ISSN 1541-308X, Physics of Wave Phenomena, 2013, Vol. 21, No. 4, pp. 251–255. © Allerton Press, Inc., 2013.

OPTICAL REFRACTION AND DIFFRACTION IN INHOMOGENEOUS MEDIA

Lateral Shift and Tunneling of Optical Beam Reflected from Gradient-Induced Inhomogeneity

D. O. Ignatyeva* **and A. P. Sukhorukov****

Lomonosov Moscow State University, Leninskie Gory 1/2, Moscow, 119991 Russia

Received June 26, 2013

Abstract—Two effects have been analytically and numerically studied: Goos–Hänchen shift, which is acquired by an optical beam reflected from a gradient inhomogeneity, and tunneling of radiation through a narrow induced inhomogeneity. A possibility of increasing the lateral beam shift in comparison with total internal reflection from a homogeneous medium is revealed. The dependence of the tunneling coefficient on the inhomogeneity parameters is determined and the critical inhomogeneity width at which the tunneling effect arises is found.

DOI: 10.3103/S1541308X1304002X

1. INTRODUCTION

To date, a number of techniques have been proposed to implement completely optical light control using different nonlinear optical mechanisms [1]. A possibility of spatial switching of light-beam propagation direction in media with nonlinearities of different types by modulating the reference-beam intensity was discussed in [2–4]. In this case, the reference radiation forms an inhomogeneity of refractive index, and, when the inhomogeneity amplitude exceeds some critical value, the signal beam undergoes total internal reflection from the reference beam. The effect of total internal reflection of a signal beam from a reference one was described in [2–4] in the framework of the geometric-optical and diffraction theories. It was noted that the ray theory may incorrectly predict the character of signal beam propagation at parameters close to critical, which separate modes of reflection from inhomogeneity and transmission through it. In this study, we analyze two effects: Goos–Hänchen shift and tunneling of radiation through an induced inhomogeneity; the difference between the geometric-optical and diffraction approaches to reflection also manifests itself in these effects.

The Goos–Hänchen effect is observed under total internal reflection of an optical beam from a boundary of a medium with a smaller refractive index; its essence is a lateral shift of the beam with respect to the propagation path predicted by geometric optics. The Goos–Hänchen effect is due to the fact that, under total internal reflection, different components of the optical-beam angular spectrum acquire different phase shifts, as a result of which the beam undergoes a shift along the interface. Another explanation of this effect implies that radiation penetrates the interface in the form of components damping in a medium with a lower optical density. The Goos–Hänchen effect was predicted for the first time for a beam reflected from a homogeneous interface between dielectric media [5]; then it was also studied for an interface with a smoothly varying refractive index [6], as well as for interfaces with metamaterials [7, 8], photonic crystals [9], metals [10], and absorbing media [11–13].

A specific feature of the problem under consideration is that the induced change in the refractive index nonmonotonically depends on the spatial coordinate. One might expect the Goos–Hänchen shift to be increased due to the increase in the penetration depth for the components exponentially decaying in the region with a lower refractive index.

We also investigated the tunneling of radiation through an induced inhomogeneity, which occurs at reflection of a signal beam from a narrow inhomogeneity (for example, the waist of a focused reference beam). In accordance with the geometric-optical and diffraction theories developed in [2–4], the signal beam propagation mode (reflection or transmission through inhomogeneity) is determined by only the amplitude of this inhomogeneity, σ_0, and is independent of its profile and width. However, when the inhomogeneity becomes sufficiently narrow, tunneling (similar to that considered in quantum mechanics [14]) occurs. The essence of this effect is that the

*E-mail: ignatyeva@physics.msu.ru

**E-mail: apsmsu@gmail.com

signal beam transmission through a narrow inhomogeneity can be observed even under the conditions of total internal reflection. Tunneling of an optical beam through an induced inhomogeneity was revealed in the numerical calculations [4, 15]. In this study, we present an analytical description of this effect. Specifically, we determined the dependence of the tunneling coefficient on the inhomogeneity parameters and found the critical inhomogeneity width at which the tunneling arises.

2. GOOS–HÄNCHEN SHIFT AT REFLECTION FROM A GAUSSIAN INHOMOGENEITY

Let us consider the case where a high-power Gaussian pump beam induces a permittivity inhomogeneity in a dielectric with a defocusing nonlinearity. In the case of a Kerr or quadratic nonlinearity, the spatial distribution of the refractive index repeats the pump beam profile and has the form

$$n(y) = n_0\left[1+\sigma(y)\right] = n_0\left\{1-\sigma_0\exp\left[-\frac{(y-w)^2}{w^2}\right]\right\}, \qquad (1)$$

where n_0 is the linear part of the refractive index, σ_0 is the amplitude of induced inhomogeneity, and w is its half-width. In the chosen coordinate system the inhomogeneity center is located on the straight line $y=w$. To simplify the analytical description of the signal beam propagation in this medium, we will approximate the inhomogeneity by a parabolic profile:

$$\begin{cases} \sigma(y) = \sigma_0\left[\dfrac{(y-w)^2}{w^2}-1\right], & |y-w|<w, \\ \sigma(y)=0, & |y-w|\geqslant w. \end{cases} \qquad (2)$$

This approximation makes it possible to obtain an analytical solution to the trajectory equation (derived, for example, in [2–4]) for a signal beam propagating through a medium with an induced inhomogeneity at a small angle θ with respect to the Ox axis:

$$y(x) = -w\cosh\frac{\sqrt{2\sigma_0}x}{w} + \frac{\theta w}{\sqrt{2\sigma_0}}\sinh\frac{\sqrt{2\sigma_0}x}{w}, \qquad (3)$$

and, thus, undergoing total internal reflection at the point

$$y_{\rm t} = w\left(1\pm\sqrt{1-\frac{\theta^2}{2\sigma_0}}\right). \qquad (4)$$

Here, the plus and minus signs correspond to the signal beam incidence from the regions with $y>2w$ and $y<0$, respectively. For definiteness, we assume the geometry of the problem under consideration to correspond to the second case. Note that the effect of total internal reflection of the beam from an induced inhomogeneity manifests itself only at initial angles below the critical value [2–4]:

$$\theta < \theta_{\rm cr} = \sqrt{2\sigma_0}. \qquad (5)$$

To calculate the shift acquired by the beam reflected from the induced inhomogeneity, it is necessary to take into account the phase acquired during the beam transmission through the gradient medium:

$$\varphi = k_0 n_0 w\sqrt{2\sigma_0}\left[\frac{\theta}{\theta_{\rm cr}} - \left(1-\frac{\theta^2}{\theta_{\rm cr}^2}\right)\ln\sqrt{\frac{1+\dfrac{\theta}{\theta_{\rm cr}}}{1-\dfrac{\theta}{\theta_{\rm cr}}}}\right]. \qquad (6)$$

Thus, the total beam shift can be found as

$$D = \frac{1}{k}\frac{\partial\varphi}{\partial\theta} = 2w\chi\ln\sqrt{\frac{1+\chi}{1-\chi}}, \qquad (7)$$

where the angular parameter $\chi=\theta/\theta_{\rm cr}$. This shift can be compared with the Goos–Hänchen shift acquired by an optical beam when the inhomogeneity has a rectangular rather than parabolic profile; this situation corresponds to reflection from a fairly thick ($kw\gg 1$) plate with a smaller refractive index $n_0(1-\sigma_0)$. In this case, the lateral shift determined from the classical formula (see, for example, [16]) has the form

$$D = \frac{2}{\theta_{\rm cr}\sqrt{1-\chi}}. \qquad (8)$$

Note that formulas (7) and (8) are applicable for the Goos–Hänchen shift only when the initial angle of signal beam inclination differs from the critical angle $\theta_{\rm cr}$ by a value exceeding the half-width of the spatial (angular) spectrum of the beam:

$$\theta_{\rm cr} - \theta > \frac{2}{ka}, \qquad (9)$$

where a is the signal (Gaussian) beam half-width.

For initial angles close to the critical value, one must take into account the partial transmission of the corresponding angular component of the signal beam through the inhomogeneity (as this was done, for example, in [17]).

Figure 1 shows the dependence of the shift for an optical beam reflected from inhomogeneities with rectangular (gray lines) and gradient (black lines) profiles. The solid lines present the results of calculation based on analytical formulas (7) and (8). The numerical calculation of the beam shift was performed by determining the difference between the geometric-optical trajectory and the position of the center of mass, $y_{\rm cm} = \int_{-\infty}^{+\infty} y|A(y)|^2dy / \int_{-\infty}^{+\infty}|A(y)|^2dy$, of the

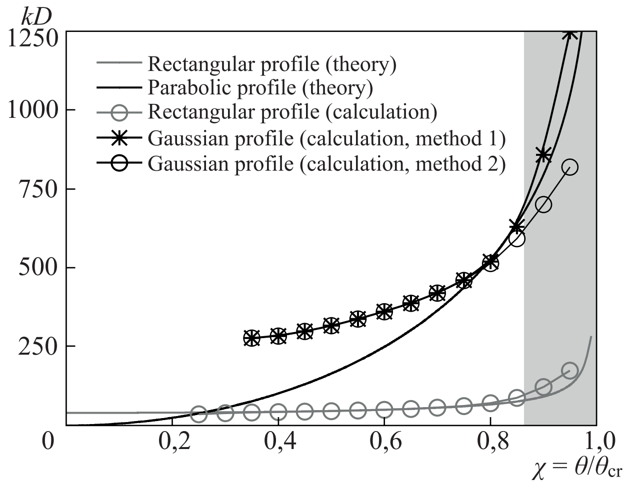

Fig. 1. Dependences of the Goos–Hänchen shift under reflection from rectangular and gradient inhomogeneities on the angular parameter χ.

entire beam (method 1) or only the reflected part (method 2). The position of the center of mass was found by solving the equation of beam diffraction in an inhomogeneous medium. The range of the angular parameters χ in which relation (9) is not fulfilled is colored gray. The normalized half-width of the inhomogeneity and signal beam turned out to be $kw = 300$; the induced inhomogeneity amplitude $\sigma_0 = 1.25 \times 10^{-3}$.

At small values of the parameter χ, corresponding to signal beam propagation in a direction almost parallel to the inhomogeneity, the beam shift at inhomogeneities of different shapes depends strongly on the refractive index gradient. In particular, at some χ values the Goos–Hänchen shift may be larger in the case of reflection from a inhomogeneity with a rectangular profile than under reflection from a parabolic inhomogeneity.

The signal beam trajectory approaches the inhomogeneity center with an increase in the parameter χ. Thus, the change in the refractive index in the region behind the turning point is smaller than in the region before it, as a result of which the exponentially decaying components penetrate deeper the region with a smaller refractive index, and the beam shift increases. Due to the trajectory bending in the inhomogeneous (gradient) medium, the beam shift may be larger than under reflection in a medium with a rectangular inhomogeneity of the same amplitude.

Note that a parabolic profile of inhomogeneity approximates well a Gaussian profile only at relatively small distances from the center. This circumstance explains the difference in the results of the analytical and numerical calculations performed at rather small values of the angular parameter χ (in this case, reflection occurs far from the inhomogeneity center).

3. TUNNELING THROUGH A NARROW INHOMOGENEITY

Tunneling is another example, in which the beam propagation dynamics predicted by geometric optics is not consistent with the solution to the wave equation. If the inhomogeneity is sufficiently wide and condition (5) is satisfied, the signal beam undergoes total internal reflection. At the same time, with a decrease in the inhomogeneity width, the fraction of radiation tunneling through the inhomogeneity constantly increases and, in the case of a very narrow inhomogeneity, complete transmission is observed [4, 15]. The signal beam tunneling is due to the leaking out of the radiation exponentially decaying in the reflection region between the turning points; thus, it depends strongly on the inhomogeneity width and profile.

To calculate approximately the tunneling coefficient through a narrow inhomogeneity, one can use the approach applied in quantum mechanics. First, turning points are determined for the central spectral component from formula (4) and then, using the quasi-classical (Wentzel–Kramers–Brillouin) approximation, one can obtain the following tunneling coefficient through the inhomogeneity [14]:

$$\tilde{T} = \frac{\exp(-2\psi)}{[1 + (1/4)\exp(-2\psi)]^2}, \qquad (10)$$

where the coefficient ψ is defined as

$$\psi = \int_{y_{t1}}^{y_{t2}} \frac{k}{\sqrt{-[\sigma(y) + \theta^2/2)]}} dy. \qquad (11)$$

The approximation of a rectangular quantum-mechanical barrier yields the following transmission coefficient:

$$\tilde{T} = \left[1 + \frac{(k^2\theta^2 + \kappa^2)^2}{4k^2\theta^2} \sinh^2 \tilde{\psi}\right]^{-1}, \qquad (12)$$

where $\tilde{\psi} = \kappa(y_{t2} - y_{t1})$, $\kappa = k\sqrt{2\sigma - \theta^2}$ is the imaginary component of the propagation constant in the region of total internal reflection.

A similar effect, referred to as frustrated total internal reflection, manifests itself in optics. Its essence is as follows: radiation incident on a thin dielectric plate under an angle exceeding the total internal reflection angle passes through this plate. The transmission coefficient through the plate depends on the radiation polarization and has the form

$$\tilde{T} = |t_{12}t_{21}|^2 \frac{\exp(-2\tilde{\psi})}{|1 - r_{12}^2 \exp(-2\tilde{\psi})|^2}, \qquad (13)$$

where t_{12}, t_{21}, and r_{12} are the corresponding amplitude coefficients of reflection and transmission

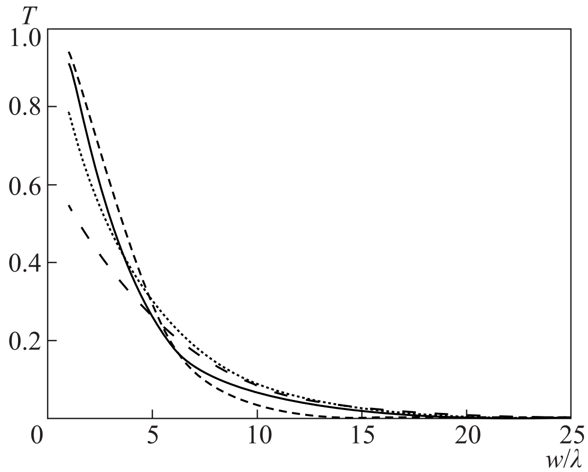

Fig. 2. Dependence of the tunneling coefficient on the inhomogeneity width: direct calculation (solid line), Wentzel–Kramers–Brillouin approximation (dashed line), approximation of a rectangular quantum-mechanical barrier (dotted line), and approximation of a homogeneous dielectric plate (short-dash line).

through an interface between two media with refractive indices n_0 (subscript 1) and $n_0(1-\sigma_0)$ (subscript 2).

Figure 2 shows a dependence of the signal-beam tunneling coefficient on the induced-inhomogeneity width. The values obtained as a result of direct simulation of the signal beam propagation via numerical solution of the diffraction equation in the presence of inhomogeneity, as well as the tunneling coefficients calculated in the above-described approximations, are presented here.

Despite the absence of complete numerical agreement between the tunneling coefficients calculated using the aforementioned techniques and the values obtained by direct calculation, we should note fairly good qualitative agreement, which manifests itself in the following. First, when the inhomogeneity is approximated by a rectangular barrier, one obtains an estimate of the tunneling coefficient with an error of 5% in the region where it significantly differs from zero (the Wentzel–Kramers–Brillouin approximation is less exact in this region). Second, all aforementioned approximations yield a rather exact estimate for the induced-inhomogeneity width at which the tunneling effect arises. According to (10), (12), and (13), tunneling occurs if the distance between the turning points satisfies the relation

$$y_{t2} - y_{t1} = \kappa^{-1}, \tag{14}$$

in other words, for the width of inhomogeneity induced by a reference beam,

$$kw < \frac{1}{2\sqrt{2\sigma_0}(1-\chi^2)}. \tag{15}$$

For an inhomogeneity with $\sigma_0 = 1.25\times10^{-3}$ and angular parameter $\chi = 0.8$, the inhomogeneity width corresponding to the occurrence of tunneling effect is 5λ (see Fig. 2).

The tunneling effect is of particular importance when the reference beam is focused to increase its intensity (and, therefore, the induced- inhomogeneity intensity). In this case, the optimal version is the reflection of the signal beam from the waist, where the inhomogeneity reaches the maximum value. Thus, to implement efficient signal-beam control, one must provide the reference-beam width in the interaction region sufficiently large to suppress the tunneling effect.

4. CONCLUSIONS

We performed an analytical and numerical study of signal beam tunneling through an induced inhomogeneity and the effect of lateral shift of a beam reflected from a gradient inhomogeneity. An analysis of the shift of a beam reflected from a gradient induced inhomogeneity showed that this shift may be sufficiently large to exceed the Goos–Hänchen shift under reflection from a homogeneous boundary due to the bending of the signal beam trajectory. The study of the tunneling effect yielded the dependence of the transmission coefficient through a narrow inhomogeneity on its parameters and revealed the critical inhomogeneity width at which this effect occurs.

ACKNOWLEDGMENTS

This study was supported by the Ministry of Education and Science of the Russian Federation (Federal Target Program "Scientific and Scientific-Pedagogical Personnel of the Innovative Russia" (agreement 14.132.21.1419) and the RFBR Projects 12-02-31396 and 12-02-90023.

D.O. Ignatyeva acknowledges the support of the Foundation of Noncommercial Programs "Dynasty".

REFERENCES

1. *Nonlinear Photonics and Novel Optical Phenomena.* Ed. by Zh. Chen and R. Morandotti (Springer, 2013).
2. V.E. Lobanov, A.A. Kalinovich, A.P. Sukhorukov, F. Bennet, and D. Neshev, "Nonlinear Reflection of Optical Beams in the Media with a Thermal Nonlinearity," Laser Phys. **19**(5), 1112 (2009).
3. V.E. Lobanov and A.P. Sukhorukov, "Repulsion and Total Reflection with Mismatched Three-Wave Interaction of Noncollinear Optical Beams in Quadratic Media," Phys. Rev. A. **84**(2), 023821 (2011).
4. D.O. Ignatyeva and A.P. Sukhorukov, "Plasmon Beams Interaction at Interface Between Metal and Dielectric with Saturable Kerr Nonlinearity," Appl. Phys. A. **109**(4), 813 (2012).

5. F. Goos and H. Hänchen, "Ein neuer und fundamentaler Versuch zur Totalreflexion," Ann. Phys. **436**(7–8), 333 (1947).
6. W. Löffler, M.P. van Exter, G.W. 't Hooft, E.R. Eliel, K. Hermans, D.J. Broer, and J.P. Woerdman, "Polarization-Dependent Goos–Hänchen Shift at a Graded Dielectric Interface," Opt. Commun. **283**, 3367 (2010).
7. De-Kui Qing and Gang Chen, "Goos–Hänchen Shifts at the Interfaces Between Left- and Right-Handed Media," Opt. Lett. **29**(8), 872 (2004).
8. Xiaolong Hu, Yidong Huang, Wei Zhang, De-Kui Qing, and Jiangde Peng, "Opposite Goos–Hänchen Shifts for Transverse-Electric and Transverse-Magnetic Beams at the Interface Associated with Single-Negative Materials," Opt. Lett. **30**(8), 899 (2005).
9. V.V. Moskalenko, I.V. Soboleva, and A.A. Fedyanin, "Surface Wave-Induced Enhancement of the Goos–Hänchen Effect in One-Dimensional Photonic Crystals," JETP Lett. **91**(8), 382 (2010).
10. M. Merano, A. Aiello, G.W. 't Hooft, M.P. van Exter, E.R. Eliel, and J.P. Woerdman, "Observation of Goos–Hänchen Shifts in Metallic Reflection," Opt. Exp. **15**(24), 15928 (2007).
11. W.J. Wild and C.L. Giles, "Goos–Hänchen Shifts from Absorbing Media," Phys. Rev. A. **25**(4), 2099 (1982).
12. J.B. Götte, A. Aiello, and J.P. Woerdman, "Loss-Induced Transition of the Goos–Hänchen Effect for Metals and Dielectrics," Opt. Exp. **16**(6), 3961 (2008).
13. Li-Gang Wang and Shi-Yao Zhu, "Large Positive and Negative Goos–Hänchen Shifts from a Weakly Absorbing Left-Handed Slab," J. Appl. Phys. **98**, 043522 (2005).
14. M. Razavy, *Quantum Theory of Tunneling* (World Scientific, 2003).
15. A.A. Kalinovich, V.E. Lobanov, A.P. Sukhorukov, and A.L. Tolstik, "Tunneling of Optical Beams Through Inhomogeneity of a Refractive Index," Bull. Russ. Acad. Sci. Phys. **74**(12), 1718 (2010).
16. M. McGuirk and C.K. Carniglia, "An Angular Spectrum Representation Approach to the Goos–Hänchen Shift," J. Opt. Soc. Am. **67**(1), 103 (1977).
17. H.M. Lai, F.C. Cheng, and W.K. Tang, "Goos–Hänchen Effect Around and Off the Critical Angle," J. Opt. Soc. Am. A. **3**(4), 550 (1986).