THE STRUCTURE OF THE INTERNAL TRANSITION LAYER IN THE REACTION-DIFFUSION PROBLEM IN THE CASE OF A BALANCED REACTION WITH A WEAK DISCONTINUITY

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One of the actual problems of the theory of singular perturbations at present is the study of nonlinear singularly perturbed partial differential equations, the solutions of which have boundary and inner layers. Such equations are of great interest both in the qualitative theory of differential equations and in many applied problems. In particular, in mathematical models of chemical kinetics, synergetics, nonlinear wave theory, biophysics and other fields of physics, where the processes under study are described by nonlinear parabolic equations with small parameters at derivatives. Solutions to such problems may contain narrow areas of rapid parameter change: boundary or internal transition layers (contrast structures) of various types – stationary or moving fronts [1].

Reaction-diffusion and reaction-diffusion-advection equations are also intensively studied due to the fact that they act as mathematical models that reveal the main mechanisms that determine the behavior of more complex physical systems. In particular, the well-known FitzHugh-Nagumo system in the stationary case can be reduced to the problem considered in the article. Also, the system of equations of the drift-diffusion model of a semiconductor with an N-shaped dependence of the drift velocity on the electric field strength is reduced to the problem posed below.

The reason for the formation of transition layers (contrasting structures) in singularly perturbed reaction-diffusion-advection models can be the fulfillment of the reaction balance condition at some point or on some curve lying in the field of consideration or advection balance, as well as the gap of coefficients by spatial coordinate [2–5].

In the works [2, 3] for the case of continuous coefficients, the so-called *critical case* was considered when the reaction balance condition is fulfilled identically, i.e. at any point in the domain. In this paper, we consider a boundary value problem for a nonlinear singularly perturbed reaction-diffusion equation in the critical case in the presence of a weak discontinuity of the reactive term. By *weak discontinuity* is meant a discontinuity of the first kind at some point, which the function of sources undergoes in the first order with respect to a small parameter.

The following problem is considered:

$$\begin{cases} N_\varepsilon u := \varepsilon^2 \dfrac{\partial^2 u}{\partial x^2} - f(u,x) - \varepsilon f_1(u,x) = 0, \quad -1 < x < 1, \\ f_1(u,x) := \begin{cases} f_1^{(+)}(u,x), & u \in I_u, \quad x > x_p \\ f_1^{(-)}(u,x), & u \in I_u, \quad x < x_p \end{cases} \\ \dfrac{\partial u}{\partial x}(-1,\varepsilon) = 0, \quad \dfrac{\partial u}{\partial x}(1,\varepsilon) = 0\,. \end{cases} \tag{1}$$

Here $0 < \varepsilon < \varepsilon_0 \ll 1$ – small parameter, $x_p \in (-1;1)$, I_u - the function $u(x,\varepsilon)$ change segment.

For this singularly perturbed reaction-diffusion equation the structure of the internal transition layer is investigated in the case of a balanced reaction with a weak discontinuity. It is shown that in the case of the balanced reaction, the presence of even a weak (asymptotically small) reaction discontinue can lead to the formation of several contrast structures of finite size, which may be stable or unstable.

The conditions under which there is a solution of the contrast structure type having an internal transition layer localized in the vicinity of the reaction break point are formulated. The existence of solutions with an internal transition layer (contrast structures) is proved, the question of their stability is investigated, and an asymptotic approximation with respect to a small parameter of this solutions is constructed. Sufficient conditions are formulated that determine either the asymptotic Lyapunov stability or the instability of each such solution.

The asymptotic approximation is constructed according to the method [6]; the asymptotic method of differential inequalities is used to prove the existence of the solution [7], as well as the asymptotic method of differential inequalities developed for problems with discontinuous nonlinearities [4, 5]; the study of stability is carried out by the method of tapering barriers [2].

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DIFFERENTIAL EQUATION OF THE NEUTRAL DELAY TYPE WITH A DISCONTINUOUS NONLINEARITY

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Consider the equation

$$\dot{x} + x = \alpha \cdot \operatorname{sign}(\dot{x}(t - T)), \tag{1}$$

where α is a nonzero constant and let S_h be the class of initial conditions defined as

$$S_h = \{\varphi(t) \in C^1[-T, 0] \;\;:\;\; \dot{\varphi}(t) > 0,\ \varphi(0) = h\}, \tag{2}$$