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Quantum Uncertainty and a Counterexample of Nonlocal Classical "Realism"

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Abstract—We analyze the scheme of an experiment in which, by examining suppression effects of the cross correlation of photons in a beamsplitter and by preparing squeezed states, it is proven that the phase difference of photons in Fock states cannot acquire a certain value, since, otherwise, the simultaneous existence of these two effects would be impossible. We show that this reveals an intrinsic inconsistency of the nonlocal classical interpretation of quantum mechanics on the basis of nonlocal classical "realism."

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INTRODUCTION

There are many quantum effects that do not have classical analogs, are paradoxical, and cannot be interpreted from the viewpoint of macroscopic "common sense." The latter term is commonly enclosed in quotation marks in order to show its inconsistency with substantially nonclassical phenomena. Researchers also call it "local realism." Local, because, in this case, models are used that obey spatiotemporal restrictions that we usually observe in the macroworld. Realism, because, as in classical physics, it is believed that physical quantities that are measured in an experiment have quite certain values prior to the moment of their measurement.

The first doubts as to the adequacy of local realism have already been created by the consideration of interference of single photons in a double-slit Jung interferometer [1] and Michelson and Mach–Zehnder double-beam interferometers (see, e.g., [2, 3] and references therein). In these experiments, an indivisible quantum is simultaneously present in two channels and interferes with itself, and, up to the moment of its detection, its particular location is not determined.

Further, the effect of three-beam interference [4] proves a priori the absence of a certain number of photons in the electromagnetic field, i.e., prior to the measurement of their number. Experiments to verify the Bell inequalities [5], who formalized the Einstein–Podolsky–Rosen paradox [6] (including the very latest ones [7, 8]) reliably refuted the local theory of hidden parameters, albeit its complete inadequacy was evident since the time of first tests [9–11] (see also [3, 12, 13]). In this case, the locality hypothesis assumes that, if two independent observers detect a pair of correlated particles, either of them his own, then the observers are not related in any way to one another, and the readings of the measuring instrument of one of them do not affect the readings of the other (see, e.g., [14, 15]). However, the validity of this assumption cannot be proven in experiments on testing the Bell theorem.

Therefore, the only "lead" of supporters of classical realism and, in fact, of the reduction of quantum theory to ordinary classical statistical physics, remains in understanding nonlocality as an unknown mysterious interaction, which is not subject to either spatial or temporal (within the light cone) restrictions. In addition, it would seem, an argument in favor of these views is the phenomenon of quantum nonlocality, which has already been experimentally proven not only for a pair or more of entangled particles, but, also, for a single photon [16–18]. In addition, the result of any experiment with quantum systems can be calculated on a computer, of course, in a probabilistic sense. The computer operates with particular values of quantities to be measured, which are certain completely prior to the moment of measurement (as in classical statistical physics). Therefore, it is rather difficult to refute nonlocal realism absolutely.

At the same time, no physicist who is engaged in specific quantum calculations, on the basis of his own experience and his inner intuition, will ever believe in nonlocal realism. Therefore, to refute it, researchers have followed the path of development of new experimental schemes that would progressively increase the absurdity of models constructed on the basis of various types of nonlocal theories of hidden parameters. Indeed, how, for example, could the effects of two-beam interference be explained taking into account experiments to verify the "deferred choice" or three-beam interference [4] within the framework of nonlocal realism? Only by nonlocal "hopping" of photons between optical channels separated from each other, even through opaque walls [19].

A significant step in testing nonlocal realism has been taken in [20–27]. In those works, objective criteria were proposed (in the form of mathematical inequalities), which allowed one, in particular, to refute one type of nonlocal theory that admitted a nonlocal connection between measuring instruments that detect a pair of quantum particles in a state that is entangled in polarization. The corresponding experiment, which was performed by the group of A. Zeilinger [23], refuted nonlocal realism of this type, though with the assumption that the Malus law is observed. However, to doubt the validity of the latter law means to further increase the degree of absurdity of the nonlocally realistic interpretation. They proceeded from the following assumptions:

(i) all measurement results are determined by the preexisting properties of particles, which are independent of the measurement;

(ii) physical states are statistical mixtures of subensembles with a certain polarization; and

(iii) the polarization is defined such that the values of the mathematical expectations taken for each subensemble obey the Malus law (i.e., follow the well-known cosine-law dependence of the intensity of the polarized beam behind the ideal polarizer).

Nevertheless, nonlocality of a stronger form, namely, the nonlocal relation between the results of measurements obtained by spaced measuring devices rather than between the devices themselves, cannot be refuted in this way [23, 28]. However, as it seems, nonlocal realism in the sense of the impossibility of abandoning the quantum superposition of all possible values of some quantity (in contrast to a certain value of it) can be refuted by the experiment described below. The meaning of this experiment is that, upon parametric creation of entangled photon pairs, two effects can be observed: the suppression of the cross correlation of photons [29] and the preparation of squeezed states of light [30–33]. The only parameter that could predetermine these observed effects prior to their measurement would be the phase difference between the photons of the entangled pair, if, of course, it exists. Thus, if we proceed from the hypothesis that hidden parameters do exist, including those that have a nonlocal nature, then the phase difference mentioned above may be such a hidden parameter in this case. However, the two effects cannot simultaneously exist at the same phase difference. Consequently, this parameter does not have a specific value, just as the Copenhagen School interprets it, and nonlocal realism receives a completely substantiated refutation.

ON THE UNCERTAINTY OF THE PHASE OF PHOTONS IN FOCK STATES

It is well known that, by virtue of the Heisenberg uncertainty principle, the phase of Fock states with a certain number of photons (including its cosine and sine, which are measurable observables and are described by Hermitian operators) is completely uncertain, i.e., is in the superposition of all its possible values from 0 to 2π (see, e.g., [4, 13] and references therein). How can this indisputable fact be interpreted by nonlocal realism? Only by the statement that the phase of a photon in some state, e.g., $|1\rangle$, still exists, but nonlocally "adapts" to a particular experimental situation, as if "knowing" in advance all the subsequent history of transformations and measurements of the photon. This can explain not only the violation of the Bell inequalities, but also all possible interference quantum effects. We will try to refute these ideas by analyzing the effects of suppression of the correlation of photocounts [29] and preparation of squeezed states upon parametric light scattering (see, e.g., [30–33]).

The effect of suppression of the correlation of photocounts is a suprising phenomenon, which demonstrates the specifics of quantum theory. It consists of the following. If a single photon is sent to one of the inputs of a 50% beamsplitter, then it will appear at one of the outputs with a probability of 1/2, thus exhibiting typically corpuscular properties. But what if either of the beamsplitter inputs does receive simultaneously a single photon? It would seem that, with probabilities of 1/4, two photons should simultaneously appear at one of the outputs, or, with a probability of 1/2, a single photon should appear at either output. In fact, this is not the case: the probability of the second event turns out to be zero, while the photons at the outputs appear only in pairs. How can this be verified? In the experiment of [29], the signals from two detectors that were installed at the outputs of the beamsplitter and operated in the regime of counting single photons were directed to the coincidence circuit (the scheme on the left). Up to the accuracy of technical noise, the signal from the coincidence circuit turned out to be zero.

Theoretically, this result can be described in the representations of both Heisenberg and Schrödinger. In the Heisenberg representation, the photon annihilation operators are introduced, which describe two input plane monochromatic modes, $\hat{a}$ and $\hat{b}$. In this case, the operators of output modes are given by $\hat{c} = (\hat{a} + \hat{b})/\sqrt{2}$ and $\hat{d} = (\hat{a} - \hat{b})/\sqrt{2}$. Further, we find the operators of photon numbers $\hat{n}_c = \hat{c}^+\hat{c}$ and $\hat{n}_d = \hat{d}^+\hat{d}$, and then, their correlation function $\langle \hat{n}_c \hat{n}_d \rangle$ by averaging over the initial state, $|1\rangle_a |1\rangle_b$. As a result, we obtain ${}_a\langle 1|\,{}_b\langle 1|\hat{c}^+\hat{c}\hat{d}^+\hat{d}|1\rangle_b |1\rangle_a = 0$.

In the Schrödinger representation, it is necessary to introduce the matrix of the beamsplitter,

$$B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} \tau & -\rho \\ \rho & \tau \end{pmatrix},$$

where ρ and τ are the amplitude reflection and transmission coefficients, respectively, which, in our case, are $1/\sqrt{2}$. The transformation of the Fock states, $|n_1\rangle$ and $|n_2\rangle$, at the inputs of the beamsplitter is described [34] by the action of the beamsplitter operator,

$$\begin{aligned} \hat{B}|n_1, n_2\rangle = \frac{1}{\sqrt{n_1 n_2}} \sum_{k_1,k_2}^{n_1,n_2} C_{k_1}^{n_1} C_{k_2}^{n_2} B_{11}^{k_1} B_{12}^{k_2} B_{21}^{n_1-k_1} B_{22}^{n_2-k_2} \\ \times \sqrt{(k_1+k_2)!(n_1+n_2-k_1-k_2)!} \\ \times |k_1+k_2, n_1+n_2-k_1-k_2\rangle. \end{aligned} \quad (1)$$

For state $|1,1\rangle$ at the input, there are two terms with states $|1,1\rangle$ at the output, but the coefficients in front of them are identical and have opposite signs: $\tau^2|1,1\rangle - \rho^2|1,1\rangle$.

How to interpret this result? According to [34], it can be regarded as a manifestation of corpuscular–wave dualism. Indeed, on the one hand, photons behave themselves as particles, demonstrating discrete photocounts, while, on the other hand, it seems that they interfere at the beamsplitter as waves with a certain phase difference. With what? Obviously, with 0 or π, so that either two or zero photons would always appear at the outputs of the beamsplitter. Thus, it is assumed, in fact, that the phase difference of photons mixed on the beamsplitter is certain. Otherwise, there will be no effect of suppression of the correlation of photocounts. Whereas, the occurrence of this certain phase difference is precisely the hidden parameter, which predetermines completely the result of the experiment; i.e., we dealing with nonlocal realism in the explicit form. Let us consider the consequences to which this interpretation leads.

How state $|1,1\rangle$ at the input of the beamsplitter can be obtained? It is very simple: this can be done as a result of parametric scattering [31–33]. This is exactly what the experimenters of [29] did. But what will happen when the signal and reference beams of the parametric process are mixed in the beamsplitter? Squeezed states of light will be prepared, which are characterized by the suppression of quantum fluctuations of one of their quadrature field components to the detriment of the other (see, e.g., [30–33] and references therein). And will this preparation be consistent with the assumption that there should always be a phase difference of 0 or π between the signal and reference beams, as it follows from the above interpretation of the result of the experiment of [29]?

Let us introduce the annihilation operators of photons of the signal and reference beams, $\hat{a}$ and $\hat{b}$. They are described by the Bogolyubov transformation of the operators of seed vacuum modes $\hat{a}_0$ and $\hat{b}_0$,

$$\hat{a} = \mu\hat{a}_0 + \nu\hat{b}_0^+, \; \hat{b} = \mu\hat{b}_0 + \nu\hat{a}_0^+. \quad (2)$$

In one of the channels, we add a phase delay. It is clear that it will not affect in any way the effect of suppression of the cross-correlation. This follows easily from the above consideration in terms of the Heisenberg representation. But how will the phase delay affect the preparation of squeezed states? The mode annihilation operator of one of the beamsplitter outputs will still be denoted as $\hat{c} = \left(\hat{a} + \hat{b}e^{i\theta}\right)/\sqrt{2}$. Correspondingly, the quadrature component is equal to $\hat{X} = \left(\hat{c} + \hat{c}^+\right)/2$. Let us find its dispersion,

$$\begin{aligned} &{}_a\langle 0|\,{}_b\langle 0|\hat{X}^2|0\rangle_b|0\rangle_a \\ &= \left(|\mu|^2 + |\nu|^2 + \mu\nu e^{i\theta} + \mu^*\nu^* e^{-i\theta}\right)/4 \\ &= (1 + 2\nu(\nu + \mu\cos\theta))/4 \end{aligned} \quad (3)$$

at real μ and ν. Here, we averaged over the initial vacuum state of seed modes, and also used the equality $|\mu|^2 - |\nu|^2 = 1$, which follows from the commutation relations $\left[\hat{a}, \hat{a}^+\right] = \left[\hat{b}, \hat{b}^+\right] = 1$.

Thus, we found that the effect of squeezing depends on phase θ. This is clear, because, at one output of the beamsplitter, the light is in a squeezed state, while, at the other output, on the contrary, its quadrature is increased by the dispersion, which corresponds to a phase change by π. But is this consistent with the assumption that the phase difference between the signal and reference beams oscillates with a probability of 1/2, taking the values of 0 or π? Yes, it is quite possible, because photons with a phase difference of 0 go into one output channel of the beamsplitter, while those with a phase difference of π fly into the other channel. How can we prove the impossibility of the simultaneous existence of the two effects?

Let us introduce the system of phase delay θ in one of the input channels of the beamsplitter (Fig. 1). The effect of squeezing of the cross-correlation does not depend on this phase delay: photons that previously traveled in pairs to the output channels of the beamsplitter will still travel in pairs. But the orientation of the ellipse of squeezing (the region of uncertainty of the quadrature components on the phase plane) will vary (see, e.g., [32, 33]). This can be registered experimentally. However, if the ellipse of squeezing has turned, then the phase difference in the channels has ceased to be 0 or π, and, therefore, the effect of suppression of the cross-correlation should, at least partially, vanish if photons are incident on the beamsplitter do interfere. And this fact can also be verified experimentally. Therefore, the effects of preparation of squeezed states and suppression of the cross-correlation of photons cannot exist simultaneously if the

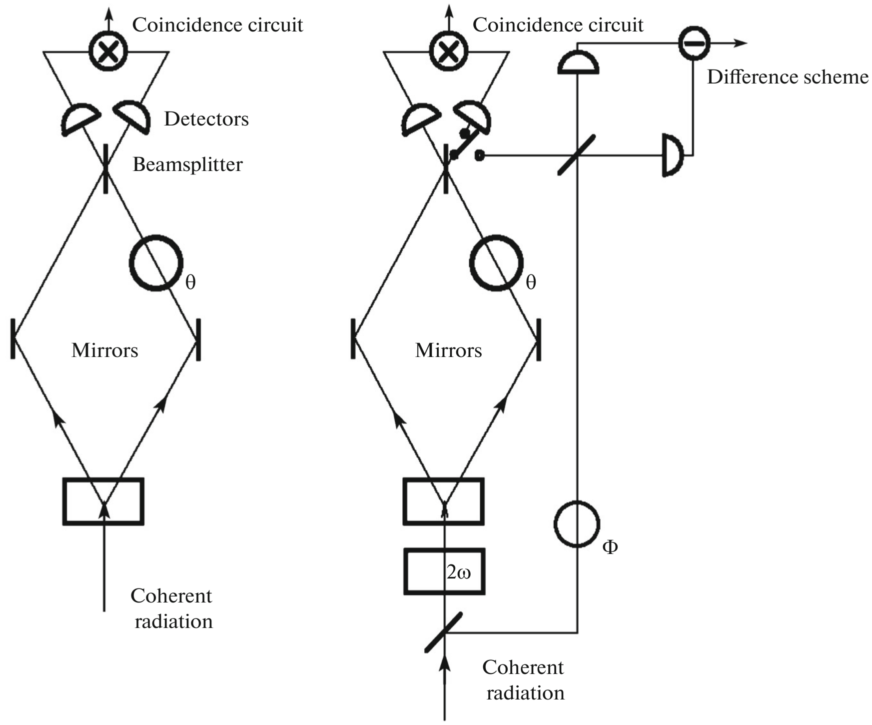

Fig. 1. Scheme of observation of the suppression of cross-correlation of photons (left) and simultaneous registration of a squeezed state (right). Under the action of coherent laser radiation, a pair of photons is generated in a nonlinear crystal. They are directed to a beamsplitter and are detected. A coincidence circuit registers the simultaneous arrival of photons at the both photodetectors (the scheme on the left). For a 50% beamsplitter, the probability of these events is zero. This means that photons arrive at the detectors only in pairs. On the right, using the scheme of a balanced homodyne detection, fluctuations of the quadrature component of the field are simultaneously recorded. To this scheme, the radiation is directed by a mode selection switch. To match the frequencies to be mixed in front of the nonlinear crystal, the frequency of the coherent radiation is doubled.

phase difference between them is certain. Thus, the basic premise about the presence of a certain phase difference of photons leads to a logical contradiction, which indicates its inadequacy.

Although the obtained result is absolutely clear and transparent, it can also be interpreted in terms of opposite mutually exclusive notions. From the point of view of the Copenhagen interpretation, no particular values of the phase difference (its sine and cosine) do exist a priori. But nonlocal realism may well cope with the situation in question in terms of only its own positions. Indeed, if there is an instantaneous nonlocal relation between all the objects that participate in the experiment, as well as between the measurement results, then the phase difference of photons of an entangled pair can acquire quite particular values, with them being such that they would correspond to the obtained result of the experiment. That is, upon registration of the effect of suppression of the cross-correlation, the phase difference will supposedly acquire one value, while, in the case of the preparation of squeezed states, it will acquire another value, depending on the detection scheme. To show the inadequacy of such an interpretation, we will modernize the experimental scheme such that these two effects could be observed simultaneously.

MODERNIZATION OF THE EXPERIMENTAL SCHEME

Let us introduce a mode switch in the scheme of the experiment (see Fig. 1, the scheme on the right), which switches the direct detection of the right detector to a balanced homodyning. The latter registers fluctuations of the quadrature component of the field (see, e.g., [32, 33] and references therein), and, if their level is below the level of the vacuum state, we can state that the squeezed state is prepared. In the first stage of the experiment, along with the fixation of the cross-correlation suppression of photons, we register the rate of photocounts of either of the photodetectors and the arrival of photons at them only in pairs. Then, we switch the scheme to the second mode, when a squeezed state is observed, and register the region of uncertainty of quadrature components (the so-called ellipse of squeezing), varying phase delay Φ in the homodyne channel. Further, as distinct from [35], we introduce phase delay θ. The ellipse of squeezing should change its orientation on the phase plane. And if this is the case, then the phase difference between photons has changed from jumps by 0 or π to some other difference. It must necessarily, at least partially, suppress the effect of cross-correlation of photocounts if we proceed from the model of interference of photons of an entangled pair with a certain phase difference. But this should change the rate of photocounts in the left detector, and photons will cease to arrive at it only in pairs. However, this will hardly occur. It is quite clear that the rate of photocounts will not change, and photons will still arrive only in pairs, since by changing the conditions of registration of radiation emerges from one channel of the beamsplitter, we cannot affect in any way the results of registration in the other channel. This merely follows from the principle of causality and elementary quantum calculations. Therefore, no particular phase difference between photons is incapable of describing the result of observations obtained using the right scheme in the figure. And this means only that such a phase difference simply does not exist, which does not fit into the frameworks of any classical realism, including the nonlocal one, precisely because of the mutually exclusive nature of the observed effects at a certain phase difference. The a priori absence of a certain value of a quantity to be measured means that this quantity is in the state of quantum superposition of its all possible values. But this fact—the quantum superposition—is just not recognized by any theory of the classical realism. These theories are focused on any alternatives that attempt to explain the results of quantum experiments without invoking the phenomenon of quantum superposition.

The rigorous substantiation requires the following formal conditions. In fact, we prove from the contrary the absence of a hidden parameter—a certain phase difference. We assume that the following postulates are correct.

1. All measurement results are determined by the preexistent properties of particles, which are independent of the measurement.

2. Physical states are statistical mixtures of subensembles of pairs of single photons with a certain phase difference.

3. Monochromatic light beams interfere upon mixing. The result of the interference depends on the phase difference mentioned above, with this dependence being such that beams with a zero phase difference (with an accuracy of up to $2\pi n$) yield a maximum of the intensity, while those with a phase difference of π (with an accuracy of up to $2\pi n$) yield a minimum. In intermediate cases, the result of the interference is such that the values of the mathematical expectations taken for each subensemble obey the law of the well-known harmonic dependence of the intensity of radiation on the phase difference.

4. Quantum squeezed states are formed by mixing the signal and reference beams of parametrically scattered light by a 50% beamsplitter. And the squeezing efficiency depends on the phase difference of the beams such that beams with a zero phase difference (with an accuracy of up to 2πn) yield a maximum of the squeezing efficiency in one output channel of the beamsplitter and a minimum in the other.

By accepting these postulates, we arrive at the above-described logical contradiction when analyzing the scheme of the experiment shown on the right in the Fig. 1.

It is interesting that, in order to refute the local theory of hidden parameters, John Bell should use only the postulate of locality and the probability theory. However, when using detectors with a nonideal quantum efficiency, the Bell theorem had to be supplemented with the description of this real detection, which can also be considered as an additional postulate [3, 12, 13]. To solve the same problem with respect to the nonlocal theory of hidden parameters in the sense of the interaction of spaced detectors, three postulates, including the law of Malus, had to be admitted. And, to prove the inadequacy of nonlocal realism in a more general sense, four postulates related to physical phenomena are already required. Therefore, the advance toward the rejection of a "stronger" nonlocality occurs at the expense of lowering the generality of the evidence. As it seems, this should not weaken their reliability, since the postulated physical phenomena with their known laws (interference of light and the preparation of squeezed states) have the irrefutable experimental confirmation.

Thus, the phase difference of a correlated photon pair indeed does not have a certain value, but, rather, is in a superposition of all its possible values from 0 to 2π. How, then, should the results of the experiment be interpreted [29], if neither a certain phase of single photons, nor their phase difference exists? It seems, the point is that, in accordance with the Feynman interpretation of quantum theory [36], it is not photons that interfere with each other, but, rather, their alternative trajectories. Indeed, how is state $|1,1\rangle$ formed at the output of the beamsplitter? It can occur in two ways: either the two photons pass through the beamsplitter, or both of them are reflected. But, in the latter case, because one of the photons is reflected from a denser medium, it acquires a phase shift π (see also [37]). The operator of the phase shift (of the accumulated phase difference), $\hat{U}_\theta = e^{-i\theta\hat{n}}$, transfers state $|1,1\rangle$ to state $-|1,1\rangle$. Therefore, the two possible alternative trajectories interfere destructively, suppressing cross-correlations. This simple and pictorial approach allows us to solve even more complex problems related to the conversion of Fock states by beamsplitters, without using complicated and cumbersome formula (1).

CONCLUSIONS

The result that was obtained in this work is important because the absence of certain values of quantities to be measured prior to the moment of their measurement is the fundamental inference of quantum theory in the Copenhagen interpretation. The experimental evidence of this fact known so far can be challenged by invoking nonlocal interactions of types that are allegedly unknown to us, which are not restricted by any areas of the light cone, and, correspondingly, by the speed of light. These are various kinds of nonlocal theories (see, e.g., [38] and references therein), which, purely in formal terms, can explain both the violation of the Bell inequalities and numerous quantum paradoxes. For example, the interference of single photons on a double-slit Jung's screen is interpreted as a nonlocal "knowledge" of a photon that passes through one slit about the existence of the other slit. In the experiment described in this work, there are solid grounds to refute statements of this kind. No nonlocal "knowledge" of the photon about its future life can explain the constancy of the rate of photocounts on the left detector and the arrival of photons on it only in pairs (Fig. 1). Therefore, the absence of a certain value of the phase difference of single photons can in no way be challenged by any hypothesis of nonlocal realism. This significantly narrows the circle of possible interpretations of quantum theory, clearly, not reducing them only to the Copenhagen interpretation. An adequate explanation can also be given in terms of the relational paradigm (see, e.g., [39]).

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