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NUCLEI
Experiment

Photoneutron Reaction Cross Sections for ^{90}Zr in Different Experiments

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Abstract—Reliability of the data on partial photoneutron reactions on ^{90}Zr obtained in the experiment carried out on the beam of bremsstrahlung was investigated using the experimental-theoretical method for partial reaction cross section evaluation based on objective physical criteria. It is found out that (γ, $1n$) and (γ, $2n$) reaction cross sections obtained using the corrections calculated via statistical theory to the neutron yield cross section $\sigma(\gamma, xn) = \sigma(\gamma, 1n) + 2\sigma(\gamma, 2n)$ satisfy physical criteria of data reliability. The integrated characteristics of the cross sections of the reactions (γ, $1n$) and (γ, $2n$) in which the distinct structural features were obtained, and the experimentally measured neutron yield cross section $\sigma(\gamma, xn)$ agree with those of evaluated cross sections. This shows that information on ^{90}Zr partial reactions competition obtained using statistical theory satisfies physical criteria of data reliability. The evaluated reaction cross sections are compared in detail with analogous data obtained before using the results of experiments carried out on the beams of quasimonoenergetic annihilation photons.

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1. INTRODUCTION

The absolute majority of cross sections of photonuclear reactions were obtained using beams of bremsstrahlung and quasimonoenergy photons originating from the annihilation in flight of relativistic positrons [1–3]. In the experiments in beams of bremsstrahlung of betatrons or microtrons, due to the continuous shape of bremsstrahlung spectrum $W(E^{\mathrm{M}}, E)$, we can directly measure the yield of the reaction:

$$Y(E^{\mathrm{M}}) = \frac{N(E^{\mathrm{M}})}{\varepsilon D(E^{\mathrm{M}})} = \alpha \int_{E_{thr}}^{E^{\mathrm{M}}} W(E^{\mathrm{M}}, E)\sigma(E)dE, \quad (1)$$

where $\sigma(E)$ is the sought cross section at a photon energy E, E_{thr} is the energy threshold of reaction, $W(E^{\mathrm{M}}, E)$ is the bremsstrahlung spectrum with an upper boundary of E^{M}, $N(E^{\mathrm{M}})$ is the number of events in the reaction, $D(E^{\mathrm{M}})$ is the dose of γ radiation, ϵ is the detector efficiency, and α is the normalizing constant [4]. The cross section of the reaction $\sigma(E)$ is determined by solving an inverse problem (1) of its development from the experimental yield $Y(E^{\mathrm{M}})$ by means of one of the dedicated methods (for instance, the Penfold–Leiss method, the least structure method, the Tikhonov regularization method, and the reduction method). In the energy range of incoming photons, in which the partial reactions (γ, $1n$), (γ, $2n$), and (γ, $3n$) compete, we determine just the cross section of neutron yield:

$$\sigma(\gamma, xn) = \sigma(\gamma, 1n) + 2\sigma(\gamma, 2n) + 3\sigma(\gamma, 3n), \quad (2)$$

which is updated by corrections calculated by the statistical theory [5], and determine the cross section $\sigma(\gamma, 2n)$ and, then, using it, the cross section of the total photoneutron reaction

$$\sigma(\gamma, sn) = \sigma(\gamma, 1n) + \sigma(\gamma, 2n) + \sigma(\gamma, 3n) \quad (3)$$

and the cross section of the (γ, $1n$) reaction.

In the experiments on beams of annihilation photons, between their pulses from the linear accelerator of electrons using special slowing-down (in a special manner capturing the neutrons from the reaction that are decelerated to the thermal energy) 4π-detectors, we directly determine the cross sections of partial reactions (γ, $1n$), (γ, $2n$), and (γ, $3n$). Because annihilation photons are accompanied by photons of their bremsstrahlung, the contribution of them to the

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cross section is eliminated by means of the difference procedure

$$Y(E^{\mathrm{M}}) = Y_{e^+}(E^{\mathrm{M}}) - Y_{e^-}(E^{\mathrm{M}}) \approx \sigma(E^{\mathrm{M}}), \quad (4)$$

due to which the experiment on direct measurement of cross sections of each of the partial reactions is carried out in three stages. Firstly, we measure yields (1) of each reaction on the positron beam $Y_{e^+}(E^{\mathrm{M}})$; then, we measure their yields $Y_{e^-}(E^{\mathrm{M}})$ (1) on the electron beam; at the third stage, under the assumption that the photon spectra of bremsstrahlung of positrons and electrons are identical, the cross sections of each partial reaction $\sigma(E^{\mathrm{M}})$ are determined as differences (4). At each stage of the experiment, we record events with one, two, and three neutrons and use the statistical analysis to get the cross sections of the $(\gamma,\, 1n)$, $(\gamma,\, 2n)$, and $(\gamma,\, 3n)$ reactions. Using these data, we simply sum to determine the cross sections of the total photoneutron reaction (3) and of the neutron yield reaction (2).

Due to cardinally different approaches to obtain information about the cross section of the reaction, the cross sections determined in experiments of different types are substantially different both in their shapes and in their absolute values [4, 6–11]. It was shown that different shapes of the cross sections determined in the experiments on bremsstrahlung beams and beams of annihilation photons are caused by different achieved effective energy resolutions [9–11]. The methods for solving the inverse problem (1) used to determine the cross sections of the reaction $\sigma(E)$ on the bremsstrahlung beam actually take into account the shape of the continuous spectrum of photons, despite all disadvantages of this approach. In addition to that, characteristic kinks in the energy dependence of the yield of the reaction $Y(E^{\mathrm{M}})$ experimentally determined with a very high statistical precision certainly testify that the cross section of the reaction has structural peculiarities [9].

At the same time, result (4) of the experiment on the beam of annihilation photons $\sigma(E^{\mathrm{M}})$ is, in its essence, not the cross section, but again just its yield: at each value of the photon energy, the determined difference $Y_{e^+}(E^{\mathrm{M}}) - Y_{e^-}(E^{\mathrm{M}})$ has contributions from all photons whose energy is higher than the threshold of the studied reaction. From the point of view of information about the cross section, both discussed experimental yields have a bad energy resolution, and, hence, their difference cannot have a well resolution. The contribution of the bremsstrahlung of positrons is not completely eliminated, and, therefore, the cross section determined in such experiment is strongly smoothed compared to the corresponding cross section determined in an experiment on the beam of bremsstrahlung [9]. As a result, almost all experimental cross sections obtained on beams of annihilation photons [1–3] have the form of one smooth resonance (two resonances in the case of deformed nuclei), unlike the cross sections obtained on beams of bremsstrahlung, in which, as a rule, we observe a large number of strongly pronounced resonances of the gross, intermediate, or fine structure.

Certainty of appearance (or, vice versa, absence) of structural peculiarities in the cross sections of reactions is very important to understand the nature of highly excited nuclear states. Multiple theoretical calculations performed in different models predict the presence of many structural peculiarities of different origin in the energy range of giant dipole resonance (GDR) [9]. The absence of such peculiarities in the cross sections of the reaction determined by the direct method in the experiments on beams of annihilation photons was and is a serious problem of describing the electromagnetic interactions of nuclei in the GDR range.

Significant discrepancies of the reaction cross sections from different experiments by the absolute value, which are more serious from the point of view of fundamental studies and various practical applications of photonuclear data, are caused by certain disadvantages of the methods for determining the multiplicity of neutrons used in both types of experiments [6–9]. It was established that both the method for separating photoneutrons by multiplicity in the experiments on beams of annihilation photons and the method for introducing corrections by the statistical theory to the cross section of neutron yield $\sigma(\gamma,\, xn)$ in experiments on beams of bremsstrahlung lead to appearance in their results of specific, frequently very substantial, systematic uncertainties, which makes the determined reaction cross sections considerably indistinguishable and violating the objective physical criteria of data integrity. In the experiments on beams of annihilation photons, it is caused by the following: when the method of neutron separation by multiplicity based on experimental data about their energies is used, formation of neutrons with close energies in reactions of different multiplicity is possible, which leads to ambiguous identification of their belonging to a particular partial reaction. In the experiments on beams of bremsstrahlung, uncertainties of such distribution can be caused by the peculiarities of the method for introducing the corrections by the statistical theory to the yield cross section $\sigma(\gamma,\, xn)$. The statistical evaporation model satisfactorily describes the processes of neutron emission from the compound nucleus almost only to incident photon energies of ∼10–15 MeV. It was demonstrated that at higher incident photon energies, at which partial reactions compete, the precision of statistical corrections by multiplicity decreases, because the processes of preequilibrium decay of the composite system, as well as the contributions of clearly nonstatistical origin, begin to make increasingly bigger difference [12–16]. It was established that in the cases of relatively light nuclei the existing systematic uncertainties of the process of distinguishing the $(\gamma, 1n)$ and $(\gamma, 2n)$ reactions are predominately associated with ignoring of contributions of the $(\gamma, 1n1p)$ reaction. The energy position and the absolute value of the cross sections of such two-nucleon reaction for the indicated nuclei, according to the calculations within CMPNR, are very close to the corresponding cross section parameters of the two-nucleon $(\gamma, 2n)$ reaction. A source of specific systematic uncertainties in this situation is the circumstance that neutrons with close energies in the $(\gamma, 1n1p)$ reaction have a multiplicity of 1, whereas those in the $(\gamma, 2n)$ reaction have a multiplicity of 2.

Concerning the discussed problems of photonuclear experiments of different types, of great interest is the problem of confidence in the shape and absolute value of the results of various studies of the photofission of ^{90}Zr nucleus to which the current work is devoted. This interest is caused by the facts that (i) this nucleus is magic by its neutron number (it has a completed neutron shell), which substantially simplifies the corresponding theoretical calculations, and (ii) for this nucleus there are the results of comprehensive experimental investigations performed on beams of both annihilation photons [17, 18] and bremsstrahlung [19–21]. In addition to that, for ^{90}Zr nucleus, using the experimental data on beams of annihilation photons, researchers estimated the cross sections of different photoneutron reactions satisfying the objective physical criteria of significance [22].

It is worth noting that the result of experiment [21] on the beam of bremsstrahlung, obtained with a step of 0.5 MeV by the incident photon energy in the range up to $E_\gamma = 22$ MeV, is in essence the cross section of the reaction $\sigma(\gamma, 1n)$. The threshold of the $^{90}\mathrm{Zr}(\gamma, 1n1p)^{88}\mathrm{Y}$ reaction is $B1n1p = 19.8$ MeV. The results of study [23], performed using the activation technique, indicate that in the photon energy range up to $\sim$30 MeV the cross section $\sigma(\gamma, 1n1p)$ has a value on the order of units of microbarns. According to the calculation results within the combined model of photonuclear reactions [24, 25], the amplitude of the cross section of the $(\gamma, 1n1p)$ reaction is by an order of magnitude less than the amplitude of the cross section of the $(\gamma, 2n)$ reaction, and the maximum of the cross section of the $(\gamma, 1n1p)$ reaction is placed in the energy axis by 4 MeV higher than the maximum of the cross section of the $(\gamma, 2n)$ reaction. Thus, we can assume that the $(\gamma, 1n1p)$ reaction must not have a significant effect on the competition of the $(\gamma, 1n)$ and $(\gamma, 2n)$ reactions on ^{90}Zr nucleus, unlike the situation observed in the cases of ^{51}V [26], ^{59}Co [15], and ^{60}Ni nuclei [14].

Table 1. Integral cross sections σ^{int} of total and partial photoneutron reactions on ^{90}Zr nucleus calculated up to photon energy $E^{\mathrm{int}} = 27.0$ MeV

Reaction	Experiment	
	Livermore [17]	Saclay [18]
(γ, xn)	1098.7 ± 4.5	1308.8 ± 3.2
(γ, sn)	1029.7 ± 5.1	1259.5 ± 3.2
$(\gamma, 1n)$	960.9 ± 4.4	1210.3 ± 3.0
$(\gamma, 2n)$	68.8 ± 2.5	49.2 ± 1.1

2. EXPERIMENTAL CROSS SECTIONS OF PHOTONEUTRON REACTIONS ON ^{90}Zr NUCLEUS

2.1. Results of Experiments on Beams of Annihilation Photons

Using the difference scheme (4), the experiments on determining the cross sections $\sigma(E^{\mathrm{M}})$ of partial $(\gamma, 1n)$ and $(\gamma, 2n)$ reactions on ^{90}Zr nucleus were carried out almost identical accelerator facilities of obtaining annihilation photons in Saclay (France) [17] and Livermore (United States) [18]. In both laboratories researchers used the same method for distinguishing the photoneutrons from different reactions by their energy based on the assumption that the only neutron from the $(\gamma, 1n)$ reaction has an energy considerably higher than the energy of each neutron from the $(\gamma, 2n)$ reaction. The neutron energy from the reaction was determined by the time of their deceleration to thermal energy in a special neutron moderator. However, the detection systems of neutrons from the reactions intended to measure their energies were substantially different. In Livermore, researchers used a system with a large number of gas discharge $^{10}\mathrm{BF}_3$ counters joined in several concentric rings of different diameter placed in paraffin. In Saclay, both the detector and the moderator were made as a liquid scintillator enriched in ^{160}Gd nuclei having a large cross section of radiation capture of thermal neutrons.

It was shown [6, 7, 9–16, 22] that the differences in the neutron detection systems are one of the causes of the fact that the cross sections of the partial $(\gamma, 1n)$ and $(\gamma, 2n)$ reactions obtained in both laboratories for the same nuclei (^{51}V, ^{75}As, ^{89}Y, ^{90}Zr, ^{115}In, $^{116-118,120,124}$Sn, ^{127}I, ^{133}Cs, ^{159}Tb, ^{165}Ho, ^{181}Ta, ^{197}Au, ^{208}Pb, ^{232}Th, and ^{238}U) substantially

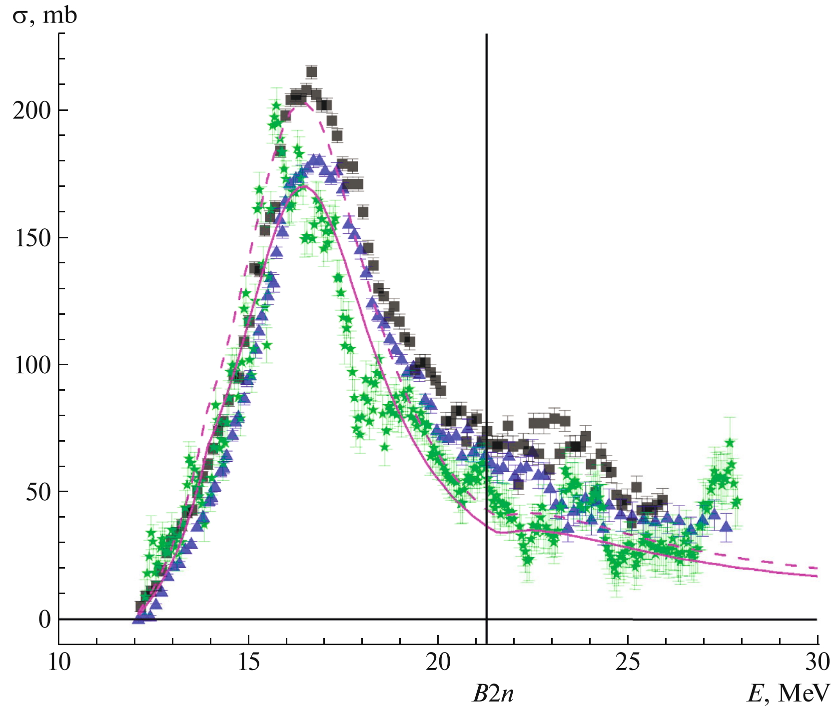

Fig. 1. Comparison of experimental cross sections of neutron yield ^{90}Zr(γ, xn) ([17], squares; [18], triangles; and [19], asterisks) against the cross section theoretically calculated within the combined model of photonuclear reactions (CMPNR) ([24, 25], before (the dashed curve) and after (the solid curve) correction (see below)).

(up to 100%) differ in the absolute values. These discrepancies of data certainly are systematic: the cross sections of the (γ, 1n) reaction have larger values in Saclay, whereas the (γ, 2n) reactions have larger values, on the contrary, in Livermore. In the discussed case of ^{90}Zr nucleus, the ratios of integral cross sections of partial reactions obtained in Saclay and Livermore are, respectively, $R^{\mathrm{int}}_{S/L}(1n) = 1.26$ and $R^{\mathrm{int}}_{S/L}(2n) = 0.73$. It was shown [6, 7, 9–16, 22] that such discrepancies are caused by systematic uncertainties of the used method for determining the multiplicity of detected neutrons and their belonging to a particular partial reaction, as well as, by a considerable dependence of the neutron detection efficiency on their energy. As a consequence of manifestations of the indicated disadvantages, the cross sections of the (γ, 1n) and (γ, 2n) reactions, as their sums ((2), (3)), differ significantly.

The discrepancies in the data for the cross section of neutron yield $\sigma(\gamma,\ xn)$ are illustrated in Fig. 1. Table 1 lists the corresponding integral cross sections of the (γ, xn) reaction, as well as those for (γ, sn), (γ, 1n), and (γ, 2n), calculated up to an incident photon energy of $E^{\mathrm{int}} = 27.0$ MeV at performing a comprehensive comparative analysis [22]. We clearly see the above mentioned differently directed discrepancies in the cross sections of the (γ, 1n) and (γ, 2n) reactions: $\sigma^{\mathrm{int}}_S(\gamma, 1n) > \sigma^{\mathrm{int}}_L(\gamma, 1n)$: 1210.3 and 960.9 mb, respectively, while $\sigma^{\mathrm{int}}_S(\gamma, 2n) < \sigma^{\mathrm{int}}_L(\gamma, 2n)$: 49.2 and 68.8 mb. Obviously, such discrepancies cannot be eliminated by traditional simple renormalization: a reduction in discrepancies of the cross sections of the (γ, 1n) reaction would be accompanied by an increase in discrepancies of the cross sections of the (γ, 2n) reaction.

2.2. Results of Experiments on Beams of Bremsstrahlung

In Fig. 1, along with the experimental data on beams of annihilation photons [17, 18] and the calculation results carried out in CMPNR [24, 25], we also present the cross section of neutron yield $\sigma(\gamma,$ $xn)$ obtained for ^{90}Zr nucleus in the experiment on the beam of bremsstrahlung [19]. The energy dependence $Y(E^{\mathrm{M}})$ of the yield of reaction (1) was measured with a step of 50 keV using $^{10}BF_3$-counters placed in the paraffin moderator. We used a multichannel method of measuring 30 statistically independent energy dependences of the neutron yield, which allowed finally reaching a high statistical precision of 0.25% at an incident photon energy of $E_\gamma = 20$ MeV and 0.10% at $E_\gamma = 28$ MeV. We obtained the cross section of the $\sigma(\gamma, xn) = \sigma(E)$ reaction from its yield $Y(E^{\mathrm{M}})$ as a solution to the inverse problem (1) by developing the cross section using the Penfold–Leiss method with a variable step of processing.

We introduced corrections on the multiplicity of photoneutrons computed by the statistical theory into the experimental cross section of the neutron yield $\sigma(\gamma,\ xn)$ in the incident photon range of energies E exceeding the reaction threshold $(\gamma,\ 2n)$ $B2n =$ 21.3 MeV and got the cross section of the total photoneutron reaction (3).

Table 2 provides a comparison of values of integral yield cross sections $\sigma^{\text{int}}(\gamma,\ xn)$ calculated for the discussed experimental [17–19] and theoretical [24, 25] cross sections. The data testify to the following:

—The best fit to the theoretically calculated cross section (978.55 MeV mb [24, 25], the dashed curve in Fig. 1) is the Saclay cross section (1025.29 MeV mb [17]), which was used earlier to perform estimation using physical criteria of significance [22].

—The cross section discussed in this work (820.29 MeV mb [19]) differs from the theoretically calculated one noticeably larger than the Livermore cross section (868.91 MeV mb [18]).

The causes of observed discrepancies in the absolute value of the yield cross sections $\sigma(\gamma,\ xn)$, obtained in experiments on the beams of annihilation photons and bremsstrahlung are not fully clear in general.

The cross section of neutron yield $\sigma(\gamma, xn)$ (2) was determined (but not reported) also in a similar experiment [20], performed on the beam of bremsstrahlung. Eight independent dependences of the neutron yield $Y(E^{\text{M}})$ measured both in experiment [19], using $^{10}BF_3$-counters placed in paraffin moderator, allowed reaching the statistical precision of 0.1% in the final result of the experiment [20]. In the same manner as in experiment [19], the cross section $\sigma(\gamma,\ xn)$ was obtained using the Penfold–Leiss method with a variable step of processing, but with a larger step in the incident photon energy (0.1 MeV). By the results of the conducted experiment [20], researchers published only the cross section of the total photoneutron $\sigma(\gamma,\ sn)$ reaction (3), obtained, as in experiment [19], from the cross section of neutron yield by introducing the corrections on the multiplicity of neutrons calculated by the statistical theory.

2.3. Structural Peculiarities of Cross Section of Total Photoneutron Reaction

Figure 2 shows both cross section of the total photoneutron $\sigma(\gamma,\ sn)$ reaction obtained on beams of bremsstrahlung [19, 20] in comparison to the cross sections obtained on beams of annihilation photons [17, 18]. In this figure, for each cross section we specify the energy positions of structural peculiarities (sometimes relatively weakly pronounced), which are discussed in each of works [17–20].

The data provided in Fig. 2 testify to the following:

—A large number of structural peculiarities of the cross section of the total photoneutron reaction were observed and discussed in each of the discussed experiments.

—The energy positions of the majority of the discussed structural peculiarities in the cross sections obtained in different experiments agree well with each other.

—The number of sufficiently pronounced structural peculiarities in the experiments on beams of bremsstrahlung exceeds the number of such peculiarities in the experiments on beams of annihilation photons.

—The structural peculiarities of the cross sections are substantially stronger pronounced in experiments on beams of bremsstrahlung, compared to the experiments on beams of annihilation photons, as a result of which many discussed structural peculiarities just slightly take their shape.

It is noted [19] that in the experimental cross sections there are 11 detected clearly exhibiting narrow resonances in the photon energy range from 12 to 17 MeV. A certain problem is the relatively weak manifestation of similar resonances in the cross section of experiment [20]. To some degree it can be caused by the fact that the energy dependence of the neutron yield $Y(E^{\text{M}})$ in experiment [19] was measured with an energy step of 50 keV, whereas in experiment [20] it was measured with an energy step of 100 keV. In addition to that, in experiment [19] the final result was obtained as a result of processing 30 independent dependences $Y(E^{\text{M}})$, whereas in experiment [20] it was obtained using just 8 dependences. Thus, a possible explanation of discrepancies in manifestation of structural peculiarities in the energy range 12–17 MeV can be the differences in the energy step and achieved statistical precision. In the energy range above 17 MeV, in which, apparently, the structural peculiarities have a larger width and are located at a larger distance from each other, the specified circumstances play a smaller role, and both cross sections [19, 20] agree well with each other.

All the above said confirms the conclusions of the studies performed earlier [9–11] that the discrepancy of the experimental results on beams of bremsstrahlung and quasimonoenergy annihilation photons is caused by the difference in the achieved efficient energy resolution.

Table 2. Centers of mass $E^{\mathrm{c.m}}$ and integral cross sections σ^{int} calculated by experimental [17–19] and theoretical [12, 24, 25] (before and after correction) cross sections of ^{90}Zr(γ, xn) reaction

Energy range	$E^{\mathrm{int}} = B2n = 21.29$ MeV		$E^{\mathrm{int}} = 26.00$ MeV	
	$E^{\mathrm{c.m}}$, MeV	σ^{int}, MeV mb	$E^{\mathrm{c.m}}$, MeV	σ^{int}, MeV mb
Experiment [17]	17.15 ± 0.15	1025.29 ± 2.11	18.51 ± 0.21	1307.80 ± 3.15
Experiment [18]	17.26 ± 0.17	868.91 ± 2.0	18.55 ± 0.39	1098.65 ± 4.49
Experiment [19]	16.87 ± 0.52	820.29 ± 6.13	18.01 ± 0.86	991.33 ± 9.73
Theory [24, 25]	16.83 ± 0.83	978.55 ± 11.72	17.83 ± 0.73	1152.29 ± 11.9
Theory, corrected	16.87 ± 0.83	820.29 ± 9.82	17.87 ± 0.73	965.93 ± 9.98

3. EVALUATION OF CROSS SECTIONS OF PHOTONEUTRON REACTION ON ^{90}Zr NUCLEUS USING EXPERIMENTAL-THEORETICAL METHOD

3.1. Objective Physical Criteria of Significance for Cross Sections of Partial Photoneutron Reactions

In the study of the above mentioned significant systematic discrepancies in the absolute value of the cross sections of partial photoneutron (γ, $1n$) and (γ, $2n$) reactions obtained in different experiments [6, 7, 9–16, 22], the objective physical criteria for integrity of the data about such cross sections [27, 28] are the ratios of the cross sections of a certain partial reaction $\sigma(\gamma, in)$ to the yield cross section $\sigma(\gamma, xn)$:

$$F_i = \sigma(\gamma, in)/\sigma(\gamma, xn) = \sigma(\gamma, in)/[\sigma(\gamma, 1n) + 2\sigma(\gamma, 2n) + 3\sigma(\gamma, 3n) + \ldots]. \quad (5)$$

There exist two rigid absolute physical criteria of significance. Firstly, the ratios F_i^{exp} must be certainly positive, because all terms of ratios (5) are the cross sections having the dimension of area. Secondly, the values of the cross sections must not exceed physical upper bounds (1.00, 0.50, 0.33, respectively, for $i = 1, 2, 3, \ldots$) according to definitions (5). On the basis of comparing the cross sections evaluated using these criteria and those obtained by the methods for reliable separation of partial reactions in activation experiments [6, 7, 9–16, 22], we established the third nonstrict criterion of significance: closeness of experimental ratios F_i^{exp} to the ratios F_i^{theor} obtained by calculations within CMPNR [24, 25]. It was established [6, 7, 9–16, 22] that the experimental cross sections of the partial reactions for which the ratios F_i^{exp} do not satisfy the physical criteria contain systematic uncertainties caused by uncertain (erroneous) identification of neutron multiplicity: a share of neutrons from one partial reaction is inadequately transferred to the other. As a result, the cross section of one of them groundlessly reduces until the appearance in it of physically forbidden negative values, whereas the cross section of the other increases up to appearance in it of uncertain values F_i^{exp} exceeding the above mentioned limiting values.

We used relation (5) to analyze the certainty of the cross sections of the partial (γ, $1n$) and (γ, $2n$) reactions on ^{90}Zr nucleus obtained in experiment [19]. As we have noted above, using its results, the cross sections of neutron yield $\sigma(\gamma, xn)$ and the cross section of the total photoneutron reaction $\sigma(\gamma, sn)$ were published. In the incident photon energy range up to $B3n = 33.6$ MeV, this allows determining the unpublished cross sections of the (γ, $1n$) and (γ, $2n$) reactions needed for analysis using the natural relations:

$$\sigma(\gamma, 2n) = \sigma(\gamma, xn) - \sigma(\gamma, sn), \quad (6)$$

$$\sigma(\gamma, 1n) = \sigma(\gamma, xn) - 2\sigma(\gamma, 2n) = \sigma(\gamma, sn) - \sigma(\gamma, 2n), \quad (7)$$

the application of which to the results of the considered experiment [19] enables obtaining the ratios $F_{1,2}^{exp}$ (5), the main physical criteria of data significance.

Figure 3 shows the ratios $F_{1,2}^{exp}$ [19] obtained in this manner compared to the ratios $F_{1,2}^{exp}$ [17, 18] and $F_{1,2}^{theor}$ [24, 25]. We clearly see that the ratios $F_{1,2}^{exp}$ [19], although significantly differing from $F_{1,2}^{exp}$ [17, 18] in shape, in essence, are oscillations within uncertainties (caused by the above discussed structural peculiarities of the yield cross section) with respect to the curve of theoretically computed ratios $F_{1,2}^{theor}$ [24, 25]. Agreement of the ratios $F_{1,2}^{exp}$ [19] and $F_{1,2}^{theor}$ indicates that the separation of the cross sections of the (γ, $1n$) and (γ, $2n$) reactions by means of introducing the corresponding corrections to the cross section of neutron yield in experiment [19] was carried out fully credibly.

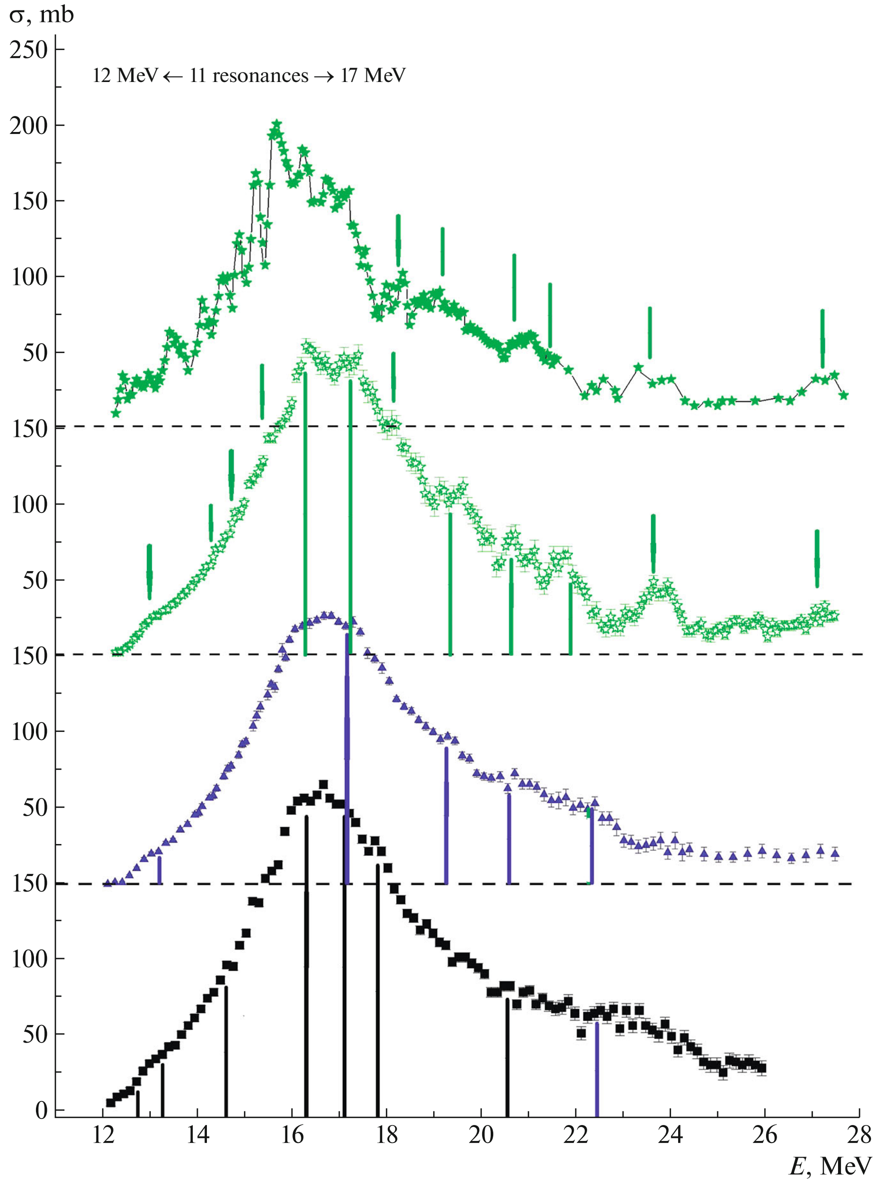

Fig. 2. Comparison of experimental ([17], squares; [18], triangles; and [19], solid asterisks, [20], hollow asterisks) cross sections of full photoneutron reaction ^{90}Zr(γ, sn). Vertical arrows (directed upwards) indicate the energy positions of structural peculiarities discussed in the corresponding works. The arrows directed downwards mean the positions of the appearing structural peculiarities not discussed in the works.

3.2. Reliable Cross Sections of Partial Photoneutron Reactions on ^{90}Zr Nucleus Corresponding to Physical Criteria

In order to evaluate the cross sections of partial reactions using the above described experimental-theoretical method under conditions of maximal closeness of the experimental [19] and theoretical [24, 25] cross sections, the latter (the solid curve in Fig. 1) was slightly corrected. On the basis of the data from Table 2, the theoretical cross section was shifted to higher energies by a value of 0.04 (16.87−16.83) MeV and multiplied by a coefficient 0.84 (820.29/978.55). We used the thus corrected cross section of neutron yield $\sigma(\gamma,\ xn)$ [19] to evaluate the cross sections of partial $(\gamma,\ 1n)$ and $(\gamma,\ 2n)$ reactions on ^{90}Zr nucleus using the experimental-theoretical method [26, 27], which is as follows. To determine the cross sections of partial reactions satisfying the criteria of data significance, the experimental cross section of neutron yield $\sigma(\gamma,\ xn)$ (2), almost independent of the issues of neutron separation by their multiplicity, because it contains all contributions of reactions with different multiplicity, is divided into contributions of partial reactions according to the ratios F_i^{theor} (5), which

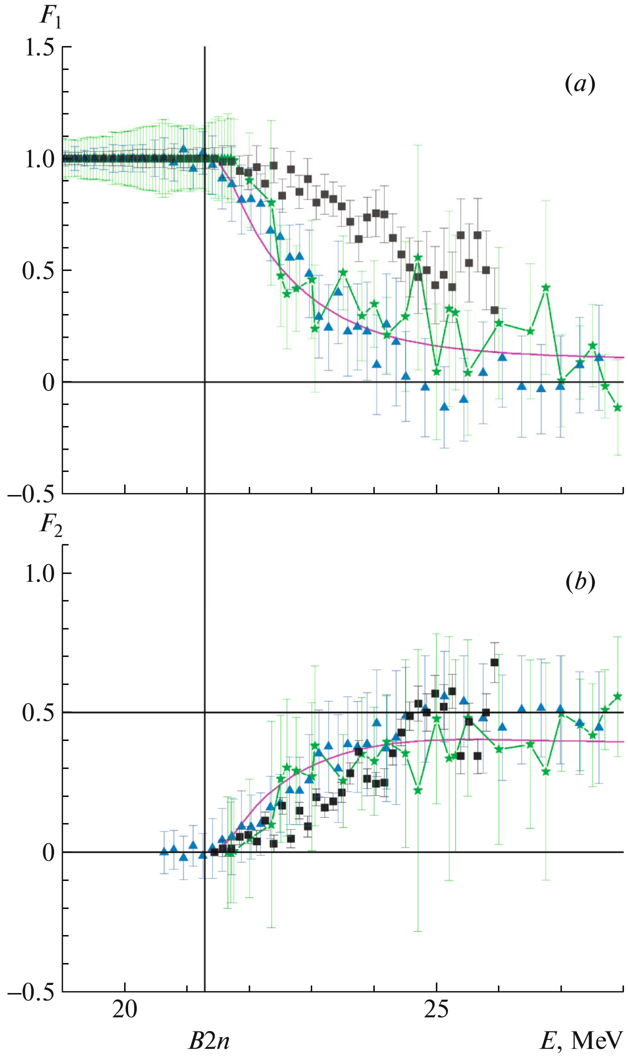

Fig. 3. Ratios (a) F_1 and (b) F_2 obtained for ^{90}Zr nucleus using experimental data ([17], squares; [18], triangles; [19], asterisks), compared to the calculation results in CMPNR ([24, 25], curves).

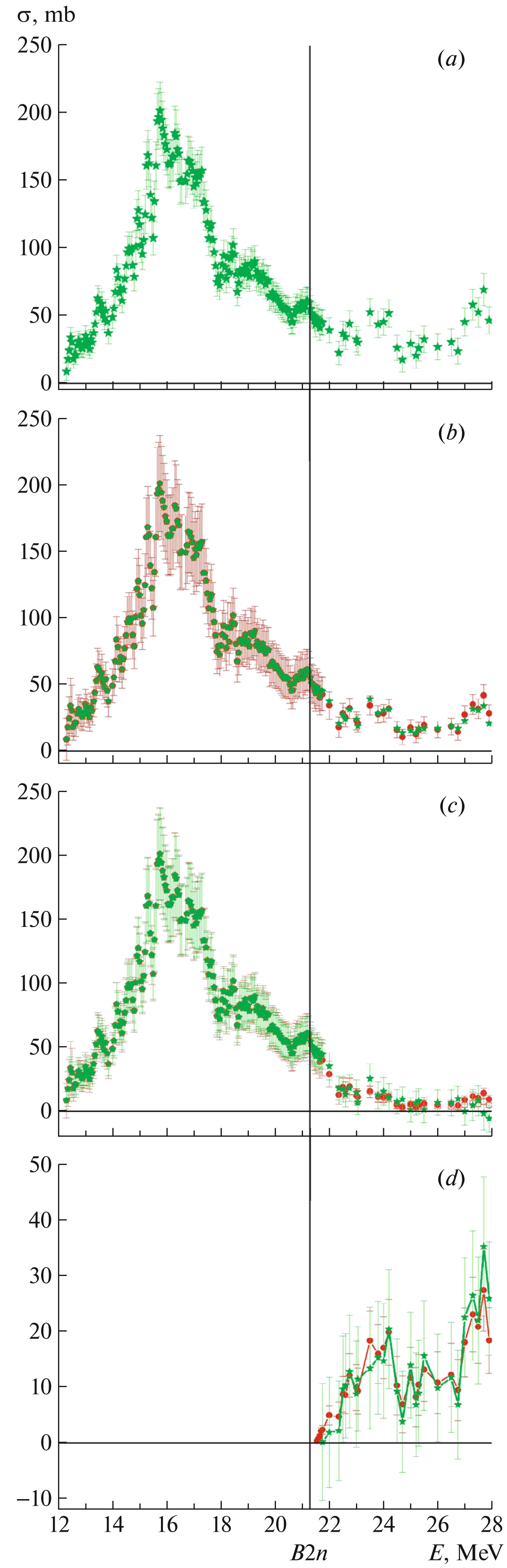

Fig. 4. Evaluated (circles) and experimental ([19], asterisks) cross sections of reactions on ^{90}Zr nucleus: (a) $\sigma(\gamma, xn)$, (b) $\sigma(\gamma, sn)$, (c) $\sigma(\gamma, 1n)$, and (d) $\sigma(\gamma, 2n)$.

also are independent of the issues of experimental neutron separation by multiplicity:

$$\sigma^{eval}(\gamma, in) = F_i^{theor} \times \sigma^{exp}(\gamma, xn). \qquad (8)$$

The cross sections of partial reactions $\sigma^{eval}(\gamma,\ in)$ evaluated in this manner are free from the discussed systematic uncertainties, because the ratios between them correspond to the ratios F_i^{theor} determined by the laws of CMPNR [24, 25] and their corresponding sum (2), the cross section of neutron yield $\sigma^{eval}(\gamma,$ $xn)$, coincides with the experimental cross section $\sigma^{exp}(\gamma,\ xn)$. Figure 4 presents the cross sections evaluated in comparison to the experimental data [19]. Additionally, for more vividness, the experimental and evaluated cross sections of the $(\gamma,\ 1n)$ and $(\gamma,\ 2n)$ reactions are presented in Fig. 5 in the incident pho-

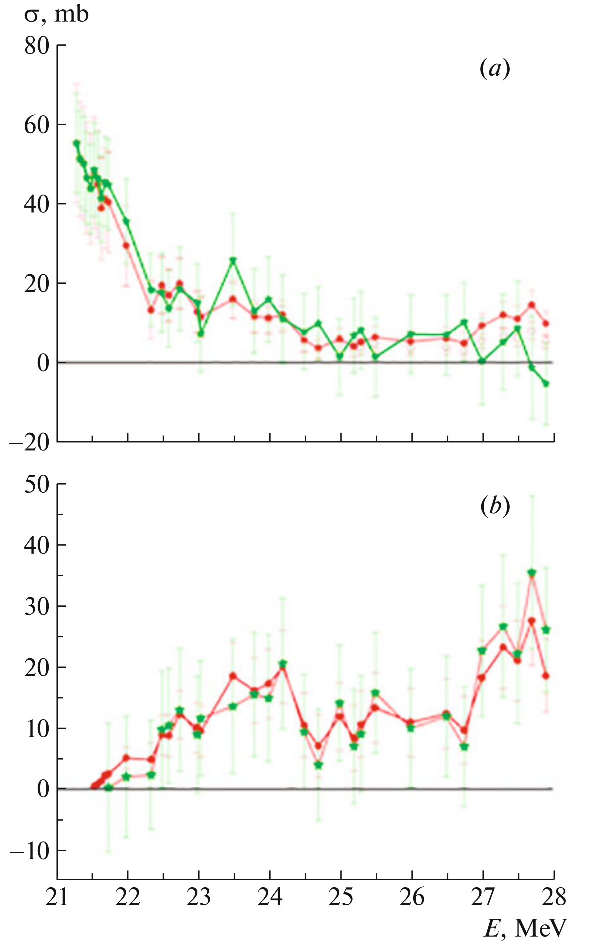

Fig. 5. Evaluated (circles) and experimental ([19], asterisks) cross sections of reactions on ^{90}Zr nucleus: (*a*) $\sigma(\gamma, 1n)$ and (*b*) $\sigma(\gamma, 2n)$ in the energy range above $B2n$.

ton energy range above $B2n$. Table 3 shows the corresponding data for the integral cross sections of the experimental and evaluated cross sections of the reactions. The data given in Figs. 4 and 5 and in Table 3 indicate that the discussed experimental cross sections of the partial reactions determined on the beam of bremsstrahlung [19] agree with the evaluated cross sections, that is, satisfy the physical criteria of significance.

This conclusion diverges from the results of earlier similar studies in the cases of ^{51}V [26], ^{59}Co [15], and ^{60}Ni [14] nuclei, which emphasize the role of the $(\gamma, 1n1p)$ reaction in these cases of relatively light nuclei. Table 4 provides a comparison of the data about energy positions and amplitudes of the $(\gamma, 1n1p)$ and $(\gamma, 2n)$ reactions calculated in CMPNR [24, 25] for the discussed nuclei and some others. We clearly see that, although in ^{51}V, ^{59}Co, and ^{60}Ni the parameters of both reactions are very close, in the case with ^{90}Zr nucleus the maximum cross section value of the $(\gamma, 1n1p)$ reaction appears to be approximately six-fold less than the maximum cross section of the $(\gamma, 2n)$ reaction, and, moreover, it appears by 4 MeV higher along the energy axis. It should be noted that the results of calculations of the of the cross section of the ^{90}Zr$(\gamma, 1n1p)^{88}$Y reaction agree with the results of its determination in the activation experiment [23]. In the discussed incident photon energy range, the cross section of this reaction is no larger than 5 mb, whereas the cross section of the ^{90}Zr$(\gamma, 2n)^{88}$Zr reaction is approximately 20 mb. This confirms the above mentioned assumptions that the $(\gamma, 1n1p)$ reaction plays a very minor role in in the case of the discussed ^{90}Zr nucleus. Thus, without a noticeable contribution of the $(\gamma, 1n1p)$ reaction, the corrections calculated by the statistical theory allow reliably determine the cross sections of the $(\gamma, 1n)$ and $(\gamma, 2n)$ reactions. It is worth emphasizing that this conclusion also confirms by the preliminary results of studying the reliability of the cross sections of partial reactions on ^{127}I, ^{165}Ho, and ^{181}Ta nuclei [29], in the cases of which the contributions of the cross section of the $(\gamma, 1n1p)$ reaction can also be ignored, according to the results of calculations in CMPNR (Table 4).

The cross section of neutron yield $\sigma^{exp}(\gamma, xn)$ [19] used in the current work has an absolute value less (Fig. 1, Table 2) than the value of the cross section $\sigma^{exp}(\gamma, xn)$ [17] used for the estimate carried out earlier [22]. In view of this, the cross sections of the $(\gamma, 1n)$ and $(\gamma, 2n)$ reactions evaluated in the current work turn out to be considerably smaller (Table 4) than the results of the previous estimates [22]. However, we should note that the previous estimates were obtained using an additional normalization of the theoretical cross section of neutron yield $\sigma^{theor}(\gamma, xn)$ [24, 25] to the experimental cross section $\sigma^{exp}(\gamma, xn)$ [17], which was a small increase in the theoretical cross section (multiplication by a factor of 1.04), whereas the current estimates were obtained by multiplying the theoretical cross section $\sigma^{theor}(\gamma, xn)$ by a factor of 0.84. Thus, the previous and new estimates may be qualitatively compared using multiplication of new estimates by a factor of $1.24 = 1.04 \times 1.0/0.84$. Table 5 shows the integral cross sections of the $(\gamma, 1n)$ and $(\gamma, 2n)$ reactions obtained in this manner and corresponding to the closeness of the cross sections of neutron yield $\sigma^{exp}(\gamma, xn)$ [19] and $\sigma^{exp}(\gamma, xn)$ [17]. These data indicates that, despite obvious discrepancies in the shape of the cross sections $\sigma^{exp}(\gamma, 1n)$ [19] and $\sigma^{exp}(\gamma, 1n)$ [17], as well as $\sigma^{exp}(\gamma, 2n)$ [19] and $\sigma^{exp}(\gamma, 2n)$ [17], the corresponding integral cross section are very close to each other. This means that the experimental-theoretical method for evaluating the cross sections of partial reactions by the data about the yield cross sections $\sigma^{exp}(\gamma, xn)$ obtained

Table 3. Integral cross sections $\sigma^{\rm int}$ (in MeV mb) calculated for evaluated cross sections of total and partial photoneutron reactions on ^{90}Zr isotope in comparison to experimental data [19]

Reaction	Evaluated data	Experiment [19]
$E^{\rm int} = B2n = 21.3$ MeV		
(γ, xn)	820.29 ± 10.76	820.29 ± 9.4
(γ, sn)	820.29 ± 10.76	820.29 ± 9.4
$(\gamma, 1n)$	820.29 ± 10.76	820.29 ± 9.4
$(\gamma, 2n)$	0	0
$E^{\rm int} = 27.0$ MeV		
(γ, xn)	1021.26 (13.32)	1021.26 (10.26)
(γ, sn)	960.65 (12.53)	965.07 (13.16)
$(\gamma, 1n)$	900.03 (11.68)	908.87 (13.18)
$(\gamma, 2n)$	60.61 (4.54)	56.2 (11.96)

on beams of both quasimonoenergy annihilation photons [17] and bremsstrahlung [19] leads to the cross sections of partial reactions that have almost identical absolute values and satisfy the physical criteria of data significance. Furthermore, the discrepancies in the shape (appearance of structural peculiarities) of the cross sections of partial reactions evaluated in this work using the yield cross section $\sigma^{exp}(\gamma,\ xn)$ [19] and the cross sections evaluated earlier using $\sigma^{exp}(\gamma, xn)$ [19] are caused by differences in the energy resolution achieved in experiments [17, 19] (see Subsection 2.3).

4. CONCLUSIONS

The performed studies allow making certain conclusions about the reliability of the cross sections of the partial reactions $(\gamma,\ 1n)$ and $(\gamma,\ 2n)$ on ^{90}Zr nucleus determined in the experiment on the beam of bremsstrahlung by introducing corrections on neutron multiplicity into the cross section of neutron yield $\sigma^{exp}(\gamma,\ xn)$ [19] calculated by the statistical theory. We showed that, in the conditions of absence of a noticeable contribution of the $(\gamma,\ 1n1p)$ reaction whose description requires departure from the purely statistical description of the photofission processes of ^{90}Zr nucleus, the corrections of this type allow obtaining the experimental data on the cross sections of partial reactions satisfying the physical criteria of significance.

In the evaluated cross sections of the $(\gamma,\ 1n)$ and $(\gamma,\ 2n)$ reactions agreeing with the experimental cross sections, we observe strongly pronounced structural peculiarities corresponding to such peculiarities in the experimental cross section of neutron yield $\sigma^{exp}(\gamma,\ xn)$ [19]. In view of this, the new evaluated cross sections of the $(\gamma,\ 1n)$ and $(\gamma,\ 2n)$ reactions significantly differ in the shape from the cross sections of the reactions evaluated earlier [22] using the yield cross section $\sigma^{exp}(\gamma,\ xn)$ [17], which includes no pronounced structural peculiarities. As noted, such discrepancies in the shape are caused by the difference in the energy resolution achieved in experiment [17, 19].

Because the used cross sections of neutron yield $\sigma^{exp}(\gamma,\ xn)$ [17] and $\sigma^{exp}(\gamma,\ xn)$ [19] are rather noticeably different also in their absolute value by unclear reasons, the new evaluated cross sections of partial reactions significantly differ in the value from the cross sections evaluated earlier [22]. However, the integral characteristics of the cross sections of the $(\gamma,\ 1n)$ and $(\gamma,\ 2n)$ reactions evaluated using the cross sections of neutron yield $\sigma^{exp}(\gamma,\ xn)$ [17] and $\sigma^{exp}(\gamma, xn)$ [19] agreeing in their absolute value are close. This indicates that the experimental-theoretical method of evaluation (8) leads to close statistically significant cross sections of partial reactions on ^{90}Zr nucleus using the experimental cross

Table 4. Comparison of energy positions $E^{\max}$ (MeV) and amplitudes $\sigma^{\max}$ (mb) of maximums of cross sections of $(\gamma,\ 1n1p)$ and $(\gamma,\ 2n)$ reactions calculated in CMPNR [24, 25]

Reaction	$(\gamma,\ 1n1p)$		$(\gamma,\ 2n)$	
Nucleus	$E^{\max}$	$\sigma^{\max}$	$E^{\max}$	$\sigma^{\max}$
^{51}V	24.4	12.6	23.6	11.9
^{59}Co	21.8	19.4	23.4	15.4
^{60}Ni	31.0	7.7	24.0	7.8
^{90}Zr	28.0*	2.4*	24.0	14.2
^{127}I	23.0	4.4	18.6	72.7
^{165}Ho	21.5	9.6	16.7	145.4
^{181}Ta	34.6	1.1	16.2	193.0

* Data agreeing with results of activation experiment [23].

Table 5. Comparison of values of integral cross sections $\sigma^{\rm int}$ (in MeV mb) calculated for evaluated cross sections of reactions on ^{90}Zr isotope at close values of $E^{\rm int}$

Reaction	Data on $\sigma^{\rm int}$ [19]	Data on $\sigma^{\rm int}$ [19] $\times 1.24$	Evaluation of $\sigma^{\rm int}$ [22] by data from [17]
	$E^{\rm int} = 27.0$ MeV		$E^{\rm int} = 27.8$ MeV
(γ, xn)	1021	1266	1309
$(\gamma, 1n)$	900	1116	1158
$(\gamma, 2n)$	61	76	71

sections of neutron yield agreeing in their absolute value obtained in the experiments on the beam of annihilation photons [17] and bremsstrahlung [19].

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflict of interest.

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