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NONLINEAR OPTICS

New Mechanism of Solitons Formation at Diffraction on a Periodic Inhomogeneity Induced in a Cubic Nonlinear Medium

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Abstract—Formation of solitons during the propagation of the initial Gaussian pulse in a Kerr medium and its diffraction on the induced periodic layer structure are described using the computer simulation. Dependence of the characteristics and the number of the resulting solitons on the parameters of the structure is demonstrated.

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1. INTRODUCTION

In the last years propagation of optical signals in photonic crystals has been extensively studied [1–8]. A photonic crystal is a medium with a periodically changing refractive index and a typical scale of the order of the wavelength [1]. This gives rise to a photonic bandgap. Varying the characteristics of the periodic inhomogeneity, one can control propagation of light in these structures. Specifically, in photonic crystals it was experimentally found that discrete multicolored solitons can be formed in periodic structures [9]. Periodic structures also allow propagation of a train of one-dimensional solitons near the first Bloch band [10]. In [11, 12] the authors proposed a new approach that led to formation of high-intensity filaments in periodic lattices due to regularization of the nonlinear self-action mechanism by the lattice-induced diffraction.

It is well known that in a cubic nonlinear medium the Gaussian pulse undergoes self-focusing due to self-action. However, it propagates with periodic oscillations [13] and does not evolve into solitons whose properties are well studied [14–19].

In this work we investigate propagation of optical signals in a cubic nonlinear medium in which a periodic inhomogeneity is induced, for example, by a laser beam. This inhomogeneity occupies a relatively small part of the medium, but conditions for propagation of solitons in it differ from those in the surrounding nonlinear medium. Due to bandgaps, there can be situations where the pulse fails to pass through this induced photonic crystal as a whole. An expectable result is its splitting into several pulses propagating with different velocities, and their shape will be similar to that of a soliton due to the self-action mechanisms.

2. FORMULATION OF THE PROBLEM

In this work splitting of a pulse into several solitons as it moves through a periodic inhomogeneity in a cubic nonlinear medium is investigated using mathematical simulation.

The complex amplitude A of an optical pulse propagating along the z coordinate within time t was described by the dimensionless equation

$$\varepsilon \frac{\partial A}{\partial t} + iD\frac{\partial^2 A}{\partial z^2} + i\beta\varepsilon A + i\alpha|A^2|A = 0, \quad (1)$$

where ε is the permittivity, $D = 1/4\pi\chi$ is the dispersion coefficient, $\beta = \pi\chi$, χ is the ratio of the optical wave frequency to the periodic structure frequency, and α is the cubic nonlinearity coefficient. Zero boundary conditions were used. The width of the calculated region was chosen such that the field near the boundaries was close to zero during calculations. At the initial moment of time the pulse was given in the Gaussian form:

$$A(t=0) = \exp\left[-\frac{(z-z_0)^2}{a_z^2} + i2\pi\chi(z-z_0)\right], \quad (2)$$

where z_0 is the pulse center coordinate and a_z is the pulse width. This pulse propagated rightward over the calculated region, and its intensity was subject

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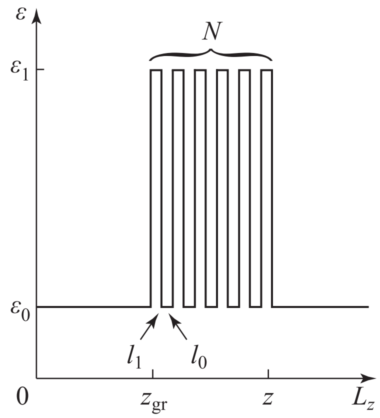

Fig. 1. Dependence of the permittivity on the z coordinate.

to oscillation due to self-action [13]. The nonlinearity coefficient α was constant in the entire medium. The permittivity varied in the following way (Fig. 1): it was equal to ε_0 over the entire nonlinear medium except the grating that began with the $z_{\rm gr}$ coordinate and consisted of N layers with $\varepsilon = \varepsilon_1$ of length l_1 alternating with layers $\varepsilon = \varepsilon_0$ of length l_0. The pulse approached the grating, passed through it, and propagated further. However, due to the bandgaps, the pulse shape changed. On leaving the grating, the pulse was divided into several solitons that moved with different velocities.

3. NUMERICAL SIMULATION OF THE SOLITON SPLITTING ON THE PERIODIC INHOMOGENEITY

To analyze soliton propagation, numerical simulation of Eq. (1) was performed with different parameters. Its results with the parameters $\chi = 1$, $\alpha = 10$, $a_z = 1$, $z_0 = 80$, $z_{\rm gr} = 100$, $N = 5$, $l_0 = l_1 = 1$, $\varepsilon_0 = 1$, and $\varepsilon_1 = 1.01$ are shown in Fig. 2: (a) the initial pulse, (b) the moment of passing through the photonic crystal, (c) after passing through the grating, the pulse begins to split, (d) the pulse is divided into several solitons moving with different velocities. The solitons are labeled with Roman numerals I, II, III, and IV. It is seen that even a small number of layers ($N = 5$) and a small difference in values ε leads to the pulse splitting, which begins in the grating and ends in the nonlinear medium with constant ε.

As is known [14], solitons in a cubic nonlinear medium have the form

$$A(z) = \frac{A_0 \exp(ivz)}{\cosh(z/l)}, \tag{3}$$

where A_0, l, and v, respectively, are the soliton amplitude, length, and velocity. The correspondence of the arising solitons with the solution (3) was analyzed using the integral metric

$$M = \frac{\displaystyle\int \left| |A_{\rm calc}|^2 - |A_{\rm anal}|^2 \right| dz}{\displaystyle\int |A_{\rm calc}|^2 \, dz}, \tag{4}$$

where $A_{\rm calc}$ is the numerical solution and $A_{\rm anal}$ is the analytic solution (3) with the amplitude and length corresponding to the numerical solution. At $M = 0$, the form of the numerical solution coincides with the soliton form. Since after the passing through the grating there are several solitons at each moment of time, integration was performed not over the entire z but in the vicinity of the soliton maximum, two its half-widths away. For each of the solitons shown in Fig. 2(d), the values of metric (4) were $M_{\rm I} = 5.012 \times 10^{-3}$, $M_{\rm II} = 4.373 \times 10^{-3}$, $M_{\rm III} = 6.448 \times 10^{-3}$, and $M_{\rm IV} = 5.254 \times 10^{-3}$. If we compare the Gaussian shape of form (2) using this metrics on the same interval, its value will be $M \approx 0.06$. Consequently, though the pulses formed after the passage through the induced grating are not exact cubic solitons of form (3), they are an order of magnitude closer to it than the Gaussian pulse with the coinciding amplitude and width.

Apart from the shape of the arising solitons, their phase was analyzed. As is evident from (3), the soliton phase linearly depends on the z coordinate. To a first approximation, the phase of the calculated solitons agrees to that, as is seen in Fig. 2. However, a more detailed analysis reveals some phase deviations. A derivative of the phase with respect to the z coordinate was taken. It turned out that the instantaneous velocity at the center of a soliton differs from the velocity at its periphery. For the first soliton, the velocity at the center was $v_{\rm m} = 5.91$, two half-widths away to the right it was $v_{\rm r} = 5.07$, and two half-widths away to the left it was $v_{\rm l} = 6.77$. This means that the right part of the soliton moves slower than its left part, and the soliton broadens. The velocity analysis of the other solitons shows that the second and the fourth solitons also broaden while the third one narrows. It is seen in Fig. 2(e) that the trajectories of the solitons are not continuous but rather consist of a series of maxima and minima, that is, solitons oscillate. This means that the form of the arising solitons does not completely coincide with the solution (3).

A series of numerical experiments was performed with different values of the inhomogeneity ε_1. The number of the arising solitons and their velocities were investigated. To find the velocities of the pulses, the derivative of the phase with respect to the z coordinate was averaged for each soliton in proportion to the intensity:

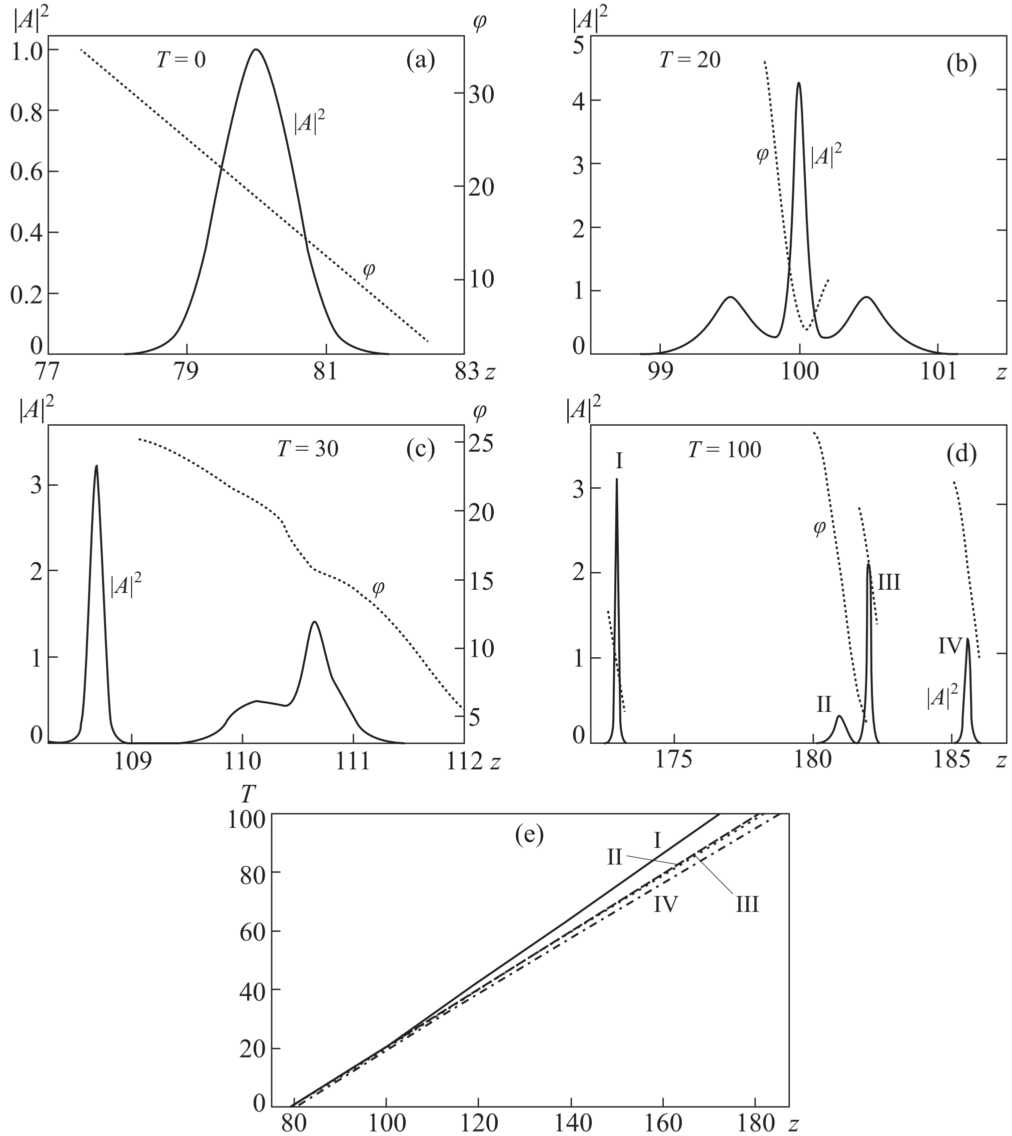

Fig. 2. Intensity and phase profiles of the pulse at $\chi = 1$, $\alpha = 10$, $a_z = 1$, $z_0 = 80$, $z_{\rm gr} = 100$, $N = 5$, $l_0 = l_1 = 1$, $\varepsilon_0 = 1$, $\varepsilon_1 = 1.01$, $T = 0$ (a), 20 (b), 30 (c), and 100 (d) and the pulse amplitude distribution (e). Roman numerals designate the arising solitons.

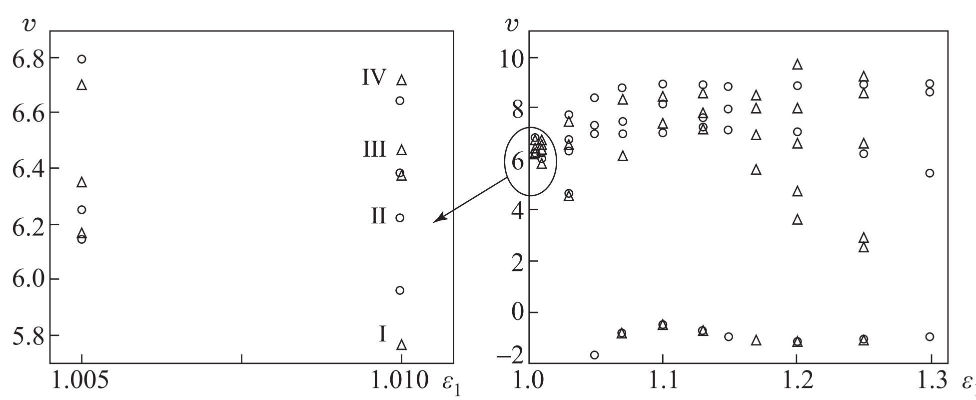

Fig. 3. Velocities of the arising solitons as a function of the inhomogeneity ε_1 at $\chi = 1$, $\alpha = 10$, $a_z = 1$, $z_0 = 80$, $z_{\rm gr} = 100$, $N = 5$, $l_0 = l_1 = 1$, and $\varepsilon_0 = 1$. The number of layers is $N = 1$ (○) and 5 (Δ). Roman numerals correspond to the solitons as in Fig. 2.

$$v = \frac{\int (\partial\varphi/\partial z)|A|^2 dz}{\int |A|^2 dz}, \tag{5}$$

and the integration was performed within two half-widths from the maximum of the soliton.

Figure 3 shows velocities as a function of the inhomogeneity ε_1. The data are for the single inhomogeneity $N = 1$ and the periodic grating of $N = 5$ layers. The velocity of the initial beam is $v_0 = 2\pi\chi \approx 6.3$. When ε_1 is not very different from $\varepsilon_0 = 1$, the velocities of the arising solitons are close to v_0, though slightly different. Some solitons are faster than the initial pulse, and some are slower. Because of this, the solitons diverge from one another by a certain distance with time. The number of solitons increases with ε_1 and their velocities become more and more different. At a certain ε_1 a soliton with a negative velocity arises, which corresponds to reflection from the grating.

The results can be experimentally verified. It is reasonable to use a pulsed laser, e.g., an ytterbium laser with a wavelength of 1064 nm, for producing the initial pulse. This laser is capable of generating pulses with the duration $T_0 \approx 100$ fs and the peak power density up to $P_0 \approx 1$ TW cm^{-2}, which is enough for effective nonlinear action in various media. A reasonable choice for the nonlinear medium is quartz glass with its dispersion coefficient $D \approx 200$ fs^2 cm^{-1} and dispersion length $L_\mathrm{d} = T_0^2/D \approx 50$ cm. The coefficient of nonlinearity in quartz is $\gamma \approx 2$ cm TW^{-1}, and the nonlinear length is $L_\mathrm{nl} = 1/\gamma P_0 \approx 0.5$ cm. The ratio of these lengths is $L_\mathrm{d}/L_\mathrm{nl} \approx 100$. In our numerical calculation, the dimensionless nonlinear and dispersion lengths are, respectively, $L_\mathrm{nl} \approx 0.1$ and $L_\mathrm{d} \approx 12.5$. Their ratio is $L_\mathrm{d}/L_\mathrm{nl} \approx 125$, which agrees with the proposed experiment.

4. CONCLUSIONS

It is shown with computer simulation that conditions for propagation of a self-focusing Gaussian pulse in a cubic nonlinear medium are violated in a periodic inhomogeneity. As a result, the pulse is divided into several solitons propagating with different velocities. It is found that the velocities of the arising solitons depend on the value of the inhomogeneity.

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REFERENCES

1. V.F. Shabanov, S.Ya. Vetrov, and A.V. Shabanov, *Optics of Real Photonic Crystals* (Siberian Branch, Rus. Acad. Sci., Novosibirsk, 2005) [in Russian].
2. J.-M. Lourtioz, H. Benisty, V. Berger, J.-M. Gerard, D. Maystre, and A. Tchelnokov, *Photonic Crystals: Towards Nanoscale Photonic Devices* (Springer, N.Y., 2005).
3. E.L. Ivchenko and A.N. Poddubny, "Resonant Three-Dimensional Photonic Crystals," Phys. Solid State. **48**(3), 581 (2006).
4. Y. Kominis and K. Hizanidis, "Lattice Solitons in Self-Defocusing Optical Media: Analytical Solutions of the Nonlinear Kronig–Penney Model," Opt. Lett. **31**(19), 2888 (2006).
5. Y. Kominis and K. Hizanidis, "Power Dependent Soliton Location and Stability in Complex Photonic Structures," Opt. Exp. **16**(6), 12124 (2008).
6. V.A. Trofimov, T.M. Lysak, O.V. Matusevich, Sheng Lan, "Parameter Control of Optical Soliton in One-Dimensional Photonic Crystal," Math. Model. Anal. **15**(4), 517 (2010).
7. J.D. Joannopoulos, S.G. Johnson, J.N. Winn, and R.D. Meade, *Photonic Crystals: Molding the Flow of Light* (Princeton University Press, Princeton, 2008).
8. Z. Chen, M. Segev, and D.N. Christodoulides, "Optical Spatial Solitons: Historical Overview and Recent Advances," Rep. Prog. Phys. **75**(8), 086401 (2012).
9. D.N. Neshev, A.A. Sukhorukov, A. Dreischuh, R. Fischer, S. Ha, J. Bolger, L. Bui, W. Krolikowski, B.J. Eggleton, A. Mitchell, M.W. Austin, and Y.S. Kivshar, "Nonlinear Spectral-Spatial Control and Localization of Supercontinuum Radiation," Phys. Rev. Lett. **99**(12), 123901 (2007).
10. X. Wang, Z. Chen, J. Wang, and J. Yang, "Observation of In-Band Lattice Solitons," Phys. Rev. Lett. **99**(24), 243901 (2007).
11. S. Suntsov, D. Abdollahpour, D.G. Papazoglou, P. Panagiotopoulos, A. Couairon, and S. Tzortzakis, "Tailoring Femtosecond Laser Pulse filamentation Using Plasma Photonic Lattices," Appl. Phys. Lett. **103**, 021106 (2013).
12. P. Panagiotopoulos, N.K. Efremidis, D.G. Papazoglou, A. Couairon, and S. Tzortzakis, "Tailoring the Filamentation of Intense Femtosecond Laser Pulses with Periodic Lattices," Phys. Rev. A. **82**(6), 061803 (2010).
13. S.A. Akhmanov, A.P. Sukhorukov, and R.V. Khokhlov, "Self-Focusing and Diffraction of Light in a Nonlinear Medium," Sov. Phys.-Usp. **10**, 609 (1968).
14. Yu.S. Kivshar and G.P. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals* (Academic Press, Amsterdam, 2003).

15. N.N. Akhmediev and A. Ankevich, *Solitons, Nonlinear Pulses and Beams* (Chapman and Hall, London, 1997).
16. B.A. Malomed, *Soliton Management in Periodic Systems* (Springer-Verlag, N.Y., 2006).
17. C.H. Tenorio, E.V. Vargas, V.N. Serkin, M.A. Granados, T.L. Belyaeva, R.P. Moreno, and L.M. Lara, "Dynamics of Solitons in the Model of Nonlinear Schrödinger Equation with an External Harmonic Potential: I. Bright Solitons," Quantum Electron. **35**(9), 778 (2005).
18. C.H. Tenorio, E.V. Vargas, V.N. Serkin, M.A. Granados, T.L. Belyaeva, R.P. Moreno, and L.M. Lara, "Dynamics of Solitons in the Model of Nonlinear Schrödinger Equation with an External Harmonic Potential: II. Dark Solitons," Quantum Electron. **35**(10), 929 (2005).
19. D.P. Caetano, S.B. Cavalcanti, J.M. Hickmann, A.M. Kamchatnov, R.A. Kraenkel, and E.A. Makarova, "Soliton Propagation in a Medium with Kerr Nonlinearity and Resonant Impurities: A Variational Approach," Phys. Rev. E. **67**(4), 046615 (2003).