

# The monoculture vs. rotation strategies in forestry: formalization and prediction by means of Markov-chain modelling

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## Abstract

The monoculture strategy of forest management, where the same tree species (e.g., *Picea abies*) is cultivated in a number of successive planting–growing–felling cycles, is generally considered to be economically efficient, yet not sustainable as it reduces biodiversity in the forest. The sound alternative suggests a long-term strategy of forest management in which different forest types rotate either with planting after clear cutting, or by natural forest succession, yet the commercial output remains dubious. We suggest an approach to formalization and modelling forest dynamics in the long-term by means of Markov chains, the monoculture strategy resulting in an absorbing chain and the rotation one in a regular chain. The approach is illustrated with a case study of Russkii Les, a managed forest located in the Moscow Region, Russia, and the nearby forest reserve having been used as a data source for undisturbed forest dynamics. Starting with conceptual schemes of transitions among certain forest types (states of the chain) in the monoculture and rotation cases, we estimated the transition probabilities by an original method based on average duration of the corresponding states and on the likelihood of alternative transitions from a state into the next one. Formal analysis of the regular chain reveals an opportunity to achieve particular management objectives within the rotation strategy, in particular, to get the distribution of forest types in accordance with an adopted hierarchy of their commercial values, i.e. more valuable types have greater shares.

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## 1. Introduction

Prediction of the future dynamics in natural resources, including forests, and comparison of alternative strategies in the nature management, are widely recognised nowadays to be a matter of modelling. Traditional (i.e. time-homogeneous) Markov-chain models have long been known as a means to describe, in formal and quantitative terms, the successional series in vegetation and their final states typical to a given bioclimatic zone or a given edaphotope. Markov models are attractive because they are simple, probabilistic and history-independent. They do not require deep insight

into the mechanisms of dynamic change, but can help to specify areas where such insight would be valuable and therefore act as both a guide and stimulant to further research.

One of the early applications of Markov models in ecology was by Williams et al. (1969) in their study of rainforest communities. Stephens and Waggoner (1970) used Markov models to predict the composition of a mixed hardwood forest in the United States. Well-known examples of the use of Markov models to characterize forest successions were given by Horn (1975), in which the succession was estimated by simple approximations of the plant-by-plant replacement process. Other studies of the use of Markov models in forests include those of Usher (1979, 1981), Tavaré (1979); Hubbell and Foster (1987); Acevedo et al. (1995); Denisenko et al. (1996); Balzter (2000) (see also reviews in Mirkin and Naumova (1984); Blatt et al. (2001); Benabdellah et al. (2003)).

Markov models were widely criticized for their inadequacies as models of natural systems (Usher (1981);

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Hobbs (1983)). Serving as a good didactic means, the traditional models did not become tools for reliable ecological prediction and made way for much more sophisticated approaches, like ‘gap’-modelling (Shugart, 1992). However, the current progress in applications of computer-aided, geo-information systems (GIS) to forest studies has motivated a new generation of Markov (and non-Markov sometimes) models to be developed, with a potential to yield real ecological predictions and to answer questions of practical importance.

Our research efforts were aimed at further development of non-absorbing-chain models able to describe management/rehabilitation successions. A class of non-absorbing-chain models has been constructed and analysed, which enables description of successions under alternative forest management sustaining biodiversity in the forest. With an aggregated model comprising 7 diverse types of forest, a hierarchy of economic values among those types is shown to be compatible with, and achievable by, an ecologically sound strategy of forest management.

Our modelling approach is based on a new method (Logofet and Lesnaya (2000); Korotkov et al. (2001); Logofet et al. (2001); Logofet and Korotkov (2002)) proposed for evaluating the transition probabilities of the model from expert-given data on duration times of successional stages, based on a fundamental result of Markov-chain theory. The method is free of the ergodic hypothesis behind, and bulky statistical routines inherent in, the former approach of landscape ecology. However, the new method requires a deeper insight into how long it takes for a particular stage of forest succession to change into the next one. To evaluate this stage duration time, a technique has been developed that uses modern views on, and the ensuing classification of, the ontogenetic states in the population of a tree edicator<sup>1</sup> species. The method and the technique, while being the core of the novel methodology of Markov-chain modelling, still leave some uncertainties in the model construction, as some of the required coefficients can only be obtained by model calibration. Supplied with the calibration techniques, the novel methodology has turned the (new generation of) Markov-chain models of forest successions from a good didactic means into a well-grounded tool for ecological predictions.

Management strategies towards plantation of a commercially profitable spruce monoculture and a long-term strategy of forest management, in which a variety of forest types are planted or naturally developed at the place of felled areas, are studied in this paper. The spruce monoculture strategy yields logically an absorbing chain in which the spruce forest type is the sole absorbing state. The alternative strategy yields, on the contrary, a regular Markov chain with a steady-state distribution (in terms of the relative area for the forest type) in

which all the types are present (Korotkov (2000)). It thus illustrates qualitatively the biodiversity paradigm in forest management and can give quantitative certainty to geobotanic views of what should be the shares of small-leaved, coniferous, and broad-leaved tree spp. in weakly disturbed coniferous-broadleaf forest.

In this paper, we suggest an approach to formalization and modelling of forest dynamics in the long-term by means of Markov chains, the monoculture strategy resulting in an absorbing chain and the rotation one in a regular chain. The approach is illustrated with a case study of Russkii Les Experimental Forestry (Moscow Region, Russia). Section 2 gives a summary of our approach in Markov-chain modelling of successions. Two models of successions are proposed in Section 3. The first model mirrors the current policy of forest management in Russkii Les. Spruce monocultures represent an absorbing state in this model. An alternative model of forest management suggests rotation of the dominant tree species in the forestry cycle and accounts for the relative economic value of different forest stands. The current area distribution among the forest types has been calculated by means of GIS-technology.

## 2. Formalism of Markov-chain models for successions

Numerous sources give detailed explanations of how regular Markov chains are formulated and some of them provide discussions with emphasis on modelling ecological processes (see, for example, Jeffers (1978); Usher (1992); Guttorp (1995)). To construct a Markov chain for successions, one identifies the states of the chain with specified stages of succession and enumerates them by natural numbers. At any time  $t$ , the chain can be in one of some finite number  $n$  of states. The set  $S = \{1, \dots, n\}$  is thereafter the set of all the chain stages, or stages of succession. The matrix of transition probabilities  $P$  provides a compact and unique description of the behaviour of a Markov chain. Each element  $p_{ij}$  in the matrix  $P$  represents the probability of the transition from a particular state  $j$  to the next state  $i$ . If the chain is in a state  $j$  at a given time moment  $t$ , then the probability that it will be in a state  $i$  at a subsequent time  $t+1$  does not depend on the chain behaviour before the time  $t$  (the Markov property, see e.g. Karlin, 1968):

$$p_{ij} = \Pr(x_{t+1} = i | x_t = j) \quad (1)$$

where  $x_t \in S$  is the state at time instant  $t$  (a first-order chain). The probabilities in each column add up to 1 (column stochastic matrix), and a row represents all possible ways of arriving at one state. A regular chain can be iterated via matrix multiplication to find the probability of each type of transition occurring in more than one time step.

The states of a Markov chain for succession are traditionally identified with the distinguishable stages of

<sup>1</sup> A dominant species rendering the most influence on community composition.

succession, while the pattern of the transition probability matrix  $P=[p_{ij}]$  (see Eq. (1)) is defined by a given scheme of successional transitions. The major problem is to estimate the entries of the transition probability matrix. Logofet and Lesnaya (2000) proposed the following method of  $p_{ij}$  estimation. If  $m_j$  is the average duration (expert-)given for stage  $j$  and expressed in the number of time steps, then the diagonal elements of matrix  $P$  are

$$p_{jj} = 1 - \frac{1}{m_j}, \quad i = 1, 2, \dots, n. \quad (2)$$

If there are  $k \geq 1$  arrows  $j \rightarrow i$  outgoing from state  $j$ , then

$$p_{ij} = \frac{1}{km_j} \quad (3)$$

(under assumption that all the transitions from state  $j$  are equiprobable<sup>2</sup>), otherwise  $p_{ij}=0$ . If we presume  $m_i > 1$ , then  $p_{ij} > 0$  for any  $j$ .

The successional system is described by a stochastic vector  $x(t)$  whose component  $x_i(t)$  shows the probability that the chain is in state  $i$  ( $i=1,2,\dots,n$ ) after  $t$  time steps ( $t=0,1,2,\dots$ ), interpreted normally as the relative area distribution among the states. Vector  $x(t)$  is called the state probability vector. The Markov property results in the well-known linear dynamic equation:

$$x(t+1) = Px(t), \quad t = 0, 1, 2, \dots, \quad (4)$$

with the general solution

$$x(t) = P^t x(0), \quad t = 0, 1, 2, \dots \quad (5)$$

(see e.g. Roberts (1976)).

The limit distribution  $x(\infty)$  to which an initial state converges (for great enough  $t$ ) can be found either in an elegant algebraic way through solution of the eigenvalue-eigenvector problem, or by mere multiplication of matrix  $P$  by itself a great enough number of times. The convergence behaviour of homogeneous chains, together with the 'equilibrium' terminology, fits surprisingly well to the Clementsian paradigm of succession theory suggesting a persistent motion to the climax state of vegetation which is in equilibrium with the environment (Logofet and Lesnaya (2000)).

In the time-homogeneous case, both the equilibrium distribution and the time characteristics (the mean first-passage time in a regular chain or the mean time before absorption in an absorbing chain) can be found by known routines and formulae of matrix algebra from the transition probability matrix.

Mathematical properties of the Markov-chain formalism in the time-heterogeneous case have been used before to

develop methods of the current-state projection and time prediction. The time-heterogeneous Markov-chain model features  $P(t)$  the transition probability matrix depending now on  $t$  unlike the fixed matrix  $P$  of the homogeneous model. It arises logically when certain key factors are known to govern the course of succession and a certain scenario is assumed to govern the course of the factors in time. The basic Eq. (4) turns now into

$$x(t+1) = P(t)x(t), \quad t = 0, 1, 2, \dots, \quad (6)$$

with the solution

$$x(t) = P(t-1)P(t-2)\dots P(0)x(0), \quad (7)$$

now losing the algebraic elegance of the homogeneous case. The projection task of the model reduces therefore to correct formalisation of the (known or hypothesised) scenarios into pertinent entries of matrix  $P(t)$ . The prediction task of the model concerning the expected time it would take to reach intermediate or terminal stages of succession is generally reduced to calculation of sojourn times.

$$N = (I - Q)^{-1} \quad (8)$$

Here  $I$  is the identity matrix. To define the matrix  $Q$  it is necessary to consider the canonical form of the matrix  $P$  which unites all  $n-s$  ergodic sets, and  $s$  transient sets, namely,

$$P = \begin{pmatrix} S & O \\ R & Q \end{pmatrix},$$

where matrix  $O$  consists entirely of 0's, the  $s \times s$  submatrix  $Q$  concerns the process as long as it stays in transient states; the  $s \times (n-s)$  matrix  $R$  deals with the transition from transient to ergodic states, and the  $(r-s) \times (r-s)$  matrix  $S$  concerns the process after it has reached an ergodic state (Kemeny and Snell (1960)).

While calculation of these characteristics in the homogeneous case was based upon the known meaning that the entries of the so-called fundamental matrix  $N$ , have in a homogeneous chain with absorbing states (Kemeny and Snell (1960)), this formula no longer makes sense for an inhomogeneous chain. However, an equivalent representation of matrix  $N$  in the form of an infinite series,

$$N = I + Q + Q^2 + Q^3 + \dots, \quad (9)$$

does suggest to use its finite sum corresponding to a given scenario of sufficiently long duration  $t_f$ ,

$$N(t_f) = I + Q(0) + Q(1)Q(0) + \dots + Q(t_f)\dots Q(1)Q(0) \quad (10)$$

as an analogue of the fundamental matrix to estimate the sojourn times. Calculation by Eq. (10) yielded matrices  $N(t_f)$  whose relative error has never, in our practice, exceeded 4% as compared with direct calculations of random-walking trajectories along a given chain with matrix  $P(t)$ . In non-Markov settings (see Korotkov et al. (2001)), it is possible to

<sup>2</sup> The assumption is not crucial for the method and can be easily relaxed to account for any knowledge on which of the alternative transitions might be more likely.

combine the Markov approach as above while the model stays within the ‘Markov’ regions of the phase space (till absorption in a subclimax state) with expert estimates for the subclimax-to-climax transition.

### 3. Absorbing/non-absorbing-chain models under alternative strategies of forest management

Field studies and the analysis of land-use history have revealed (Offman (1999); Ponomarenko et al. (1999); Smirnova et al. (1999); Zaugolnova (2000)) that there are only successional communities in the current vegetation cover, whereas the climax and subclimax ones are completely absent on the territory of Russkii Les Experimental Forestry and adjacent Prioksko-Terrasnyi Biosphere Reserve founded in 1945 (Zabelina et al. (1998)).

In Russkii Les Experimental Forestry, the cycle of forestry actions now begins with planting forest cultures of coniferous species onto felled areas and terminates with final felling of the mature stands. The present-day policy of forest management tends to formation of primarily spruce monocultures (for faster gains in commercial wood) featuring poor composition of potential community-forming species. This kind of forest management can result in a drastic decrease in floristic diversity and a drastic disturbance of the natural processes in forest ecosystems (Chumachenko et al. (2003); Denisenko et al. (2002); Smirnova (2004)).

To construct a model means practically to determine the transition matrix in quantitative terms. The tradition of landscape ecology in solving this problem is to estimate each transition probability as the frequency of the corresponding event registered in observations. It has been found however (see Logofet and Lesnaya (2000), and references therein) that modern observations cannot meet the formal requirement for the number of ‘repetitions of the trial’ which is needed for reliable substitution of the event frequency for the event probability, and the necessity of another methodology thus becomes evident.

Particular schemes have been elaborated for the stand dynamics under alternative scenarios of forest management. Danki forest district in Russkii Les Experimental Forestry has been studied as it represents all the main types of forest typical of the Moscow Region. This forestry borders with Prioksko-Terrasnyi Biosphere Reserve, which has similar forest types and therefore some data for that reserve has been used to calibrate the model. The classification of forest stands is done for the dominant tree species and age classes, which determine the ages of intermediate and principal fellings.

The cartographic database for Danki forest district in Russkii Les Experimental Forestry has been developed by means of TopoL<sup>®</sup> (TopoL Software, 1998), a universal software tool to create geo-information systems. TopoL<sup>®</sup>

enables one to work both with raster data and with cartographic vector data, associating the data with semantic information not only locally but also as external databases (Greshnov and Starostenko, 1999; Korotkov et al., 2000; Starostenko, 2000). Cartographical materials are all presented in the uniform system of Gauss-Kruger coordinates. Topographic maps of the scale 1:10,000 provide the cartographic basis (the raster layer), and the vector layer with forest strata boundaries (at the same scale) is referred to the basis. Forest maps are correspondingly related with an accuracy of 10–20 meters. The cartographic database has been created by D.A. Starostenko and V.N. Korotkov on the basis of forest inventory data presented by State Forest Inventory Enterprise ‘Moslesproekt’. Each polygon (forest stratum) has an associated external database with characteristics of the forest ecosystem state, including data on the tree, undergrowth, and underwood layers and the grass cover. The databases are compiled of data from a number of forest inventories and complemented with data from geobotanic field studies.

Based on the databases of forest inventories of ‘Danki’ forest district, the following 7 types of tree stands, which are the most widespread in forest tracts, have been specified: spruce forest, pine forest, birch forest, aspen forest, lime forest, oak forest of seedling origin, oak forest of coppice-growth origin (Fig. 1(A)). These forest types are divided further into young stands, middle-aged stands, maturity-approaching stands, mature stands and overmature stands (Fig. 1(B)). The maturity ages for different types of forest are defined by administrative bodies of forest management. When stands come to the age of maturity, they are clear cut, which may happen in one or two cuts.

The following scenarios are considered to describe the forest management cycles:

- Scenario 1 spruce-targeted management-mirrors the current tendencies in forest management since only spruce monocultures have been created on felled areas in the recent 20–30 years (Fig. 2);
- Scenario 2 ecologically sound management with rotation of dominant tree species and conservation of biodiversity (Fig. 3);

and the corresponding models have been constructed and analysed by both graph-theoretic and mathematical means.

Stages of succession were specified in the conceptual scheme of succession transitions. When we use a scenario with given clear-cut durations and with given area ratios between the types of forest which regenerates after the clear-cuts, all the model parameters can be evaluated explicitly, and the transition probabilities can be calculated without uncertainties using Eqs. (2) and (3). To estimate the model parameters for the natural course of succession we use the calibrated model (as an initial estimation) for adjacent Prioksko-Terrasnyi Biosphere Reserve (Logofet and Korotkov (2002)), which has been calibrated to fit its



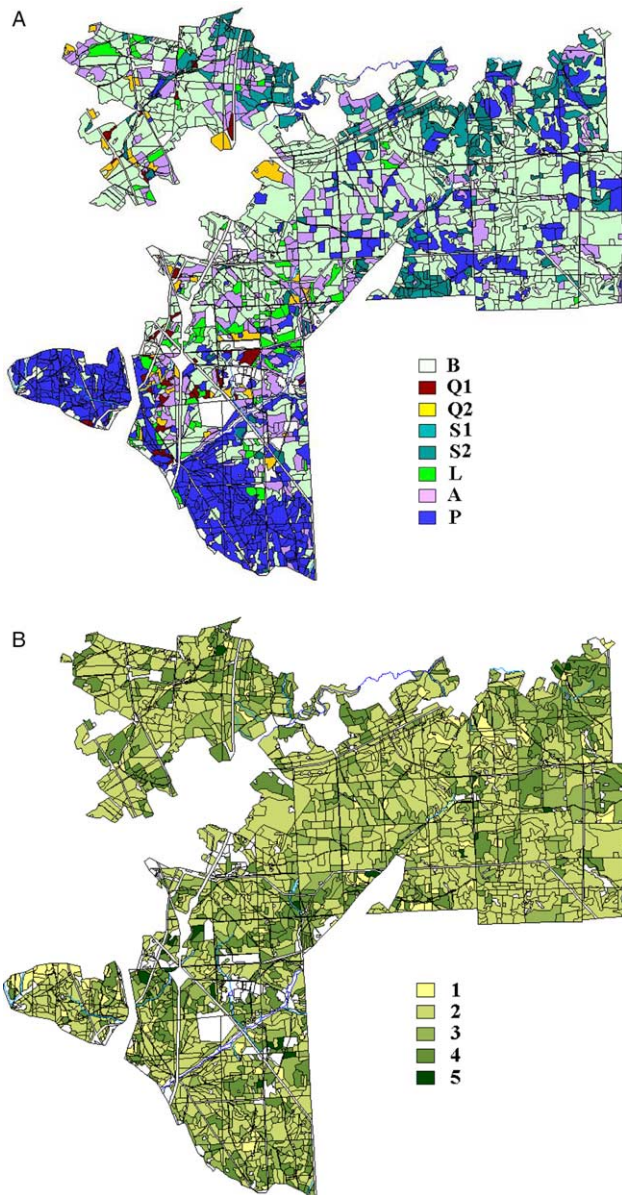


Fig. 1. Map of Danki forest district in Russkii Les Experimental Forestry. (A) Spatial distribution of the forest types enlisted: B—birch forest, Q1—oak forest of seedling origin, Q2—oak forest of coppice -growth origin, S1—spruce forest cultures, S2—spruce forest of natural origin, L—lime forest, A—aspens forest, P—pine forest. (B) Age gradation of stands: 1—young stands, 2—middle-aged stands, 3—maturity-approaching stands, 4—mature stands, 5—overmature stands.

observed state variables in terms of the relative area distribution among the specified forest types. The idea of calibration (Logofet and Korotkov (2002)) is simple: the 1955 data were taken as the initial state vector  $x(0)$ . This vector was projected by Eq. (7), to the year 2000. Taking the 1999 data as a reference vector the uncertain parameters of the model were chosen in a way that minimises deviation of the model projection from the reference vector. Since the method allows one to restore the stage duration data from the diagonal entries of a given matrix  $P(1)$  in a unique way, this was done for the calibrated matrix.

Our original method of chain construction using given or estimated average duration for stages and using given or estimated likelihood ratios between the alternative transitions (types of new plantations in the case of Russkii Les, Scenario 2) gives a certain advantage in formalizing tasks for forest management and in the task of the natural regeneration modelling, when it is impossible to construct a plausible model without calibration. Validation, as the next step in model development, has to wait for the next forest inventory data, while some calibrated parameters will have still to be modified to be in conformity with a future scenario of Russkii Les management.

The scenario-1 model could hardly (and has not) reveal(ed) anything new in the fate of the forest, but it served a basis for special methodology studies. With a simple forestry cycle, it has been studied whether the time-homogeneous or hetero-homogeneous model formulations should be applied and when a non-Markov scheme can be recommended as a formalism for forestry dynamics in the long term.

Two versions have been suggested for this scenario. The first one implies analysis of natural regeneration under the forest canopy. Clear cuts with undergrowth conservation are modelled for the forests with spruce undergrowth. If the spruce undergrowth is preserved on a clear-cut area, then duration of stay at the young stage can decrease by 10–20 years. When the natural regeneration of spruce is absent on the cut area, a monoculture of this tree species is being created. The second version implies clear cuts of mature stands without undergrowth conservation, followed by spruce monoculture plantation on the clear-cut. Young growth tending is assumed in both versions to decrease the share of small- and broad-leaved tree species in the composition of stands under formation.

Fig. 4 represents the graph of a Markov-chain model that corresponds to the long-term scheme of spruce-targeted forest management presented in Fig. 2. It features the tendencies typical to spruce silviculture, and the formal analysis shall naturally ‘reveal’ the logical fate of the system to be monocultural. Table 1 shows a logical way to aggregate certain vertices of the graph (or the states of the scheme) into those corresponding to the forest types. Thereafter, the condensation of the graph (i.e. the factor-graph on strongly connected components of the original graph) coincides with the aggregation (Fig. 5; see, e.g. Harary et al., 1965, for the formal graph-theoretic notions mentioned). It shows apparently that spruce forest (S1) is the only absorbing state in the chain, and this conclusion is illustrated by model calculations too (see Table 2), with a time-homogeneous Markov-chain model constructed for the scheme of Fig. 5 by a method discussed in the previous chapter. Time predictions can be obtained by a standard technique for absorbing Markov-chain models.

Fig. 6 represents the graph of a Markov-chain model that corresponds to the long-term scheme of ecologically sound forest management presented in Fig. 3 (Scenario 2).

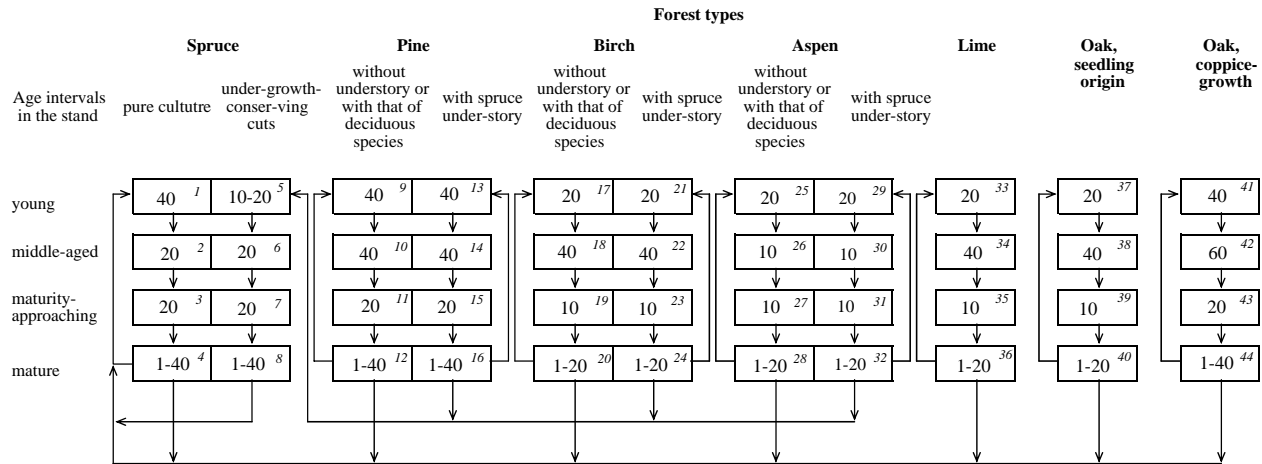


Fig. 2. A scheme of transitions in the long-term forest dynamics under spruce-targeted management with spruce undergrowth conservation.

It features rotation of the dominant tree species after each forest cut; hence the components of different forest types now become interrelated. All the forest types enlisted, except for Q2 (see Table 3), belong to an ergodic set F of the chain states, so that standard predictions of the time-homogeneous, regular Markov-chain model (see Logofet and Lesnaya (2000)) can be obtained by the standard methods. However, to reveal a feature of principal importance, we consider an aggregated model where all the age states of a particular forest type are combined into the one representing that particular forest (Table 3). The aggregated graph is shown in Fig. 7.

The corresponding Markov-chain model describes dynamics of the 7D-vector

$$x(t)[Q1\ P\ S\ B\ L\ A\ Q2]^T \quad (11)$$

governed by Eq. (4). The pattern of the transition matrix matches the graph in Fig. 7(b), while the entries are

calculated as suggested by Eqs. (2), (3) and the footnote given there. The durations of states required in the method are defined by 7 parameters  $m_j$  ( $j=1, \dots, 7$ ), which may vary as shown in Table 3 (mind numeration!). The idea of dominant species rotation generates alternative transitions from each state of the chain to some 2 or 3 other ones. The likelihood of those alternative transitions has been chosen in proportion to relative economic values of the forest types, i.e. the economic hierarchy

$$Q1)P)S)B)L)A)Q2 \quad (12)$$

which is adopted in the Russkii Les case study: the more valuable an alternative type is, the more probable is the transition to that type.

For the purpose of illustration, we fix the likelihood ratio at 4:2:1 for any 3 alternative transitions and at 2:1 for any 2.

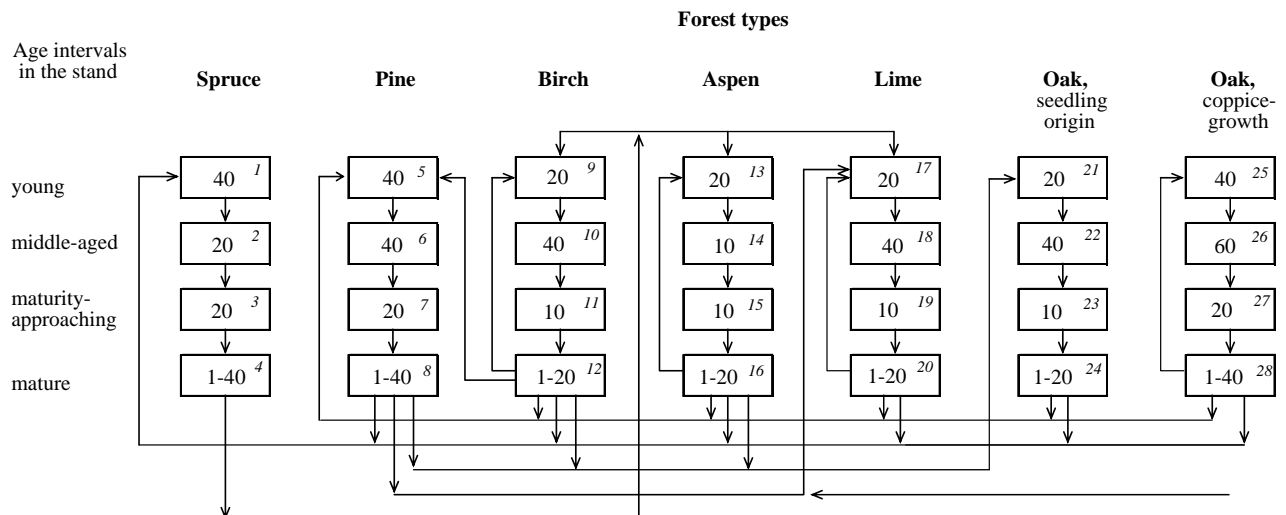


Fig. 3. A scheme of transitions in the long-term forest dynamics under ecologically sound management with rotation of dominant tree species.

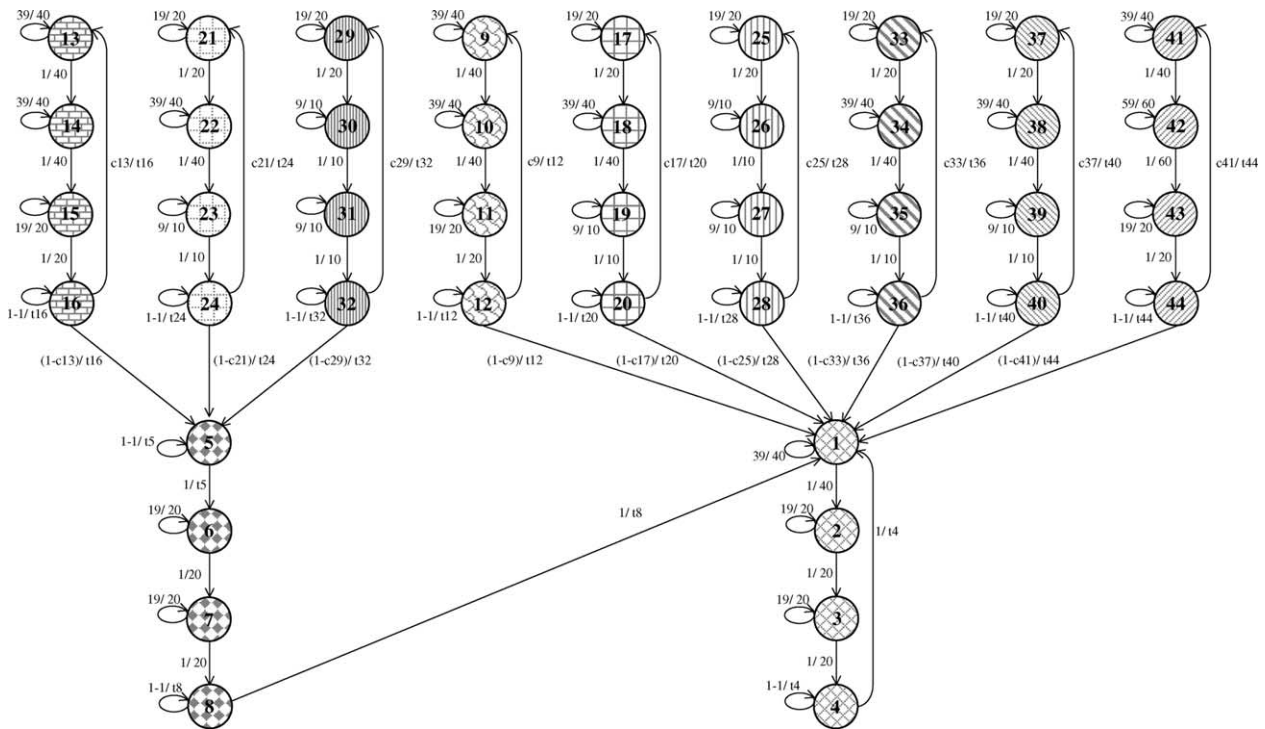


Fig. 4. Directed graph of a Markov chain model corresponding to the forest management scheme presented in Fig. 2. Parameters:  $tS$  ( $S=4,8,\dots,44$ ) means the average duration of stage  $S$ ;  $cS$  ( $S=9,13,17,\dots,41$ ) a portion of the mature area that is not rotated after felling.

The transition probability matrix then takes on the form

$$\mathbf{P} = \begin{bmatrix} (m_1 - 1)/m_1 & 4/7/m_2 & 0 & 4/7/m_4 & 0 & 4/7/m_6 & 0 \\ 2/3/m_1 & (m_2 - 1)/m_2 & 0 & 2/7/m_4 & 2/3/m_5 & 2/7/m_6 & 2/3/m_7 \\ 1/3/m_1 & 2/7/m_2 & (m_3 - 1)/m_3 & 1/7/m_4 & 1/3/m_5 & 1/7/m_6 & 1/3/m_7 \\ 0 & 0 & 4/7/m_3 & (m_4 - 1)/m_4 & 0 & 0 & 0 \\ 0 & 1/7/m_2 & 2/7/m_3 & 0 & (m_5 - 1)/m_5 & 0 & 0 \\ 0 & 0 & 1/7/m_3 & 0 & 0 & (m_6 - 1)/m_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (m_7 - 1)/m_7 \end{bmatrix}$$

It has been proved algebraically that the limit vector  $\mathbf{x}(\infty) = \mathbf{e}$  (the normalised dominant eigenvector of matrix  $P$ ) takes on a 6-parameter form shown in the left column of Table 4. Thereby, the hierarchy (12) holds true, indeed, in the area distribution since we have

$$296m_1 > 294m_2 > 245m_3 > 140m_4 > 70m_5 > 35m_6$$

for any feasible choice of parameters  $m_j$ . Vectors  $e$  are calculated for the minimal and maximal values of state durations  $m_j$  just illustrate the hierarchy numerically (Table 4).

The strict-hierarchy property in the equilibrium area distribution can be generalised for any likelihood ratio  $h_1:h_2:1$  where  $h_1 > \theta_1 h_2$ ,  $h_2 > \theta_2 1$  with  $\theta_1 = 80/71$ ,  $\theta_2 \approx 1.34$ .

Note also that the observed distribution is not hierarchical, nor can it be produced by a feasible choice of parameters even when excluding its last non-zero component. As a practical recommendation, a kind of non-linear optimal problem might be posed to find out the values of  $m_j$  and  $h_1:h_2:1$  which would minimise the difference between

the current vector  $\mathbf{x}(0)$  and the equilibrium one  $e$ . To solve the problem numerically would, however, require a software for constrained non-linear optimisation.

As regards time prediction in a non-absorbing Markov-chain model, a standard method is applied to calculate the matrix,  $M=[m_{ij}]$ , of the mean first passage time  $m_{ij}$ : if the chain starts from state  $j$ , then how long does it take to reach state  $i$  (Kemeny and Snell, 1960). The  $m_{ij}$  values (rounded to integers) of the mean first passage time matrices for the minimal duration of the stages (see Fig. 3) are:

$$M_{\min} = \begin{bmatrix} 3 & 194 & 279 & 166 & 294 & 136 & 294 \\ 220 & 3 & 296 & 239 & 170 & 209 & 170 \\ 339 & 327 & 5 & 358 & 289 & 328 & 289 \\ 707 & 695 & 368 & 10 & 657 & 696 & 657 \\ 1503 & 1491 & 1164 & 1522 & 20 & 1492 & 1453 \\ 2917 & 2905 & 2578 & 2936 & 2867 & 71 & 2867 \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

Table 1  
Graph condensation and model aggregation for the original scheme which represents 'a spruce-targeted forest management'

Vertex designation	Meaning	Forest type-age states	Total duration, yr	
			min	max
S1	Spruce forest with no under-wood	1–4	81	90
S2	Spruce forest	5–8	51	70
P1	Pine forest with no underwood or with deciduous underwood	9–12	101	110
P2	Pine forest with spruce under-wood	13–16	101	110
B1	Birch forest with no underwood or with deciduous underwood	17–20	71	80
B2	Birch forest with spruce under-wood	21–24	71	80
A1	Aspen forest with no under-wood or with deciduous under-wood	25–28	41	50
A2	Aspen forest with spruce underwood	29–32	41	50
L	Lime forest	33–36	71	80
Q1	Oak forest of seedling origin	41–44	121	130
Q2	Oak forest of coppice-growth origin	37–40	71	80

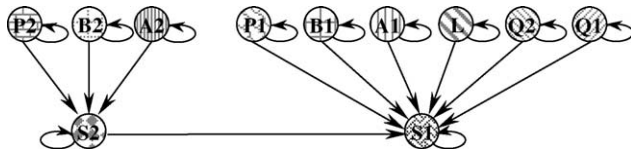


Fig. 5. Graph-theoretic manipulations with the model graph presented in Fig. 4: the aggregated graph (factor-graph) on the components specified in Table 1.

Coming back to the disaggregated model of Fig. 6, we can see that the hierarchal limit distribution among forest types is split up further among the age states in proportion to their age intervals (see Fig. 8).

The scenario-2 model has been thoroughly studied and it has demonstrated an advantage of the new methodology in a vivid way. With an economic hierarchy of the forest types adopted, it has been mathematically proved that their

equilibrium distribution can almost always follow the same hierarchy in terms of the relative area if the after-cut management retains certain (mathematically found) proportions among the alternative forest types to be developed on the cut areas. Thus, the ecological paradigm has been shown to be compatible with an economic one.

So, the model demonstrates that, with an ecologically sound strategy of forest management, the relative area distribution of forest types under sustainable management follows the economic hierarchy:  $e_1 > e_2 > \dots > e_7$ . The economic idea has thus appeared to be qualitatively compatible with, and integrated into, the ecological one. Quantitative aspects of such an integration can be developed further, along the above approach, into a computer-aided (more specifically, a GIS- and Markov-chain-model-aided), decision-support system for silviculture.

#### 4. Summary and conclusions

Mathematical properties of the Markov-chain formalism in the time-heterogeneous case have been used before to develop methods of the current-state projection and time prediction. In this paper a study of further versions of the Markov-chain models has been commenced, in which non-absorbing chains are found capable of describing the cycling management/rehabilitation successions. Our method is free of the ergodic hypothesis but requires data on how long it takes for a particular stage of forest succession to change into the next one.

Simulation experiments have been conducted with regard to the model calibrated for the Russkii Les study. Two scenarios have been analysed. The first scenario - spruce-targeted management- represents the current tendencies in forest management since only spruce monocultures have been created on felled areas in the recent 20–30 years. With a simple forest cycle it has been studied whether the time-homogeneous or heterogeneous model formulations should be applied and when a non-Markov setting can be recommended as the formalisms for forestry dynamics in the long term. The second scenario is forest

Table 2  
Model dynamics for the initial distribution,  $x(0)$ , of forest types which corresponds to the current area data

Chain state	Relative area distribution, $x(t)$ , after $t$ years:									
	$t$	0	1	10	100	200	300	400	600	$\infty$
Q1		0.0114	0.0113	0.0105	0.0050	0.0022	0.0009	0.0004	0.0001	0
P1		0.1520	0.1505	0.1376	0.0562	0.0208	0.0077	0.0028	0.0004	0
P2		0.0543	0.0538	0.0491	0.0201	0.0074	0.0027	0.0010	0.0001	0
S1		0.0504	0.0594	0.1382	0.6779	0.9018	0.9704	0.9908	0.9990	1
S2		0.0541	0.0580	0.0864	0.0951	0.0341	0.0102	0.0030	0.0003	0
B1		0.2688	0.2650	0.2333	0.0651	0.0158	0.0038	0.0009	0.0001	0
B2		0.2373	0.2340	0.2059	0.0575	0.0139	0.0034	0.0008	0.0000	0
L		0.0353	0.0348	0.0306	0.0085	0.0021	0.0005	0.0001	0.0000	0
A1		0.0729	0.0711	0.0569	0.0062	0.0005	0.0000	0.0000	0.0000	0
A2		0.0433	0.0422	0.0338	0.0037	0.0003	0.0000	0.0000	0.0000	0
Q2		0.0202	0.0199	0.0175	0.0049	0.0012	0.0003	0.0001	0.0000	0



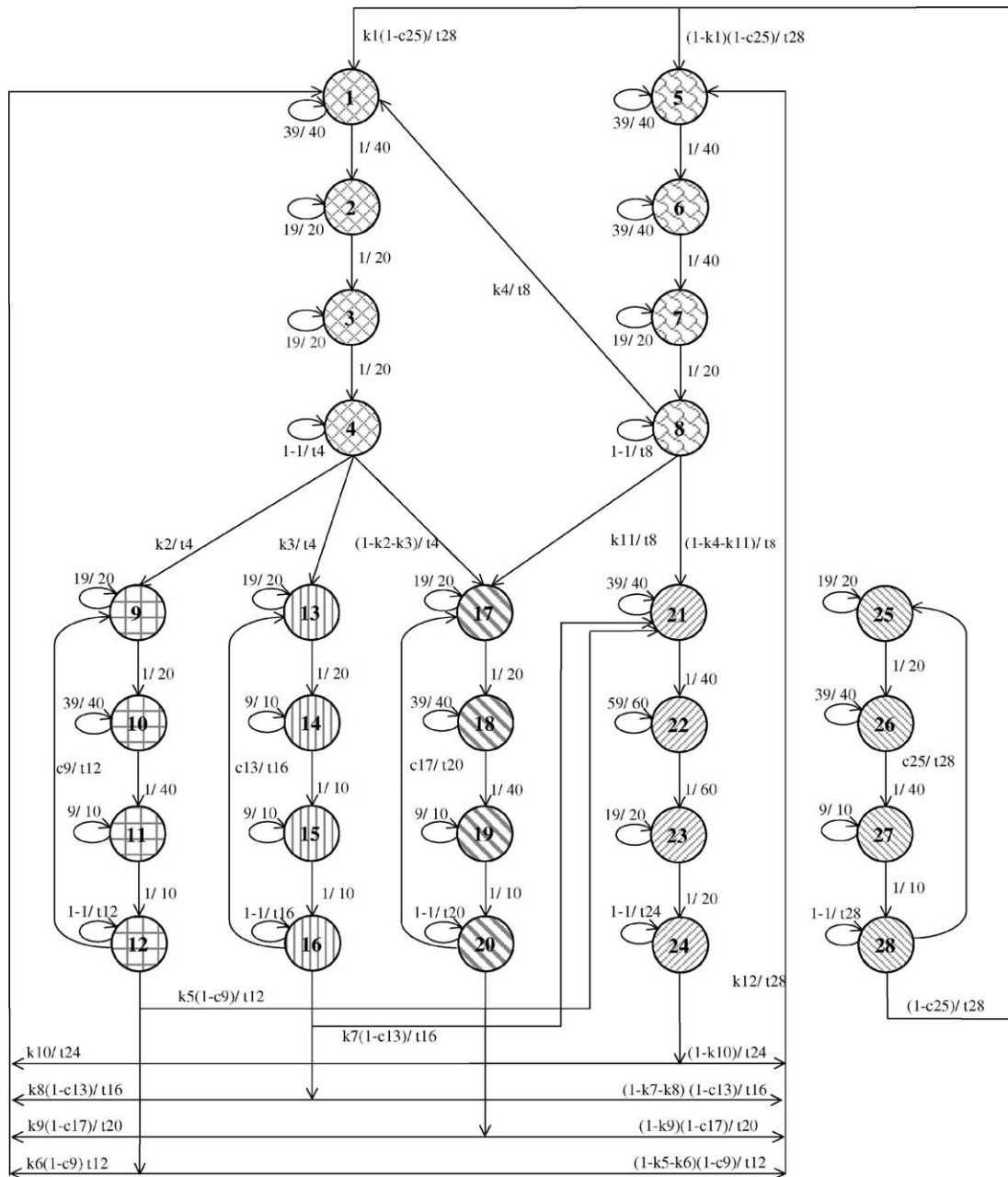


Fig. 6. Directed graph of a Markov chain model corresponding to the forest management scheme presented in Fig. 3. Parameters:  $tS$  ( $S=4,8,\dots,28$ ) means the average duration of stage  $S$ ;  $cS$  ( $S=9,13,17,\dots,25$ ) is a portion of the forest area that is not rotated after cut;  $k\#$  ( $\#=1,2,\dots,12$ ) is a likelihood coefficient for an alternative transition.

Table 3

Graph condensation and model aggregation for the original scheme which represents an 'ecologically sound forest management'

Vertex designation	Meaning	Forest type-age states	Total duration, yr	
			Min.	Max.
F	Mixed forest	1–24	$\infty$	$\infty$
S	Spruce forest	1–4	81	90
P	Pine forest	5–8	101	110
B	Birch forest	9–12	71	80
A	Aspen forest	13–16	41	50
L	Lime forest	17–20	71	80
Q1	Oak forest of seedling origin	21–24	121	130
Q2	Oak forest of coppice-growth origin	25–28	71	80

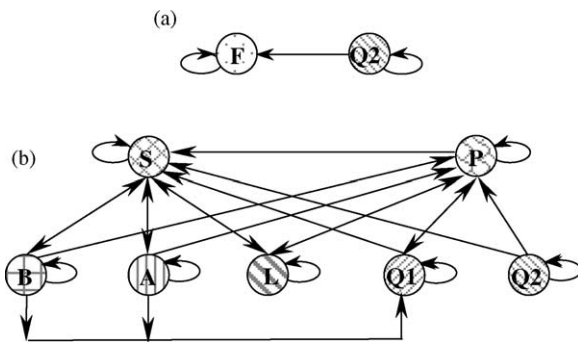


Fig. 7. Graph-theoretic manipulations with the model graph presented in Fig. 5. (A) the condensation on strongly connected components; (B) the aggregated graph (factor-graph) on the components specified in Table 3.

Table 4  
Relative area distributions among the forest types specified in Table 3

Model distribution $x(\infty)$ in a general form, with $\sum 296$ $m_1 + 294m_2 + 245m_3 + 140m_4 + 70$ $m_5 + 35m_6$	Calculated at $m_{\min}$ . (%)	Calculated at $m_{\max}$ . (%)	Observed (%)
Q1 296 $m_1/\Sigma$	35.2	34.5	1.2
P 294 $m_2/\Sigma$	29.2	29.0	20.6
S 245 $m_3/\Sigma$	19.5	19.8	10.5
B 140 $m_4/\Sigma$	9.8	10.1	50.6
L 70 $m_5/\Sigma$	4.9	5.0	3.5
A 35 $m_6/\Sigma$	1.4	1.6	11.6
Q2 0	0.0	0.0	2.0

management with rotation of dominant tree species after each forest cut and conservation of biodiversity. The likelihood of those alternative transitions has been chosen in proportion to relative economic values of the forest types: the more valuable an alternative type is, the more probable is the transition to that type. With the assumed economic hierarchy of the forest types it has been mathematically proven that their equilibrium distribution can almost always follow the same hierarchy in terms of the relative area if the after-cut management retains particular proportions among the alternative forest types to be developed on the cut areas.

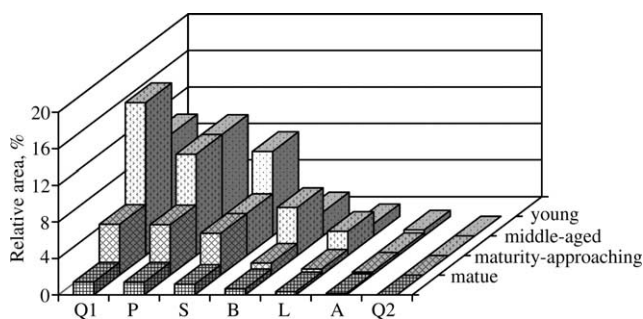


Fig. 8. Limit vector of the disaggregated model of Fig. 3 calculated for the average  $m$ -values between  $m_{\min}$  and  $m_{\max}$ .

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