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★**Applied algebraic dynamics.**
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Besides presenting results obtained by the authors and recent developments in algebraic dynamics, this book aims at applying methods and results from algebraic dynamics to concrete problems arising from various branches of science. By algebraic dynamics here, the authors mean the study of the theory of iterations of self-mappings on a given set $\mathbb{S}$ which is usually endowed with a certain algebraic structure. Let $f\colon \mathbb{S} \to \mathbb{S}$ be a self-map on $\mathbb{S}$. When there is a topology or measure defined on $\mathbb{S}$, one is interested in the asymptotic behavior or distribution of the orbit $x_0, x_1 = f(x_0), \dots, x_{i+1} = f(x_i), \dots$ for $x_0 \in \mathbb{S}$. Such a consideration is, in particular, suitable for concrete problems as discussed in this book.

The need for the investigation of algebraic dynamics comes from their importance in number theoretical problems arising from iterations of rational functions, especially Diophantine problems related to the iterations of morphisms on varieties defined over a number field [see J. H. Silverman, *The arithmetic of dynamical systems*, Springer, New York, 2007; MR2316407 (2008c:11002)]. The dynamical systems addressed in the book under review are mainly dynamical systems in finite groups and rings, as well as spaces defined over p-adic fields. Although many of the dynamical systems discussed in the book, in particular p-adic dynamics, have number theoretical features, the authors focus on their ergodic properties, which are the main theme of the book. Especially, p-adic ergodic theory and its applications are emphasized in the book.

As in many other fields in mathematics, problems and phenomena investigated in the theory of dynamical systems have their roots in other areas of science. Readers may be amazed to learn that the fields to which the theory of algebraic dynamics can be applied can range from computer science, cryptography and theoretical physics to other branches of science such as cognitive science, psychology, neurophysiology and genetics. As applications of algebraic dynamics are one of the main purposes in the book, there are potential readers whose backgrounds are not in mathematics. The authors minimize the prerequisites for reading the book—elementary number theory and basic notions from algebra and analysis. As for p-adic analysis, the main tool that is used throughout the book, there is a chapter entirely devoted to the introduction of this subject. Overall, the authors have made the book self-contained.

This book is divided into three parts. The first two parts provide the theoretical background for the whole book. The last part consists of ten chapters discussing possible applications of results from Parts I and II to various fields of science.

Basic results and notions in number theory, algebra and p-adic fields are reviewed in Chapter 1 (which precedes Part I). This chapter serves as a sort of reference for later chapters. After reviewing these basic materials, the following four chapters comprise the first part of the book. In Part I, two main topics are discussed in detail. The first concerns ergodic polynomial transformations on various types of finite, commutative algebraic structures (groups and rings) and the second topic is measure preserving and ergodic transformations on normed spaces over the ring of p-adic integers $\mathbb{Z}_p$.

Chapters 2 and 3 focus on the key notions and results in p-adic ergodic theory and p-adic analysis. In Chapter 2, ergodic transformations between finite algebraic structures are discussed in detail. From this perspective, the authors are concerned with the classifications of finite abelian groups as well as finite commutative rings that have ergodic polynomial transformations. Chapter 3 concentrates on the theory and basic results in p-adic analysis, which is the main tool that is used throughout the whole book. The authors chose to restrict their attention to functions (mappings) defined on complete normed vector spaces over p-adic fields $\mathbb{Q}_p$. Several classes of functions (mappings) are further emphasized. Among those is the class of compatible functions which are functions satisfying some uniform congruence conditions. The relationship between compatibility and differentiability as well as Lipschitz conditions is discussed thoroughly.

The main subject in Chapter 4 is ergodicity and measure preservation of transformations over p-adic spaces. Attention is paid to functions defined in Chapter 3, such as compatible functions, 1-Lipschitz and differentiable transformations that are measure preserving or ergodic. Characterizations of measure preserving or ergodic transformations on higher-dimensional normed space $\mathbb{Z}_p^n$ are also established.

In Chapter 5, dynamical systems associated to the monomial transformation $x \mapsto x^n$, $n \geq 2$, are studied in depth. The main topic to investigate here is the number of cycles in $\mathbb{Q}_p$ of these transformations as p varies over all prime numbers. Exact formulae in terms of n, r and p for the number of $\mathbb{Q}_p$-rational r-periodic points for f are obtained. Using these formulae, for fixed integers n and r the distribution of r-cycles in $\mathbb{Q}_p$ as p varies is computed. Furthermore, for the same monomial dynamics the authors also introduce the notion of fuzzy cycles and calculate the number of fuzzy cycles.

There are two chapters in Part II. The main theme of this part is similar to that of Part I. Instead of looking at commutative algebraic structures as in Part I, the authors give more attention here to noncommutative groups having ergodic polynomial transformations. Because the underlying algebraic structures are noncommutative, the definition of polynomial mappings needs to be clarified. Noncommutative differential calculus and basics of polynomial mappings on groups are discussed in Chapter 6. Then, in Chapter 7, characterizations of solvable groups having ergodic polynomial transformation are given. Furthermore, ergodic polynomials over profinite groups are also considered. Results in this part can be viewed as generalizations of results in Chapters 2 and 4.

Part III presents another focus of this book—applications of algebraic dynamics developed in previous chapters. There are ten chapters in this part and every chapter contains an independent subject. The topics touched upon in the book range from information processing like cryptography and pseudo-random generators to cognitive science, psychology, etc. Although the applied areas look diverse, they have a lot of common features that can be explained and understood by means of algebraic dynamics. As indicated by the authors in the preface, p-adic ergodic theory such as that developed in Chapter 4 has important applications in computer science and automata theory. Ideas and techniques of p-adic ergodic theory have valuable applications in cryptography. Several stream ciphers and cryptographic primitives are developed with these ideas. Readers can find these interesting applications in Chapters 8 to 11. Another interesting application, as proposed by the second author of this book in [*Non-Archimedean analysis: quantum paradoxes, dynamical systems and biological models*, Kluwer Acad. Publ., Dordrecht, 1997; MR1746953 (2001h:81004)], is to apply dynamical systems in the finite ring $\mathbb{Z}_m$ for modeling of cognitive processes as in psychology. Also, dynamical systems on the projective limit structure on $\mathbb{Z}_m$ have potential applications to the study of mental information as well as to genetics. These interesting discussions can be found in Chapters 14, 15 and 16. As Part III covers a variety of subjects, readers can easily find topics or chapters that are interesting to them.

Besides number theory, the field of p-adic dynamical systems also arises from the study of p-adic theoretical physics. This book indicates more potential interactions between algebraic dynamics/p-adic dynamics and other fields of science. This is a book suitable for researchers in mathematics as well as scientists from other disciplines.

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