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Collinear Diffraction of Gaussian Optical Beams by Successive Acoustic Pulses

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The peculiarities of collinear acoustooptical diffraction of Gaussian optical beams by two successive acoustic pulses of finite dimensions with an arbitrary temporal envelope is theoretically investigated. The system of two first-order equations that bound Fourier-spectra of transmitted and diffracted light beams and of acoustic pulse propagating in the same direction is deduced. The filter transmission curves during the collinear diffraction by two acoustic pulses with temporal envelope described by Gaussian and sinc (*x*) functions are calculated. The dependence of acoustooptical cell transmission bandwidth on interpulse distance, and on both pulse and crystal length variation is studied. The possibility of the spectral analysis of optical radiation with a help of acoustic pulses is discussed.

Key words: collinear Bragg diffraction, strong acoustooptical interaction, acoustooptic tunable filters, spectral analysis of optical radiation, successive acoustic pulses.

1. INTRODUCTION

Acoustooptical tunable filters utilizing a collinear interaction are one of the most promising [1–3]. They are widely used in various fields of acoustoelectronics, optics, spectroscopy, and laser technology. The most important characteristic of a filter is the bandwidth which is often required to be the narrowest possible. This bandwidth is in inverse proportion to the interaction length, and because of that, filters are made as long as possible. Filters up to 8 inches in length are made with bandwidths down to 1 Å. The tuning of a collinear filter bandwidth can be accomplished by means of finite dimension acoustic pulses. By varying the pulse duration, form, or using linear frequency modulated pulse, one can alter the filter transmission characteristics substantially.

The utilization of an acoustic pulse for the purpose of optical radiation spectral analysis makes it possible to improve the transmission function in comparison to a case of continuous acoustic signal; however, this analysis is restricted by the time during which an acoustic pulse passes through an acoustooptical cell. The collinear interaction, under some conditions, permits the performance of a continuous spectral analysis of optical radiation with the help of successive acoustic pulses.

In the case of collinear interaction, the diffraction takes place the entire time unless an acoustic pulse propagates inside a crystal. Therefore, if the interpulse distance *S* does not exceed the crystal length L, the diffracted light is generated continuously. In order to solve this problem, it is necessary to investigate the simplest model of light diffraction by two successive pulses. In such situation, a period of time can exist when simultaneous acoustooptical diffraction by each pulse occurs. This results in a considerable distortion of the diffracted optical pulse formed at the output of the cell.

An analysis of literature reveals some investigations dedicated to the diffraction of light by acoustic pulses [4, 5]; even though all authors considered only orthogonal acoustooptical interaction. In addition, this interaction is mathematically treated with the help of a planewave approximation, which is the milestone of all acoustooptical theories [6]. Due to the long length and small transverse dimensions of acoustooptical cells, the plane-wave approximation is not valid for the description of collinear diffraction. Hence, it is necessary to consider divergent beams of finite dimensions. The theory of the strong acoustooptical interaction of Gaussian beams was proposed in [7–9]. This paper offers a theoretical investigation of the collinear diffraction of a bounded light beam by three-dimensional successive acoustic pulses of finite length.

2. THEORY OF STRONG COLLINEAR ACOUSTOOPTICAL INTERACTION

One of the acoustic wave equation solutions in an anisotropic medium is the slightly divergent Gaussian beam [10]. If this beam propagates in the x direction without an acoustic energy walk-off then the deformations field can be described by the following function

$$G(x,y,z) = \frac{1}{1-jDx}\exp - \left\{\frac{y^2+z^2}{R^2(1-jDx)}\right\}, \tag{1}$$

where $D = 2/(KR^2)$ is the divergency of a pulse in the transverse y and z directions; R is the initial transverse dimensions of a beam. The term $(1 - jDx)$ describes the alterations of the beam's phase and radius as it propagates along the x coordinate.

Two equal pulses of acoustical deformations propagating along the x direction in a medium without an acoustic energy walk-off may be written as

$$a_o a(x,y,z,t) = a_o G(x,y,z) V(x,t) \exp\{j(\Omega t - Kx)\} + c.c., \tag{2}$$

where $V(x, t)$ is the acoustic pulses temporal envelope: the function describing the form of two propagating pulses; a_o is an input acoustic wave amplitude; Ω and K are the frequency and the wave number of a pulse; c.c. is a complex conjugate of the previous term.

The acoustic pulses propagation in a medium is accompanied by a wave of elastic deformation determined by the tensor $S_{lm}a(x, y, z, t)$. The wave of elastic deformation alters the refractive index of the medium; this is associated with the elastooptical effect which is described by the p_{jklm} tensor [11, 12]. The permeability tensor variation under the influence of the acoustic field deformation has the form $\Delta\varepsilon_{jk} = -N_j^2 N_k^2 p_{jklm} S_{lm}$, where N_j and N_k are the principal refractive indices of the medium, and j, k, l, m are the coordinate indices.

The optical field vector $\vec{E}$ in a medium disturbed by sound propagation should satisfy the following equation

$$\mathrm{rotrot}\vec{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\hat{\varepsilon}_o\vec{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Delta\hat{\varepsilon}(a\vec{E}) = 0, \tag{3}$$

where $\hat{\varepsilon}_o$ is the permeability tensor without sound, $\Delta\hat{\varepsilon}$ is the variation of tensor $\hat{\varepsilon}_o$ caused by sound propagation. The value of $\Delta\hat{\varepsilon}$ is proportional to a_o. Taking into account beams of finite dimensions means that rotrot $\vec{E} \neq -\nabla^2\vec{E}$ because graddiv $\vec{E} \neq 0$ even in isotropic medium, and certainly in the anisotropic one [9].

As it is known [6], the polarization of transmitted light is orthogonal to the one of diffracted light in the case of collinear diffraction. Hence, the optical flux in the interaction region may be presented as a sum of two orthogonally polarized beams propagating along the same x direction as the acoustic pulse Eq. (2)

$$\vec{E} = E_t(x,y,z,t)\vec{e}_t \exp\left[j(k_t x - \omega_t t)\right] + E_d(x,y,z,t)\vec{e}_d \exp\left[j(k_d x - \omega_d t)\right], \tag{4}$$

where $\vec{e}_t$ and $\vec{e}_d$ are the unit polarization vectors, $E_t(x, y, z, t)$ and $E_d(x, y, z, t)$ are the amplitudes of the optical beams which are varying slowly with time and x-coordinate, ω, n, and $k = \omega n/c$ are the frequency, refraction index and wave number of an optical wave; here and below index t refers to the transmitted component, while index d means the distracted component.

Substituting the optical field taken in the form of Eq. (4) in a wave equation Eq. (3), one can obtain a couple of scalar equations that bound the spectra of optical field amplitudes $U_t(k_y, k_z, x, t)$ and $U_d(k_y, k_z, x, t)$. A detailed deduction of these equations is described in the Appendix. Finally we get

$$\begin{aligned} &j\frac{\partial U_d}{\partial x} + \frac{k_y^2}{2k_d}U_d = \\ &\quad q_1 \exp\left(-j\eta x\right)V(x,t)\iint A(K_y,K_z,x)U_t(k_y+K_y,k_z+K_z,x,t)dK_y dK_z, \end{aligned} \tag{5}$$

$$\begin{aligned} &j\frac{\partial U_t}{\partial x} + \frac{k_z^2}{2k_t}U_t = \\ &\quad q_2 \exp\left(j\eta x\right)V(x,t)\iint A^*(K_y,K_z,x)U_d(k_y-K_y,k_z-K_z,x,t)dK_y dK_z. \end{aligned} \tag{6}$$

Here $A(K_y, K_z, x) = \pi R^2 \exp\{-(K_y^2 + K_z^2)R^2(1 - jDx)/4\}$ is the Fourier transform of Eq. (1) with respect to y and z, $\eta = k_t + K - k_d$ is the phase mismatch parameter, k_y, k_z and K_y, K_z are, correspondingly, the transverse components of the optical and acoustical wave-vectors.

We represent U_t and U_d as follows

$$U_t(k_y,k_z,x,t) = f_t(x,t)\exp\{jxk_z^2/2k_t\}\exp\{-(k_y^2+k_z^2)r_t^2(x)/4\}, \tag{7}$$

$$U_d(k_y,k_z,x,t) = f_d(x,t)\exp\{jxk_y^2/2k_d\}\exp\{-(k_y^2+k_z^2)r_d^2(x)/4\}. \tag{8}$$

where $f_t(x, t)$ and $f_d(x, t)$ are the axial amplitudes of the optical beams (measured at $k_y = k_z = 0$), $r_t(x)$ and $r_d(x)$ are the radii of these beams. In the case of collinear acoustooptical interaction the alteration of optical beams' radii is negligibly small; moreover, the radii of the diffracted r_d and the transmitted r_t are linked by the expression: $r_d = r_t/\sqrt{1 + r_t^2/R^2}$.

After substitution of Eqs. (7) and (8) into Eqs. (5) and (6) we can integrate over K_y, K_z coordinates analytically. As a result, neglecting the dependence $r(x)$, we obtain the system of two first-order equations describing the collinear diffraction of slightly divergent light beam by two acoustic pulses of an arbitrary form under the conditions of strong interaction

$$\frac{\partial f_d}{\partial x} = -jq_1 f_t(x,t)V(x,t)\frac{\exp\{-jx\eta\}}{(1-jDx)+\rho_t^2(x)}, \tag{9}$$

$$\frac{\partial f_t}{\partial x} = -jq_2 f_d(x,t)V(x,t)\frac{\exp\{jx\eta\}}{(1+jDx)+\rho_d^2(x)}, \tag{10}$$

where $\rho = r/R$. For further calculations we will assume that $q = q_1 = q_2$. The system of Eqs. (9) and computations. If we consider two Gaussian acoustic pulses of $2l$ length propagating along x-direction at v speed, then the function $V(x, t)$ may be determined as

$$v(x,t) = \exp\left\{-\frac{(vt-x)^2}{l^2}\right\} + \exp\left\{-\frac{(vt-x-S)^2}{l^2}\right\}\exp\{j\varphi(S)\}, \tag{11}$$

where $\varphi(S) = KS + \varphi_o$ is the phase difference between two pulses, φ_o is the initial phase of the second pulse.

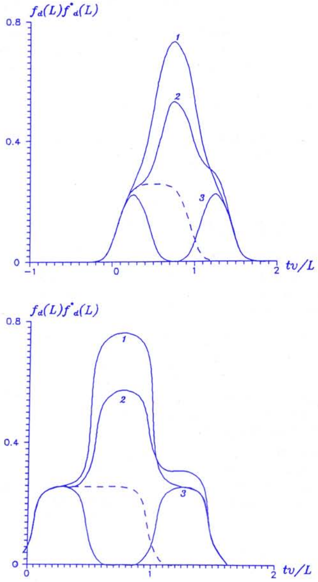

Fig. 1. The dependence of diffracted pulse form $f_d(L)f_d^*(L)$ on the magnitude of phase difference between acoustic pulses.
$S/L = 0.5$; $DL = 1$; $l/L = 0.2$ (a); $l/L = 0.1$ (b); $\varphi(S) = 0$ (curve 1), $\varphi(S) = \pi/2$ (curve 2), $\varphi(S) = \pi$ (curve 3).

3. CALCULATION OF DIFFRACTED LIGHT CHARACTERISTICS

The interaction region lies within $0 < x < L$, so it is natural to introduce dimensionless coordinate x/L and time tv/L. The variation of diffracted light axial amplitude $f_d(L)f_d^*(L)$ at the acoustoptic cell output with time tv/L describes the diffracted pulse form. All calculations of this pulse form were carried out at zero mismatch ($\eta L = 0$).

Figure 1 displays the dependence of diffracted pulse form on the magnitude of phase difference between acoustic pulses. Here the crystal length exceeds the interpulse distance twice and the acoustic pulses duration five times. The dashed curve denotes the diffracted pulse form in the case of a single acoustic pulse. Curve 1 refers to a diffraction by two acoustic pulses with zero phase difference between them. We can see more than double the growth in amplitude in comparison to the dashed curve; furthermore, the diffracted pulse duration increases considerably, as the full duration of diffracted pulse is defined by the length of two acoustic pulses and the distance between them.

The exaggeration of phase difference up to $\pi/2$ (curve 2) reduces the diffraction effectivity and deforms the curve form; however, the diffracted pulse duration remains constant. If two acoustic pulses are out of phase by π (curve 3), the diffraction by one pulse is in antiphase to the diffraction of the other pulse; this results in the disintegration of the diffracted signal into two separate pulses. Figure 1(b) describes the same situation but for an acoustic pulse two times shorter. As seen, the results practically repeat the previous curves, but the diffracted pulses have more pronounced flat top.

The dependence of diffraction effectivity which is proportional to the $f_d(L)$ value on the phase difference between acoustic pulses is depicted in Fig. 2 for the case of $S = L$. We can see that the optimal diffraction efficiency can be acquired unless the phase difference does not exceed 0.47π. As soon as $|\varphi| = \pi/2$, a substantial decay of efficiency takes place, and when two acoustic pulses are out of phase by π, this efficiency decreases more than three times.

Based on the results obtained, further computations were carried out at zero phase difference $\varphi = 0$. Figure 3(a) demonstrates the diffracted pulse form at the different interpulse distance S. The

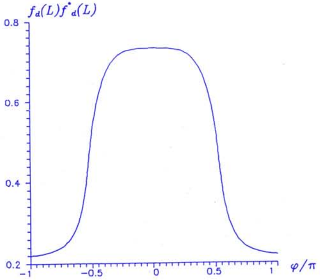

Fig. 2. The dependence of diffraction effectivity on the phase difference φ/π between acoustic pulses.
$S = L$; $DL = 1$; $l/L = 0.1$.

variation of this parameter alters the relationship between the durations of two acoustooptical diffractions: the diffraction by one and by two acoustic pulses. Curve 1 refers to the case of two acoustic pulses with a small interpulse distance $S \ll L$; therefore, the diffraction effectivity is close to maximum, whereas the diffracted pulse duration is close to minimum. In $S = L/2$ (curve 2) a period of time appears when the diffraction by only one acoustic pulse occurs. As a result of this, the calculated curve attains the twin-peaked form. The case of equality of interpulse distance and crystal length $S = L$ corresponds to a diffraction of light by two consecutive single acoustic pulses.

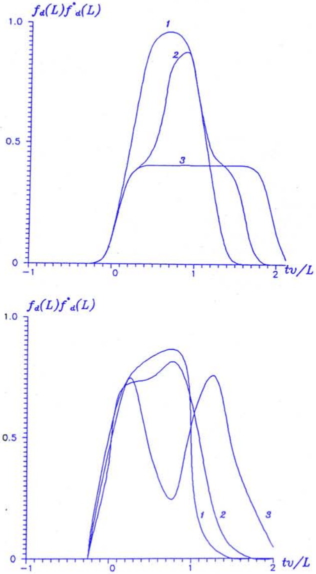

Fig. 3. The dependence of diffracted pulse form $f_d(L)f_d^*(L)$ on the interpulse distance S/L.
$l/L = 0.2$; $DL = 1$ (a); $DL = 5$ (b); $S/L = 0.25$ (curve 1), $S/L = 0.5$ (curve 2). $S/L = 1$ (curve 3).

If the divergency of acoustic pulses is large ($DL = 5$), and the input acoustic power is increased ten times (see Fig. 3(b)), the diffracted pulse form deforms considerably in comparison to the corresponding curves of Fig. 3(a); however, the full duration of diffracted pulse remains constant. Whether the interpulse distance is of acoustic pulse length order $S = L$, the diffracted pulse is a single whole (curves 1 and 2); otherwise, it is disintegrated into two separate pulses (curve 3).

Figure 4 shows the diffracted pulse form at a different length of feeding acoustic pulse in the case of $S = L$. Curves 1–3 in Fig. 4 describe the situation when $2l < L$; this means that the consecutive acoustic pulses do not overlap. As seen, the diffracted pulse form in this case turns out to be constant, i.e., it does not depend on an acoustic pulse length. The flat top of a diffracted pulse is observed for all curves, since a diffraction by a single pulse took place most of the time. However, the diffracted light intensity increases noticeably with the increment of acoustic pulse duration. It is obvious, that the flat top of a diffracted pulse can be transformed into a continuous light of the same amplitude if we continuously feed acoustic pulses at the distance of crystal length. In the case of overlapping acoustic pulses $2l > L$ the flat top of the diffraction pulse vanishes. It is illustrated by the dashed curve in Fig. 4(a) calculated for the situation of $l = L = S$. Here, the maximal effectivity is achieved at four times lower the level of the input acoustic power.

Similar to Fig. 3(b) the growth of acoustic pulses divergency ($DL = 5$) results in a significant distortion of the diffracted pulse. As noticed in Fig. 4(b), the flat top mentioned deteriorates because of the acoustic amplitude decay along the crystal length. Unless $l > 0.4L$, the diffracted pulse remains a single whole; otherwise, it is disintegrated into two successive pulses. Thus, the considerable divergency of acoustic pulse eliminates the possibility of transforming a diffracted optical pulse into a continuous light, even if we feed an infinite sequence of acoustic pulses at $S = L$.

4. CALCULATION OF COLLINEAR FILTER TRANSMISSION FUNCTIONS

It has already been noted that the bandwidth of a collinear filter is one of the most important characteristics. For monochromatic sound this bandwidth is defined by crystal length and has a usual dependence of signal on mismatch parameter: sinc $(\eta L/2\pi)$, i.e., contains the central maximum and side lobes. The utilization of successive acoustic pulses of different duration can alter the bandwidth considerably. To carry out a continuous spectrum analysis of optical radiation, according to the results obtained (see Fig. 4(a)), the interpulse distance should match the crystal length, $S = L$.

The mathematical treatment of a collinear acoustooptical diffraction allowing for finite dimensions beams makes it possible to define the diffraction effectivity not by the ratio of incident and diffracted beams power densities but by the ratio of power fluxes of these beams, as it is always done experimentally. The power flux in an optical beam can be calculated either proceeding from a transversal optical field distribution or from its Fourier-spectrum and distribution over k_y, k_z (Parceval's theorem).

A power flux at the input of the acoustooptic cell is determined as follows $P_o = 0.5\exp\{-(k_y^2+k_z^2)r_t^2/2\}\, dk_y dk_z$, because $f_t(0) = 1$; while at the output of a crystal it may be regained as $P = 0.5 f_d(L) f_d^*(L)\exp\{-(k_y^2+k_z^2)\, r_d^2/2\}\, dk_y dk_z$, where $f_d(L)$ should be used in the form of Eq. (9). The ratio P/P_o characterizes an acoustooptical diffraction effectivity.

The character of a collinear filter transmission function alters considerably with a dimensionless time tv/L; this is the important peculiarity of a diffraction of light by successive acoustic pulses. If $tv/L = 0.5$, an alignment of the first pulse and the crystal centers takes place. The moment $tv/L = 1$ refers to a symmetrical case when the centers of both pulses are situated at crystal edges; that is the first pulse half-left and the second pulse half-entered the crystal. All further calculations of the transmission functions were carried out at a small divergency of an acoustic pulse ($DL = 1$) and the input acoustic field amplitude qL was chosen so, that the maximal optical energy transfer between the transmitted and diffracted light would take place at the acoustooptic cell output ($x = L$).

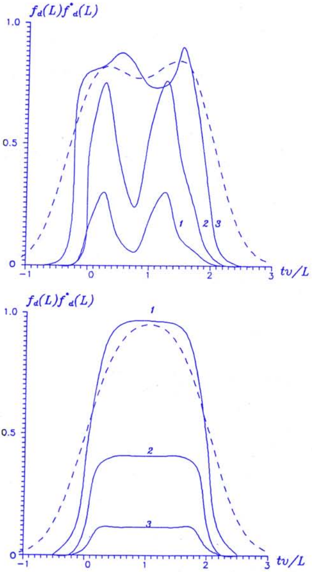

Fig. 4. The dependence of diffracted pulse form $f_d(L)f_d^*(L)$ on the acoustic pulse length l/L.
$S/L = 1$; $DL = 1$ (a); $DL = 5$ (b); $l/L = 0.4$ (curve 1), $l/L = 0.2$ (curve 2) $l/L = 0.1$ (curve 3).

Fig. 5a shows the collinear filter transmission functions during a diffraction of light by two Gaussian acoustic pulses with a duration equal to a crystal length: $l = L/2$. The dashed curve corresponds to the moment $tv/L = 0.5$ when the first pulse accommodates the whole crystal completely. As it can be seen, this curve defines an optimal transmission function of a filter: the magnitude of side lobes amplitude does not exceed 4% at close to maximal diffraction effectivity. The displacement of a pulse against the crystal center (solid and dotted curves) makes the sides lobes

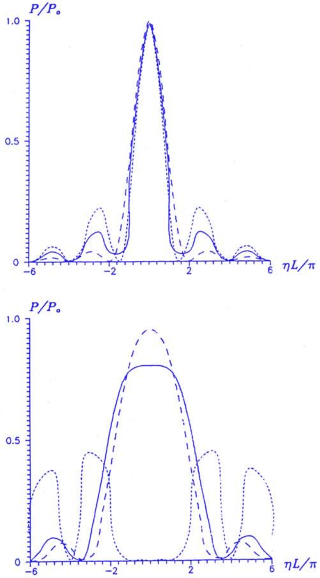

Fig. 5. Collinear filter transmission curves for the Gaussian acoustic pulse. $l/L = 0.5$; $S/L = 1$; $\varphi(S) = 0$ (a); $\varphi(S) = \pi$ (b); $tv/L = 0.5$ (dashed curve); $tv/L = 0.75$ (solid curve); $tv/L = 1$ (dotted curve).

increase substantially reaching the 20% level at the moment $tv/L = 1$. However, the bandwidth of a collinear filter in this case remains constant at any moment of dimensionless time tv/L. Thus, the utilization of Gaussian acoustic pulses of long duration permits the performance of a continuous spectral analysis of optical radiation, unless the level of side lobes does not play a considerable role. Otherwise, this analysis should be carried out periodically excluding the periods when the side lobes exceed the permissible level.

pulses are situated symmetrically ($tv/L = 1$), a collinear filter does not transmit the central frequency signal at all because of the subtraction of the diffraction results by two antiphase acoustic pulses. The shift of a pulse position from $tv/L = 0.5$ to $tv/L = 1$ leads to an increase of side lobes up to the 50% level. Thus, the application of antiphase acoustic pulses for the purpose of optical radiation spectral analysis deteriorates collinear filter characteristics.

The transmission function can be modified not only by means of pulse-to-crystal ratio variation, but by the acoustic pulse form rearrangement as well. Hence, a collinear filter transmission function characteristics investigation may be interesting in the case of acoustic pulses of a non-Gaussian form. As an example, we chose a pulse of finite duration with a temporal envelope described by function:

$$V(x,t) = \begin{cases} 0, & |x| > l \\ \mathrm{sinc}\{-(vt-x)/l\} + \mathrm{sinc}\{-(vt-x-S)/l\}\exp\{-j\varphi(S)\}, & |x| < l \end{cases} \tag{12}$$

Fig. 6 presents the results of computer simulation using acoustic pulse defined by the $V(x, t)$ function mentioned.

The maximal spectral resolution during the diffraction of light by successive acoustic pulses can be obtained unless the length of the pulse and of the crystal is the same; this situation is displayed in Fig. 6(a). The dashed, solid, and dotted curves of Fig. 6 refer to the same moments of dimensionless time tv/L as in the corresponding curves of Fig. 5. In contrast to the transmission curves achieved for a Gaussian acoustic pulse of the same duration (see Fig. 5(a)), here a collinear filter bandwidth is 1.5 times wider; although the displacement of a pulse does not result in the appearance of important side lobes. Another peculiarity in the case of non-Gaussian pulse is the substantial decrease in the central frequency transmission coefficient as the pulse shifts towards the edge of the crystal; at the moment $tv/L = 1$ the transmission curve acquires two maxima.

The increase of interpulse distance twice corresponds to the situation when only one acoustic pulse is completely or partially accommodated in the crystal at any moment. The graphs of Fig. 6(b) are plotted under such an assumption. As seen from the results presented, the displacement of a pulse relative to the central position leads to some dilation of transmission bandwidth accompanied with the side lobes increment. Nevertheless, the decline of the central frequency transmission coefficient is less significant in comparison to the previous case of two non-Gaussian pulses of the same duration shown in Fig. 6(a).

The investigations carried out testify that the simultaneous diffraction of light by two acoustic pulses is always accompanied by the considerable distortion of a collinear filter transmission curve The utilization of pulses with the duration much less than the crystal length (broad transmission bandwidth) makes it possible to perform the spectral analysis of optical radiation during the whole time, unless a complete pulse is accommodated inside the crystal. To avoid the distortions it is necessary to disconnect the instrumentation at the period of a transient process. Otherwise, whether the pulse duration is close to the crystal length (narrow transmission bandwidth), the measurements can be carried out only during the time when the shift of the pulse center relative to the crystal center is not significant.

5. APPENDIX

Let us assume that vector $\vec{e_t}$ is directed along 0y, while vector $\vec{e_d}$ is directed along 0z. Considering this condition and substituting the optical field taken in the form of Eq. (4) in a wave equation Eq. (3) and neglecting the values of $\partial^2 E_t/\partial x^2$ and $\partial^2 E_d/\partial x^2$ one can obtain the following vectoral relation

$$\begin{aligned}
&\left[\vec{e}_x\left(\frac{\partial^2 E_t}{\partial x\partial y}+jk_t\frac{\partial E_t}{\partial y}\right)-\vec{e}_y\left(\frac{\partial^2 E_t}{\partial z^2}+2jk_t\frac{\partial E_t}{\partial x}\right)+\vec{e}_z\left(\frac{\partial^2 E_t}{\partial y\partial z}\right)\right]\exp\left[j(k_t x-\omega_t t)\right]+\\
&\left[\vec{e}_x\left(\frac{\partial^2 E_d}{\partial x\partial z}+jk_d\frac{\partial E_d}{\partial z}\right)-\vec{e}_z\left(\frac{\partial^2 E_d}{\partial y^2}+2jk_d\frac{\partial E_d}{\partial x}\right)+\vec{e}_y\left(\frac{\partial^2 E_d}{\partial y\partial z}\right)\right]\exp\left[j(k_d x-\omega_d t)\right]=\\
&=\frac{(\omega_t+\Omega)^2}{c^2}\Delta\hat{\varepsilon}\vec{e}_t G(x,y,z)V(x,t)E_t(x,y,z,t)\exp\{j[(k_t+K)x-(\omega_t+\Omega)t]\}+\\
&+\frac{(\omega_t-\Omega)^2}{c^2}\Delta\hat{\varepsilon}\vec{e}_t G^*(x,y,z)V(x,t)E_t(x,y,z,t)\exp\{j[(k_t-K)x-(\omega_t-\Omega)t]\}+\\
&+\frac{(\omega_d+\Omega)^2}{c^2}\Delta\hat{\varepsilon}\vec{e}_d G(x,y,z)V(x,t)E_d(x,y,z,t)\exp\{j[(k_d+K)x-(\omega_d+\Omega)t]\}+\\
&+\frac{(\omega_d-\Omega)^2}{c^2}\Delta\hat{\varepsilon}\vec{e}_d G^*(x,y,z)V(x,t)E_d(x,y,z,t)\exp\{j[(k_d-K)x-(\omega_d-\Omega)t]\}
\end{aligned} \tag{A.1}$$

where $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$ are the unit vectors along $0x$, $0y$ and $0z$ axes. As seen from Eq. (A.1), if $k_t < k_d$ (it is equal to $n_t < n_d$) then the upshifted diffraction occurs. In this case $\omega_d = \omega_t + \Omega$ and in the right side of Eq. (A.1) only the first and the fourth item should be left. Whenever the situation of $n_t > n_d$ takes place, the downshifted diffraction occurs in which $\omega_d = \omega_t - \Omega$ and in the right side of Eq. (A.1) only the second and the third item should be left.

As the relation Eq. (A.1) should be satisfied at any moment of time t we set equal to each other the terms at $\exp\{j(k_t x - \omega_t t)\}$ and at $\exp\{j(k_d x - \omega_d t)\}$ on both sides of the equation. Hence, two vectoral equations can be obtained from (A.1) in the condition of the upshifted diffraction

$$\begin{aligned}
&\vec{e}_x\left(\frac{\partial^2 E_t}{\partial x\partial y}+jk_t\frac{\partial E_t}{\partial y}\right)-\vec{e}_t\left(\frac{\partial^2 E_t}{\partial z^2}+2jk_t\frac{\partial E_t}{\partial x}\right)+\vec{e}_d\left(\frac{\partial^2 E_t}{\partial y\partial z}\right)=\\
&\quad=\frac{\omega_t^2}{c^2}\Delta\hat{\varepsilon}\vec{e}_d G^*(x,y,z)V(x,t)E_d(x,y,z,t)\exp\{-j\eta x\},
\end{aligned} \tag{A.2}$$

$$\begin{aligned}
&\vec{e}_x\left(\frac{\partial^2 E_d}{\partial x\partial z}+jk_t\frac{\partial E_d}{\partial z}\right)-\vec{e}_d\left(\frac{\partial^2 E_d}{\partial y^2}+2jk_t\frac{\partial E_d}{\partial x}\right)+\vec{e}_t\left(\frac{\partial^2 E_d}{\partial y\partial z}\right)=\\
&\quad=\frac{\omega_d^2}{c^2}\Delta\hat{\varepsilon}\vec{e}_t G(x,y,z)V(x,t)E_t(x,y,z,t)\exp\{j\eta x\}.
\end{aligned} \tag{A.3}$$

Here $\eta = k_t + K - k_d$ is the phase mismatch. In order to transfer from vectoral to scalar equations one should multiply the acquired vectoral expressions Eq. (A.2) by vector $\vec{e}_t$ and Eq. (A.3) by vector $\vec{e}_d$ in a scalar form and obtain, as a result, the folowing system

$$2jk_t\frac{\partial E_t}{\partial x}+\frac{\partial^2 E_t}{\partial z^2}=2k_t q_2\exp\{j\eta x\}G^*(x,y,z)V(x,t)E_d(x,y,z,t), \tag{A.4}$$

$$2jk_d\frac{\partial E_d}{\partial x}+\frac{\partial^2 E_d}{\partial z^2}=2k_d q_1\exp\{-j\eta x\}G(x,y,z)V(x,t)E_t(x,y,z,t). \tag{A.5}$$

Here $q_1 = k_d(\vec{e}_d\Delta\hat{\varepsilon}\,\vec{e}_t)/n_d^2$, $q_2 = k_t(\vec{e}_t\Delta\hat{\varepsilon}\,\vec{e}_d)/n_t^2$. To solve the system of Eqs. (A.4) and (A.5), we perform a two-dimensional Fourier transformation of functions E_t, E_d, and G in the yz plane. The relation between U_t and E_t, as well as between U_d and E_d, can be determined as

$$U(k_y, k_z, x, t) = (2\pi)^{-1} \int\int E(x, y, z, t) \exp\{-j(k_y y + k_z z)\} dy dz, \tag{A.6}$$

and between G and A as

$$A(K_y, K_z, x) = (2\pi)^{-1} \int\int G(x, y, z) \exp\{-j(K_y y + K_z z)\} dy dz. \tag{A.7}$$

After these transformations we finally get the system of Eqs. (5) and (6).

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