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Diffraction-Induced Spatially Confined Laser Pulse Splitting in Photonic Crystals

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Abstract—It is shown theoretically that the effect of diffraction-induced optical pulse splitting in a linear photonic crystal under the conditions of the Laue transmission geometric scheme of Bragg diffraction can be realized for a spatially confined laser pulse in a hundred-period multilayer structure.

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INTRODUCTION

The theory developed some time ago for the Bragg diffraction of short optical pulses in resonant nonlinear [1] and linear [2, 3] photonic crystals (PCs) in the Laue transmission scheme made it possible to predict the effects of diffraction-induced pulse splitting. At the same time, it was supposed that in the case of linear PCs, the pulses were plane quasi-monochromatic wave packets with an unbounded wave front in their cross sections. Moreover, the corresponding expressions for linear PCs were expanded in terms of a series of small parameters in an approach similar those developed in X-ray optics [4, 5]. Further research showed that due to the diffraction character of light in PCs, it was incorrect to perform expansions in terms of small parameters in most cases of practical interest. This work presents a more rigorous theory of this phenomenon for the diffraction of a pulse constrained both in time and space in a linear PC.

THEORETICAL

Let us consider a one-dimensional crystal consisting of optically isotropic, periodically alternating layers with thicknesses d_1, d_2 and refraction coefficients n_1, n_2. The layers lie perpendicular to the PC surface. A random light pulse falls on the surface at an angle θ to its normal. The electric field of the pulse is denoted as

$$E_{in}(\mathbf{r},t) = A_{in}(\mathbf{r},t)\exp(i\mathbf{k}_0\mathbf{r} - i\omega_0 t), \qquad (1)$$

where A_{in} is generally a complex, slowly changing amplitude; ω_0 is the central frequency of the signal; $k_0 = |\mathbf{k}_0| = \omega_0/c = 2\pi/\lambda_0$, λ_0 is the central wave length; c is the speed of light in a vacuum; $k_{0x} = k_0\sin\theta$, $k_{0z} = k_0\cos\theta$; and the axes x and z are directed along the PC surface and inward of the crystal along the normal to the surface, respectively. For the sake of simplicity, we consider a pulse with s-polarization of the field. The field inside of the PC obeys the equation

$$\Delta E(\mathbf{r},t) - \varepsilon(x)c^{-2}\partial^2 E(\mathbf{r},t)/\partial t^2 = 0, \qquad (2)$$

where the dielectric permittivity $\varepsilon(x) = n^2(x)$ and $\Delta = \partial^2/\partial x^2 + \partial^2/\partial z^2$ is the Laplacian. Let us present the refraction coefficient of the medium $n(x)$ in the form $n(x) = n_e + \Delta n(x)$, where $n_e = (n_1 d_1 + n_2 d_2)/d = n_2 + \delta_0\xi$ is the average refraction coefficient, $\delta_0 = n_1 - n_2$ is the modulation of the refraction coefficient, $\xi = d_1/d$, and $d = d_1 + d_2$ is the period of the structure. In layers with thickness d_1, the function $\Delta n(x) = \delta_0(1 - \xi)$; in layers with thickness d_2, the function $\Delta n(x) = -\delta_0\xi$.

Let us present the field (1) of an incident pulse on the PC surface $z = 0$ in the form of a two-dimensional Fourier expansion, i.e., as a set of plane monochromatic waves with amplitudes $E_{in}(k_x, \omega)$, frequencies $\omega = \omega_0 + \Omega$, and wave vectors $\mathbf{k}$, the components of which are defined by the expressions $k_x = k_{0x} + K$, $k_z = [(\omega/c)^2 - k_x^2]^{1/2}$:

$$E_{in}(x,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_{in}(k_x,\omega)\exp(ik_x x - i\omega t)dk_x d\omega, \qquad (3)$$

where the spectral-angular amplitudes

$$E_{in}(k_x,\omega) \equiv A_{in}(K,\Omega) = (1/4\pi^2)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_{in}(x,t)\exp(-iKx + i\Omega t)dxdt. \qquad (4)$$

We consider a PC with a low value for the modulation of the refraction coefficient ($\delta_0 \ll n_e$) and the diffraction of a pulse in a two-wave approximation, in which there are two “strong” waves for each spectral-angular field component: transmitted (E_0) and diffracted (E_h) waves with wave vectors $\mathbf{q}_0$ and $\mathbf{q}_h = \mathbf{q}_0 + \mathbf{h}$, where $\mathbf{h}$ is the reciprocal lattice vector. In this case, the complete field $E(\mathbf{r},t) = E_0(\mathbf{r},t) + E_h(\mathbf{r},t)$ is a coherent superposition of the transmitted and diffracted pulses in the region $0 \le z \le L$, where L is the thickness of the PC. Allowing for the continuity of the tangent components of the wave vectors $q_{0x} = k_x$ on the surface $z = 0$, we search for the field in the PC as

$$E_g(x,z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_g(K,\Omega)\exp[i(k_x - g)x + iq_{0z}z - i\omega t]dKd\Omega, \quad (5)$$

where $g = 0, h$; $h = 2k_0\sin\theta_B = 2\pi/d$ is the module of the reciprocal lattice vector, the components of which are $h_x = -h$, $h_z = 0$; θ_B is the Bragg angle for radiation with central frequency ω_0; and $q_{0z} = q_{hz}$ is the value of the z-components of the wave vectors in the PC, as defined from the wave equation (see (12)).

Dielectric permittivity $\varepsilon(x)$ in the two-wave approximation is denoted as [4]

$$\varepsilon(x) = \chi_0 + \chi_h\exp(-ihx) + \chi_{-h}\exp(ihx), \quad (6)$$

where χ_0, χ_h, and χ_{-h} are the Fourier components of the dielectric permittivity, as defined by the following expressions [2, 3]: $\chi_0 = n_e^2$,

$$\chi_h = i(n_e\delta_0/\pi)[1 - \exp(i2\pi\xi)],$$
$$\chi_{-h} = -i(n_e\delta_0/\pi)[1 - \exp(-i2\pi\xi)].$$

Substituting (5) and (6) into wave Eq. (2), we obtain the following set of dynamic equations for the field amplitudes A_0 and A_h in (5):

$$(k^2n_e^2 - k_x^2 - q_{0z}^2)A_0 + k^2\chi_{-h}A_h = 0, \quad (7.1)$$

$$[k^2n_e^2 - (k_x - h)^2 - q_{0z}^2]A_h + k^2\chi_hA_0 = 0, \quad (7.2)$$

where $k = \omega/c$.

Let us introduce the following convenient designations:

$$\gamma_0 = (k^2n_e^2 - k_x^2)^{1/2}/k, \ \alpha = h(2k_x - h)/k^2, \quad (8)$$

$$\beta = (q_{0z}^2 - k^2\gamma_0^2)/k^2, \quad (9)$$

where the function $\alpha(K, \Omega)$ defines the degree of deviation from the precise Bragg condition $\alpha = 0$. Set (7) can then be simply denoted as:

$$\beta A_0 - \chi_{-h}A_h = 0, \quad (10.1)$$

$$(\beta - \alpha)A_h - \chi_hA_0 = 0. \quad (10.2)$$

Since the solution for system (10) is nontrivial for amplitudes A_0 and A_h, we obtain a quadratic equation in terms of the variable β. This equation has two solutions:

$$\beta_{1,2} = (1/2)[\alpha \pm (\alpha^2 + 4\chi_h\chi_{-h})^{1/2}]. \quad (11)$$

Taking (9) into account, we finally obtain the following final expression for the desired value of the z-component of wave vectors in the PC:

$$q_{0z}^{(1,2)} = k(\gamma_0^2 + \beta_{1,2})^{1/2}. \quad (12)$$

In addition, Eq. (10.1) implies a simple connection between the amplitudes of the fields A_h and A_0: $A_{hj} = R_jA_{0j}$, where $j = 1, 2$; $R_{1,2} = \beta_{1,2}/\chi_{-h}$. It should be emphasized that in contrast to [2, 3], solution (12) and values (8) and (9) are precise without having to expand them in terms of the parameters K/k_0, Ω/ω_0 and $(\theta - \theta_B)/\theta_B$.

The Fourier amplitudes of the fields $A_{0,h}(K, \Omega)$ are defined from the continuity condition of the electric and magnetic fields on the input surface of the PC $z = 0$:

$$A_{in} + A_S = A_{01} + A_{02}, \quad (13.1)$$

$$k_z(A_{in} - A_S) = q_{0z}^{(1)}A_{01} + q_{0z}^{(2)}A_{02}, \quad (13.2)$$

$$R_1A_{01} + R_2A_{02} = 0, \quad (13.3)$$

where A_S is the amplitude of a specularly reflected wave. Taking the reflection of the transmitted and diffracted pulses from the lower surface of the PC into account does not change the main features of diffraction-induced pulse splitting. (This will be in the subject of a separate work.) The solution for set (13) is written as

$$A_{01} = -[(1 + R_S)R_2/R_{12}]A_{in}, \quad (14)$$

$$A_{02} = [(1 + R_S)R_1/R_{12}]A_{in}, \quad (15)$$

where $R_{12} = R_1 - R_2$, $R_S = A_S/A_{in} = (k_z - f_S)/(k_z + f_S)$ is the Fresnel reflection coefficient, modified due to diffraction; and $f_S = (q_{0z}^{(2)}R_1 - q_{0z}^{(1)}R_2)/R_{12}$. Finally, we obtain the following expression for the complete field in the PC:

$$E(x, z, t) = [A_0(x, z, t) + A_h(x, z, t)\exp(-ihx)]P(x, t), \quad (16)$$

where $P(x, t) = \exp(ik_{0x}x - i\omega_0t)$,

$$A_g(x,z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B_gA_{in}(K,\Omega)\exp(iKx - i\Omega t)dKd\Omega, \quad (16.1)$$

$$B_g = A_{in}^{-1}\sum_{j=1,2} A_{gj}\exp(iq_{0z}^{(j)}z). \quad (16.2)$$

Here, $B_0 = T(z)$ and $B_h = R(z)$ are the amplitude coefficients of the transmission and reflection of plane waves in a PC layer with thickness z, respectively.

There are thus four waves for each plane-wave component of the field: two waves with the wave vectors $\mathbf{q}_0^{(1,2)}$ for the transmitted wave and two waves with the wave vectors $\mathbf{q}_h^{(1,2)}$ for the diffracted wave. Since they have pairwise identical x and z components, their sum represents two standing waves along the x axis with different phase and group velocities along z axis, owing to the differences in the effective refraction coefficients $n_{ej}(K, \Omega) = (n_e^2 + \beta_j)^{1/2}$ and in their dispersion, which as a rule considerably exceeds the material dispersion of PC layers. As a result of the superposition of the waves in dependence on the value of index j, a field of each type is mainly localized in the PC layers with a lower or higher refraction coefficient n_j and the absorption coefficients $\mu_j = 2k_0\mathrm{Im}(n_j)$, respectively (the so-called Borrmann and anti-Borrmann modes).

Figure 1 shows the intensities of both $I_j(x) = |A_j(x)|^2$ modes in different z sections of the PC, measured in extinction depth units $\Lambda = \lambda_0\gamma_{0r}/2|\chi_h|$, where $\gamma_{0r} =$

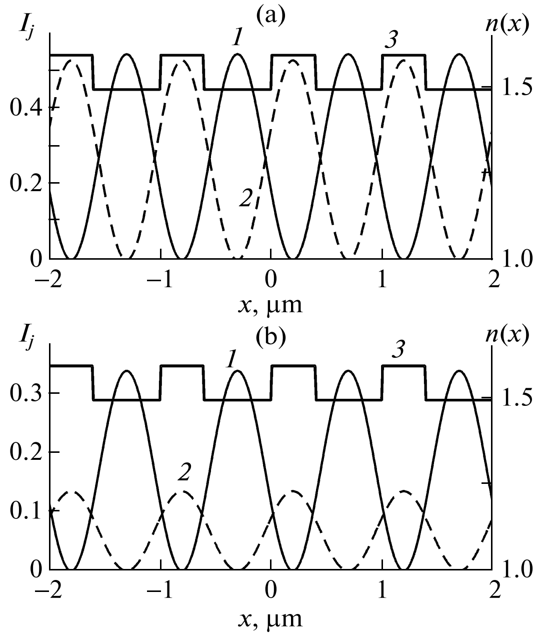

Fig. 1. Borrmann (*1*) and anti-Borrmann (*2*) modes at depth $z = 10\Lambda$ (a) and $z = 300\Lambda$ (b); (*3*) is a profile of the real part of the refraction index. The parameters of the PC were $d = 1$ μm, $\xi = 0.4$, $n_1 = 1.5 + i \times 10^{-5}$, $\delta_0 = 0.1 + i \times 5 \times 10^{-5}$; extinction depth $\Lambda = 7.8$ μm; wavelength $\lambda_0 = 1$ μm, $\theta = \theta_B = 30°$.

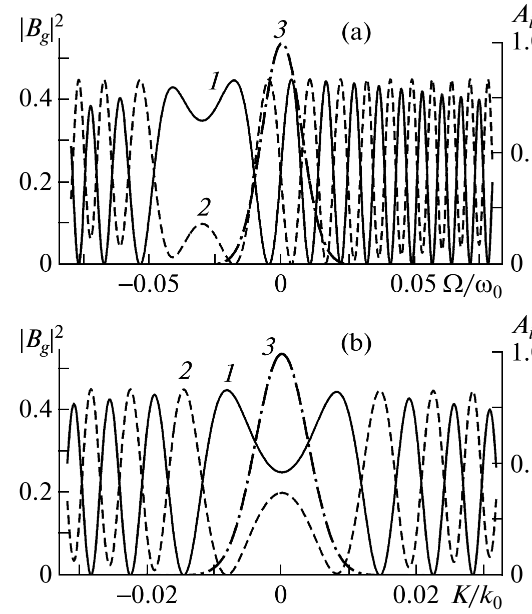

Fig. 2. The spectral (a) and angular (b) dependences of the reflection (*1*) and transmission (*2*) coefficients and the incident radiation spectra, normalized to unity (*3*). Width of the photonic crystal $z = 1$ mm; extinction depth $\Lambda = 7.43$ μm; transversal pulse width $r_0 = 50$ μm; pulse duration $\tau_0 = 0.1$ ps.

$\mathrm{Re}(n_e^2 - \sin^2\theta)^{1/2}$. At a low enough z, the quantity Λ is equal to the period of the so-called pendulum solution, i.e., to the effect of a complete transfer of transmitted wave energy (E_0) to that of diffracted wave (E_h), and vice versa. It can be seen in from Fig. 1b that with an increase in z, the anti-Borrmann mode (curve *2*) is suppressed as a result of its localization in layers with a high absorption coefficient.

The intensities of the transmitted ($g = 0$) and diffracted ($g = h$) pulses are defined by the expressions $I_g(x, z, t) = |A_g(x, z, t)|^2$, which are snapshots of them at $t = \mathrm{const}$.

RESULTS AND DISCUSSION

Let us discuss the optimum conditions for realizing the diffraction-induced splitting of incident pulses. Expressions (14)–(16) imply that effective Bragg pulse diffraction in a PC is defined by the condition $|\alpha(K, \Omega)| \le \alpha_B = 2|\chi_h|$, which imposes a restriction on the spectral ($\Delta\Omega_0$) and angular (ΔK_0) widths of spectrum $A_{in}(K, \Omega)$ of the incident pulse and on the deviation of incidence angle θ from the Bragg angle θ_B. In addition, a substantial role is played by the correlation between widths $\Delta\Omega_0$, ΔK_0 and the values of widthwise oscillation periods $\Delta\Omega_z$, ΔK_z of reflection and transmission coefficients (16.2) (see below).

In the case of narrow frequency spectrum ($\Delta\Omega_0 \ll \omega_0$), (8) implies that $\alpha(K) = 2\sin2\theta_B[\Delta\theta + K/(k_0\cos\theta_B)]$, where $\Delta\theta = \theta - \theta_B$. At the same time, the angular width of the Bragg reflection $\Delta K_B = k_0|\chi_h|/2\sin\theta_B$. For wide quasi-monochromatic pulse ($\Delta K_0 \ll k_{0x}$), the spectral pulse amplitude $A_{in}(K, \Omega) \approx A_{in}(\Omega)\delta(K - \Omega\sin\theta_B/c)$; therefore, the offset parameter $\alpha(\Omega) = 2\sin2\theta_B[\Delta\theta + (\Omega/\omega_0)\tan\theta_B]$. This implies that the spectral width of the Bragg reflection $\Delta\Omega_B = \omega_0|\chi_h|/2\sin^2\theta_B$.

Let the PC be subjected to a pulse with Gaussian amplitude distribution

$$A_{in}(x, 0, t) = A\exp[-(x\cos\theta/r_0)^2 - (t - x\sin\theta/c)^2/\tau_0^2], \quad (17)$$

where r_0 is the transverse size of the pulse and τ_0 is the pulse duration. If the longitudinal size of the pulse $\tau_0 c \ll r_0$, then the spectral width $\Delta\Omega_0 \approx 2/\tau_0$ and the angular width $\Delta K_0 \approx 2\cos\theta/r_0$. This implies that the condition $|\alpha| \le \alpha_B$ is true for pulses with duration $\tau_0 > 4\sin^2\theta_B/\omega_0|\chi_h|$ and width $r_0 > \lambda_0\sin2\theta_B/\pi|\chi_h|$. For example, if $\lambda_0 \approx 1$ μm, $|\chi_h| \approx 0.1$ and $\theta_B \sim 30°$, then $\tau_0 > 5$ fs, $r_0 > 3$ μm. However, the presence of short-period oscillations (at $z \gg \Lambda$) in coefficients R and T (16.2) with periods $\Delta\Omega_z = 4\pi\omega_0(\Lambda/z) \ll \Delta\Omega_B$ and $\Delta K_z = 2\Delta K_B(2\Lambda/z)^{1/2} \ll \Delta K_B$ leads to an increase in the acceptable values of τ_0 and r_0 by factors of approximately (Λ/z) and $(\Lambda/z)^{1/2}$, respectively.

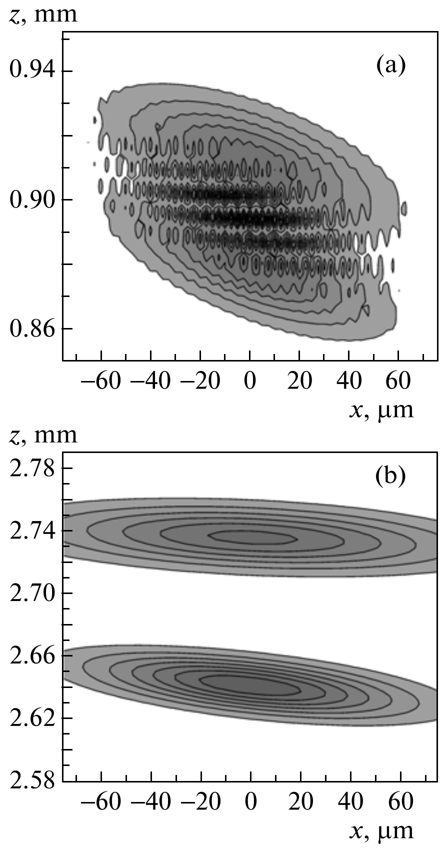

Fig. 3. Spatial intensity distribution of a complete field $I(x, z)$ (a) and diffracted pulses $|E_h(x, z)|^2$ at different moments in time: $t = 4$ ps (a), $t = 12$ ps (b). Parameters of the incident pulse: $r_0 = 50$ μm; $\tau_0 = 0.1$ ps.

Figure 2 shows the diagrams of the spectral (Fig. 2a) and angular (Fig. 2b) dependences of the reflection $|R|^2$ and transmission $|T|^2$ coefficients and the spectrum of the incident pulse $|A_{in}|$. The following PC parameters are used here and below: $d = 1$ μm, $\xi = 0.5$, $n_2 = 1.5 + i \times 10^{-5}$, $\delta_0 = 0.1 + i \times 10^{-5}$, and $\lambda_0 = 1$ μm. The displacement of the oscillation structure in Fig. 2a by the magnitude $\Omega = -\omega_0(|\chi_h|/2\sin^2\theta_B)^2$ is determined by the condition $d\varphi/d\Omega = 0$ for reaching the minimum phase difference $\varphi(\Omega) = (q_{0z}^{(1)} - q_{0z}^{(2)})z$ in the expressions for coefficients $T(\Omega, z) \sim \cos\varphi$ and $R(\Omega, z) \sim \sin\varphi$ (16.2), which are true for $\Omega \ll \Delta\Omega_B$.

The difference in the group velocities of $\upsilon_{gr}^{(j)} = (dq_{0z}^{(j)}/d\omega)^{-1}$ for waves of the first and second type lead us to conclude that after a certain time t, the pulses will diverge by a considerable distance, $2\tau_0 c/\gamma_{0r} = |\upsilon_{gr}^{(1)} - \upsilon_{gr}^{(2)}|t$ at depth $z_0 = tc/\gamma_{0r}$. Allowing for the expansion of $q_{0z}^{(j)}$ (12) by the small parameter β_j/γ_0^2, it is thus easy to obtain $z_0 \approx 2\tau_0 c\gamma_{0r}/|\chi_h|$. This distance decreases with a shortening of the pulse duration and with an increase in the modulation depth of the refraction index. For example, if $\tau_0 = 0.1$ ps, the splitting of the incident pulse into two pulses inside the PC will occur at a comparatively small depth, $z_0 \approx 0.94$ mm.

The pulse propagation dynamic in the PC thus unfolds as follows. At the initial moments of time, the pulse in the PC propagates as a uniform whole until a certain depth $z < z_0$ is reached and a pendulum solution is observed. The intensity of the total field in the PC, $|E_0 + E_h|^2$, is a two-dimensional periodical distribution with a period d along the PC surface and a period Λ along the z axis. In the region $z \approx z_0$, the pulse begins to split into two pulses (see Fig. 3a) that propagate independently at different group velocities. Figure 3a shows the oscillation structure in the reciprocal intersection region of both pulses.

Each of the split pulses contains only one mode (Borrmann or anti-Borrmann) and is the sum of spatially and temporaly coinciding transmitted and diffracted pulses. The distance between the pulses grows linearly with an increase in depth z (Fig. 3b). The pulses propagate perpendicularly to the PC surface in a channel with a width of $2r_0/\cos\theta$; this is explained by the returning influence of diffraction-induced reflection from the periodic layers of the PC on the field.

At the PC output, the pulses split spatially. As a result, two transmitted pulses propagate along the direction $\mathbf{k}_0$ and two diffracted pulses propagate along the direction $\mathbf{k}_h$.

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