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Second order edgeworth and cornish-fisher expansions for statistics based on random means

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For distribution of normalized random means based on samples with random size we give asymptotic expansions with Student or Laplace limit laws. Samples of random size and non-normal limit laws occur e.g. in insurance, economics, biology and for modeling city-size growth or high-frequency stock index returns, see [1], [2].

Let $X_1, X_2, \dots$ be i. i. d. random variables (r.v.) with $\mathbb{E}\,|X_1|^5 < \infty$, $\mathbb{E}\,(X_1) = \mu$, $0 < \mathrm{Var(X_1)} = \sigma^{-2}$, skewness $\lambda_3 = \sigma^3\,\mathbb{E}\,(X_1-\mu)^3$ and kurtosis $\lambda_4 = \sigma^4\,\mathbb{E}\,(X_1-\mu)^4$ and suppose that r.v. X_1 admits Cramér's condition: $\limsup_{|t|\to\infty} \left|\mathbb{E}e^{itX_1}\right| < 1$. We denote the mean $T_m = (X_1+\cdots+X_m)/m$, $m = 1, 2, ...$ Then one has

$$\sup_x |\mathbb{P}(\sigma\sqrt{m}(T_m-\mu)\le x) - \Phi_{m,2}(x)| \le C_1 m^{-3/2},$$

where $\Phi_{m,2}(x)$ is the second order Edgeworth expansion with normal limit law.

Consider now a random mean T_{N_n} with a random sample size $N_n = N_n(r)$ of observations $X_1, X_2, ...$ and N_n is independent of them, where $N_n(r)$ is a negative binomial distributed (shifted by 1) r.v. such that

$$\mathbb{P}(N_n(r)=i) = \Gamma(i+r-1)((i-1)!\,\Gamma(r))^{-1}\,(1/n)^r\,(1-1/n)^{i-1}, \quad r>0, \quad i,n\in\mathbb{N}:=\{1,2,...\}.$$

Let $g(n) = \mathbb{E}\big(N_n(r)\big) = r(n-1)+1$, then $\mathbb{P}(N_n(r)/\mathbb{E}(N_n(r)) \le x)$ tends to the gamma distribution $G_{r,r}(x)$ having density $g_{r,r}(x) = r^r\,x^{r-1}e^{-rx}/\Gamma(r)$ and the limit distribution of $\mathbb{P}\left(\sigma\sqrt{g(n)}(T_{N_n(r)}-\mu)\le x\right)$ is the Student t - law $S_{2r}(x)$ with density $s_{2r}(x) = \Gamma(r+1/2)(\sqrt{2r\pi}\,\Gamma(r))^{-1}\left(1+x^2/(2r)\right)^{-(r+1/2)}$, see [1] or [2] and references therein.

Lemma. *Suppose $r\ge 1$. For $x>0$ and all $n\ge 2$ there exists a real number $C_2(r)>0$ such that*

$$\sup_{x\ge 0}\left|\mathbb{P}(N_n(r)\le g(n)x) - G_{r,r}(x) + g_{r,r}(x)\big((x-1)(r-2) - 2Q_1(g(n)x)\big)/(2r(n-1))\right| \le C_2(r)\,n^{-\min\{r,2\}},$$

where $Q_1(y) = 1/2 - (y-[y])$ and $[y]$ is the integer part of y with $y-1<[y]\le y$.

Using Theorem 3.1 of [1] and the second order Edgeworth type expansions of T_m and $N_n(r)$ we get new expansion for the random mean $T_{N_n}(r)$ with $r>1$, which allows also to obtain the Cornish-Fisher expansion, see [3]. For shortage of space the results are given only for $r=2$.

Theorem. *Let $r=2$. Under the mentioned conditions there exists a constant $C>0$ such that*

$$\sup_x\left|\mathbb{P}\left(\sigma\sqrt{2n-1}(T_{N_n}-\mu)\le x\right) - S_4(x) + \left(A_1(x)\frac{1}{\sqrt{2n-1}} + A_2(x)\frac{1}{2n-1}\right)s_4(x)\right| \le C\,n^{-3/2}.$$

with $A_1(x) = \lambda_3\left(x^2-2\right)/9$ and $A_2(x) = x\left(10\lambda_3^2/(9\,(4+x^2)) - \lambda_4/6\right)$.

Let x_α and u_α be α-quantiles of $\sigma\sqrt{g(n)}\,(T_{N_n}-\mu)$ and the Student t-distribution $S_4(x)$, respectively. Then

$$x_\alpha = u_\alpha - A_1(u_\alpha)\frac{1}{\sqrt{2n-1}} + \left(\frac{s_4'(u_\alpha)}{2s_4(u_\alpha)}A_1^2(u_\alpha) + A_1'(u_\alpha)A_1(u_\alpha) - A_2(u_\alpha)\right)\frac{1}{2n-1} + \mathcal{O}\left(n^{-3/2}\right), \quad n\to\infty.$$

Similar results are obtained for the case, when the random size N_n is the maximum of n i.i.d. discrete Pareto r. v. In this case the Laplace law is the limit distribution of the normalized random mean T_{N_n}.

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Sailboat Trajectory Optimization

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We study the optimal strategy for a sailboat to reach an upwind island under the hypothesis that the wind direction fluctuates according to a Brownian motion. The problem is singular because we assume that there is no loss of time when tacking. We exhibit the optimal strategy. The proof of optimality, since the HJB equation does not admit a closed form solution, involves an intricate estimate of derivatives of the value function. Finally we explicitly provide the asymptotic behavior of the value function and we give some new insights on the stochastic flow of a reflected SDE.

Concentration inequalities for Harris recurrent Markov chains

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Concentration inequalities are a powerful tool to control the tail probability that a random variable X exceeds some prescribed value t. They are a crucial step in deriving many results in numerous fields such as statistics, learning theory, discrete mathematics, statistical mechanics, information theory or convex geometry. The purpose of this talk is to present Bernstein type inequality for unbounded classes of functions $\mathcal{F}$ and Hoeffding type functional inequality for Harris recurrent Markov chains. To avoid some complicated mixing conditions, we make use of the well-known regeneration properties of Markov chains. It is noteworthy that when deriving exponential inequalities for Markov chains (or any other process with some dependence structure) one can not expect to recover fully the classical results from the i.i.d. case. The goal is then to get some counterparts of the inequalities for i.i.d. random variables with some extra terms that appear in the bound as a consequence of a Markovian structure of the considered process. Our inequalities allow to obtain fast rates of convergence in mathematical statistics. Moreover, all constants involved in our bounds of the considered inequalities are given in an explicit form which can be advantageous in practical considerations. Firstly, we present the theory for regenerative Markov chains, next we show how to generalize these results and establish exponential bounds for Harris recurrent case.

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