2md + \sigma)} \sin(2m\varphi_1 - \varphi_2) \right\},$$

$$\varepsilon_{G_3} = 0,$$

где

$$\omega_{G_3} = \omega_{G_3}^{(0)} = G \frac{A-B}{AB} \frac{1+\kappa^2}{\kappa^2+\lambda^2}, \quad \omega_{G_1}^{(0)} = G \frac{\pi(A-B)}{2AB\mathbf{K}} \frac{\sqrt{1+\kappa^2}}{\kappa\sqrt{\kappa^2+\lambda^2}},$$

$$\kappa^2 = \frac{C(A-B)}{A(B-C)}, \qquad n_1 = \frac{I_2(A-C)}{2AC} \frac{\pi\kappa}{\sqrt{(1+\kappa^2)(\kappa^2+\lambda^2)}K(\lambda)}.$$

1. Ряды Фурье функций : $e^{-2i\omega}$, $e^{2i\omega}$, $\cos 2\omega$, $\sin 2\omega$, $\cos^2 \omega$, $\sin^2 \omega$, $\cos 2g$, $\sin 2g$, $\cos^2 g$, $\sin^2 g$.

2. Ряды Фурье функций: $e^{-4i\omega}$, $e^{4i\omega}$, $\cos 4\omega$, $\sin 4\omega$, $\cos^2 2\omega$, $\sin^2 2\omega$, $\cos 4g$, $\sin 4g$, $\cos^2 2g$, $\sin^2 2g$.

3. Ряды Фурье функций: $\cos\theta$, $\sin\theta$, $\cos^2 \dfrac{\theta}{2}$, $\sin^2 \dfrac{\theta}{2}$.

4. О разложениях скоростей $\dot{l}$, $\dot{L}$, $\dot{g}$, $\dot{\theta}$ **и правых частей уравнений Андуайе для задачи Эйлера-Пуансо.**

5. Ряды Фурье функций: $\cos^2 l$, $\sin^2 l$, $\cos 2l$, $\sin 2l$.

6. Ряды Фурье функций: $\cos^2 \theta$, $\sin^2 \theta$, $\cos 2\theta$, $\sin 2\theta$.

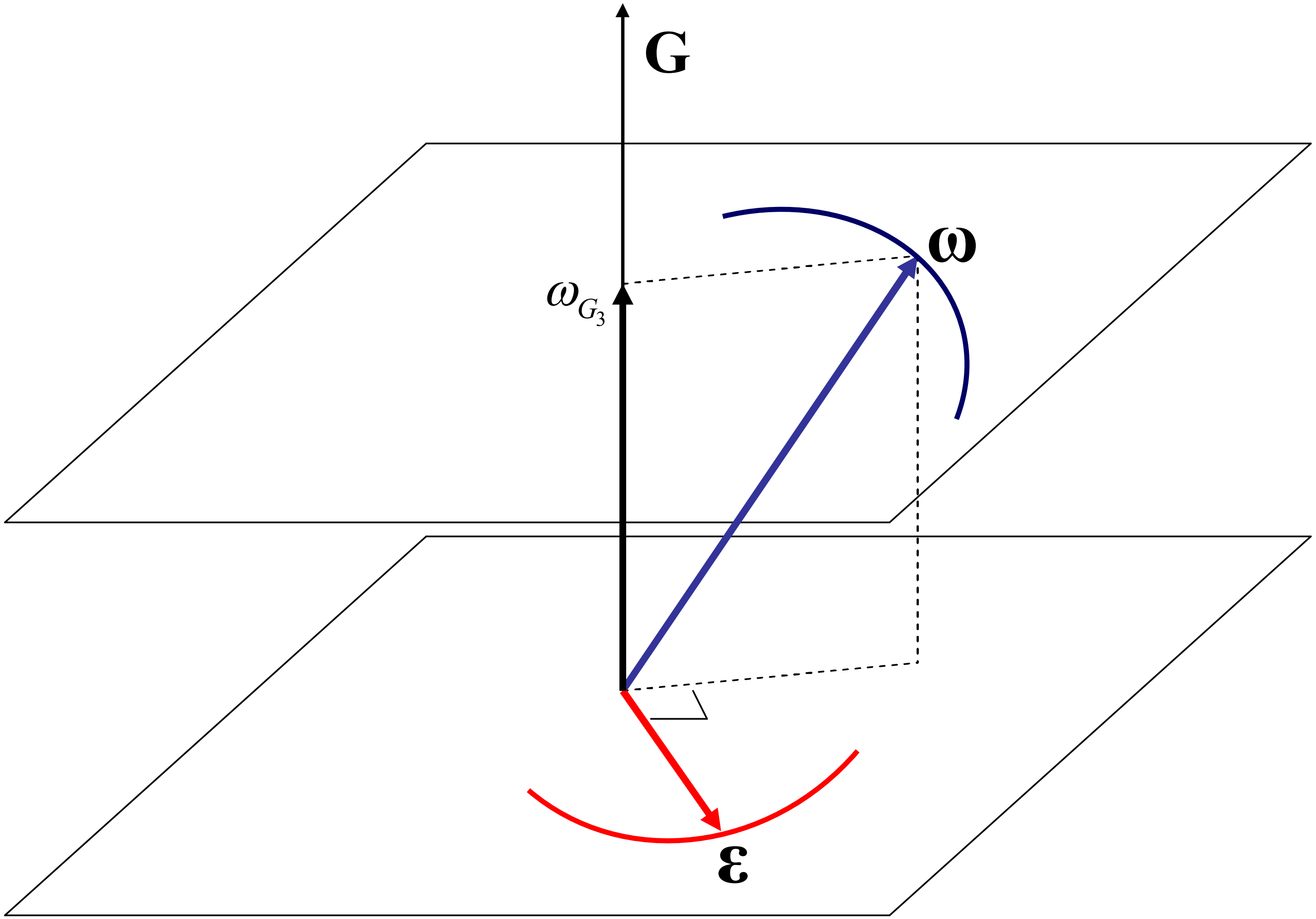
7. Ряды Фурье функций: $\cos\theta\cos^2 l$, $\cos\theta\sin^2 l$, $\cos\theta\cos 2l$, $\cos\theta\sin 2l$.

8. Ряды Фурье функций для переменных: g, $\dot{g}$, $\ddot{g}$.

9. Ряды Фурье функций для переменных: l, $\dot{l}$, $\ddot{l}$.

10. Ряды Фурье функции $\sin\theta\sin l\cos l$ **и переменных** θ, $\dot{\theta}$, $\ddot{\theta}$.

11. Решение задачи Эйлера-Пуансо в рядах Фурье для переменных Андуайе.



G
ω
ω_{G_3}
ε

Гессиан задачи:

$$\Delta = \begin{vmatrix} \dfrac{\partial^2 F_0}{\partial I_1^2} & \dfrac{\partial^2 F_0}{\partial I_1 \partial I_2} \\ \dfrac{\partial^2 F_0}{\partial I_2 \partial I_1} & \dfrac{\partial^2 F_0}{\partial I_2^2} \end{vmatrix} = \begin{vmatrix} \dfrac{\partial n_1}{\partial I_1} & \dfrac{\partial n_1}{\partial I_2} \\ \dfrac{\partial n_2}{\partial I_1} & \dfrac{\partial n_2}{\partial I_2} \end{vmatrix}$$

$$F_0 = \frac{I_2^2}{2A}\left[1 + \frac{A-C}{C}\frac{\kappa^2}{\lambda^2+\kappa^2}\right],$$

$$n_1 = -\frac{1}{G\mathbf{J}(\lambda)}\frac{\partial F_0}{\partial \lambda}, \quad n_2 = \frac{\partial F_0}{\partial G} + \frac{\mathbf{\Lambda}(\lambda)}{G\mathbf{J}(\lambda)}\frac{\partial F_0}{\partial \lambda} \ .$$

$$\Delta = \frac{\pi^2 (A-C)}{4AC^2\left(1+\kappa^2\right)\lambda^2\lambda'^2\mathbf{K}^3}\left\{\lambda^2\left\{\kappa^2\mathbf{K} + \left[1 + (C-A)/A\right]\mathbf{E}\right\} + \kappa^2\left(\mathbf{E}-\mathbf{K}\right)\right\} > 0$$

16.8. Соотношения между углами θ, γ и θ_ω в задаче Эйлера-Пуансо для осесимметричного тела. Ряды невозмущенного движения сфероида. $C > A$

$$\tan\theta_\omega = \sqrt{\frac{1+e}{1-e}}\tan\theta \qquad 0 < e = \frac{C^2 - A^2}{A^2 + C^2} \qquad \sigma = \frac{\sqrt{1+e}-\sqrt{1-e}}{\sqrt{1+e}+\sqrt{1-e}} = \frac{C-A}{C+A}$$

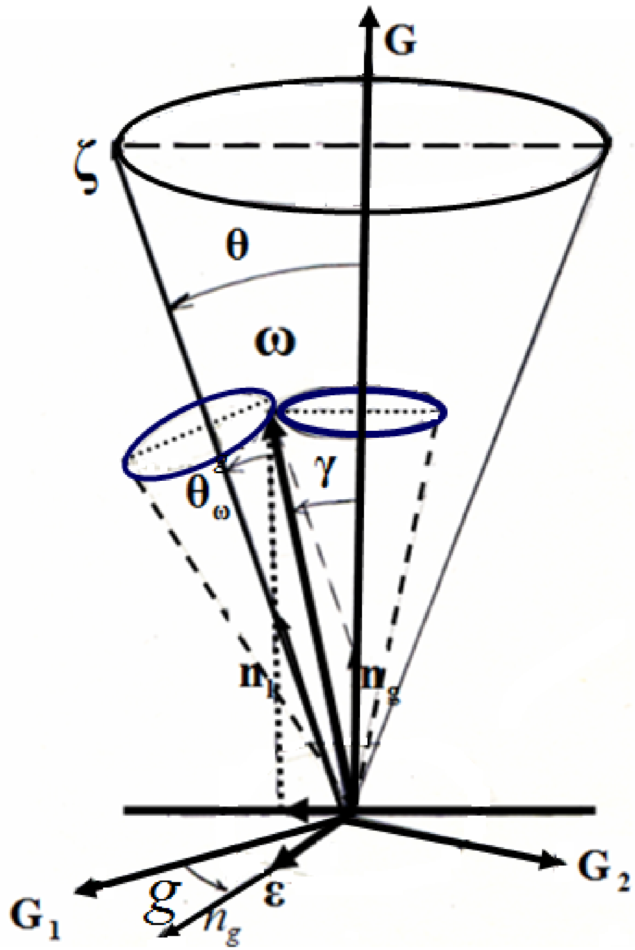
$$\theta_\omega - \theta = 2\sum_{m=1}^{\infty}\frac{\sigma^m \sin m\theta}{m},$$

$$\theta - \theta_\omega = 2\sum_{m=1}^{\infty}\frac{(-1)^m \sigma^m \sin m\theta}{m},$$

$$\cos\theta_\omega = -\sigma + \left(1-\sigma^2\right)\sum_{m=1}^{\infty}\sigma^{m-1}\cos m\theta,$$

$$\sin\theta_\omega = \left(1-\sigma^2\right)\sum_{m=1}^{\infty}\sigma^{m-1}\sin m\theta,$$

$$\cos\theta = \sigma + \left(1-\sigma^2\right)\sum_{m=1}^{\infty}(-1)^{m-1}\sigma^{m-1}\cos m\theta_\omega .$$



30.1. Уравнения возмущенного вращательного движения деформируемого тела.

$$H = H_0 + H_1$$

$$\frac{d\varphi_1}{dt} = \frac{\partial H}{\partial I_1}, \quad \frac{dI_1}{dt} = -\frac{\partial H}{\partial \varphi_1},$$

$$\frac{d\varphi_2}{dt} = \frac{\partial H}{\partial I_2}, \quad \frac{dI_2}{dt} = -\frac{\partial H}{\partial \varphi_2},$$

$$\frac{d\varphi_3}{dt} = \frac{\partial H}{\partial I_3}, \quad \frac{dI_3}{dt} = -\frac{\partial H}{\partial \varphi_3},$$

$$H_0 = \frac{I_2^2}{2A}\left[1 + \frac{A-C}{C}\frac{\kappa^2}{\lambda^2 + \kappa^2}\right] = H_0\left(I_1, I_2\right)$$

$$H_1 = H - H_0 = H_1\left(I_i, \varphi_i, t\right)$$

Уравнения возмущенного вращательного движения в переменных:

$$G = I_2,\ \lambda = \lambda(I_1 / I_2),\ \rho = \rho(I_3 / I_2),\ \varphi = \varphi_1,\ \psi = \varphi_2,\ h = \varphi_3 .$$

$$\dot{G} = \frac{\partial W}{\partial \psi},$$

$$\dot{\lambda} = -\frac{1}{G\mathbf{J}(\lambda)}\frac{\partial W}{\partial \varphi} + \frac{\mathbf{\Lambda}(\lambda)}{G\mathbf{J}(\lambda)}\frac{\partial W}{\partial \psi},$$

$$\dot{\rho} = \frac{1}{G} ctg\,\rho \frac{\partial W}{\partial \psi} - \frac{1}{G}\operatorname{cosec}\rho \frac{\partial W}{\partial h},$$

$$\dot{\varphi} = \Omega(G,\lambda) + \frac{1}{G\mathbf{J}(\lambda)}\frac{\partial W}{\partial \lambda},$$

$$\dot{\psi} = \omega(G,\lambda) - \frac{\mathbf{\Lambda}(\lambda)}{G\mathbf{J}(\lambda)}\frac{\partial W}{\partial \lambda} - \frac{1}{G} ctg\,\rho \frac{\partial W}{\partial \rho},$$

$$\dot{h} = \frac{1}{G}\operatorname{cosec}\rho \frac{\partial W}{\partial \rho},$$

$$W = -H_1 = H_0 - H = W(\lambda, \rho, \varphi, \psi, h, t)$$

$$\Omega(G,\lambda) = \frac{\pi G}{2\mathbf{K}(\lambda)}\frac{A-C}{AC}\frac{\kappa}{\sqrt{(1+\kappa^2)(\lambda^2+\kappa^2)}}$$

$$\omega(G,\lambda) = \frac{G}{C}\left(1 - \frac{A-C}{A}\frac{\mathbf{\Pi}(\kappa^2,\lambda)}{\mathbf{K}(\lambda)}\right)$$

$$\mathbf{\Lambda}(\lambda) = \frac{2\sqrt{1+\kappa^2}}{\pi\kappa\sqrt{\kappa^2+\lambda^2}}\left\{(\kappa^2+\lambda^2)\mathbf{\Pi}\left(\frac{\pi}{2},\kappa^2,\lambda\right) - \lambda^2\mathbf{K}(\lambda)\right\}$$

$$\mathbf{J}(\lambda) = \frac{2\kappa\lambda\sqrt{1+\kappa^2}}{\pi(\kappa^2+\lambda^2)^{3/2}}\mathbf{K}(\lambda)$$

Уравнения возмущенного вращательного движения в неинерциальной системе отсчета

$$\dot{G} = \frac{\partial U}{\partial \psi},$$

$$\dot{\lambda} = -\frac{1}{G\mathbf{J}(\lambda)}\frac{\partial U}{\partial \varphi} + \frac{\mathbf{\Lambda}(\lambda)}{G\mathbf{J}(\lambda)}\frac{\partial U}{\partial \psi},$$

$$\dot{\rho} = \sin\pi_1 \sin(h-\Pi_1)\frac{d\Pi_1}{dt} + \cos(h-\Pi_1)\frac{d\pi_1}{dt} + \frac{1}{G} ctg\,\rho\frac{\partial U}{\partial \psi} - \frac{1}{G}\cos ec\,\rho\frac{\partial U}{\partial h},$$

$$\dot{\varphi} = \Omega(G,\lambda) + \frac{1}{G\mathbf{J}(\lambda)}\frac{\partial U}{\partial \lambda},$$

$$\dot{\psi} = \omega(G,\lambda) + \cos ec\,\rho \sin\pi_1 \cos(h-\Pi_1)\frac{d\Pi_1}{dt} - \sin(h-\Pi_1)\frac{d\pi_1}{dt} - \frac{\mathbf{\Lambda}(\lambda)}{G\mathbf{J}(\lambda)}\frac{\partial U}{\partial \lambda} - \frac{1}{G} ctg\,\rho\frac{\partial U}{\partial \rho},$$

$$\dot{h} = (1-\cos\pi_1)\frac{d\Pi_1}{dt} - \cot\rho\left[\sin\pi_1\cos(h-\Pi_1)\frac{d\Pi_1}{dt} - \frac{d\pi_1}{dt}\sin(h-\Pi_1)\right] + \frac{1}{G}\csc\rho\frac{\partial U}{\partial \rho},$$

$$U = U\left(\lambda, \rho, \varphi, \psi, h, t\right)$$

30.2. Возмущения первого порядка во вращательном движении слабодеформируемой планеты в переменных действие-угол.

Невозмущенное эйлеровское (чандлеровско-эйлеровское) движение.

$$H_1 = 0,\ R_E = 0 \text{ и } U = 0$$

$$G = G_0,\ \rho = \rho_0, \lambda = \lambda_0,$$

$$\varphi = \Omega(G_0, \lambda_0)t + \varphi_0, \psi = \omega(G_0, \lambda_0)t + \psi_0,\ h = h_0$$

$G_0,\ \rho_0, \lambda_0, \varphi_0,\ \psi_0,\ h_0$ - начальные условия задачи

К вычислению возмущений первого порядка

$$G_1 = \int \frac{\partial U}{\partial \psi} dt,$$

$$\rho_1 = \frac{1}{G} \cot \rho \int \frac{\partial U}{\partial \psi} dt - \frac{1}{G} \csc \rho \int \frac{\partial U}{\partial h} dt,$$

$$\lambda_1 = -\frac{1}{G\mathbf{J}(\lambda)} \int \frac{\partial U}{\partial \varphi} dt + \frac{\mathbf{\Lambda}(\lambda)}{G\mathbf{J}(\lambda)} \int \frac{\partial U}{\partial \psi} dt,$$

$$h_1 = \frac{1}{G} \csc \rho \int \frac{\partial U}{\partial \rho} dt,$$

$$\varphi_1 = \frac{\partial \Omega(G,\lambda)}{\partial G} \int G_1 dt + \frac{\partial \Omega(G,\lambda)}{\partial \lambda} \int \lambda_1 dt + \frac{1}{G\mathbf{J}(\lambda)} \int \frac{\partial U}{\partial \lambda} dt,$$

$$\psi_1 = \frac{\partial \omega(G,\lambda)}{\partial G} \int G_1 dt + \frac{\partial \omega(G,\lambda)}{\partial \lambda} \int \lambda_1 dt - \frac{\mathbf{\Lambda}(\lambda)}{G\mathbf{J}(\lambda)} \int \frac{\partial U}{\partial \lambda} dt - \frac{1}{G} ctg\, \rho \int \frac{\partial U}{\partial \rho} dt$$

$$\Omega(G,\lambda) = \frac{\pi G}{2\mathbf{K}(\lambda)} \frac{A-C}{AC} \frac{\kappa}{\sqrt{(1+\kappa^2)(\lambda^2+\kappa^2)}}, \quad \omega(G,\lambda) = \frac{G}{C}\left(1 - \frac{A-C}{A} \frac{\mathbf{\Pi}(\kappa^2,\lambda)}{\mathbf{K}(\lambda)}\right).$$

$$U = \sum_{\mathbf{v};k_1,k_2} U_{\mathbf{v};k_1,k_2}(\lambda,\rho) \cos(\Theta_{\mathbf{v}} + k_1\varphi_1 + k_2\varphi_2)$$

30.2.1. Возмущения модуля углового момента G.

$$\delta G = \sum_{\mathbf{v}} \sum_{\mu=\pm1} G_{\mathbf{v};0,\mu} \cos\left(\Theta_{\mathbf{v}} + \mu\varphi_2\right) + \sum_{\mathbf{v}} \sum_{\mu=\pm1} G_{\mathbf{v};0,2\mu} \cos\left(\Theta_{\mathbf{v}} + 2\mu\varphi_2\right) +$$

$$+\sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm1;\\ \varepsilon=\pm1}} G_{\mathbf{v};2\varepsilon m,\mu} \cos\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + \mu\varphi_2\right) + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm1;\\ \varepsilon=\pm1}} G_{\mathbf{v};2\varepsilon m,2\mu} \cos\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + 2\mu\varphi_2\right)$$

$$G_{\mathbf{v};0,\mu} = P\frac{\mu f_{0,1}(\lambda)R_{\nu;\mu}(\rho)}{\Omega_{\mathbf{v}} + \mu\omega}, \qquad G_{\mathbf{v};0,2\mu} = 2P\frac{\mu b_{0,2}(\lambda)R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\mu\omega},$$

$$G_{\mathbf{v};2\varepsilon m,\mu} = P\frac{\mu f_{2m,\varepsilon\mu}(\lambda)R_{\mathbf{v};\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega}, \qquad G_{\mathbf{v};2\varepsilon m,2\mu} = 2P\frac{\mu b_{2m,2\varepsilon\mu}(\lambda)R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega}.$$

30.2.2. Возмущения эксцентриситета фазовой траектории λ.

$$\delta\lambda = \sum_{\mathbf{v}} \sum_{\mu=\pm1} \lambda_{\mathbf{v};0,\mu} \cos\left(\Theta_{\mathbf{v}} + \mu\varphi_2\right) + \sum_{\mathbf{v}} \sum_{\mu=\pm1} \lambda_{\mathbf{v};0,2\mu} \cos\left(\Theta_{\mathbf{v}} + 2\mu\varphi_2\right) +$$

$$+\sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm1;\\ \varepsilon=\pm1}} \lambda_{\mathbf{v};2\varepsilon m,\mu} \cos\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + \mu\varphi_2\right) + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm1;\\ \varepsilon=\pm1}} \lambda_{\mathbf{v};2\varepsilon m,2\mu} \cos\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + 2\mu\varphi_2\right)$$

$$\lambda_{\mathbf{v};0,\mu} = \frac{P\mathbf{\Lambda}\left(\lambda\right)}{G\mathbf{J}\left(\lambda\right)} \cdot \frac{\mu f_{0,1}(\lambda) R_{\mathbf{v};\mu}(\rho)}{\Omega_{\mathbf{v}} + \mu\omega},$$

$$\lambda_{\mathbf{v};0,2\mu} = 2\frac{P\mathbf{\Lambda}\left(\lambda\right)}{G\mathbf{J}\left(\lambda\right)} \cdot \frac{\mu b_{0,2}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\mu\omega},$$

$$\lambda_{\mathbf{v};2\varepsilon m,\mu} = \frac{P}{G\mathbf{J}\left(\lambda\right)} \cdot \frac{\mu\left[\mathbf{\Lambda}\left(\lambda\right) - 2\varepsilon m\right] f_{2m,\varepsilon\mu}(\lambda) R_{\nu;\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega},$$

$$\lambda_{\mathbf{v};2\varepsilon m,2\mu} = \frac{2P}{G\mathbf{J}\left(\lambda\right)} \cdot \frac{\mu\left[\mathbf{\Lambda}\left(\lambda\right) - \varepsilon m\right] b_{2m,2\varepsilon\mu}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega}.$$

30.2.3. Возмущения наклонности вектора кинетического момента ρ .

$$\delta\rho = \sum_{\mathbf{v}} \rho_{\mathbf{v};0,0} \cos\left(\Theta_{\mathbf{v}}\right) + \sum_{\mathbf{v}} \sum_{\mu=\pm1} \rho_{\mathbf{v};0,\mu} \cos\left(\Theta_{\mathbf{v}} + \mu\varphi_2\right) + \sum_{\mathbf{v}} \sum_{\mu=\pm1} \rho_{\mathbf{v};0,2\mu} \cos\left(\Theta_{\mathbf{v}} + 2\mu\varphi_2\right) +$$

$$+ \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm1;\\ \varepsilon=\pm1}} \rho_{\mathbf{v};2\varepsilon m,\mu} \cos\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + \mu\varphi_2\right) + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm1;\\ \varepsilon=\pm1}} \rho_{\mathbf{v};2\varepsilon m,2\mu} \cos\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + 2\mu\varphi_2\right)$$

$$\rho_{\mathbf{v};0,0} = 2\frac{P}{G}\csc\rho\,\frac{\nu_5\left(b_{0,0}R_{\mathbf{v}}^{(22)} + c_{0,0}R_{\mathbf{v}}^{(33)}\right)}{\Omega_{\mathbf{v}}},$$

$$\rho_{\mathbf{v};0,\mu} = \frac{P}{G}\csc\rho\,\frac{\left(\mu\cos\rho + \nu_5\right) f_{0,1}(\lambda) R_{\nu;\mu}(\rho)}{\Omega_{\mathbf{v}} + \mu\omega},$$

$$\rho_{\mathbf{v};0,2\mu} = \frac{P}{G}\csc\rho\,\frac{\left(2\mu\cos\rho + \nu_5\right) b_{0,2}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\mu\omega},$$

$$\rho_{\mathbf{v};2\varepsilon m,\mu} = \frac{P}{G}\csc\rho\,\frac{\left(\mu\cos\rho + \nu_5\right) f_{2m,\varepsilon\mu}(\lambda) R_{\mathbf{v};\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega}$$

$$\rho_{\mathbf{v};2\varepsilon m,2\mu} = \frac{P}{G}\csc\rho\,\frac{\left(2\mu\cos\rho + \nu_5\right) b_{2m,2\varepsilon\mu}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega}.$$

30.2.4. Возмущения долготы восходящего узла плоскости Андуайе ортогональной вектору кинетического момента

$$\delta h = \sum_{\mathbf{v}} h_{\mathbf{v};0,0}\sin\left(\Theta_{\mathbf{v}}\right) + \sum_{\mathbf{v}}\sum_{\mu=\pm1} h_{\mathbf{v};0,\mu}\sin\left(\Theta_{\mathbf{v}} + \mu\varphi_2\right) + \sum_{\mathbf{v}}\sum_{\mu=\pm1} h_{\mathbf{v};0,2\mu}\sin\left(\Theta_{\mathbf{v}} + 2\mu\varphi_2\right) +$$

$$+\sum_{\mathbf{v}}\sum_{m=1}^{\infty}\sum_{\substack{\mu=\pm1;\\ \varepsilon=\pm1}} h_{\mathbf{v};2\varepsilon m,\mu}\sin\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + \mu\varphi_2\right) + \sum_{\mathbf{v}}\sum_{m=1}^{\infty}\sum_{\substack{\mu=\pm1;\\ \varepsilon=\pm1}} h_{\mathbf{v};2\varepsilon m,2\mu}\sin\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + 2\mu\varphi_2\right)$$

$$h_{\mathbf{v};0,0} = \frac{P}{G}\csc\rho\,\frac{\left(1+\delta-3c_{0,0}\right)}{\Omega_{\mathbf{v}}}\cdot\frac{\partial R_{\mathbf{v}}^{(22)}}{\partial\rho},$$

$$h_{\mathbf{v};0,\mu} = \frac{P}{G}\csc\rho\,\frac{f_{0,1}(\lambda)}{\Omega_{\mathbf{v}}+\mu\omega}\cdot\frac{\partial R_{\nu;\mu}(\rho)}{\partial\rho},$$

$$h_{\mathbf{v};0,2\mu} = \frac{P}{G}\csc\rho\,\frac{b_{0,2}(\lambda)}{\Omega_{\mathbf{v}}+\mu\omega}\cdot\frac{\partial R_{\mathbf{v};2\mu}(\rho)}{\partial\rho},$$

$$h_{\mathbf{v};2\varepsilon m,\mu} = \frac{P}{G}\csc\rho\,\frac{f_{2m,\varepsilon\mu}(\lambda)}{\Omega_{\mathbf{v}}+2\varepsilon m\Omega+\mu\omega}\cdot\frac{\partial R_{\mathbf{v};\mu}(\rho)}{\partial\rho},$$

$$h_{\mathbf{v};2\varepsilon m,2\mu} = \frac{P}{G}\csc\rho\,\frac{b_{2m,2\varepsilon\mu}(\lambda)}{\Omega_{\mathbf{v}}+2\varepsilon m\Omega+\mu\omega}\cdot\frac{\partial R_{\mathbf{v};2\mu}(\rho)}{\partial\rho}.$$

30.2.5. Возмущения переменной φ_1.

$$\delta\varphi_1 = \sum_{\mathbf{v}} \varphi_{\mathbf{v};0,0} \sin\left(\Theta_{\mathbf{v}}\right) + \sum_{\mathbf{v}} \sum_{\mu=\pm1} \varphi_{\mathbf{v};0,\mu} \sin\left(\Theta_{\mathbf{v}} + \mu\varphi_2\right) + \sum_{\mathbf{v}} \sum_{\mu=\pm1} \varphi_{\mathbf{v};0,2\mu} \sin\left(\Theta_{\mathbf{v}} + 2\mu\varphi_2\right) +$$

$$+ \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm1;\\ \varepsilon=\pm1}} \varphi_{\mathbf{v};2\varepsilon m,\mu} \sin\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + \mu\varphi_2\right) + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm1;\\ \varepsilon=\pm1}} \varphi_{\mathbf{v};2\varepsilon m,2\mu} \sin\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + 2\mu\varphi_2\right)$$

$$\varphi_{\mathbf{v};0,0} = \frac{2P}{G\mathbf{J}\left(\lambda\right)} \cdot \frac{\partial b_{0,0}}{\partial\lambda} \cdot \frac{R_{\mathbf{v};0}(\rho)}{\Omega_{\mathbf{v}}},$$

$$\varphi_{\mathbf{v};0,\mu} = \frac{P}{G\mathbf{J}\left(\lambda\right)} \left\{ (\Omega_{\mathbf{v}} + \mu\omega) \frac{\partial f_{0,1}(\lambda)}{\partial\lambda} + \mu\left[\Omega\mathbf{J}\left(\lambda\right) + \Omega_{\lambda}\mathbf{\Lambda}\left(\lambda\right)\right] f_{0,1}(\lambda) \right\} \times \frac{R_{\mathbf{v};\mu}(\rho)}{\left(\Omega_{\mathbf{v}} + \mu\omega\right)^2},$$

$$\varphi_{\mathbf{v};0,2\mu} = \frac{P}{G\mathbf{J}\left(\lambda\right)} \left\{ (\Omega_{\mathbf{v}} + 2\mu\omega) \frac{\partial b_{0,2}(\lambda)}{\partial\lambda} + 2\mu\left[\Omega\mathbf{J}\left(\lambda\right) + \Omega_{\lambda}\mathbf{\Lambda}\left(\lambda\right)\right] b_{0,2}(\lambda) \right\} \times \frac{R_{\mathbf{v};2\mu}(\rho)}{\left(\Omega_{\mathbf{v}} + 2\mu\omega\right)^2},$$

$$\varphi_{\mathbf{v};2\varepsilon m,\mu} = \frac{P}{G\mathbf{J}\left(\lambda\right)} \left\{ (\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega) \frac{\partial f_{2m,\varepsilon\mu}(\lambda)}{\partial\lambda} + \mu\left(\Omega\mathbf{J}\left(\lambda\right) + \Omega_{\lambda}\left[\mathbf{\Lambda}\left(\lambda\right) - 2\varepsilon m\right]\right) f_{2m,\varepsilon\mu}(\lambda) \right\} \times \frac{R_{\mathbf{v};\mu}(\rho)}{\left(\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega\right)^2},$$

$$\varphi_{\mathbf{v};2\varepsilon m,2\mu} = \frac{P}{G\mathbf{J}\left(\lambda\right)} \left\{ (\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega) \frac{\partial b_{2m,2\varepsilon\mu}(\lambda)}{\partial\lambda} + 2\mu\left(\Omega\mathbf{J}\left(\lambda\right) + \Omega_{\lambda}\left[\mathbf{\Lambda}\left(\lambda\right) - \varepsilon m\right]\right) b_{2m,2\varepsilon\mu}(\lambda) \right\} \times \frac{R_{\mathbf{v};2\mu}(\rho)}{\left(\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega\right)^2}.$$

30.2.6. Возмущения переменной φ_2.

$$\delta\varphi_2 = \sum_{\mathbf{v}} \psi_{\mathbf{v};0,0} \sin\left(\Theta_{\mathbf{v}}\right) + \sum_{\mathbf{v}} \sum_{\mu=\pm 1} \psi_{\mathbf{v};0,\mu} \sin\left(\Theta_{\mathbf{v}} + \mu\varphi_2\right) + \sum_{\mathbf{v}} \sum_{\mu=\pm 1} \psi_{\mathbf{v};0,2\mu} \sin\left(\Theta_{\mathbf{v}} + 2\mu\varphi_2\right) +$$

$$+ \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1;\\ \varepsilon=\pm 1}} \psi_{\mathbf{v};2\varepsilon m,\mu} \sin\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + \mu\varphi_2\right) + \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm 1;\\ \varepsilon=\pm 1}} \psi_{\mathbf{v};2\varepsilon m,2\mu} \sin\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + 2\mu\varphi_2\right)$$

$$\psi_{\mathbf{v};0,0} = -\frac{2P}{G\mathbf{J}(\lambda)} \left\{ \mathbf{\Lambda}(\lambda) \frac{\partial b_{0,0}}{\partial \lambda} \cdot \frac{R_{\mathbf{v};0}(\rho)}{\Omega_{\mathbf{v}}} + \cot\rho \frac{\mathbf{J}(\lambda)\left(1+\delta-3c_{0,0}\right)}{2\Omega_{\mathbf{v}}} \frac{\partial R_{\mathbf{v}}^{(22)}}{\partial \rho} \right\}$$

$$\psi_{\mathbf{v};0,\mu} = \frac{P}{G\mathbf{J}(\lambda)} \left[\left\{ -\mathbf{\Lambda}(\lambda)(\Omega_{\mathbf{v}} + \mu\omega) \frac{\partial f_{0,1}(\lambda)}{\partial \lambda} + \mu\left[\omega\mathbf{J}(\lambda) + \omega_\lambda \mathbf{\Lambda}(\lambda)\right] f_{0,1}(\lambda) \right\} \times \frac{R_{\mathbf{v};\mu}(\rho)}{(\Omega_{\mathbf{v}} + \mu\omega)^2} - \cot\rho \frac{\mathbf{J}(\lambda) f_{0,1}(\lambda)}{\Omega_{\mathbf{v}} + \mu\omega} \cdot \frac{\partial R_{\mathbf{v};\mu}(\rho)}{\partial \rho} \right]$$

$$\psi_{\mathbf{v};0,2\mu} = \frac{P}{G\mathbf{J}(\lambda)} \left[\left\{ -\mathbf{\Lambda}(\lambda)(\Omega_{\mathbf{v}} + 2\mu\omega) \frac{\partial b_{0,2}(\lambda)}{\partial \lambda} + 2\mu\left[\omega\mathbf{J}(\lambda) + \omega_\lambda \mathbf{\Lambda}(\lambda)\right] b_{0,2}(\lambda) \right\} \times \frac{R_{\mathbf{v};2\mu}(\rho)}{(\Omega_{\mathbf{v}} + 2\mu\omega)^2} - \cot\rho \frac{\mathbf{J}(\lambda) b_{0,2}(\lambda)}{\Omega_{\mathbf{v}} + \mu\omega} \cdot \frac{\partial R_{\mathbf{v};2\mu}(\rho)}{\partial \rho} \right]$$

$$\psi_{\mathbf{v};2\varepsilon m,\mu} = \frac{P}{G\mathbf{J}(\lambda)} \left[\left\{ -\mathbf{\Lambda}(\lambda)(\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega) \frac{\partial f_{2m,\varepsilon\mu}(\lambda)}{\partial \lambda} + \mu\left(\omega\mathbf{J}(\lambda) + \omega_\lambda \left[\mathbf{\Lambda}(\lambda) - 2\varepsilon m\right]\right) f_{2m,\varepsilon\mu}(\lambda) \right\} \times \right.$$

$$\left. \times \frac{R_{\mathbf{v};\mu}(\rho)}{(\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega)^2} - \cot\rho \frac{\mathbf{J}(\lambda) f_{2m,\varepsilon\mu}(\lambda)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega} \cdot \frac{\partial R_{\mathbf{v};\mu}(\rho)}{\partial \rho} \right]$$

$$\psi_{\mathbf{v};2\varepsilon m,2\mu} = \frac{P}{G\mathbf{J}(\lambda)} \times \left[\left\{ -\mathbf{\Lambda}(\lambda)(\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega) \frac{\partial b_{2m,2\varepsilon\mu}(\lambda)}{\partial \lambda} + 2\mu\left(\omega\mathbf{J}(\lambda) + \omega_\lambda \left[\mathbf{\Lambda}(\lambda) - \varepsilon m\right]\right) b_{2m,2\varepsilon\mu}(\lambda) \right\} \times \right.$$

$$\left. \times \frac{R_{\mathbf{v};2\mu}(\rho)}{(\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega)^2} - \cot\rho \frac{\mathbf{J}(\lambda) b_{2m,2\varepsilon\mu}(\lambda)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega} \cdot \frac{\partial R_{\mathbf{v};2\mu}(\rho)}{\partial \rho} \right]$$

Вспомогательные формулы

$$\sigma = \frac{\pi}{2\mathbf{K}} F(\arctan\frac{\kappa}{\lambda}, \lambda'), \quad d = \frac{\pi \mathbf{K}'}{2\mathbf{K}}$$

$$\frac{\partial\sigma}{\partial\lambda} = -\frac{\pi^2}{4\lambda\lambda'^2\mathbf{K}^2}\mathbf{\Lambda_0}(n,\lambda), \quad n = \arctan\frac{\kappa}{\lambda},$$

$$\mathbf{\Lambda_0}(n,\lambda) = \frac{1}{\pi}\left\{F(n,\lambda)[\mathbf{E}(\lambda) - \mathbf{K}(\lambda)] + \mathbf{K}(\lambda)E(n,\lambda)\right\},$$

$$\frac{\partial d}{\partial\lambda} = -\frac{\pi}{2\lambda\lambda'^2\mathbf{K}^2}[\mathbf{E}'\mathbf{K} + \mathbf{E}\mathbf{K}' - \mathbf{K}\mathbf{K}'] = -\frac{\pi}{2\lambda\lambda'\mathbf{K}^2}.$$

$$\Omega_\lambda = \frac{C-A}{AC}\frac{\kappa}{\sqrt{1+\kappa^2}}\frac{(\kappa^2+\lambda^2)\mathbf{E} - \lambda'^2\kappa^2\mathbf{K}}{\mathbf{K}^2\lambda\lambda'^2(\kappa^2+\lambda^2)^{3/2}},$$

$$\omega_\lambda = \frac{C-A}{AC}\frac{\left[\lambda^2\mathbf{KE} - (\kappa^2+\lambda^2)\mathbf{\Pi E} + \lambda'^2\kappa^2\mathbf{\Pi K}\right]}{\mathbf{K}^2\lambda\lambda'^2(\kappa^2+\lambda^2)}$$

$$R_{\mathbf{v};2\mu}(\rho) = R_{\mathbf{v}}^{(22)} + \mu R_{\mathbf{v}}^{(12)} = -A_{\mathbf{v}}^{(0)}\sin^2\rho + 2\sin\rho(\cos\rho - \mu)A_{\mathbf{v}}^{(1)} - \frac{1}{2}A_{\mathbf{v}}^{(2)}(1 - \mu\cos\rho)^2,$$

$$R_{\nu;\mu}(\rho) = R_{\nu}^{(23)} + \mu R_{\nu}^{(13)} = -A_{\nu}^{(0)}\sin 2\rho + 2A_{\nu}^{(1)}(\cos 2\rho - \mu\cos\rho) + A_{\nu}^{(2)}\sin\rho(\cos\rho - \mu),$$

$$R_{\nu;0}(\rho) = R_{\nu}^{(22)} - 2R_{\nu}^{(33)} = -A_{\nu}^{(0)}(1 - 3\cos^2\rho) + 3\sin 2\rho A_{\nu}^{(1)} + \frac{3}{2}A_{\nu}^{(2)}\sin^2\rho.$$

$$R_{\mathbf{v};2\mu}(\rho) = R_{\mathbf{v}}^{(22)} + \mu R_{\mathbf{v}}^{(12)} = -A_{\mathbf{v}}^{(0)} \sin^2 \rho + 2\sin\rho(\cos\rho - \mu) A_{\mathbf{v}}^{(1)} - \frac{1}{2} A_{\mathbf{v}}^{(2)} (1 - \mu\cos\rho)^2 ,$$

$$R_{\nu;\mu}(\rho) = R_{\nu}^{(23)} + \mu R_{\nu}^{(13)} = -A_{\nu}^{(0)} \sin 2\rho + 2A_{\nu}^{(1)} (\cos 2\rho - \mu\cos\rho) + A_{\nu}^{(2)} \sin\rho(\cos\rho - \mu) ,$$

$$R_{\nu;0}(\rho) = R_{\nu}^{(22)} - 2R_{\nu}^{(33)} = -A_{\nu}^{(0)} (1 - 3\cos^2\rho) + 3\sin 2\rho A_{\nu}^{(1)} + \frac{3}{2} A_{\nu}^{(2)} \sin^2 \rho .$$

$$\frac{\partial R_{\mathbf{v};2\mu}(\rho)}{\partial\rho} = -A_{\mathbf{v}}^{(0)} \sin 2\rho + 2(\cos 2\rho - \mu\cos\rho) A_{\mathbf{v}}^{(1)} - A_{\mathbf{v}}^{(2)} (1 - \mu\cos\rho)\mu\sin\rho ,$$

$$\frac{\partial R_{\mathbf{v};\mu}(\rho)}{\partial\rho} = -2A_{\mathbf{v}}^{(0)} \cos 2\rho + 2A_{\mathbf{v}}^{(1)} (\mu\sin\rho - \sin 2\rho) + A_{\mathbf{v}}^{(2)} (\cos 2\rho - \mu\cos\rho) ,$$

$$\frac{\partial R_{\mathbf{v};0}(\rho)}{\partial\rho} = 6\cos 2\rho A_{\nu}^{(1)} + \left[-3A_{\nu}^{(0)} + \frac{3}{2} A_{\nu}^{(2)} \right] \sin 2\rho ,$$

$$\frac{\partial R_{\mathbf{v}}^{(22)}}{\partial\rho} = -\left(A_{\mathbf{v}}^{(0)} - \frac{1}{2} A_{\mathbf{v}}^{(2)} \right) \sin 2\rho + 2A_{\mathbf{v}}^{(1)} \cos 2\rho .$$

$$b_{0,0} = \frac{1}{2}(1+\delta) - \frac{\delta(1+\kappa^2)}{2(\kappa^2+\lambda^2)} + \frac{D\mathbf{E}}{2\mathbf{K}(\kappa^2+\lambda^2)},$$

$$c_{0,0} = \frac{\delta(1+\kappa^2)}{\kappa^2+\lambda^2} - \frac{D\mathbf{E}}{\mathbf{K}(\kappa^2+\lambda^2)}.$$

$$\frac{\partial b_{0,0}}{\partial \lambda} = \frac{b_1\mathbf{K}^2 + b_2\mathbf{E}\mathbf{K} + b_3\mathbf{E}^2}{2(\kappa^2+\lambda^2)^2\lambda\lambda'^2\mathbf{K}^2},$$

$$\frac{\partial c_{0,0}}{\partial \lambda} = -2\frac{\partial b_{0,0}}{\partial \lambda} = -\frac{b_1\mathbf{K}^2 + b_2\mathbf{E}\mathbf{K} + b_3\mathbf{E}^2}{(\kappa^2+\lambda^2)^2\lambda\lambda'^2\mathbf{K}^2}.$$

$$b_1 = \lambda'^2\left[2\delta(1+\kappa^2)\lambda^2 - D(\kappa^2+\lambda^2)\right], \quad b_2 = 2D\kappa^2\lambda'^2,$$

$$b_3 = -D(\kappa^2+\lambda^2), \qquad D = \delta(1+\kappa^2) - \kappa^2,$$

$$b_{0,1} = -\frac{\pi^2 \mathbf{M}}{2\mathbf{K}^2(\kappa^2+\lambda^2)} \cdot \frac{1}{\cosh(\sigma)},$$

$$b_{0,2} = \frac{\pi^2 \mathbf{M}}{2\mathbf{K}^2(\kappa^2+\lambda^2)} \cdot \frac{1}{\sinh(2\sigma)},$$

$$b_{2m,0} = \frac{\pi^2 D}{2\mathbf{K}^2(\kappa^2+\lambda^2)} \cdot \frac{m}{\sinh(2md)},$$

$$b_{2m,\varepsilon\mu} = -\varepsilon\mu \frac{\pi^2 (mD+\varepsilon\mu\mathbf{M})}{2\mathbf{K}^2(\kappa^2+\lambda^2)} \cdot \frac{1}{\cosh(2md-\varepsilon\mu\sigma)},$$

$$b_{2m,2\varepsilon\mu} = -\frac{\pi^2 (mD+2\varepsilon\mu\mathbf{M})}{4\mathbf{K}^2(\kappa^2+\lambda^2)} \cdot \frac{1}{\sinh\left[2(md-\varepsilon\mu\sigma)\right]};$$

$$\sigma = \frac{\pi}{2\mathbf{K}} F(\arctan\frac{\kappa}{\lambda}, \lambda'), \quad d = \frac{\pi \mathbf{K}'}{2\mathbf{K}}$$

$$\frac{\partial f_{0,1}}{\partial \lambda} = \frac{A_{0,1}}{\cosh(\sigma)} + B_{0,1} \frac{\sinh(\sigma)}{\cosh^2(\sigma)},$$

$$\frac{\partial b_{0,2}}{\partial \lambda} = \frac{A_{0,2}}{\sinh(2\sigma)} + B_{0,2} \frac{\cosh(2\sigma)}{\sinh^2(2\sigma)},$$

$$\frac{\partial b_{2m,0}}{\partial \lambda} = \frac{A_{2m,0}}{\cosh(2md)} + B_{2m,0} \frac{\sinh(2md)}{\cosh^2(2md)},$$

$$\frac{\partial f_{2m,\varepsilon\mu}}{\partial \lambda} = \frac{A_{2m,\varepsilon\mu}}{\cosh(2md-\varepsilon\mu\sigma)} + B_{2m,\varepsilon\mu} \frac{\sinh(2md-\varepsilon\mu\sigma)}{\cosh^2(2md-\varepsilon\mu\sigma)},$$

$$\frac{\partial b_{2m,2\varepsilon\mu}}{\partial \lambda} = \frac{A_{2m,2\varepsilon\mu}}{\sinh\left[2(md-\varepsilon\mu\sigma)\right]} + B_{2m,2\varepsilon\mu} \frac{\cosh\left[2(md-\varepsilon\mu\sigma)\right]}{\sinh^2\left[2(md-\varepsilon\mu\sigma)\right]};$$

$$A_{0,1} = -\frac{\pi\kappa\sqrt{1+\kappa^2}\left[\delta(\lambda^2-\kappa^2)\lambda'^2\mathbf{K} + (1-\delta)(\kappa^2+\lambda^2)\mathbf{E} + 2D\lambda^2\lambda'^2\mathbf{\Pi}\right]}{2\lambda\lambda'^2(\lambda^2-\kappa^2)\mathbf{K}^2(\kappa^2+\lambda^2)^{3/2}} + \frac{\pi^2\mathbf{M}\left[\kappa^2\lambda'^2\mathbf{K} + (\kappa^2+\lambda^2)\mathbf{E}\right]}{\lambda\lambda'^2(\kappa^2+\lambda^2)^2\mathbf{K}^3},$$

$$A_{0,2} = \frac{\pi\kappa\sqrt{1+\kappa^2}\left[\delta(\lambda^2-\kappa^2)\lambda'^2\mathbf{K} + (1-\delta)(\kappa^2+\lambda^2)\mathbf{E} + 2D\lambda^2\lambda'^2\mathbf{\Pi}\right]}{2\lambda\lambda'^2(\lambda^2-\kappa^2)\mathbf{K}^2(\kappa^2+\lambda^2)^{3/2}} - \frac{\pi^2\mathbf{M}\left[\kappa^2\lambda'^2\mathbf{K} + (\kappa^2+\lambda^2)\mathbf{E}\right]}{\lambda\lambda'^2(\kappa^2+\lambda^2)^2\mathbf{K}^3},$$

$$A_{2m,\varepsilon\mu} = -\frac{\pi\varepsilon\mu\kappa\sqrt{1+\kappa^2}\left[\delta(\lambda^2-\kappa^2)\lambda'^2\mathbf{K} + (1-\delta)(\kappa^2+\lambda^2)\mathbf{E} + 2D\lambda^2\lambda'^2\mathbf{\Pi}\right]}{2\lambda\lambda'^2(\lambda^2-\kappa^2)(\kappa^2+\lambda^2)^{3/2}\mathbf{K}^2} + \frac{\pi^2(\varepsilon\mu mD + \mathbf{M})\left[-\kappa^2\lambda'^2\mathbf{K} + (\kappa^2+\lambda^2)\mathbf{E}\right]}{\lambda\lambda'^2(\kappa^2+\lambda^2)^2\mathbf{K}^3},$$

$$A_{2m,2\varepsilon\mu} = -\frac{\pi\varepsilon\mu\kappa\sqrt{1+\kappa^2}\left[\delta(\lambda^2-\kappa^2)\lambda'^2\mathbf{K} + (1-\delta)(\kappa^2+\lambda^2)\mathbf{E} + 2D\lambda^2\lambda'^2\mathbf{\Pi}\right]}{2\lambda\lambda'^2(\lambda^2-\kappa^2)(\kappa^2+\lambda^2)^{3/2}\mathbf{K}^2} + \frac{\pi^2(mD + 2\varepsilon\mu\mathbf{M})\left[-\kappa^2\lambda'^2\mathbf{K} + (\kappa^2+\lambda^2)\mathbf{E}\right]}{\lambda\lambda'^2(\kappa^2+\lambda^2)^2\mathbf{K}^3},$$

$$B_{0,1} = -\frac{\pi^4\mathbf{M}\mathbf{\Lambda_0}(\kappa^2,\lambda)}{8\lambda\lambda'^2(\kappa^2+\lambda^2)\mathbf{K}^4}, \qquad B_{0,2} = \frac{\pi^4\mathbf{M}\mathbf{\Lambda_0}(\kappa^2,\lambda)}{4\lambda\lambda'^2(\kappa^2+\lambda^2)\mathbf{K}^4}, \qquad B_{2m,0} = \frac{\pi^3 Dm^2}{2\lambda\lambda'^2(\kappa^2+\lambda^2)\mathbf{K}^4},$$

$$B_{2m,\varepsilon\mu} = \frac{\pi^3(\varepsilon\mu mD + \mathbf{M})\left[-4m + \varepsilon\mu\pi\mathbf{\Lambda_0}(n,\lambda)\right]}{8\lambda\lambda'^2(\kappa^2+\lambda^2)\mathbf{K}^4}, \qquad B_{2m,2\varepsilon\mu} = \frac{\pi^3(mD + 2\varepsilon\mu\mathbf{M})\left[-2m + \varepsilon\mu\pi\mathbf{\Lambda_0}(n,\lambda)\right]}{4\lambda\lambda'^2(\kappa^2+\lambda^2)\mathbf{K}^4};$$

30.3. Возмущенное вращательное движение небесного тела в случае малых значений параметра λ.

$$U = U_0 + \lambda U_1 + \lambda^2 U_2$$

$$U_0 = \frac{3}{4} n^2 (A - C) \sum_{\mathbf{v}} \left\{ R_{\nu}(\rho, \delta, \kappa) \cos \Theta_{\mathbf{v}} + \right.$$

$$\left. + B_{2,2}^{(0)} \left[R_{\mathbf{v},2}(\rho) \cos(\Theta_{\mathbf{v}} + 2\varphi_1 + 2\varphi_2) + R_{\mathbf{v},-2}(\rho) \cos(\Theta_{\mathbf{v}} - 2\varphi_1 - 2\varphi_2) \right] \right\},$$

$$\delta G = P B_{2,2}^{(0)} \sum_{\mathbf{v}} \left[\frac{R_{\mathbf{v},2}(\rho)}{\Omega_{\mathbf{v}} + 2\Omega + 2\omega} \cos(\Theta_{\mathbf{v}} + 2\varphi_1 + 2\varphi_2) - \frac{R_{\mathbf{v},-2}(\rho)}{\Omega_{\mathbf{v}} - 2\Omega - 2\omega} \cos(\Theta_{\mathbf{v}} - 2\varphi_1 - 2\varphi_2) \right],$$

.

$$P = \frac{3}{2} n^2 (A - C), \; B_{2,2}^{(0)} = \frac{1}{2} \delta \sqrt{1 + \kappa^2} \, .$$

$$\frac{(\delta G)_{CH}}{(\delta G)_{EU}} = \frac{\sqrt{1 + \kappa_{CH}^2}}{\sqrt{1 + \kappa_{EU}^2}} = 1.001348$$

30.3.2. Возмущения наклонности вектора кинетического момента ρ.

$$\dot{\rho} = \frac{1}{G} ctg\,\rho \frac{\partial U_0}{\partial \varphi_2} - \frac{1}{G} \cos ec\,\rho \frac{\partial U_0}{\partial h} \qquad \dot{\rho} = \frac{\lambda}{G} \csc\rho \left(\cos\rho \frac{\partial U_1}{\partial \varphi_2} - \frac{\partial U_1}{\partial h} \right)$$

$$\begin{aligned}
\delta\rho = {} & 2\frac{P}{G}\csc\rho \sum_{\mathbf{v}} \frac{\nu_5\left(b_{0,0}R_{\mathbf{v}}^{(22)} + c_{0,0}R_{\mathbf{v}}^{(33)}\right)}{\Omega_{\mathbf{v}}} \cos(\Theta_{\mathbf{v}}) + \\
& + \frac{P}{G}\csc\rho \sum_{\mathbf{v}} \sum_{\mu=\pm1} \frac{\left(\mu\cos\rho + \nu_5\right) f_{0,1}(\lambda) R_{\nu;\mu}(\rho)}{\Omega_{\mathbf{v}} + \mu\omega} \cos\left(\Theta_{\mathbf{v}} + \mu\varphi_2\right) + \\
& + \frac{P}{G}\csc\rho \sum_{\mathbf{v}} \sum_{\mu=\pm1} \frac{\left(2\mu\cos\rho + \nu_5\right) b_{0,2}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\mu\omega} \cos\left(\Theta_{\mathbf{v}} + 2\mu\varphi_2\right) + \\
& + \frac{P}{G}\csc\rho \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm1;\\ \varepsilon=\pm1}} \frac{\left(\mu\cos\rho + \nu_5\right) f_{2m,\varepsilon\mu}(\lambda) R_{\nu;\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + \mu\omega} \cos\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + \mu\varphi_2\right) + \\
& + \frac{P}{G}\csc\rho \sum_{\mathbf{v}} \sum_{m=1}^{\infty} \sum_{\substack{\mu=\pm1;\\ \varepsilon=\pm1}} \frac{\left(2\mu\cos\rho + \nu_5\right) b_{2m,2\varepsilon\mu}(\lambda) R_{\mathbf{v};2\mu}(\rho)}{\Omega_{\mathbf{v}} + 2\varepsilon m\Omega + 2\mu\omega} \cos\left(\Theta_{\mathbf{v}} + 2\varepsilon m\varphi_1 + 2\mu\varphi_2\right).
\end{aligned}$$

30.3.3. Возмущения переменной λ .

$$\dot{\lambda} = \frac{1}{G}\left[\frac{\mathbf{\Lambda}\left(\lambda\right)}{\mathbf{J}\left(\lambda\right)} - \frac{1}{\mathbf{J}\left(\lambda\right)}\right]\frac{\partial U_0}{\partial \varphi_1} = \frac{\lambda}{2}\left[1 + \frac{\kappa^2 - 6}{2\sqrt{1+\kappa^2}}\right]\frac{\partial U_0}{\partial \varphi_1}$$

$$\delta\lambda = \frac{1}{2}\lambda P L_{2,2}(\delta,\kappa^2)\sum_{\mathbf{v}}\left[\frac{R_{\mathbf{v},2}(\rho)}{\Omega_{\mathbf{v}} + 2\Omega + 2\omega}\cos(\Theta_{\mathbf{v}} + 2\varphi_1 + 2\varphi_2) - \frac{R_{\mathbf{v},-2}(\rho)}{\Omega_{\mathbf{v}} - 2\Omega - 2\omega}\cos(\Theta_{\mathbf{v}} - 2\varphi_1 - 2\varphi_2)\right],$$

$$L_{2,2}(\delta,\kappa^2) = \frac{1}{2}\delta\sqrt{1+\kappa^2}\left[1 + \frac{\kappa^2 - 6}{2\sqrt{1+\kappa^2}}\right].$$

Возмущения от дополнительной части возмущающей функции порядка λ

$$U_1 = \frac{1}{2} P \sum_{\mathbf{v}} \Big\{ B_{2,1}^{(1)} \Big[R_{\mathbf{v},1}(\rho) \cos(\Theta_{\mathbf{v}} + 2\varphi_1 + \varphi_2) + R_{\mathbf{v},-1}(\rho) \cos(\Theta_{\mathbf{v}} - 2\varphi_1 - \varphi_2) \Big] +$$

$$+ B_{0,1}^{(1)} \Big[R_{\mathbf{v},1}(\rho) \cos(\Theta_{\mathbf{v}} + \varphi_2) + R_{\mathbf{v},-1}(\rho) \cos(\Theta_{\mathbf{v}} - \varphi_2) \Big] \Big\} ,$$

$$P = \frac{3}{2} n^2 (A - C) , \qquad B_{0,1}^{(1)} = \frac{2\mathbf{M}^{(0)}}{\kappa^3}, \qquad B_{2,1}^{(1)} = \frac{A - B}{4\kappa A} \left[-1 + \frac{A}{C - A} \sqrt{1 + \kappa^2} \right];$$

$$\delta\lambda = -\frac{1}{2} P L_{2,1}(\delta, \kappa) \sum_{\mathbf{v}} \left[\frac{R_{\mathbf{v},1}(\rho)}{\Omega_{\mathbf{v}} + 2\Omega + \omega} \cos(\Theta_{\mathbf{v}} + 2\varphi_1 + \varphi_2) - \frac{R_{\mathbf{v},-1}(\rho)}{\Omega_{\mathbf{v}} - 2\Omega - \omega} \cos(\Theta_{\mathbf{v}} - 2\varphi_1 - \varphi_2) \right]$$

$$-\frac{1}{2} P L_{0,1}(\delta, \kappa) \sum_{\mathbf{v}} \left\{ \left[\frac{R_{\mathbf{v},1}(\rho)}{\Omega_{\mathbf{v}} + \omega} \cos(\Theta_{\mathbf{v}} + \varphi_2) - R_{\mathbf{v},-1}(\rho) \frac{R_{\mathbf{v},-1}(\rho)}{\Omega_{\mathbf{v}} - \omega} \cos(\Theta_{\mathbf{v}} - \varphi_2) \right] \right.,$$

$$L_{2,1}(\delta, \kappa) = -\frac{\kappa}{4(1+\kappa^2)} \left[\delta (1 + \kappa^2 + \sqrt{1+\kappa^2}) - \kappa^2 \right], \quad L_{0,1}(\delta, \kappa) = \frac{1}{\kappa(1+\kappa^2)} \left[\delta (1 + \kappa^2 - \sqrt{1+\kappa^2}) - \kappa^2 \right].$$

30.3.4. Возмущения угловой переменной φ.

$$\dot{\varphi} = \frac{3\kappa^2}{4\lambda\sqrt{1+\kappa^2}} n^2 (A-C) \sum_{\mathbf{v}} \Big\{ B_{2,1}^{(1)} \Big[R_{\mathbf{v},1}(\rho)\cos(\Theta_{\mathbf{v}} + 2\varphi_1 + \varphi_2) + R_{\mathbf{v},-1}(\rho)\cos(\Theta_{\mathbf{v}} - 2\varphi_1 - \varphi_2) \Big] +$$

$$+ \frac{3\kappa^2}{4\lambda\sqrt{1+\kappa^2}} n^2 (A-C) B_{0,1}^{(1)} \sum_{\mathbf{v}} \Big[R_{\mathbf{v},1}(\rho)\cos(\Theta_{\mathbf{v}} + \varphi_2) + R_{\mathbf{v},-1}(\rho)\cos(\Theta_{\mathbf{v}} - \varphi_2) \Big] \Big\},$$

$$\delta\varphi = P\Phi_{2,1}(\delta,\kappa) \sum_{\mathbf{v}} \left[\frac{R_{\mathbf{v},1}(\rho)}{\Omega_{\mathbf{v}} + 2\Omega + \omega} \sin(\Theta_{\mathbf{v}} + 2\varphi_1 + \varphi_2) + \frac{R_{\mathbf{v},-1}(\rho)}{\Omega_{\mathbf{v}} - 2\Omega - \omega} \sin(\Theta_{\mathbf{v}} - 2\varphi_1 - \varphi_2) \right]$$

$$+ P\Phi_{0,1}(\delta,\kappa) \sum_{\mathbf{v}} \left\{ \left[\frac{R_{\mathbf{v},1}(\rho)}{\Omega_{\mathbf{v}} + \omega} \sin(\Theta_{\mathbf{v}} + \varphi_2) + R_{\mathbf{v},-1}(\rho) \frac{R_{\mathbf{v},-1}(\rho)}{\Omega_{\mathbf{v}} - \omega} \sin(\Theta_{\mathbf{v}} - \varphi_2) \right] \right.,$$

$$\Phi_{2,1}(\delta,\kappa) = \frac{\kappa^2}{2\lambda\sqrt{1+\kappa^2}} B_{2,1}^{(1)}, \quad P = \frac{3}{2} n^2 (A-C), \quad \Phi_{0,1}(\delta,\kappa) = \frac{\kappa^2}{2\lambda\sqrt{1+\kappa^2}} B_{0,1}^{(1)};$$

30.3.5. Возмущения угловой переменной ψ .

$$\delta\psi = -\frac{\kappa^2}{\lambda\sqrt{1+\kappa^2}}\int U_1 dt - \frac{1}{G} ctg\rho \int \frac{\partial U_0}{\partial \rho} dt$$

30.3.6. Возмущения долготы восходящего узла плоскости Андуайе h.

$$\delta h = \frac{1}{G}\csc\rho\int\frac{\partial U_0}{\partial\rho}dt + \frac{\lambda}{G}\csc\rho\int\frac{\partial U_1}{\partial\rho}dt$$

$$\delta h = \frac{1}{2}\lambda\frac{P}{G}\csc\rho B_{2,1}^{(1)}\sum_{\nu}\left[\frac{R'_{\nu,1}(\rho)}{\Omega_\nu + 2\Omega + \omega}\sin(\Theta_\nu + 2\varphi_1 + \varphi_2) + \frac{R'_{\nu,-1}(\rho)}{\Omega_\nu - 2\Omega - \omega}\sin(\Theta_\nu - 2\varphi_1 - \varphi_2)\right] +$$

$$+\frac{1}{2}\lambda\frac{P}{G}\csc\rho B_{0,1}^{(1)}\sum_{\nu}\left[\frac{R'_{\nu,1}(\rho)}{\Omega_\nu + \omega}\sin(\Theta_\nu + \varphi_2) + \frac{R'_{\nu,-1}(\rho)}{\Omega_\nu - \omega}\sin(\Theta_\nu - \varphi_2)\right],$$

$$B_{2,1}^{(1)} = -\frac{1}{4\kappa}\left[\delta(1+\kappa^2+\sqrt{1+\kappa^2}) - \kappa^2\right], \qquad B_{0,1}^{(1)} = \frac{1}{\kappa^3}\left[\delta(1+\kappa^2-\sqrt{1+\kappa^2}) - \kappa^2\right].$$

30.4. Вековые возмущения Чандлеровского-эйлеровского движения небесных тел (Земли).

$$\dot{G} = 0,$$
$$\dot{\lambda} = 0,$$
$$\dot{\rho} = \sin\pi_1 \sin(h - \Pi_1)\frac{d\Pi_1}{dt} + \cos(h - \Pi_1)\frac{d\pi_1}{dt},$$
$$\dot{\varphi} = \Omega\left(G, \lambda\right) + \frac{1}{G\mathbf{J}\left(\lambda\right)}\frac{\partial\left\langle U\right\rangle}{\partial\lambda},$$

$$\dot{\psi} = \omega\left(G, \lambda\right) + \cos ec\rho \sin\pi_1 \cos(h - \Pi_1)\frac{d\Pi_1}{dt} - \sin(h - \Pi_1)\frac{d\pi_1}{dt} - \frac{\mathbf{\Lambda}\left(\lambda\right)}{G\mathbf{J}\left(\lambda\right)}\frac{\partial\left\langle U\right\rangle}{\partial\lambda} - \frac{1}{G}ctg\rho\frac{\partial\left\langle U\right\rangle}{\partial\rho},$$

$$\dot{h} = (1 - \cos\pi_1)\frac{d\Pi_1}{dt} - \cot\rho\left[\sin\pi_1\cos(h - \Pi_1)\frac{d\Pi_1}{dt} - \frac{d\pi_1}{dt}\sin(h - \Pi_1)\right] + \frac{1}{G}\csc\rho\frac{\partial\left\langle U\right\rangle}{\partial\rho},$$

$$\left\langle U\right\rangle = U = P\left[b_{0,0}(\lambda)R^{(22)}_{0.0.0.0.0}(\rho) + c_{0,0}(\lambda)R^{(33)}_{0.0.0.0.0}(\rho)\right], \ P = \frac{3}{2}n^2(A - C);$$

$$b_{0,0} = \frac{[\delta(1+\kappa^2) - \kappa^2]\mathbf{E} + [\delta(\lambda^2 - 1) + \kappa^2 + \lambda^2]\mathbf{K}}{2\mathbf{K}(\kappa^2 + \lambda^2)}, \qquad c_{0,0} = \frac{-\mathbf{E}D + \delta(1+\kappa^2)\mathbf{K}}{\mathbf{K}(\kappa^2 + \lambda^2)},$$

$$R^{(22)}_{0,0,0,0,0} = -A^{(0)}_{0,0,0,0,0}\sin^2\rho - \frac{1}{2}(1 + \cos^2\rho)A^{(2)}_{0,0,0,0,0} + A^{(1)}_{0,0,0,0,0}\sin 2\rho,$$

$$R^{(33)}_{0,0,0,0,0} = -A^{(0)}_{0,0,0,0,0}\cos^2\rho - \frac{1}{2}(1 + \sin^2\rho)A^{(2)}_{0,0,0,0,0} - A^{(1)}_{0,0,0,0,0}\sin 2\rho.$$

$$\left\langle \dot{h} \right\rangle = \frac{1}{G} \csc\rho \frac{\partial \left\langle U \right\rangle}{\partial \rho},$$

$$\left\langle \dot{\varphi} \right\rangle = \frac{1}{G\mathbf{J}\left(\lambda\right)} \frac{\partial \left\langle U \right\rangle}{\partial \lambda},$$

$$\left\langle \dot{\psi} \right\rangle = -\frac{\mathbf{\Lambda}\left(\lambda\right)}{G\mathbf{J}\left(\lambda\right)} \frac{\partial \left\langle U \right\rangle}{\partial \lambda} - \frac{1}{G} \cot\rho \frac{\partial \left\langle U \right\rangle}{\partial \rho},$$

$$\left\langle U \right\rangle = P\left[b_{0,0}(\lambda) R^{(22)}_{0.0.0.0.0}(\rho) + c_{0,0}(\lambda) R^{(33)}_{0.0.0.0.0}(\rho) \right],$$

$$b_{0,0} = \frac{[\delta(1+\kappa^2) - \kappa^2]\mathbf{E} + [\delta(\lambda^2 - 1) + \kappa^2 + \lambda^2]\mathbf{K}}{2\mathbf{K}(\kappa^2 + \lambda^2)},$$

$$c_{0,0} = \frac{-\mathbf{E}D + \delta(1+\kappa^2)\mathbf{K}}{\mathbf{K}(\kappa^2 + \lambda^2)},\ D = \delta(1+\kappa^2) - \kappa^2.$$

30.4.1. Гравитационная составляющая чандлеровского движения полюса.

$$\dot{\varphi} = \frac{3\pi n^2 (A-C)\left(b_1\mathbf{K}^2 + b_2\mathbf{EK} + b_3\mathbf{E}^2\right)}{8G\kappa\lambda^2\lambda'^2\sqrt{(1+\kappa^2)(\kappa^2+\lambda^2)}\mathbf{K}^3}\left[-A_0^{(0)}(1-3\cos^2\rho) + 3\sin 2\rho A_0^{(1)} + \frac{3}{2}A_0^{(2)}\sin^2\rho\right]$$

$$b_1 = \lambda'^2\left[2\delta(1+\kappa^2)\lambda^2 - D(\kappa^2+\lambda^2)\right],\ b_2 = 2D\kappa^2\lambda'^2,$$
$$b_3 = -D(\kappa^2+\lambda^2),\ D = [\delta(1+\kappa^2) - \kappa^2].$$

$$\dot{\varphi} = \frac{3n^2(A-C)(2-D)}{4\omega C\sqrt{1+\kappa^2}}\left[-A_{\mathbf{0}}^{(0)}(1-3\cos^2\rho) + 3\sin 2\rho A_{\mathbf{0}}^{(1)} + \frac{3}{2}A_{\mathbf{0}}^{(2)}\sin^2\rho\right]$$

30.4.2. Постоянная прецессии.

$$\dot{h} = \frac{3n^2(A-C)}{4\omega C \sin\rho}\left[1+\delta-\frac{3\delta(1+\kappa^2)}{\kappa^2+\lambda^2}+\frac{3D\mathbf{E}}{\mathbf{K}(\kappa^2+\lambda^2)}\right]\left\{-\left(A_{\mathbf{0}}^{(0)}-\frac{1}{2}A_{\mathbf{0}}^{(2)}\right)\sin 2\rho + 2A_{\mathbf{0}}^{(1)}\cos 2\rho\right\}$$

$$\dot{h} = \frac{3n^2(A-C)(\delta-2)}{4\omega C \sin\rho}\left\{-\left(A_{\mathbf{0}}^{(0)}-\frac{1}{2}A_{\mathbf{0}}^{(2)}\right)\sin 2\rho + 2A_{\mathbf{0}}^{(1)}\cos 2\rho\right\},$$

$$\Delta\dot{h} = \frac{9\lambda^2}{8\kappa^2}\cdot\frac{n^2(A-C)}{\omega C\sin\rho}\left[2-\delta(1+\kappa^2)+\kappa^2\right]\left\{-\left(A_{\mathbf{0}}^{(0)}-\frac{1}{2}A_{\mathbf{0}}^{(2)}\right)\sin 2\rho + 2A_{\mathbf{0}}^{(1)}\cos 2\rho\right\}.$$

Вклад чандлеровского движения полюса в постоянную прецессии.

$$p = p_0(1-0.222921\times 10^{-11})$$

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