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Inverse Problem for Recovering of Meta-Atom Characteristics by Transmittance and Reflectance of a Metafilm

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Abstract—In this paper, an algorithm for designing of the metafilms consisting of nonidentical spherical particles with given spectral properties is proposed. A solution is constructed analytically, and the algorithm can be easily implemented in natural experiment.

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INTRODUCTION

Metamaterials are periodic composite materials that consist of subwavelength polarizable particles (meta-atoms) with a selective frequency resonance response to electric and magnetic fields.

Currently the design of two-dimensional metamaterials (metafilms) is among the most actual problems of applied optics. Many works using numerical algorithms [1–3] are dedicated to the calculation of their electrodynamic characteristics (i.e., to the solution of the direct problem). As an alternative to numerical modeling the analytical approach was proposed in [4–6] in which coefficients of transmission T and reflection R of a metafilm are calculated via known surface susceptibility density matrix $\|\chi_s\|$. This approach is much less labor-consuming.

Existing algorithms for the metafilms design with given electrodynamic characteristics in a wide range of angles of incidence θ can be realized by the multiple solution of the direct problem with directional change of geometric parameters. Such repeated calculations are extremely labor-consuming, therefore the creation of more effective algorithm for solving the inverse problem is an actual task.

ALGORITHM FOR THE DIRECT PROBLEM

In this work, we use an analytical approach consisting of three stages. In the first stage, the tensors for the electric $\hat{\alpha}_e$ and magnetic $\hat{\alpha}_m$ polarizabilities of individual meta-atoms are determined. Due to the spherical symmetry of the particles, their main components are equal and can be calculated as

$$\alpha_e = \frac{6\pi i}{k_0^3} a_1; \quad \alpha_m = \frac{6\pi i}{k_0^3} b_1, \tag{1}$$

where k_0 is the free-space wavenumber; i is an imaginary unit; and a_1 and b_1 are the first partial amplitudes of Mie's scattering theory, which are expressed by terms of Riccati–Bessel functions. Nondiagonal elements equal zero. Formulas (1) were obtained using dipole approximation and can be applied even if the meta-atoms do not meet the quasistationarity condition [9–10].

In the second stage, we consider the surface susceptibility density of the metafilm $\|\chi_s\|$. Its components are [4]

$$\chi_{es}^{xx} = \frac{N\langle\alpha_e\rangle}{1 - \dfrac{N\langle\alpha_e\rangle}{4r}}; \quad \chi_{ms}^{yy} = \frac{N\langle\alpha_m\rangle}{1 - \dfrac{N\langle\alpha_m\rangle}{4r}}, \tag{2}$$

$$\chi_{es}^{zz} = \frac{N\langle\alpha_e\rangle}{1 - \dfrac{N\langle\alpha_e\rangle}{2r}} = \frac{4r\chi_{es}^{xx}}{4r - \chi_{es}^{xx}}. \tag{3}$$

In (2)–(3), angle brackets $\langle\,\rangle$ mean the averaging over the metafilm surface; $N = 1/l^2$ is the concentration of particles; l is the period of the square lattice in whose sites the meta-atoms are located; and $r \approx 0.6956l$ is the so-called impact domain radius [11].

Calculations of transmission T and reflection R coefficients for the metafilm are based on the fields joining above and below it, with the effective boundary conditions taking into account [12]. In the case of oblique incidence (TM-polarized wave) on the metafilm consisting of spherical particles (see Fig. 1), they are as follows [13]

$$T(\theta) = \frac{1 + e(\theta)m(\theta)}{1 - e(\theta)m(\theta) - e(\theta) + m(\theta)};$$
$$R(\theta) = \frac{e(\theta) + m(\theta)}{1 - e(\theta)m(\theta) - e(\theta) + m(\theta)}. \tag{4}$$

Here the relations were used

$$e(\theta) = -i\frac{k_0\cos\theta}{2}\chi_{es}^{xx};$$
$$m(\theta) = i\frac{k_0}{2\cos\theta}\left(\chi_{ms}^{yy} + \chi_{es}^{zz}\sin^2\theta\right). \quad (5)$$

The time dependence is $e^{i\omega t}$, where ω is the angular frequency of the incident wave.

ALGORITHM THE INVERSE PROBLEM

Meta-atom properties can be reconstructed through known values of the transmission and reflection coefficients of a metafilm. By carring out a sufficient number of measurements at several fixed values of θ (e.g., one at normal incidence and one at oblique incidence), we obtain a system of equations for elements of the matrix $\|\chi_s\|$. In the case of normal incidence ($\theta = \theta_0 \equiv 0$) this system can be reversed [14]:

$$e(0) = \frac{R(0)+T(0)-1}{R(0)+T(0)+1};\quad m(0) = -\frac{R(0)-T(0)+1}{R(0)-T(0)-1}. \quad (6)$$

Thus, two components of surface susceptibility density have been found out

$$\chi_{es}^{xx} = \frac{2i}{k_0}e(0);\quad \chi_{ms}^{yy} = \frac{2i}{k_0}m(0), \quad (7)$$

the remaining component is found by solving Eqs. (4) for oblique incidence angle θ_1

$$\chi_{es}^{zz} = -\frac{\chi_{ms}^{yy}}{\sin^2\theta_1} + \frac{2i}{k_0}\frac{e(\theta_1)\cos\theta_1}{\sin^2\theta_1}. \quad (8)$$

However, for spherical particles electrical surface susceptibility densities χ_{es}^{xx} and χ_{es}^{zz} are interconnected (3), it is sufficient to use only a single pair of coefficients, one incidence angle θ_0.

It should be noted that for this algorithm it is not important which method (numerical or experimental) is used to determine $T(\theta_i)$ and $R(\theta_i)$, where $i = 0.1$. However, at normal incidence it is difficult to separate the incident and reflected components in the experiment. It may be more beneficial to carry out the measurements for χ_{ee}^{xx} and χ_{mm}^{yy} at oblique incidence with two different nonzero angles[15].

The elements of $\|\chi_s\|$ matrix are constant values for specific configuration of metafilm structure. Once χ_{es}^{xx}, χ_{ms}^{yy}, and χ_{es}^{zz} are found out, they are substituted into Eq. (4), and the values of $T(\theta)$ and $R(\theta)$ are calculated, but now for arbitrary incidence angle θ.

If we need to obtain electrodynamic characteristics for a structure composed of the same resonators but located with another period, at first we will calculate its polarizability. By using (6)–(7) and Eq. (2), we easily obtain

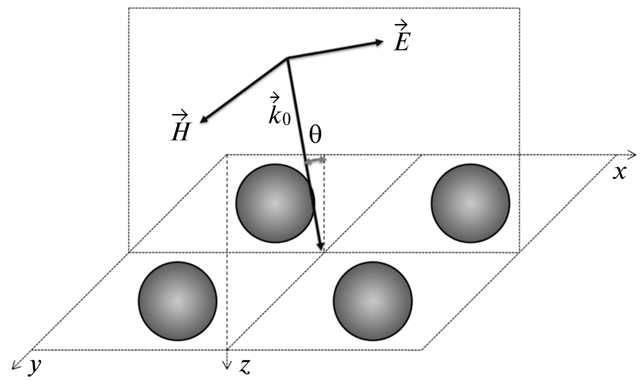

Fig. 1. Oblique incidence of TM-polarized wave on a metafilm composed of spherical particles.

$$\langle\alpha_e\rangle = \frac{4r(R+T-1)}{N(R+T-1) - 2ik_0rN(R+T+1)};$$
$$\langle\alpha_m\rangle = \frac{4r(R-T+1)}{N(R-T+1) - 2ik_0rN(R-T-1)}. \quad (9)$$

The averaging in (9) takes the meta-atoms size distribution into account that is induced by technological manufacturing process, and small variations of dielectric permeability of material. Nevertheless, the model will be correct, if the deviations from the average value are not too great.

Now, substituting $\langle\alpha_e\rangle$ and $\langle\alpha_m\rangle$ into (2)–(3) and using the resulting values of χ_{es}^{xx}, χ_{ms}^{yy} and χ_{es}^{zz} we calculate spectra $T(\theta, l)$ and $R(\theta, l)$ for arbitrary incidence angle θ and period l (4). In this manner the particular inverse problem when the incident, transmitted, and reflected waves have the same polarization, is solved. The relations (9) are exactly the same for a TE-polarized wave.

When polarizability tensors $\hat{\alpha}_e$ and $\hat{\alpha}_m$ are in a general form and there is a rotation of polarization plane for the interacting with the metafilm waves, for obtaining components via known spectra of transmission and reflection it is possible to use the system of the equations derived from [16]. That system is more complex than (4), and more pairs of T and R are required to solve it; however, it is also linear.

CONCLUSIONS

An algorithm that can be used for the design of metafilms with given electrodynamic characteristics (from spherical resonators) is proposed. It takes into account the meta-atom nonidentty (insignificant variations in geometric and material parameters). Application of analytical formulas for determination of polarizability matrix components for constituent particles via transmittance and reflectance spectra allows to sufficiently increase the productivity of numerical calculations of metafilm properties. The algorithm can be generalized for the case of more complicated geometry of particles, for instance, bianisotropic U-shaped resonators.

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