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A Permanent Spherical Magnet with Inhomogeneous Magnetization

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Abstract—Inhomogeneously distributed magnetization that induces a strong magnetic field in a predetermined local area is established in finite permanent magnets. The maximum possible magnetic induction in the vicinity of a magnet's center is found analytically for a perfect spherical magnet.

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INTRODUCTION

Advanced magnetic materials allow the creation of permanent magnets with high values of residual induction B_r and coercive force H_c. Their maximum values have been established for sintered Nd–Fe–B systems: B_r = 14200 G and H_c = 12500 E [1]. The magnetization vector is thus

$$I = \frac{B_r}{4\pi} = 1131 \text{ G}. \quad (1)$$

The greatest possible magnetic field that can be induced by this substance upon homogeneous magnetization in an infinite sample is estimated at

$$B_r = 4\pi I. \quad (2)$$

The magnetic field in a narrow transverse slit of a homogeneously magnetized toroid has a similar value B of induction. Equation (2) is usually used to estimate the maximum magnetic induction of the field that can be generated by a magnetic material with a definite value I of magnetization [2].

The aim of this work was to determine the possibility of using magnetic matter (permanent magnets) to obtain a magnetic field greater than (2), and to establish its maximum value, assuming that each element in a specimen is a permanent magnet with infinite coercive force and magnetic moment

$$p_m = I\Delta V, \quad (3)$$

where ΔV is the magnet's volume.

The solution to this problem could be obvious, since the magnetic field induced by an infinitely large number of magnets (magnetic dipoles) at a given point can reach any high value [3]. If the volume occupied by magnetic matter is limited, a magnetic field greater than (2) can also be attained in some domains. For example, the magnetic field at the edges of a homogeneously magnetized rectangular magnet has an infinitely high value [4]. However, if the magnet's volume is limited and there are no singularities in the magnetization distribution, the solution to our problem is not obvious.

In reality, the above ideal conditions do not exist. Any magnet has finite coercive force, and any magnetization distribution is inhomogeneous. This limits our ability to obtain magnetic fields greater than (2). In practice, a permanent spherical magnet with a diameter of 12 cm allows us to obtain magnetic fields on the order of 50000 G in a gap with a diameter of 0.15 mm [5]. This is almost 4x greater than the value determined by Eq. (2).

ORIENTATION OF THE MAGNETIC MOMENT THAT INDUCES THE GREATEST MAGNETIC FIELD AT A PREDETERMINED POINT IN A SPECIFIED DIRECTION

Let a dipole with magnetic momentum p_m, oriented at angle β relative to radius vector $\vec{r}$, and drawn to point z by axis coordinate Z, be at point y on coordinate axis Y (Fig. 1).

The strength of magnetic field $\vec{H}$ induced by dipole $\vec{p}_m$ at the point with radius vector $\vec{r}$ is

$$\vec{H} = \frac{3(\vec{p}_m\vec{r})\vec{r}}{r^5} - \frac{\vec{p}_m}{r^3}. \quad (4)$$

In accordance with Eq. (4), for the strength vector component of a magnetic field along coordinate axis Z, we have

$$H_z = \frac{3p_m \cos\beta\cos\alpha}{r^3} - \frac{p_m\cos(\alpha+\beta)}{r^3}, \quad (5)$$

where α is the angle between the vector $\vec{r}$ and axis Z. Using the relation

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \quad (6)$$

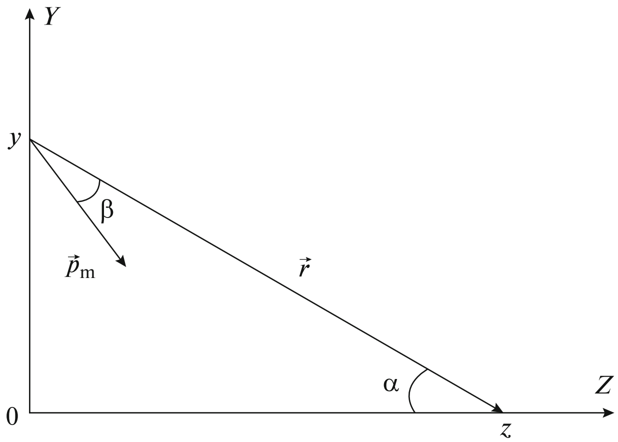

Fig. 1. Location and orientation of magnetic dipole $\vec{p}_m$ relative to radius vector $\vec{r}$ of the point of observation in the determined Cartesian system.

for field (5), we obtain

$$H_z = (2\cos\alpha\cos\beta + \sin\alpha\sin\beta)\frac{p_m}{r^3}. \quad (7)$$

We now find at what angle β the orientation of magnetic momentum vector $\vec{p}_m$ of magnetic field (7) projected onto axis Z will be greatest.

By differentiating Eq. (7) with respect to variable β, the equation for the unknown angle is

$$-2\cos\alpha\sin\beta + \sin\alpha\cos\beta = 0. \quad (8)$$

The tangent of the unknown angle can be derived from Eq. (8):

$$\tan\beta = \frac{1}{2}\tan\alpha. \quad (9)$$

In light of solution (9) and relations

$$\cos\beta = \frac{1}{\sqrt{1+\tan^2\beta}}, \quad \sin\beta = \frac{\tan\beta}{\sqrt{1+\tan^2\beta}}, \quad (10)$$

we obtain the following maximum value of the magnetic field strength projection

$$H_z = \sqrt{3\cos^2\alpha + 1}\,\frac{p_m}{r^3}. \quad (11)$$

GREATEST POSSIBLE FIELD GENERATED BY A SPHERICAL MAGNET

Based on the results of the previous section of this paper, we derive the greatest possible magnetic field induced by the spherical magnet. We consider the spherical layer with radius r shown in Fig. 2. We select a narrow ring with radius $r\sin\alpha$ within this layer. The magnetic moments of this ring then generate a magnetic field that is strongest at the sphere's center (at point 0), assuming that these magnetic moments are deflected from the radius of the sphere toward axis Z

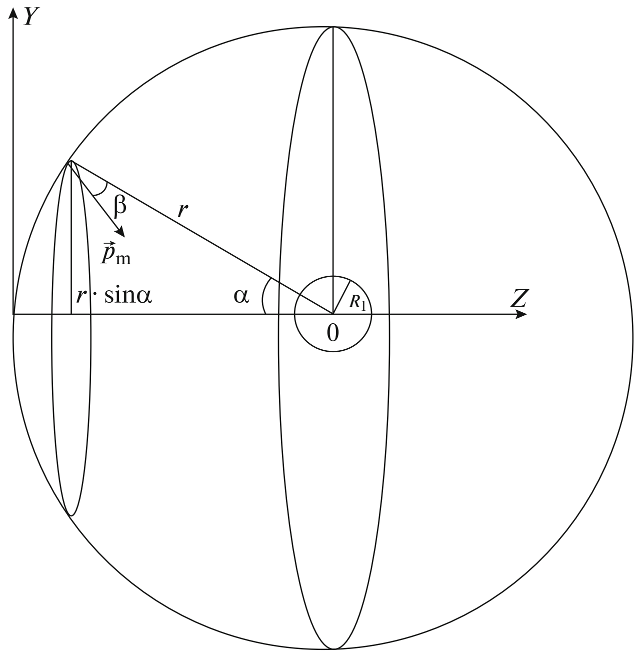

Fig. 2. Scheme of a ball magnet with a spherical cavity at its center and the geometric magnitudes used in calculating the strength of the magnetic field at the ball's center.

at angle β determined by relationship (9). The magnetic field at the sphere's center is determined by Eq. (11), where

$$p_m = I2\pi r\sin\alpha\, r d\alpha dr. \quad (12)$$

The magnetic field determined by the magnetic moments of the whole spherical layer is equal to the integral

$$H_z = 2\int_0^{\pi/2}\sqrt{3\cos^2\alpha+1}\,\frac{I2\pi r\sin\alpha\, r d\alpha dr}{r^3} = 4\pi\frac{Idr}{r}\int_0^{\pi/2}\sqrt{3\cos^2\alpha+1}\sin\alpha d\alpha. \quad (13)$$

Integral (13) is taken in elementary functions, and the trivial algebraic operations for field (13) result in the expression

$$H_z = 12\pi\frac{Idr}{r}\left(\frac{1}{\sqrt{3}} + \frac{1}{6}\ln(2+\sqrt{3})\right). \quad (14)$$

Integrating relation (14) over all spherical layers for a spherical magnet with radii of R_1 to R_2, we obtain the expression for the greatest possible magnetic field strength:

$$H_z = \frac{12\pi I}{\sqrt{3}}\left(\frac{1}{\sqrt{3}} + \frac{1}{6}\ln(2+\sqrt{3})\right)\ln\left(\frac{R_2}{R_1}\right). \quad (15)$$

According to Eq. (15) and in light of Eq. (2) for the maximum magnetic induction of a spherical magnet, we obtain

$$B = \left(1 + \frac{1}{2\sqrt{3}}\ln(2+\sqrt{3})\right)\ln\left(\frac{R_2}{R_1}\right)B_r = 1.38\ln\left(\frac{R_2}{R_1}\right)B_r. \quad (16)$$

For the parameters of the spherical magnet used in [5] in particular ($R_2 = 6$ cm and $R_1 = 0.075$ cm), we have

$$B = 6.05B_r = 86000 \text{ G} = 8.6 \text{ T}. \quad (17)$$

This value is almost 1.5 times greater than the record one found experimentally for a permanent magnet with inhomogeneously distributed magnetization and similar parameters in [5] and is almost twice that numerically calculated in [6].

CONCLUSIONS

For a spherical magnet with a diameter of 1 m, we can obtain a magnetic field with an induction of $B = 9.53B_r = 135000$ G = 13.5 T in a cavity with a diameter of 1 mm. This is comparable to the constant magnetic fields obtained in modern superconductive coils.

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