

New Types of Spatial Solitons in Photorefractive Crystals

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Abstract—New types of “bright,” “dark,” “grey,” and multicomponent photorefractive spatial solitons have been found. The paths of their propagation (self-bending) and specific spatial scales of light field distribution (fanning), resulting from self-action, have been calculated. A possibility of a new nonlinear-optical effect—the formation of spatial shock wave—has been shown. All the analytical results have been confirmed by computer simulation.

INTRODUCTION

One of the most exciting fields of modern laser physics is self-organization in systems consisting of a nonlinear medium and a light field. For Kerr-type nonlinearity, self-consistent solutions (solitons) of such problems have been studied rather well. The concepts of one- and two-component solitons as self-consistent spatially localized solutions of many nonlinear problems have solidly clamped in quite different fields of modern physics: fiber and nonlinear optics, physics of one-dimensional (1D) chains and two-dimensional (2D) atomic planes in ferromagnetics, conjugated polymers, HTSCs, etc. In this way, results of recent investigations of solitons multisolitons, and of soliton pairs in photorefractive crystals (PRCs), are of great importance. Starting from the pioneer paper (M. Segev *et al.*, 1992, *Phys. Rev. Lett.*, **68**, 923), related to PRCs with drift (local) nonlinearity, bright, dark, gray, vector, vortex, and multi solitons, and their propagation and interaction, were intensively studied. It was shown that one can write a stable soliton-like waveguide in PRC and capture a low-intensity light beam (M. Morin *et al.*, 1995, *Opt. Lett.*, **20**, 2066). Stable bright–bright, bright–dark, and dark–dark pairs of incoherent spatial solitons were also observed (Z. Chen, *et al.*, 1996, *Opt. Lett.*, **21**, 1436).

Our main goal is to present some new types of stable soliton-like laser beams in PRC with drift and diffusion nonlinear response.

THE MODEL

Our model is based on the well-known 2D system of material equations for the internal electric field $E_{sc}(x, z)$ in PRC

$$\begin{cases} \frac{\partial n}{\partial t} = \frac{\partial N_d^+}{\partial t} - \frac{1}{e} \frac{\partial j}{\partial x} \\ \frac{\partial N_d^+}{\partial t} = s(I + I_0)(N_d - N_d^+) - \gamma_R n N_d^+ \\ j = e\mu n(E_0 + E_{sc}) - \mu\Theta \frac{\partial n}{\partial x} \\ \frac{\partial E_{sc}}{\partial x} = \frac{4\pi e}{\epsilon}(n + N_a - N_d^+). \end{cases}$$

Here n , N_d , N_d^+ , and N_a are free carriers, donors, ionized donors, and acceptors' densities, respectively; e and μ are free carriers charge and mobility; j is the current density; s is the photoionization cross section; I is the light intensity; sI_0 describes the dark photoionization; γ_R is the two-body recombination constant; E_0 is the external electric field applied to PRC along the x -axis; ϵ and Θ are the dielectric constant and temperature.

Propagating light beam with the complex amplitude $A(x, z)$ and the wave number k was described by a shortened wave equation

$$i(\partial A / \partial z) = (1/2k)(\partial^2 A / \partial x^2) + k(\delta\eta / \eta)A,$$

with no regard for absorption. Here, $\delta\eta = -(r_{\text{eff}}\eta^3/2) \times E_{sc}(x, z)$ is the nonlinear addition to the refractive index η ; r_{eff} is the electro-optical constant. Taking into account redistribution of $I(x, z)$ and $E_{sc}(x, z)$, all the equations form a full self-consistent problem.

SOLUTION OF MATERIAL EQUATIONS

We use a steady-state solution of the system of material equations for the case $N_a \gg n$, $I_0 \gg I$, and $\partial E / \partial x \ll 1$

$$E_{sc} = 1/[aI_0(\chi + 1)(\lambda_1 - \lambda_2)]$$

$$\times \sum_{m=0}^{\infty} (\lambda_1 / \lambda_2^{m+1} - \lambda_2 / \lambda_1^{m+1}) \partial^m I / \partial x^m,$$

$$a = \epsilon / (4\pi e N_a), \quad \chi = N_a / (N_d - N_a),$$

$$\lambda_{1,2} = eE_0 / (2\Theta) \pm \sqrt{[eE_0 / (2\Theta)]^2 + e/[a\Theta(\chi + 1)]}.$$

If $E_0 \sim 10$ kV/cm, $\Theta \sim 300$ K, and the spatial scale of changing light field is not less than the wavelength, and only two first terms of the series, which are proportional to I and $\partial I / \partial x$, can be held. Furthermore, these terms will be called the local and nonlocal components of PRC nonlinear response.

Relation between spatial distributions of E_{sc} and I is much more evident in spectral representation where

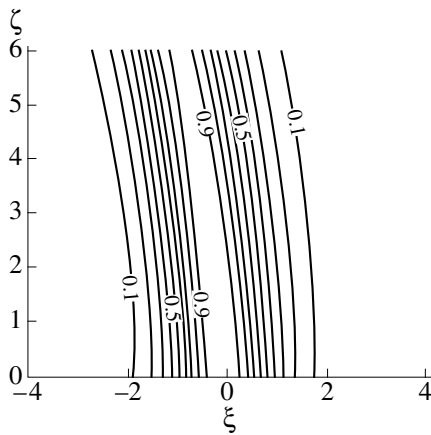


Fig. 1. Self-bending of the one-component bright soliton in PRC: $\gamma = 0.1$.

PRC plays a role of a spatial filter with the transfer function

$$T(\kappa) = -E_0[1 + i\kappa\Theta/(eE_0)]/\{I_0[1 - i\kappa aE_0(\chi + 1) + (a\kappa^2\Theta/e)(\chi + 1)]\}$$

regarding to the spatial spectrum $I(\kappa)$ of $I(x)$. The cases of pure drift and spatial diffusion can be described as the limiting cases for $\Theta \rightarrow 0$ and $E_0 \rightarrow 0$,

$$T_{dr}(\kappa) = -E_0/\{I_0[1 - i\kappa aE_0(\chi + 1)]\},$$

$$T_{df}(\kappa) = -i\kappa\Theta/\{eI_0[1 + (a\kappa^2\Theta/e)(\chi + 1)]\}.$$

Both expressions can be simplified by expanding into a power series in κ and taking into account only the terms up to the first order.

ONE-COMPONENT SOLITONS

For Kerr-type nonlinearity, one usually looks for the solutions as

$$A(x, z) = Y(x)\exp(-ivz).$$

Here, $Y(x)$ describes the field distribution and v determines its nonlinear phase shift. Restricting our consideration to two first terms of the power series and normalizing the equation, we can obtain

$$d^2\rho/d\xi^2 + 2\gamma\rho^2 d\rho/d\xi + 2(\rho^2 - \beta)\rho = 0.$$

Here, $\xi = x/x_0$, $\zeta = z/L_d$, $\rho(\xi) = Y(\xi)/\sqrt{R/I_0}$, x_0 is the spatial scale of the problem, $L_d = kx_0^2$ is the diffraction length, $R = L_d/L_r$, $L_r = 1/(\alpha E_0)$ is the nonlinear refraction length, $\alpha = kr_{\text{eff}}\eta^2/2 > 0$, $\beta = L_d v$, $\gamma = -2(\alpha k E_0/R) \times [\Theta/(eE_0) + a(\chi + 1)E_0]x_0$ describes a nonlocality.

When $\gamma = 0$, the self-consistent solution looks like the well-known one-component bright soliton $\rho(\xi) = \sqrt{2\beta}/\cosh(\sqrt{2\beta}\xi)$, but it is not the case. However,

assuming $\gamma < 1$ and $\beta = 1/2$, one can find the solution in automodel form

$$\rho_s(\xi, \zeta) = K \operatorname{sech}[K(\xi - V\zeta)] \exp[i\Phi(\xi, \zeta)],$$

$$\Phi(\xi, \zeta) = -V\xi + (1/2)(V^2 - K^2)\zeta.$$

One can show that, in the first order on γ , $K = \text{const}$ and

$$dV/d\zeta = -(8/15)\gamma K^2.$$

This means that the one-component solution's "top" moves along the parabolic trajectory $\xi_s = -(4/15)\gamma K^2 \zeta^2$. Figure 1 shows intensity isolines of the one-component soliton on the (ξ, ζ) -plane.

MODULATION INSTABILITY

Another trivial solution of the problem is a plane wave with the amplitude $\rho(\xi) = \rho_0$ for $\beta = \rho^2$. Let us consider the stability of this solution regarding to small ($\delta\rho_0 \ll 1$) harmonic perturbation $\delta\rho(\xi) = \delta\rho_0 \cos(\Omega\xi)$ at spatial frequency $\Omega = \kappa x_0$. Using the linearization technique developed by Bespalov and Talanov, one can show that the perturbation amplitude will exponentially increase with the rate

$$g = (1/2)\operatorname{Im}\sqrt{\Omega^2(\Omega^2 - 4\rho_0^2) + i\gamma\Omega}$$

with increasing ζ . In the case of Kerr-type nonlinearity ($\gamma \rightarrow 0$), the growth takes place only in the limited band of spatial frequencies $0 < \Omega < \Omega_b = 2\rho_0$, and the maximal value of g is realized for $\Omega = \Omega_m = \sqrt{2}\rho_0$. The nonlocal component extends the instability band up to $\Omega \rightarrow \infty$. However, even when $\gamma \approx 1$, the band width is very close to Ω_b . The modulation instability is illustrated by Fig. 2.

So, in PRC, the plane waves are instable about any small random perturbation. Propagating through PRC, such waves split into rather thin filaments (farming). The filaments' average frequency and thickness are determined by $\Lambda = 2\pi/\Omega_m$ and the inverse width of the instability band.

SPATIAL SHOCK WAVES

Let us analyze a new kind of soliton-like solution—photorefractive spatial shock waves—on a phase portrait, i.e., let us use an analogy between $\rho(\xi)$ and nonlinear oscillations in the potential

$$U(\rho) = \rho^4/2 - \beta\rho^2$$

with the nonlinear damping constant described by the term $2\gamma\rho^2(d\rho/d\xi)$. Two $U(\rho)$ minima with the coordinates $\rho_{1,2} = \pm\sqrt{\beta}$ are focuses and a single maximum $\rho = d\rho/d\xi = 0$ is a nodal point. Nearby the nodal point, linearization results in equation

$$d^2\rho/d\xi^2 - 2\beta\rho = 0.$$

Its solution $\rho(\xi) \propto \exp(\sqrt{2\beta}\xi)$ grows with increasing ξ .

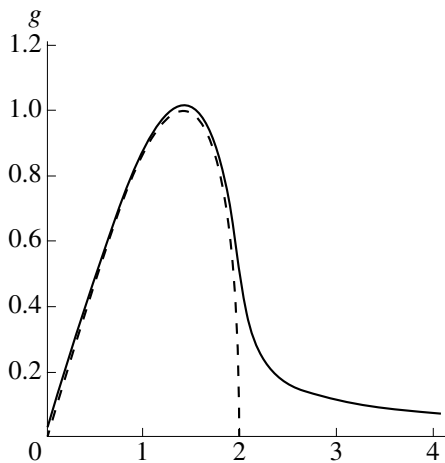


Fig. 2. Harmonic perturbation gain vs. its normalized spatial frequency: Kerr-type nonlinearity (dashed line) and PRC with $R = 1$, $\gamma = 1$ (solid line).

Nearby the focus, the same procedure gives us the equation

$$d^2 u / d\xi^2 + 2\gamma\beta du / d\xi + 4\beta u = 0$$

of damped oscillations of the shift $u(\xi) = \rho(\xi) - \rho_{1,2}$ from the equilibrium. Its solution looks like periodic ($\gamma < 2/\sqrt{\beta}$, $\omega = \sqrt{4\beta - (\gamma\beta)^2}$) or aperiodic ($\gamma \geq 2/\sqrt{\beta}$) going to $\rho_{1,2}$ with $\xi \rightarrow \infty$.

The presented approximations were confirmed by a numerical integration. The phase trajectory, shown in Fig. 3, is a spiral, twisted to ρ_1 . The spatial shock wave profile $\rho(\xi)$ demonstrates all the main specific features of solutions of this type. A sharp exponential front gives way to a plateau with damped oscillations.

STEPLIKE INTENSITY PROFILE

Numerical integrating with boundary conditions, corresponding to the spatial shock wave, has shown that this solution is stable and that its spatial profile does not change with propagation through PRC. So, the spatial shock wave belongs to a new type of soliton-like solutions with separating variables.

To illustrate the importance of such solutions, we present here the data of computer simulation of the evolution of the laser beam with the steplike spatial profile $q(\xi, 0) = (1/2)[1 + \tanh(\xi)]$. Figure 4 shows $|q(\xi)|^2$ in a number of consequent PRC crosssections for $\gamma = 0.1$.

As radiation propagates, a diffraction peak appears on a front of the intensity distribution. Due to the local component of nonlinearity, this peak gradually transforms into a bright spatial soliton. Direction of its propagation gradually shifts from the ξ -axis, due to self-bending. As radiation moves away from the input plane, next and next spatial solitons split from the front of the intensity distribution. At the same time, the spatial distribution of the remainder beam part always

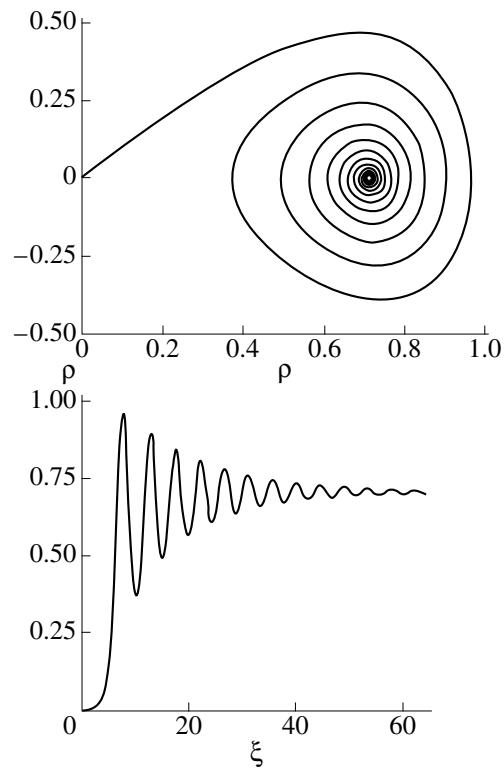


Fig. 3. Intensity profiles of spatial shock wave for $R = 1$, $\beta = 0.5$, and $\gamma = 0.15$ on a phase portrait (top) and in coordinate space (bottom).

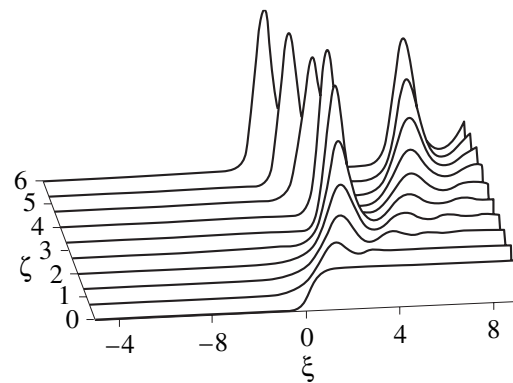


Fig. 4. Excitation of bright solitons and spatial shock waves by radiation with steplike distribution of intensity, $\gamma = 0.1$.

looks like a spatial shock wave. So, the elemental self-consistent solutions of the problem give us great insight into the main results of our computer simulation.

TWO-COMPONENT SOLITONS

Let us try now to find two-component solutions with separating variables,

$$A(x, z) = Y_1(x) \exp(-iv_1 z) + Y_2(x) \exp(-iv_2 z).$$

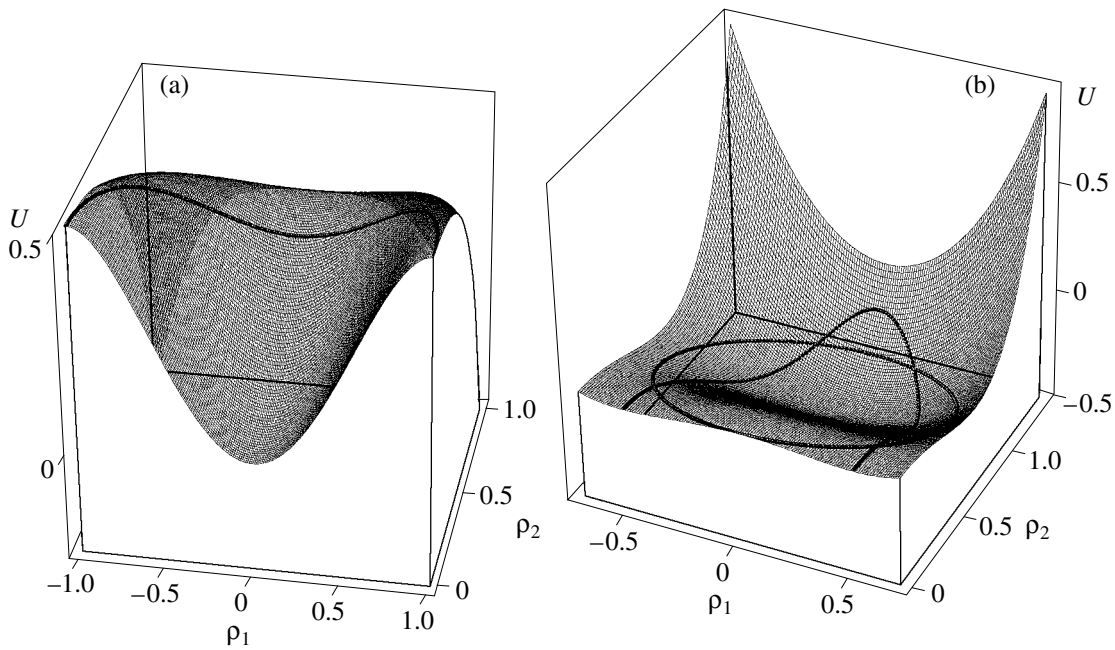


Fig. 5. $U(\rho_1, \rho_2)$ for defocusing (left figure) and self-focusing (right figure) cases; $\beta_1 = 1$, $\beta_2 = 0.25$, two-component solitons' trajectories are shown by solid lines.

Here, $Y_{1,2}(x)$ and $v_{1,2}$ define the incoherent components' spatial profiles and nonlinear phase shifts. Such a form of solution results in the normalized system of equations:

$$d^2 \rho_{1,2} / d\xi^2 \pm 2[(\rho_1^2 + \rho_2^2) - \beta_{1,2}] \rho_{1,2} = 0,$$

where $\beta_{1,2} = L_d v_{1,2}$, “ \pm ” correspond to self-focusing and defocusing cases.

As previously, one may use an analogy between $\rho_{1,2}(\xi)$ character and nonlinear oscillations of two coupled oscillators in the potential

$$U(\rho_1, \rho_2) = \pm (1/2)(\rho_1^2 + \rho_2^2) \mp (\beta_1 \rho_1^2 + \beta_2 \rho_2^2).$$

When $\beta_1 = \beta_2 = \beta$, two-component solitons look like stable pairs of two mutually incoherent bright (self-focusing case),

$$\rho_1 = [\sqrt{2\beta} / \cosh(\sqrt{2\beta}\xi)] \cos(\alpha),$$

$$\rho_2 = [\sqrt{2\beta} / \cosh(\sqrt{2\beta}\xi)] \sin(\alpha),$$

or dark (self-defocusing case)

$$\rho_1 = \sqrt{\beta} \tanh(\sqrt{2\beta}\xi) \cos(\alpha),$$

$$\rho_2 = \sqrt{\beta} \tanh(\sqrt{2\beta}\xi) \sin(\alpha),$$

one-component solitons. When $\beta_1 > \beta_2$, stable pairs can be formed by incoherent bright and dark soliton both in the self-focusing case,

$$\rho_1 = \pm \sqrt{2\beta_1 - \beta_2} / \cosh[\sqrt{2(\beta_1 - \beta_2)}\xi],$$

$$\rho_2 = \pm \sqrt{\beta_2} \tanh[\sqrt{2(\beta_1 - \beta_2)}\xi],$$

and in the self-defocusing case

$$\rho_1 = \pm \sqrt{\beta_1} \tanh[\sqrt{2(\beta_1 - \beta_2)}\xi],$$

$$\rho_2 = \pm \sqrt{2\beta_2 - \beta_1} / \cosh[\sqrt{2(\beta_1 - \beta_2)}\xi].$$

Such two-component solitons' trajectories start ($\xi \rightarrow -\infty$) from one of the point of unstable equilibrium and come ($\xi \rightarrow +\infty$) to the symmetrically located point (Fig. 5).

In the self-focusing case, we succeeded in finding the quite new two-component solution with a limited energy. Its trajectory starts from and returns to the same point (0, 0) on the plane (ρ_1, ρ_2) . For $\beta_1 = 4\beta_2$, this new two-component solution can be written as

$$\rho_1 = \pm \sqrt{6\beta_2} / \cosh^2(\sqrt{2\beta_2}\xi),$$

$$\rho_2 = \pm \sqrt{6\beta_2} \sinh(\sqrt{2\beta_2}\xi) / \cosh^2(\sqrt{2\beta_2}\xi).$$

Figure 6 illustrates spatial profiles of symmetrical and asymmetrical components of new soliton and its stable propagation through PRC with the length of about 5 cm. Figure 7 demonstrates the decay of the components without nonlinear interaction. Figure 8 illustrates the stability of the new soliton's spatial structure on perturbations of input profiles of both components by a Gaussian noise. Increase of the noise level results in the soliton decay (Fig. 9). Figures 10 and 11 prove its stability regarding collisions with the same two-component and one-component bright solitons.

Computer simulation enables us to follow the two-component soliton's spatial structure with changing β_1/β_2 and to confirm its stability.

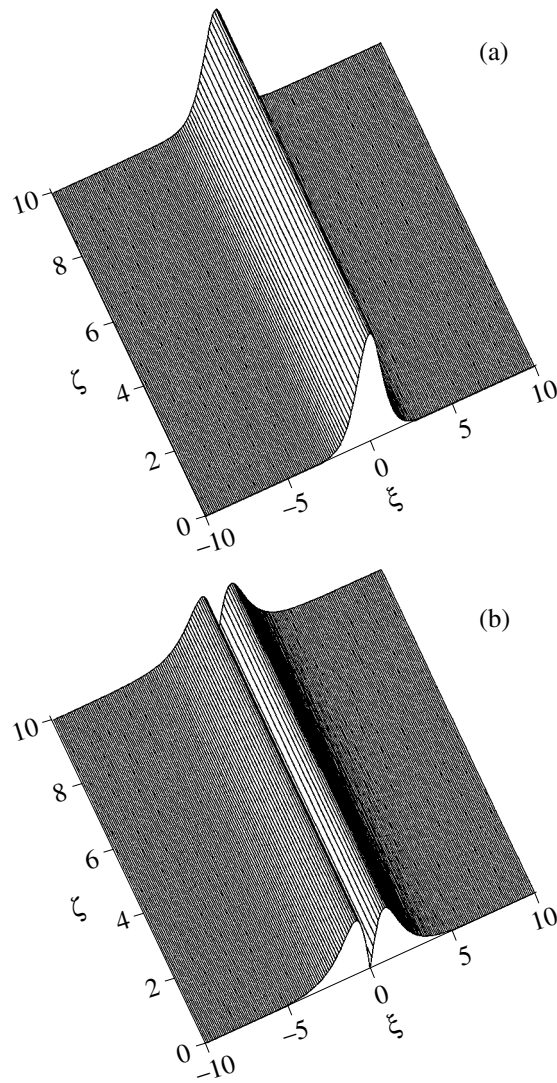


Fig. 6. Spatial profiles (a) $|\rho_1(\xi)|^2$ and (b) $|\rho_2(\xi)|^2$ of two-component soliton and its stable propagation along the ξ -axis, $\beta_1 = 1$, $\beta_2 = 0.25$.

MULTICOMPONENT SOLITONS

Now we will consider much more complicated multicomponent solitons, composed by more than two incoherent self-consistent light components. All such components have a limited energy and spatially localized profiles of eigent high-order modes of their common nonlinear waveguide, written in PRC. In the case of a two-component soliton, the nonlinear waveguide profile is determined by the following spatial distribution of η :

$$\Delta\eta \propto \Delta\eta_{\max} / \cosh^2(\xi/\xi_0).$$

Here, $\Delta\eta_{\max} = 6\beta_2$ and $\xi_0 = 1/\sqrt{2\beta_2}$. Let us suppose that all the solutions of a new class write a nonlinear waveguide with an analogous spatial profile. However, we will consider $\Delta\eta_{\max}$ and ξ_0 as parameters that have to be determined. Then, at the first step, our problem

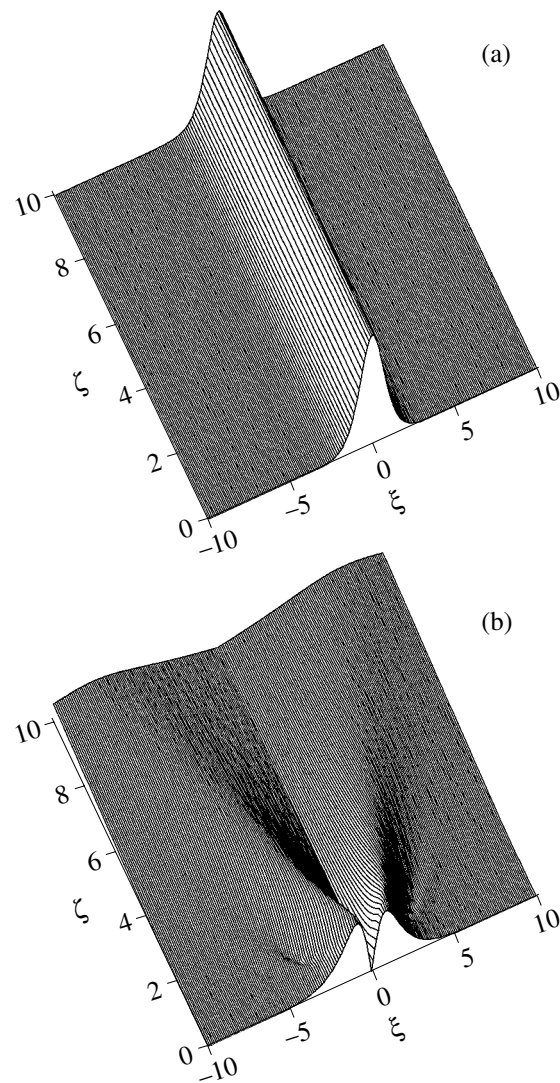


Fig. 7. Evolution of spatial profiles (a) $|\rho_1(\xi)|^2$ and (b) $|\rho_2(\xi)|^2$ of two-component soliton without cross-modulation coupling: $\beta_1 = 1$, $\beta_2 = 0.25$.

can be reduced to calculating eigent modes of an optical waveguide with the specified spatial profile. At the second step, we require that the nonlinear waveguide, written in PRC by the soliton components, should have, namely, the used η profile. This requirement separates self-consistent solutions from all the solutions, which were found on the first step, and enables us to determine the constants of corresponding expansion in a basis of the waveguide eigenfunctions.

A character of multicomponent solutions is significantly simplified for special discrete relations between $\Delta\eta_{\max}$ and ξ_0 . In such cases, the system, which must be solved, can be written in the form

$$d^2 \rho_i^{(n)} / d\xi^2 + 2[\beta_0 n(n+1) / \cosh^2(\sqrt{2\beta_0}\xi) - \beta_i] \rho_i^{(n)} = 0,$$

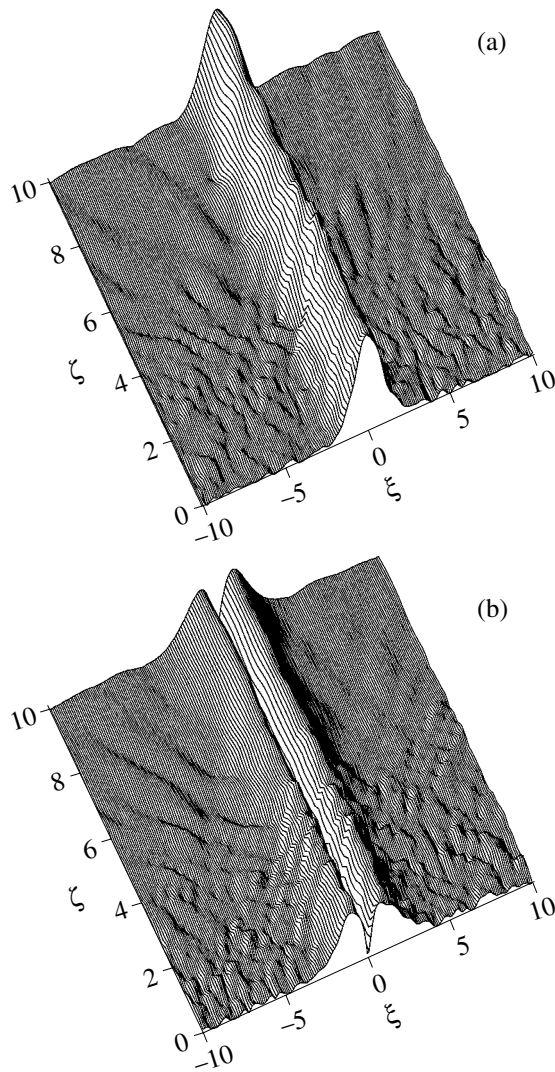


Fig. 8. Stability of spatial profiles (a) $|\rho_1(\xi)|^2$ and (b) $|\rho_2(\xi)|^2$ of two-component soliton. The components are perturbed by a 10% Gaussian noise; $\beta_1 = 1$, $\beta_2 = 0.25$.

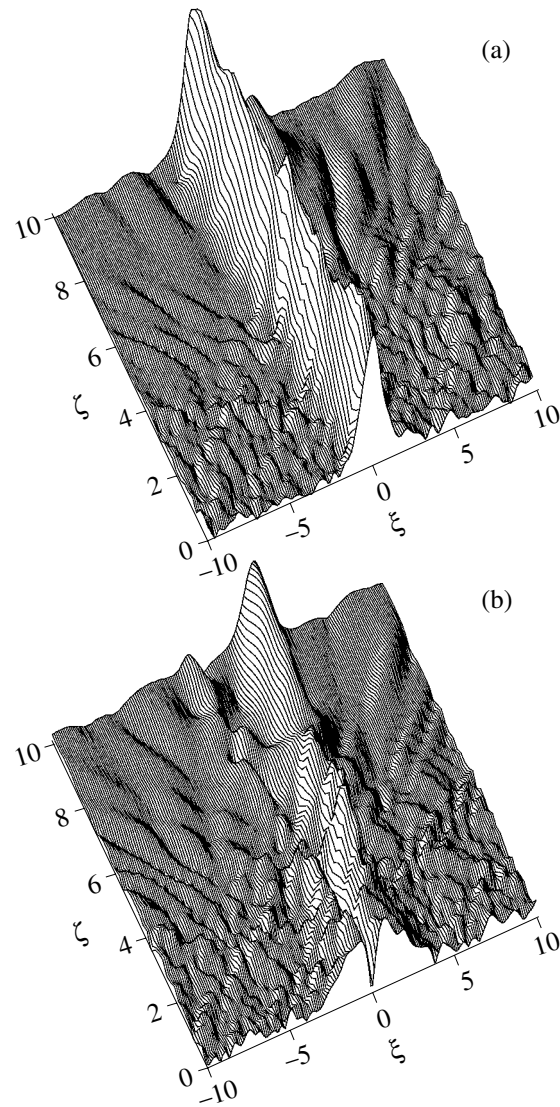


Fig. 9. Decay of two-component soliton. Light field components (a) $|\rho_1(\xi)|$ and (b) $|\rho_2(\xi)|$ are perturbed by a 20% Gaussian noise; $\beta_1 = 1$, $\beta_2 = 0.25$.

where $n = 1, 2, \dots$ and $i = 1, 2, \dots, n$. The condition of self-consistent multicomponent soliton $[\rho_1^{(n)}, \rho_2^{(n)}, \dots, \rho_n^{(n)}]$ can be determined as

$$\sum_{i=1}^n [\rho_i^{(n)}]^2 = \beta_0 n(n+1) / \cosh^2(\sqrt{2\beta_0}\xi).$$

Let us write only some solutions of this class for $n = 1, 2$, and 3 :

$$\begin{cases} \rho_1^{(1)} = \sqrt{2\beta_0} / \cosh(\sqrt{2\beta_0}\xi), & \beta_1 = \beta_0; \\ \rho_1^{(2)} = \sqrt{6\beta_0} / \cosh^2(\sqrt{2\beta_0}\xi), & \beta_1 = 4\beta_0 \\ \rho_2^{(2)} = \sqrt{6\beta_0} \sinh(\sqrt{2\beta_0}\xi) / \cosh^2(\sqrt{2\beta_0}\xi) \\ \beta_2 = \beta_0; \end{cases}$$

$$\begin{cases} \rho_1^{(3)} = (3/2)\sqrt{5\beta_0} / \cosh^3(\sqrt{2\beta_0}\xi), & \beta_1 = 9\beta_0 \\ \rho_2^{(2)} = \sqrt{30\beta_0} \sinh(\sqrt{2\beta_0}\xi) / \cosh^3(\sqrt{2\beta_0}\xi) \\ \beta_2 = 4\beta_0 \\ \rho_3^{(3)} = (1/2)\sqrt{3\beta_0} \\ \times [4 - 5 / \cosh^2(\sqrt{2\beta_0}\xi)] / \cosh(\sqrt{2\beta_0}\xi), & \beta_3 = \beta_0. \end{cases}$$

The first solution is nothing more than a one-component bright soliton, the second one corresponds to the two-component soliton, and the third one is a next-order soliton-like solution. Figure 12 illustrates the spatial profiles of all its components and their stable propagation through PRC.

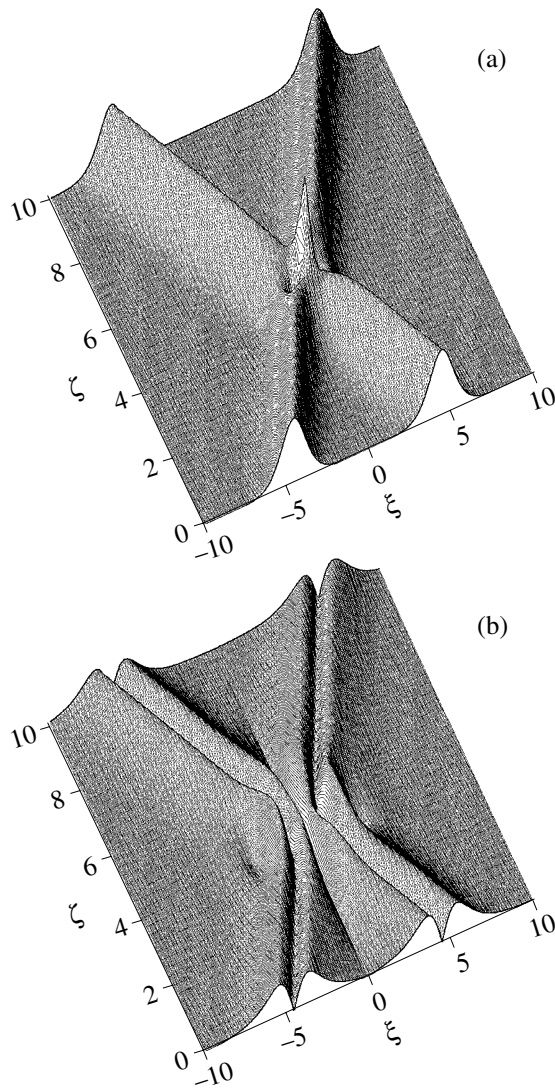


Fig. 10. Stability of spatial profiles (a) $|p_1(\xi)|^2$ and (b) $|p_2(\xi)|^2$ of two-component soliton. Crossing with the same two-component soliton; $\beta_1 = 1$, $\beta_2 = 0.25$.

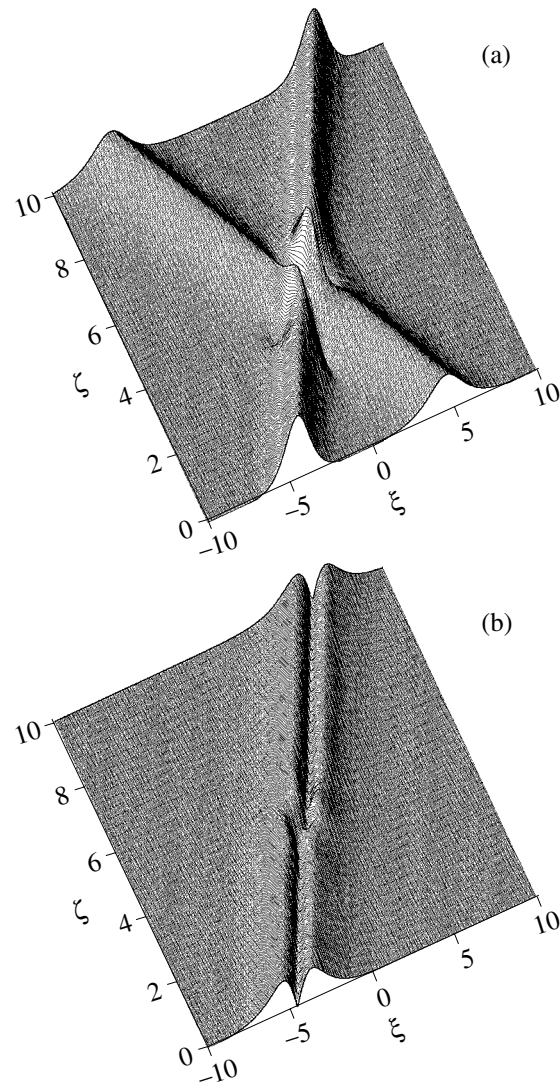


Fig. 11. Stability of spatial profiles (a) $|p_1(\xi)|^2$ and (b) $|p_2(\xi)|^2$ (right figure) of two-component soliton. Crossing with bright one-component soliton; $\beta_1 = 1$, $\beta_2 = 0.25$.

CONCLUSIONS AND FINAL REMARKS

Starting from a quite realistic model of PRC nonlinear response, we have particularized the broad physical outlines of modulation instability (fanning) and self-bending of spatial solitons.

As far as we know, we are the first to have obtained the solutions that look like spatial shock waves in PRC. We have shown that such elemental solutions are stable and can be clearly followed from the evolution of input radiation with unspecified spatial profile.

We have found new multicomponent solitons, corresponding to stable propagation of two and more self-consistent mutually incoherent spatially localized light beams with limited energy. Such solitons open new possibilities in the changing relation between the maximal change of PRC refractive index and written nonlinear waveguide width.

Multicomponent soliton-like solutions of nonlinear Schrödinger equation should be of a very general character, because this equation properly takes into account the first (cubic) term in the expansion of nonlinear force in a standard wave equation. Of great importance, also, is a spatial-temporal analogy, which enables one to extend all the main specific features of spatial solitons on the time axis. In many cases, by this equation, one can describe stable propagation of spatially localized wave packets of electronic wave functions.

One can neglect the interference of soliton components in some cases. First, namely such a situation occurs in media with a rather slow rise time of nonlinear response and different soliton components carrier frequencies. Second, two-component solitons can be formed by cross-polarized light beams. Figure 13 illustrates an experiment where intensity of a pulse (two-

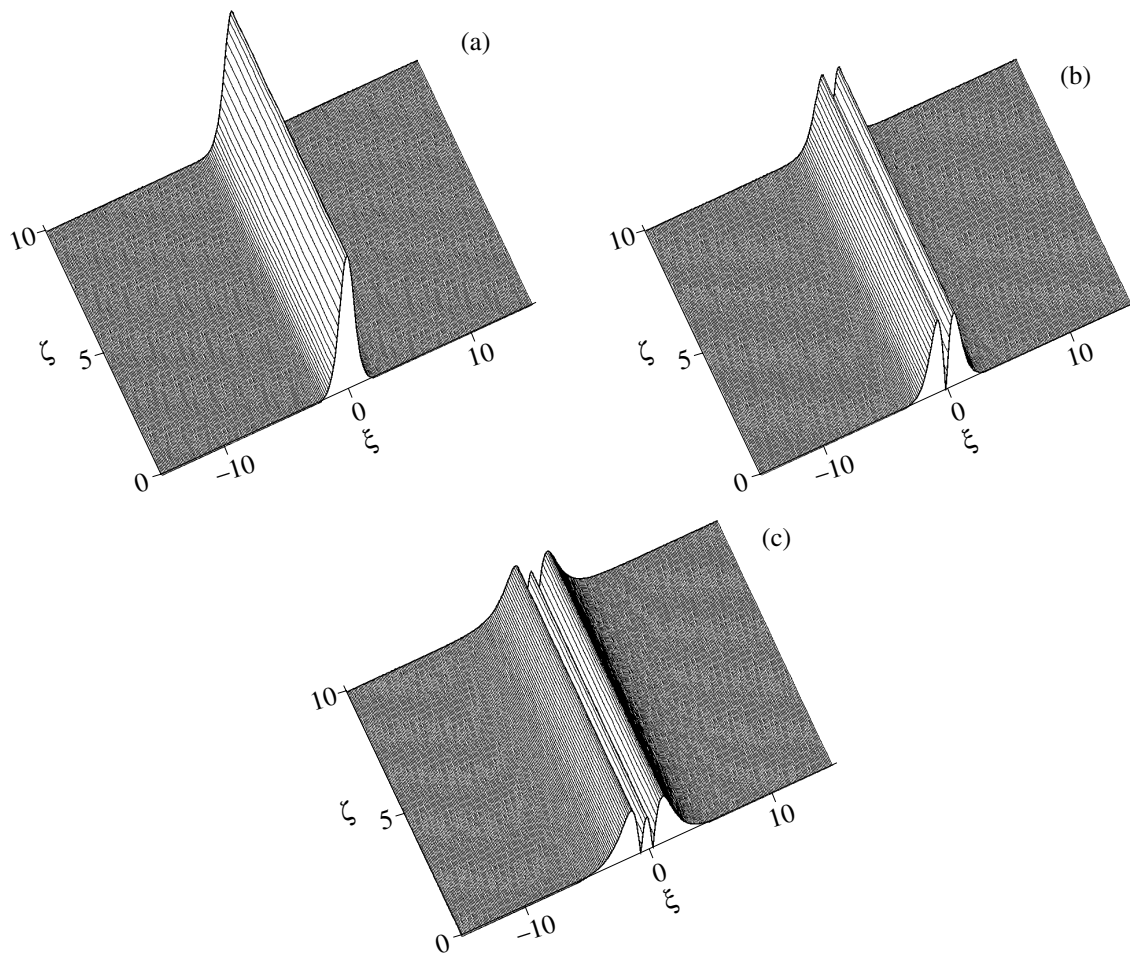


Fig. 12. Spatial profiles $|\rho_1^{(3)}(\xi)|^2$ (upper left figure), $|\rho_2^{(3)}(\xi)|^2$ (upper right figure), and $|\rho_3^{(3)}(\xi)|^2$ (center figure) of three-component soliton and its propagation along the ζ -axis; $\beta_0 = 1$.

component soliton) propagating through a single-mode waveguide is measured by a photodetector with a polarizer.

Multicomponent solutions of the nonlinear Schrödinger equation may be important in the physics

of ferromagnetics, conjugated polymers, and HTSCs. Here, an idea about incoherent (by a fast phase relaxation and different carrier frequencies), but bounded and stable, electronic wave packets, propagating along 1D atomic chains or 2D isolated atomic planes, can be very fruitful. Formation of incoherent wave packets from coherent superconducting pairs may explain our recent experiment with cooled Y-Ba-Cu-O films, where we observed an energy gap during 3-ns time-delay after excitation by a picosecond pulse with photon energy of ~ 2 eV (A.N. Zherikhin, *et al.*, 1994, *Physica C*, **221**, 311).

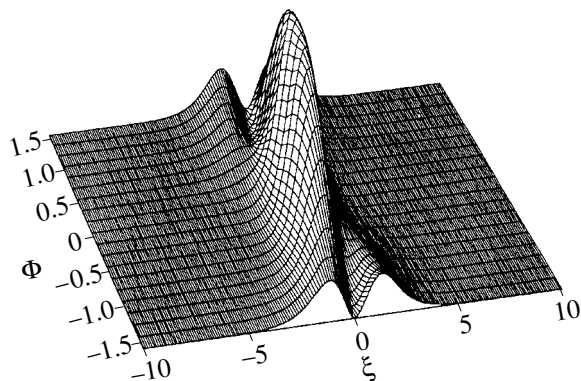


Fig. 13. Intensity profile of two-component soliton laser pulse, propagating through a single-mode waveguide. Φ is the angle of polarizer orientation.

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