Mathematical model of micropolar anisotropic (orthotropic) multilayered thin plates

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Abstract

Micropolar theory of elasticity is a structural phenomenological model of rigid deformable bodies with strongly expressed internal structure [1]. From this point of view the construction of theories of micropolar anisotropic elastic multilayered thin bars, plates and shells is actual. In paper [2] general applied theories of micropolar elastic isotropic thin shells, plates and bars are constructed on the basis of mathematically confirmed hypotheses method.

In this paper this approach is developed and on the basis of accepted hypotheses for the whole package of multilayered plate general applied theory of micropolar orthotropic elastic multilayered thin plates of symmetric structure is constructed for plane stress state and bending.

1 Problem statement

A plate of constant thickness $2h$ composed of an odd number of homogeneous mic- ropolar orthotropic layers is considered. Layers which are symmetrically located with respect to the coordinate plane $\alpha_3 = 0$ have the same thickness and physical-mechanical properties. The coordinate plane $\alpha_3 = 0$ is the middle plane of the middle layer and the whole package of multilayered plate.

We start from the basic equations of the spatial static problem of linear micro- polar theory of elasticity for orthotropic body with free fields of displacements and rotations:

Equilibrium equations [3]:

$$\frac{1}{H_1H_2}\frac{\partial}{\partial\alpha_1}\left(H_2\sigma_{11}^i\right)+\frac{1}{H_1H_2}\frac{\partial}{\partial\alpha_2}\left(H_1\sigma_{21}^i\right)+\frac{\partial\sigma_{31}^i}{\partial\alpha_3}+\frac{1}{H_1H_2}\frac{\partial H_1}{\partial\alpha_2}\sigma_{12}^i-\frac{1}{H_1H_2}\frac{\partial H_2}{\partial\alpha_1}\sigma_{22}^i=0,$$

$$\frac{1}{H_1H_2}\frac{\partial}{\partial\alpha_1}\left(H_2\sigma_{12}^i\right)+\frac{1}{H_1H_2}\frac{\partial}{\partial\alpha_2}\left(H_1\sigma_{22}^i\right)+\frac{\partial\sigma_{32}^i}{\partial\alpha_3}+\frac{1}{H_1H_2}\frac{\partial H_2}{\partial\alpha_1}\sigma_{21}^i-\frac{1}{H_1H_2}\frac{\partial H_1}{\partial\alpha_2}\sigma_{11}^i=0,$$

$$\frac{1}{H_1H_2}\frac{\partial}{\partial\alpha_1}\left(H_2\sigma_{13}^i\right)+\frac{1}{H_1H_2}\frac{\partial}{\partial\alpha_2}\left(H_1\sigma_{23}^i\right)+\frac{\partial\sigma_{33}^i}{\partial\alpha_3}=0, \tag{1}$$

$$\frac{1}{H_1H_2}\frac{\partial}{\partial\alpha_1}\left(H_2\mu_{11}^i\right)+\frac{1}{H_1H_2}\frac{\partial}{\partial\alpha_2}\left(H_1\mu_{21}^i\right)+\frac{\partial\mu_{31}^i}{\partial\alpha_3}+\frac{1}{H_1H_2}\frac{\partial H_1}{\partial\alpha_2}\mu_{12}^i-$$
$$-\frac{1}{H_1H_2}\frac{\partial H_2}{\partial\alpha_1}\mu_{22}^i+\left(\sigma_{23}^i-\sigma_{32}^i\right)=0,$$

$$\frac{1}{H_1H_2}\frac{\partial}{\partial\alpha_1}\left(H_2\mu_{12}^i\right)+\frac{1}{H_1H_2}\frac{\partial}{\partial\alpha_2}\left(H_1\mu_{22}^i\right)+\frac{\partial\mu_{32}^i}{\partial\alpha_3}+\frac{1}{H_1H_2}\frac{\partial H_2}{\partial\alpha_1}\mu_{21}^i-$$
$$-\frac{1}{H_1H_2}\frac{\partial H_1}{\partial\alpha_2}\mu_{11}^i+\left(\sigma_{31}^i-\sigma_{13}^i\right)=0,$$

$$\frac{1}{H_1H_2}\frac{\partial}{\partial\alpha_1}\left(H_2\mu_{13}^i\right)+\frac{1}{H_1H_2}\frac{\partial}{\partial\alpha_2}\left(H_1\mu_{23}^i\right)+\frac{\partial\mu_{33}^i}{\partial\alpha_3}+\sigma_{12}^i-\sigma_{21}^i=0.$$

Here H_1, H_2 are Lame's coefficients in the curvilinear orthogonal system of coordinates; $\hat{\sigma}^i, \hat{\mu}^i$ are asymmetric tensors of force and moment stresses of the i - th layer. Number of layers is equal to $2n+1$.

Physical relations [3]:

$$\gamma_{11}^i=a_{11}^i\sigma_{11}^i+a_{12}^i\sigma_{22}^i+a_{13}^i\sigma_{33}^i,\ \gamma_{22}^i=a_{12}^i\sigma_{11}^i+a_{22}^i\sigma_{22}^i+a_{23}^i\sigma_{33}^i,\ \gamma_{33}^i=a_{13}^i\sigma_{11}^i+a_{23}^i\sigma_{22}^i+a_{33}^i\sigma_{33}^i,$$

$$\gamma_{12}^i=a_{77}^i\sigma_{12}^i+a_{78}^i\sigma_{21}^i,\ a_{21}^i=a_{78}^i\sigma_{12}^i+a_{88}^i\sigma_{21}^i,\ \gamma_{13}^i=a_{56}^i\sigma_{31}^i+a_{66}^i\sigma_{13}^i,\ \gamma_{31}^i=\widetilde{a}_{55}^i\sigma_{31}^i+a_{56}^i\sigma_{13}^i,$$

$$\gamma_{23}^i=a_{44}^i\sigma_{23}^i+a_{45}^i\sigma_{32}^i,\ \ \gamma_{32}^i=a_{45}^i\sigma_{23}^i+a_{55}^i\sigma_{32}^i, \tag{2}$$

$$\chi_{11}^i=b_{11}^i\mu_{11}^i+b_{12}^i\mu_{22}^i+b_{13}^i\mu_{33}^i,\ \chi_{22}^i=b_{12}^i\mu_{11}^i+b_{22}^i\mu_{22}^i+b_{23}^i\mu_{33}^i,\ \chi_{33}^i=b_{13}^i\mu_{11}^i+b_{23}^i\mu_{22}^i+b_{33}^i\mu_{33}^i,$$

$$\chi_{12}^i=b_{77}^i\mu_{12}^i+b_{78}^i\mu_{21}^i,\ \chi_{21}^i=b_{78}^i\mu_{12}^i+b_{88}^i\mu_{21}^i,\ \chi_{13}^i=b_{56}^i\mu_{31}^i+b_{66}^i\mu_{13}^i,\ \chi_{31}^i=\widetilde{b}_{55}^i\mu_{31}^i+b_{56}^i\mu_{13}^i,$$

$$\chi_{23}^i=b_{44}^i\mu_{23}^i+b_{45}^i\mu_{32}^i,\ \ \chi_{32}^i=b_{45}^i\mu_{23}^i+b_{55}^i\mu_{32}^i.$$

Here $\hat{\gamma}^i, \hat{\chi}^i$ are asymmetric tensors of deformations and bending-torsions, $\hat{a}^i, \hat{b}^i$ are tensors of elastic constants for micropolar orthotropic material of the i - th layer.

Geometric relations [3]:

$$\gamma_{11}^i=\frac{1}{H_1}\frac{\partial V_1^i}{\partial\alpha_1}+\frac{1}{H_1H_2}\frac{\partial H_1}{\partial\alpha_2}V_2^i,\ \ \gamma_{22}^i=\frac{1}{H_2}\frac{\partial V_2^i}{\partial\alpha_2}+\frac{1}{H_1H_2}\frac{\partial H_2}{\partial\alpha_1}V_1^i,\ \ \gamma_{33}^i=\frac{\partial V_3^i}{\partial\alpha_3},$$

$$\gamma_{12}^i=\frac{1}{H_1}\frac{\partial V_2^i}{\partial\alpha_1}-\frac{1}{H_1H_2}\frac{\partial H_1}{\partial\alpha_2}V_1^i-\omega_3^i,\ \ \gamma_{21}^i=\frac{1}{H_2}\frac{\partial V_1^i}{\partial\alpha_2}-\frac{1}{H_1H_2}\frac{\partial H_2}{\partial\alpha_1}V_2^i+\omega_3^i,$$

$$\gamma_{13}^i=\frac{1}{H_1}\frac{\partial V_3^i}{\partial\alpha_1}+\omega_2^i,\ \ \ \gamma_{31}^i=\frac{\partial V_1^i}{\partial\alpha_3}-\omega_2^i,\ \ \gamma_{23}^i=\frac{1}{H_2}\frac{\partial V_3^i}{\partial\alpha_2}-\omega_1^i,\ \ \gamma_{32}^i=\frac{\partial V_2^i}{\partial\alpha_3}+\omega_1^i,$$

$$\chi_{11}^i=\frac{1}{H_1}\frac{\partial\omega_1^i}{\partial\alpha_1}+\frac{1}{H_1H_2}\frac{\partial H_1}{\partial\alpha_2}\omega_2^i,\ \ \chi_{22}^i=\frac{1}{H_2}\frac{\partial\omega_2^i}{\partial\alpha_2}+\frac{1}{H_1H_2}\frac{\partial H_2}{\partial\alpha_1}\omega_1^i,\ \ \chi_{33}^i=\frac{\partial\omega_3^i}{\partial\alpha_3}, \tag{3}$$

$$\chi_{12}^i=\frac{1}{H_1}\frac{\partial\omega_2^i}{\partial\alpha_1}-\frac{1}{H_1H_2}\frac{\partial H_1}{\partial\alpha_2}\omega_1^i,\ \ \chi_{21}^i=\frac{1}{H_2}\frac{\partial\omega_1^i}{\partial\alpha_2}-\frac{1}{H_1H_2}\frac{\partial H_2}{\partial\alpha_1}\omega_2^i,$$

$$\chi_{13}^i=\frac{1}{H_1}\frac{\partial\omega_3^i}{\partial\alpha_1},\ \ \chi_{31}^i=\frac{\partial\omega_1^i}{\partial\alpha_3},\ \ \chi_{23}^i=\frac{1}{H_2}\frac{\partial\omega_3^i}{\partial\alpha_2},\ \ \chi_{32}^i=\frac{\partial\omega_2^i}{\partial\alpha_3}.$$

It is assumed that the layers of the plate are rigidly connected with each other and work together without slide. Conditions of conjugation of layers for displacements and rotations are written as follows:

$$V_1^i=V_1^{i+1},\ \ V_2^i=V_2^{i+1},\ \ V_3^i=V_3^{i+1},\ \ \omega_1^i=\omega_1^{i+1},\ \ \omega_2^i=\omega_2^{i+1},\ \ \omega_3^i=\omega_3^{i+1}. \tag{4}$$

Conditions of the contact between the layers for force and moment stresses are written as follows:

$$\sigma_{31}^i=\sigma_{31}^{i+1},\ \ \sigma_{32}^i=\sigma_{32}^{i+1},\ \ \sigma_{33}^i=\sigma_{33}^{i+1},\ \ \mu_{31}^i=\mu_{31}^{i+1},\ \ \mu_{32}^i=\mu_{32}^{i+1},\ \ \mu_{33}^i=\mu_{33}^{i+1}. \tag{5}$$

It is assumed that the following conditions are satisfied on the planes $\alpha_3 = \pm h_n$ of the plate:

$$\sigma_{31}^{n}\big|_{\alpha_3=h_n} = q_1^+,\ \sigma_{32}^{n}\big|_{\alpha_3=h_n} = q_2^+,\ \sigma_{33}^{n}\big|_{\alpha_3=h_n} = q_3^+,\ \mu_{31}^{n}\big|_{\alpha_3=h_n} = m_1^+,\ \mu_{32}^{n}\big|_{\alpha_3=h_n} = m_2^+,$$

$$\mu_{33}^{n}\big|_{\alpha_3=h_n} = m_3^+,\ \sigma_{31}^{-n}\Big|_{\alpha_3=-h_n} = -q_1^-,\ \sigma_{32}^{-n}\Big|_{\alpha_3=-h_n} = q_2^{-n},\ \sigma_{33}^{-n}\Big|_{\alpha_3=-h_n} = -q_3^-, \tag{6}$$
$$\mu_{31}^{-n}\Big|_{\alpha_3=-h_n} = m_1^-,\quad \mu_{32}^{-n}\Big|_{\alpha_3=-h_n} = -m_2^-,\quad \mu_{33}^{-n}\Big|_{\alpha_3=-h_n} = -m_3^+.$$

Following three types of boundary conditions are considered on the surface $\sum$ of the plate: 1) Force and moment stresses are given;2) points of the surface $\sum$ are fixed; 3) three-dimensional mixed boundary conditions of hinged support are given.

It is assumed that the thickness $2h$ of the plate is small compared with typical radii of curvature of the middle plane.

2 The construction of model of micropolar orthotropic elastic multilayered plate of symmetric structure

Considering that the method of hypotheses, along with extremely visibility, very quickly and relatively simply leads to final results for engineering practice, the model of micropolar orthotropic elastic multilayered plates will be constructed on the basis of this method. Following hypotheses are formulated for the construction of the mathematical model of micropolar orthotropic elastic multilayered plates composed of an odd number of layers, which are symmetrically located with respect to the middle plane (these are generalized hypotheses of single layered micropolar isotropic plates [2]:
1. During the deformation initially straight and normal to the middle plane of the plate fibers rotate freely at an angle as a whole rigid body, without changing their length and without remaining perpendicular to the deformed middle plane.
The formulated hypothesis is mathematically written as follows:

$$V_1^i = u_1(\alpha_1,\alpha_2) + \alpha_3\Psi_1(\alpha_1,\alpha_2),\ V_2^i = u_2(\alpha_1,\alpha_2) + \alpha_3\Psi_2(\alpha_1,\alpha_2),$$
$$V_3^i = w(\alpha_1,\alpha_2). \tag{7}$$
$$\omega_1^i = \Omega_1(\alpha_1,\alpha_2),\ \omega_2^i = \Omega_2(\alpha_1,\alpha_2),\ \omega_3^i = \omega_3(\alpha_1,\alpha_2) + \alpha_3\iota(\alpha_1,\alpha_2). \tag{8}$$

Thus, tangential displacements and normal rotation are changed by a linear law along the plate thickness. Bending and tangential rotations do not depend on coordinate α_3.It should be noted that part (7) of the accepted hypothesis, in essence, is Timoshenko's hypothesis in the classical theory of shells and plates. Here, like in paper [2], hypothesis (9), (10) in full we shall call Timoshenko's generalized for micropolar case kinematic hypothesis.
2. In the generalized Hook's law (2) force stress σ_{33}^i and moment stresses μ_{31}^i, μ_{32}^i in each layer can be neglected respectively in relation to the force stresses σ_{11}^i, σ_{22}^i and moment stresses μ_{13}^i, μ_{23}^i.
3. During the determination of the deformations, bending-torsions, force and moment stresses in each layer, first for the force stresses σ_{31}^i, σ_{22}^i and moment stress μ_{33}^i we'll take:

$$\sigma_{31}^i = \overset{o^i}{\sigma}_{31}(\alpha_1,\alpha_2),\ \sigma_{32}^i = \overset{o^i}{\sigma}_{32}(\alpha_1,\alpha_2),\ \mu_{33}^i = \overset{o^i}{\mu}_{33}(\alpha_1,\alpha_2). \tag{9}$$

After determination of mentioned quantities, values of σ_{31}^i, σ_{32}^i and μ_{33}^i in each layer will be finally defined by the addition to the correspondent values (9) summed up, obtained by integration of the first, second and sixth equilibrium equations of (1), for which the condition will be required, that quantities, averaged along the layer's thickness, are equal to zero.

With the help of the accepted hypotheses displacements, rotations, deformations, bending-torsions, force and moment stresses will be determined and conditions (4), (5) will be satisfied. In order to bring the three-dimensional problem of the mic- ropolar theory of elasticity (1)-(6) to two-dimensional, instead of the components of the tensors of force and moment stresses statically equivalent to them integral characteristics-forces T_{11}, T_{22}, S_{12}, S_{21}, N_{13}, N_{23}, N_{31}, N_{32}, moments L_{13}, L_{23}, M_{11}, M_{22}, H_{12}, H_{21}, L_{11}, L_{22}, L_{33}, L_{12}, L_{21} and hypermoments Λ_{13}, Λ_{23} are introduced:

$$T_{11} = 2\left[\int_0^{h_1} \sigma_{11}^1 d\alpha_3 + \sum_{i=1}^{n} \int_{h_i}^{h_{i+1}} \sigma_{11}^{i+1} d\alpha_3\right],\ S_{12} = 2\left[\int_0^{h_1} \sigma_{12}^1 d\alpha_3 + \sum_{i=1}^{n} \int_{h_i-1}^{h_i} \sigma_{12}^{i} d\alpha_3\right],$$

$$L_{11} = 2\left[\int_0^{h_1} \mu_{11}^1 d\alpha_3 + \sum_{i=1}^{n} \int_{h_i-1}^{h_i} \mu_{11}^{i} d\alpha_3\right], L_{12} = 2\left[\int_0^{h_1} \mu_{12}^1 d\alpha_3 + \sum_{i=1}^{n} \int_{h_i-1}^{h_i} \mu_{12}^{i} d\alpha_3\right], \quad (10)$$

$$M_{11} = 2\left[\int_0^{h_1} \alpha_3\sigma_{11}^1 d\alpha_3 + \sum_{i=1}^{n} \int_{h_i-1}^{h_i} \alpha_3\sigma_{11}^{i} d\alpha_3\right], H_{12} = 2\left[\int_0^{h_1} \alpha_3\sigma_{12}^1 d\alpha_3 + \sum_{i=1}^{n} \int_{h_i-1}^{h_i} \alpha_3\sigma_{12}^{i} d\alpha_3\right],$$

$$N_{13} = 2\left[\int_0^{h_1} \sigma_{13}^1 d\alpha_3 + \sum_{i=1}^{n} \int_{h_i-1}^{h_i} \sigma_{13}^{i} d\alpha_3\right],\ \Lambda_{13} = 2\left[\int_0^{h_1} \alpha_3\mu_{13}^1 d\alpha_3 + \sum_{i=1}^{n} \int_{h_i-1}^{h_i} \alpha_3\mu_{13}^{i} d\alpha_3\right],$$

$$L_{13} = 2\left[\int_0^{h_1} \mu_{13}^1 d\alpha_3 + \sum_{i=1}^{n} \int_{h_i-1}^{h_i} \mu_{13}^{i} d\alpha_3\right], L_{33} = 2\left[\int_0^{h_1} \mu_{23}^1 d\alpha_3 + \sum_{i=1}^{n} \int_{h_i-1}^{h_i} \mu_{33}^{i} d\alpha_3\right]; (1 \leftrightarrow 2).$$

Here $\Delta h_{i+1} = h_{i+1} - h_i$ is the thickness of the i-th layer; $2h_1$-of the middle plane.

On the basis of the accepted hypotheses the basic system of equations of mic- ropolar orthotropic elastic multilayered thin plates with free fields of displacements and rotations will be split into two independent systems of equations (the system of plane stress state and the system of bending).

Equations of plane stress state of micropolar orthotropic elastic multilayered thin plates with symmetric structure:

Equilibrium equations:

$$\frac{1}{A_1}\frac{\partial T_{11}}{\partial \alpha_1} + \frac{1}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1}(T_{11} - T_{22}) + \frac{1}{A_2}\frac{\partial S_{21}}{\partial \alpha_2} + \frac{1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2}(S_{21} + S_{12}) = -(q_1^+ + q_1^-),$$

$$\frac{1}{A_2}\frac{\partial T_{22}}{\partial \alpha_2} + \frac{1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2}(T_{22} - T_{11}) + \frac{1}{A_1}\frac{\partial S_{12}}{\partial \alpha_1} + \frac{1}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1}(S_{12} + S_{21}) = -(q_2^+ + q_2^-), \quad (11)$$

$$\frac{1}{A_1}\frac{\partial L_{13}}{\partial \alpha_1} + \frac{1}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1}L_{13} + \frac{1}{A_2}\frac{\partial L_{23}}{\partial \alpha_2} + \frac{1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2}L_{23} + S_{12} - S_{21} = -(m_3^+ + m_3^-).$$

Physical relations:

$$T_{11} = C_{11}\Gamma_{11} + C_{12}\Gamma_{22},\ T_{22} = C_{22}\Gamma_{22} + C_{12}\Gamma_{11},\ S_{12} = C_{88}\Gamma_{12} + C_{78}\Gamma_{21},$$

$$S_{21} = C_{77}\Gamma_{21} + C_{78}\Gamma_{12},\ L_{13} = d_{66}k_{13},\ L_{23} = d_{44}k_{23}, \tag{12}$$

where

$$C_{11} = 2\left[\Delta h_1 \frac{a_{22}^1}{a_{11}^1 a_{22}^1 - \left(a_{12}^1\right)^2} + \sum_{i=1}^{n} \Delta h_{i+1} \frac{a_{22}^{i+1}}{a_{11}^{i+1} a_{22}^{i+1} - \left(a_{12}^{i+1}\right)^2}\right], \quad \ldots$$

$$d_{66} = 2\left[\Delta h_1 \frac{1}{b_{66}^1} + \sum_{i=1}^{n} \Delta h_{i+1} \frac{1}{b_{66}^{i+1}}\right], \quad \ldots. \tag{13}$$

Geometric relations:

$$\Gamma_{11} = \frac{1}{A_1}\frac{\partial u_1}{\partial \alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} u_2,\ \Gamma_{22} = \frac{1}{A_2}\frac{\partial u_2}{\partial \alpha_2} + \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} u_1,$$

$$\Gamma_{12} = \frac{1}{A_1}\frac{\partial u_2}{\partial \alpha_1} - \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} u_1 - \Omega_3,\ \Gamma_{21} = \frac{1}{A_2}\frac{\partial u_1}{\partial \alpha_2} - \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} u_2 + \Omega_3,$$

$$k_{13} = \frac{1}{A_1}\frac{\partial \Omega_3}{\partial \alpha_2},\ k_{23} = \frac{1}{A_1}\frac{\partial \Omega_3}{\partial \alpha_2}, \tag{14}$$

$$k_{11} = \frac{1}{A_1}\frac{\partial \Omega_1}{\partial \alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}\Omega_2,\ k_{22} = \frac{1}{A_1}\frac{\partial \Omega_2}{\partial \alpha_2} + \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_2}\Omega_1,\ k_{33} = \iota.$$

“Softened" boundary conditions on the boundary contour of the middle plane of the plate are the followings:

$$T_{11} = T_{11}^* \text{ or } u_1 = u_1^*,\ S_{12} = S_{12}^* \text{ or } u_2 = u_2^*,\ L_{13} = L_{13}^* \text{ or } \kappa_{13} = \kappa_{13}^*. \tag{15}$$

Equations of bending of micropolar orthotropic elastic multilayered thin plates with symmetric structure:

Equilibrium equations:

$$\frac{1}{A_1}\frac{\partial N_{13}}{\partial \alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_{21}}{\partial \alpha_1} N_{13} + \frac{1}{A_2}\frac{\partial N_{23}}{\partial \alpha_2} - \frac{T_{11}}{R_1} - \frac{T_{22}}{R_2} = q_3^+ + q_3^-,$$

$$N_{31} - \frac{1}{A_1}\frac{\partial M_{11}}{\partial \alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}(M_{22} - M_{11}) - \frac{1}{A_2}\frac{\partial H_{21}}{\partial \alpha_2} -$$

$$-\frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}(H_{21} + H_{12}) = h(q_1^+ - q_1^-),$$

$$N_{32} - \frac{1}{A_2}\frac{\partial M_{22}}{\partial \alpha_2} + \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}(M_{11} - M_{22}) - \frac{1}{A_1}\frac{\partial H_{12}}{\partial \alpha_1} -$$

$$-\frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}(H_{12} + H_{21}) = h(q_2^+ - q_2^-), \tag{16}$$

$$\frac{1}{A_1}\frac{\partial L_{11}}{\partial \alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}(L_{11} - L_{22}) + \frac{1}{A_2}\frac{\partial L_{21}}{\partial \alpha_2} + \frac{1}{A_1 A_2}\frac{\partial H_1}{\partial \alpha_2}(L_{21} + L_{12}) +$$

$$+N_{23} - N_{32} = -(m_1^+ + m_1^-),$$

$$\frac{1}{A_2}\frac{\partial L_{22}}{\partial \alpha_2} + \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}(L_{22} - L_{11}) + \frac{1}{A_1}\frac{\partial L_{12}}{\partial \alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}(L_{12} + L_{21}) +$$

$$+N_{31} - N_{13} = -(m_2^+ + m_2^-),$$

$$L_{33} - \frac{1}{A_1}\frac{\partial \Lambda_{13}}{\partial \alpha_1} - \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}\Lambda_{13} - \frac{1}{A_2}\frac{\partial \Lambda_{23}}{\partial \alpha_2} - \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}\Lambda_{23} +$$
$$+ H_{12} - H_{21} = h(m_3^+ - m_3^-).$$

Physical relations:

$$M_{11} = D_{11}K_{11} + D_{12}K_{22},\;\; M_{22} = D_{22}K_{22} + D_{12}K_{11},$$
$$H_{12} = D_{88}K_{12} + D_{78}K_{21},\;\; H_{21} = D_{77}K_{21} + D_{78}K_{12},\;\; N_{13} = \widetilde{C}_{55}\Gamma_{13} + C_{56}\Gamma_{31},$$
$$N_{31} = C_{66}\Gamma_{31} + C_{56}\Gamma_{13},\;\; N_{23} = C_{55}\Gamma_{23} + C_{45}\Gamma_{32},\;\; N_{32} = C_{44}\Gamma_{32} + C_{45}\Gamma_{23}, \tag{17}$$
$$L_{11} = d_{11}k_{11} + d_{12}k_{22} + d_{13}k_{33},\;\; L_{22} = d_{22}k_{22} + d_{21}k_{11} + d_{23}k_{33},$$
$$L_{33} = d_{33}k_{33} + d_{31}k_{11} + d_{32}k_{22},\, L_{12} = d_{88}k_{12} + d_{78}k_{21},\;\; L_{21} = d_{77}k_{21} + d_{78}k_{12},$$
$$\Lambda_{13} = \lambda_{66}l_{13},\;\; \Lambda_{23} = \lambda_{44}l_{23},$$

where

$$D_{11} = 2\left[\frac{h_i^3 - h_{i-1}^3}{3}\frac{a_{22}^1}{a_{11}^1 a_{22}^1 - \left(a_{12}^1\right)^2} + \sum_{i=1}^{n}\frac{h_i^3 - h_{i-1}^3}{3}\frac{a_{22}^{i+1}}{a_{11}^{i+1}a_{22}^{i+1} - \left(a_{12}^{i+1}\right)^2}\right], \quad \ldots$$

$$C_{44} = 2\left[\Delta h_1 \frac{a_{44}^1}{a_{44}^1 a_{55}^1 - \left(a_{45}^1\right)^2} + \sum_{i=1}^{n}\Delta h_{i+1}\frac{a_{44}^{i+1}}{a_{44}^{i+1}a_{55}^{i+1} - \left(a_{45}^{i+1}\right)^2}\right], \quad \ldots$$

$$d_{11} = 2\left[\Delta h_1 \frac{b_{22}^1 b_{33}^1 - \left(b_{23}^1\right)^2}{\Delta^i} + \sum_{i=1}^{n}\Delta h_{i+1}\frac{b_{22}^{i+1}b_{33}^{i+1} - \left(b_{23}^{i+1}\right)^2}{\Delta^{i+1}}\right], \quad \ldots \tag{18}$$

$$\lambda_{66} = 2\left[\frac{h_i^3 - h_{i-1}^3}{3}\frac{1}{b_{66}^1} + \sum_{i=1}^{n}\frac{h_i^3 - h_{i-1}^3}{3}\frac{1}{b_{66}^{i+1}}\right], \quad \ldots.$$

Geometric relations:

$$K_{11} = \frac{1}{A_1}\frac{\partial \Psi_1}{\partial \alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}\Psi_2,\;\; K_{22} = \frac{1}{A_2}\frac{\partial \Psi_2}{\partial \alpha_2} + \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}\Psi_1,$$
$$\Gamma_{23} = -\vartheta_2 - \Omega_1,\;\; \Gamma_{32} = \Psi_2 + \Omega_1,\;\; \Gamma_{13} = -\vartheta_1 + \Omega_2,\;\; \Gamma_{31} = \Psi_1 - \Omega_2,$$
$$\vartheta_2 = -\frac{1}{A_2}\frac{\partial w}{\partial \alpha_2} + \frac{u_2}{R_2},\;\; \vartheta_1 = -\frac{1}{A_1}\frac{\partial w}{\partial \alpha_1} + \frac{u_1}{R_1},$$
$$K_{12} = \frac{1}{A_1}\frac{\partial \Psi_2}{\partial \alpha_1} - \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}\Psi_1 - \iota,\;\; K_{21} = \frac{1}{A_2}\frac{\partial \Psi_1}{\partial \alpha_1} - \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}\Psi_2 + \iota, \tag{19}$$
$$k_{12} = \frac{1}{A_1}\frac{\partial \Omega_2}{\partial \alpha_1} - \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}\Omega_1,\;\; k_{21} = \frac{1}{A_2}\frac{\partial \Omega_1}{\partial \alpha_2} - \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}\Omega_2,$$
$$l_{13} = \frac{1}{A_1}\frac{\partial \iota}{\partial \alpha_1},\;\; l_{23} = \frac{1}{A_2}\frac{\partial \iota}{\partial \alpha_2},$$
$$k_{11} = \frac{1}{A_1}\frac{\partial \Omega_1}{\partial \alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}\Omega_2,\;\; k_{22} = \frac{1}{A_1}\frac{\partial \Omega_2}{\partial \alpha_2} + \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_2}\Omega_1,\;\; k_{33} = \iota.$$

“Softened" boundary conditions are the followings:

$$N_{13} = N_{13}^* \text{ or } w = w^*,\ M_{11} = M_{11}^* \text{ or } K_{11} = K_{11}^*,\ H_{12} = H_{12}^* \text{ or } K_{12} = K_{12}^*,$$

$$L_{11} = L_{11}^* \text{ or } \kappa_{11} = \kappa_{11}^*,\ L_{12} = L_{12}^* \text{ or } \kappa_{12} = \kappa_{12}^*,\ \Lambda_{13} = \Lambda_{13}^* \text{ or } l_{13} = l_{13}^*. \tag{20}$$

On the basis of the constructed models of plane stress state (11)-(15) and bending (16)-(20) concrete problems of statics, free and forced vibrations of multilayered micropolar orthotropic plates will be studied.

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