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CONTROL THEORY

On the Analogue of the Isidori Relative Degree Definition for Linear Dynamical MIMO Systems[1]

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Abstract—This paper deals with the inversion problem for linear time-invariant dynamical systems. Earlier in this formulation the discussed issue has been addressed and solved for invertible systems that meet the definition of Isidori relative degree. But in fact, this problem is well-posed for a wider range of linear dynamical systems. In this paper we examine a particular case based on a definition that is similar to the definition of relative degree by Isidori. Presented here columnwise relative degree and its properties allow to consider and effectively solve the inversion problem for invertible systems that meet this definition.

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1. STATEMENT OF THE PROBLEM

The inversion problem for linear dynamical MIMO systems has been studied and partly solved earlier [1, 3, 4] in the following setting. We consider a linear (without loss of generality, discrete) dynamical "square" MIMO system of generic form:

$$\begin{aligned} x^{t+1} &= Ax^t + B\xi^t, \\ y^t &= Cx^t, \quad t = 0, 1, 2, \dots, \end{aligned} \tag{1}$$

with variables $x^t \in \mathbb{R}^n$, $y^t \in \mathbb{R}^l$, $\xi^t \in \mathbb{R}^l$ and with constant matrices A, B, C of appropriate sizes. We assume that the matrices B and C have full rank. The inversion problem is to find the unknown input signal ξ (probably with a delay), using its output y and constant parameters of the system, but making no use of the initial state vector values.

We study "square" systems, that is, systems with equal numbers of inputs and outputs because, on the one hand, the condition that the number of outputs is greater than or equal to the number of inputs is a necessary condition for invertibility, and in this sense the coincidence of these numbers is the most complicated case in the class of invertible systems. On the other hand, some of the definitions used below make sense only for square systems.

Note that invertible systems are those for which the inversion problem is well-posed, implying that the existence of a unique (in the sense of asymptotic convergence) solution can be established. Referring to some of the published results leads to the conclusion that the class of invertible systems contains a subset of multiple systems for which the inversion algorithms [3] are not applicable. Such systems we defined as "invertible", but not "constructively invertible" systems [4]. This set of systems is formed by invertible systems that do not meet the Isidori relative degree definition. It is natural to investigate the question of solving the inversion problem for such systems.

In a number of cases, the inversion problem can be reduced to the case of constructively invertible systems [4], regarding that a nonsingular linear transformation of only outputs of the system (the matrix C) may convert an invertible system to the Isidori relative degree compliant form. The algorithm of such transformation has been suggested by the author earlier. As an example, for any MIMO 3-d order linear system with 2 inputs and 2 outputs a possibility of such transformation has been proved [5]. Here we'll show that this is not the case in general. Also one of possible workarounds will be suggested.

A significant term for the following considerations is the term of relative degree, introduced by the following definition.

Definition 1 (Isidori). A vector $r = (r_1, r_2, \dots, r_l)$ is referred to as the vector of relative degree of system (1) if the following conditions are satisfied simultaneously:

(1) $C_iA^jB = 0, j = 1, 2, \dots, r_i - 2$; $C_iA^{r_i - 1}B \neq 0$, for all $i = 1, 2, \dots, l$.

(2) $$\det H(r_1, r_2, \dots, r_l) = \det\begin{pmatrix} C_1A^{r_1 - 1}B \\ \dots \\ C_lA^{r_l - 1}B \end{pmatrix} \neq 0,$$

where the C_i are the rows of the matrix C.

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Note that assumptions 1 and 2 in Definition 1 may be inconsistent; therefore, there exist systems for which this definition is not satisfied (examples can be found, e.g., in [1]). It is these systems that are considered in the present paper from the viewpoint of their constructive invertibility.

Definition 2. The Rosenbrock matrix of system (1) is the following block matrix depending on the parameter z:

$$R(z) = \begin{pmatrix} zI - A & -B \\ C & 0 \end{pmatrix}. \tag{2}$$

Definition 3. The invariant zeros of system (1) are all values of z for which the Rosenbrock matrix (2) of the system is not of full rank.

As it follows from [4], invertibility criteria for square systems is the absence of unstable invariant zeros. Invertible systems for which the definition of Isidori relative degree is satisfied can be inverted with the use of algorithms [3].

Remark 1. A nonsingular linear transformation of state vector doesn't affect invariant zeros and relative degree. A nonsingular linear transformation of only outputs may affect the Isidori relative degree definition compliance.

As it was stated above, for a 3-d order MIMO system, i.e. system (1) under $n = 3$, $l = 2$ a nonsingular outputs transformation may reduce the system to the Isidori relative degree compliant form [5]. Let us show that this is not the general case.

Proposition 1. *A nonsingular linear output transformation, reducing a system to the Isidori relative degree compliant form, is unavailable for some* LTI *systems.*

In order to prove this, it is sufficient to consider the following 4-dimensional LTI system with 2 inputs and 2 outputs reduced to the canonical controllability form [1]:

$$A = \begin{pmatrix} 0 & a_{12} & 0 & a_{14} \\ 1 & a_{22} & 0 & a_{24} \\ 0 & a_{32} & 0 & a_{34} \\ 0 & a_{42} & 1 & a_{44} \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \tag{3}$$

$$C = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

So it is an actual problem how to invert such systems.

2. COLUMN-WISE RELATIVE DEGREE DEFINITION

One of possible approaches is to consider an analogue of the Isidori relative degree definition that we shall introduce here as the column-wise relative degree.

Definition 4. A vector $r = (r_1, r_2, \ldots, r_l)$ is referred to as the vector column-wise relative degree of system (1) if the following conditions are fulfilled simultaneously:

(1) $CA^jB_i = 0, j = 1, 2, \ldots, r_i - 2$; $CA^{r_i - 1}B_i \neq 0$, for all $i = 1, 2, \ldots, l$.

(2) $\det H(r_1, r_2, \ldots, r_l) = \det(CA^{r_1 - 1}B_1 \ldots CA^{r_l - 1}B_l) \neq 0$, where the B_i are the columns of the matrix B.

Note that assumptions of this definition also may be inconsistent for some systems similarly to the case of the Isidori definition, and it is easy to prove this by constructing examples. Besides, there are examples of systems for which the Isidori relative degree definition is not satisfied, but the columnwise relative degree definition still holds. To confirm this fact, consider the following 4-dimensional system with 2 inputs and 2 outputs as a special case of the above example (3):

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \tag{4}$$

$$C = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

which Rosenbrock determinant equals to

$$|R(z)| = \begin{vmatrix} z & -1 & 0 & -1 & -1 & 0 \\ -1 & z-1 & 0 & -1 & 0 & 0 \\ 0 & -1 & z & -1 & 0 & -1 \\ 0 & -1 & -1 & z-1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{vmatrix} = -1. \tag{5}$$

This system doesn't have invariant zeros, and therefore is invertible. As it was proved above, the system can not be transformed to satisfy the Isidori definition. On the other hand, it is easy to derive that the column-wise relative degree exists and equals to $r = (3, 1)$.

3. INVERSION PROBLEM IN THE CASE OF ABSENT ZERO DYNAMICS

Let us consider the inversion problem for n-dimensional system with l inputs and l outputs. Assume that its column-wise relative degree equals to $r = (r_1, r_2, \ldots, r_l)$. Then the following assertion (similar to derived in [1, pp. 88–89] for the Isidori definition) is correct.

Proposition 2. *For a system* (1) *under* $\operatorname{rank}B = \operatorname{rank}C = l$ *with column-wise relative degree vector* $r = (r_1, r_2, \ldots, r_l)$ *the columns* A^jB_i, $j = 0, 1, \ldots, r_i - 1$, $i = 1, 2, \ldots, l$ *are linearly independent.*

This assertion is proved similarly to the analogous statement in [1, pp. 88–89].

Applying Proposition 2, one may take $|r| = r_1 + r_2 + \ldots + r_l$ linearly independent columns $[B_1, AB_1, \ldots, A^{r_1-1}B_1, B_2, \ldots, A^{r_2-1}B_2, \ldots, B_l, \ldots, A^{r_l-1}B_l]$ as a part of the transformation matrix M for a state space transformation $z = M^{-1}x$. The $n - |r|$ columns of this matrix can be taken from the kernel of C to satisfy nonsingularity of M. Of $|r|$ fixed above columns $|r| - l$ already belong to the kernel of C, the dimension of which is $n - l$. It follows from here that kernel of C contains $(n - |r|)$-dimensional linear subspace, independent with earlier fixed columns to provide $n - |r|$ linearly independent vectors, which will be added to the transformation matrix. One may check that this is always possible.

Definition 5. Columns of the matrix M, containing $A^{r_i-1}B_i$, $i = 1, 2, \ldots, l$, are referred to as the axial group of the matrix M. The corresponding columns of the matrices $\tilde{C} = CM$ and $\tilde{A} = M^{-1}AM$ also are referred to as the axial. The remaining columns (for the matrix $\tilde{A}$ as well) are referred to as the non-axial. State-space variables corresponding to the axial columns, are also referred to as the axial.

Remark 2. After mentioned above linear transformation in the transformed output matrix $\tilde{C} = CM$ its l axial columns contain columns of the matrix H, occurred in Definition 4. Its other columns are zeros due to a transformation construction, so we may consider that the measured output y provides l axial states $\bar{z}$, computed as $\bar{z} = H^{-1}y$.

Definition 6. Columns of the matrix M, containing the vectors $B_i = 1, 2, \ldots, l$, are referred to as the terminal group of the matrix M. The remaining columns of the matrix M are referred to as the non-terminal. The corresponding to the terminal columns of the matrix M rows of the matrices $\tilde{B} = M^{-1}B$ and $\tilde{A} = M^{-1}AM$ also are referred to as the terminal. The remaining rows are referred to as the non-terminal. The same terms we shall use for the corresponding to the matrix rows equations of the system.

Remark 3. After mentioned above linear transformation the transformed input matrix $\tilde{B} = M^{-1}B$ contains the identity l-dimensional submatrix in the terminal rows. Its remaining rows are zeros. This follows from the equality $M^{-1}M = I$ and the fact that l terminal columns of the matrix M are formed by the columns of B.

Consider the matrix $\tilde{A} = M^{-1}AM$. First, assume that $n = |r|$. The column-wise equality is the following:

$$\begin{aligned}\tilde{A} &= M^{-1}AM \\ &= [B_1|AB_1|\ldots|A^{r_1-1}B_1|\ldots|B_l|AB_l|\ldots|A^{r_l-1}B_l]^{-1} \\ &\times [AB_1|A^2B_1|\ldots|A^{r_1}B_1|\ldots|AB_l|A^2B_l|\ldots|A^{r_l}B_l]. \end{aligned} \tag{6}$$

Note that left multiplying by the matrix A of the matrix M causes a left shift of the non-terminal columns. Applying this to the equality $M^{-1}M = I$ we derive that in the product $M^{-1}AM$ at new positions of shifted columns one will find the columns of the identity matrix with ones in the rows, corresponding to the old (before shift) positions of the columns. Those are $n - l$ non-terminal rows of the matrix $\tilde{A}$. Thus, at the intersection of $n - l$ non-terminal rows of $\tilde{A}$ and its $n - l$ non-axial columns one will find the identity submatrix. Generic structure of the matrices $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ is given below (here the stars are placeholders for probably nonzero elements):

$$\tilde{A} = \begin{pmatrix} \tilde{A}_{11} & \tilde{A}_{12} & \ldots & \tilde{A}_{1l} \\ \tilde{A}_{21} & \tilde{A}_{22} & \ldots & \tilde{A}_{2l} \\ \ldots & \ldots & \ldots & \ldots \\ \tilde{A}_{l1} & \tilde{A}_{l2} & \ldots & \tilde{A}_{ll} \end{pmatrix}, \quad \tilde{A}_{ij} \in \mathbb{R}^{r_i \times r_j}, \tag{7}$$

$$\tilde{A}_{ij} = \begin{pmatrix} 0 & 0 & \ldots & 0 & * \\ \delta_{ij} & 0 & \ldots & 0 & * \\ 0 & \delta_{ij} & \ldots & 0 & * \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \delta_{ij} & * \end{pmatrix}, \quad \delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j, \end{cases} \tag{8}$$

$$\begin{gathered}\tilde{B} = \begin{pmatrix} \tilde{B}_{11} & \ldots & \tilde{B}_{1l} \\ \tilde{B}_{21} & \ldots & \tilde{B}_{2l} \\ \ldots & \ldots & \ldots \\ \tilde{B}_{l1} & \ldots & \tilde{B}_{ll} \end{pmatrix}, \quad \tilde{B}_{ij} \in \mathbb{R}^{r_i \times 1}, \\ \tilde{B}_{ij} = \begin{pmatrix} \delta_{ij} \\ 0 \\ 0 \\ \ldots \\ 0 \end{pmatrix}, \quad \delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j, \end{cases}\end{gathered} \tag{9}$$

Table

$\oplus$	$*\ldots*$	$\mathbf{z}^{\mathbf{t}}_{\mathbf{r_1}}$	$\ldots z^t_{j+1}$	z^t_{j+2}	$\ldots$	$z^t_{j+r_i-1}$	$\mathbf{z}^{\mathbf{t}}_{\mathbf{j+r_i}}$	$\ldots*\ldots*$	$\mathbf{z}^{\mathbf{t}}_{\mathbf{n}}$	$\boldsymbol{\xi}^{\mathbf{t}}_{\mathbf{i}}$
$=$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
z^{t+1}_{j+1}	$0\ldots0$	$*$	$\ldots0$	0	$\ldots$	0	$*$	$\ldots0\ldots0$	$*$	1
z^{t+1}_{j+2}	$0\ldots0$	$*$	$\ldots1$	0	$\ldots$	0	$*$	$\ldots0\ldots0$	$*$	0
z^{t+1}_{j+3}	$0\ldots0$	$*$	$\ldots0$	1	$\ldots$	0	$*$	$\ldots0\ldots0$	$*$	0
$\ldots$	$0\ldots0$	$*$	$\ldots0$	0	$\ldots$	0	$*$	$\ldots0\ldots0$	$*$	0
$\mathbf{z}^{\mathbf{t+1}}_{\mathbf{j+r_i}}$	$0\ldots0$	$*$	$\ldots0$	0	$\ldots$	1	$*$	$\ldots0\ldots0$	$*$	0

(11)

$$\tilde{C} = \left(\tilde{C}_1\ \tilde{C}_2\ \ldots\ \tilde{C}_l \right), \quad \tilde{C}_l \in \mathbb{R}^{l \times r_i},$$

$$\tilde{C}_i = \begin{pmatrix} 0 & 0 & \ldots & * \\ 0 & 0 & \ldots & * \\ 0 & 0 & \ldots & * \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & * \end{pmatrix}. \tag{10}$$

The non-axial states are unknown, but can be easily found by solving (with a delay) $n - l$ non-terminal equations. As it follows from the block structure of the matrices $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, transformed system breaks down into l block subsystems, terminated by the terminal equations. Note that the axial states can be found by inverting outputs (see Remark 2) and their gain products can be computed while non-terminal rows of $\tilde{B}$ are zeros, which eliminates the influence of the unknown input in all equations of this block except the first one (terminal). Gain coefficients near the non-axial states form the identity submatrix. For each of these block subsystems the left-hand side of the last equation is formed by the axial state, so it can be computed.

As an example, consider one of those subsystems (with index i). It contains r_i equations (from which $r_i - 1$ are non-terminal), starting from $j + 1$ $\left(\text{where } j = \sum_{k=1}^{i-1} r_k\right)$ and ending at $j + r_i$, and can be represented in a form of table.

The top line are the state variables, which correspond to the coefficients of the matrix and one of the input components (last column), which is important for the current block. The left column presents the left-hand side variables. In the coefficients' matrix the stars are placeholders for certain, but probably nonzero values. They correspond to the axial states (in bold) and are available at the moment t. For example, the last line of the table corresponds to the following equation:

$$z^{t+1}_{j+r_i} = A_{j+r_i, r_1} z^t_{r_1} + \ldots + z^t_{j+r_i-1} + A_{j+r_i, j+r_i} z^t_{j+r_i} \ \ldots$$
$$\ldots + A_{j+r_i, n} z^t_n, \tag{12}$$

where all parameters except $z^t_{j+r_i-1}$ are available.

Reverse iterations are performed diagonally from bottom to top in the following way. After solving the last block equation (its left-hand side is defined) one obtains the second to last unknown state ($z^t_{j+r_i-1}$) (near the unity element of a submatrix), thus making possible to compute the left-hand side for the second to last equation at the preceding moment of time. Solving it, in turn, provides one more non-axial state (with a growing time delay), and so on. After reaching the first block equation the process successfully terminates and provides all the non-axial states due to the shifted identity form of the block submatrix. It is worth noting that the non-axial state variable that appears in the left-hand side of the equation k in block i, can be found with a delay $r_i - k$.

This allows to formulate the following result.

Proposition 3. *In this reasoning subsystem of the non-terminal equations can be solved with respect to the non-axial states.*

From the way of transformation construction it follows that size of the block $\tilde{A}_{ii}$ equals to a component of the vector r with the same index. Since the left-hand side of the last block equation is formed by the axial state and the first block equation is terminal, the following assertion is correct.

Proposition 4. *If under the considered conditions the left-hand side of the terminal equation is formed by axial state variable, then the corresponding block is one-dimensional and its vector relative degree component equals to* 1.

Corollary 1. *Component of the state vector, displaying the left-hand side of the kth equation in ith block, can be found with a delay* $r_i - k$.

To obtain the unknown input one need to know states near nonzero coefficients and the left-hand side of the terminal equations in two gradual moments of time. The terminal equation is the first one in block. As it follows from the block structure of the matrices $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, nonzero coefficients of the matrix $\tilde{A}$ in the terminal rows belong only to the axial columns with defined state variables. Its left-hand side value is a component of the state vector, and according to the previous considerations it can be computed with a delay $r_i - 1$. Consequently, the ith input component can be computed with a delay r_i. Thus, the following assertion is correct.

Proposition 5. *Under the discussed conditions the system is constructively invertible with respect to the unknown input. Moreover, the ith input component is computed with a delay* r_i.

Let us note that the considered case of $n = |r|$ corresponds to the systems with the absence of zero dynamics.

4. INVERSION PROBLEM FOR SYSTEMS WITH STABLE ZERO DYNAMICS

Now we shall consider the case $n > |r|$. Here some additional columns for the matrix M are taken from the kernel of C. We shall place those vectors in the last $n - |r|$ columns of M.

Definition 7. The mentioned above columns of the matrix M, as well as the corresponding columns of the matrices $\tilde{A}$ and $\tilde{C}$ are referred to as the additional. The last $n - |r|$ equations of the system with the matrix $\tilde{A}$ are also referred to as the additional. The remaining (not additional) equations are referred to as the basic, as well as the remaining columns of the matrices M, $\tilde{A}$ and $\tilde{C}$. Analogously, the first $|r|$ states are referred to as the basic, and the remaining as the additional.

Remarks 2 and 3 under such transformation are kept correct, therefore $n - |r|$ equations are kept non-terminal. Let's investigate what happens to the matrix A.

$$\begin{aligned}\tilde{A} &= M^{-1}AM \\ &= [B_1|AB_1|\ldots|A^{r_1-1}B_1|\ldots|B_l|AB_l|\ldots|A^{r_l-1}B_l|q_1|\ldots|q_{n-|r|}|]^{-1} \\ &\times [AB_1|A^2B_1|\ldots|A^{r_1}B_1|\ldots|AB_l|A^2B_l|\ldots|A^{r_l}B_l|Aq_1|\ldots|Aq_{n-|r|}].\end{aligned} \tag{13}$$

Here, $q_1, q_2, \ldots, q_{n-|r|}$ are additional columns of the matrix M. Left multiplication by A causes a left shift of the basic non-terminal columns of M. Taking account of $M^{-1}M = I$ we derive that in the product $M^{-1}AM$ at new positions of shifted columns one will find the columns of the identity matrix with ones in the basic non-terminal rows of $\tilde{A}$. Thus, at the intersection of the basic non-terminal rows of $\tilde{A}$ and its basic non-axial columns one will find the identity submatrix. In additional equations, which are non-terminal, non-zero coefficients of the matrix $\tilde{A}$ are only in the axial and additional columns. By the way, axial and additional columns in all the equations may contain non-zero coefficients. Thus, additional equations can be rewritten as the following subsystem:

$$(z')^{k+1} = \bar{A}(z')^k + \bar{B}\bar{z}^k = \bar{A}(z')^k + \bar{B}H^{-1}y^k, \tag{14}$$

where z' are the additional states, the matrix $\bar{A} \in R^{(n-|r|)\times(n-|r|)}$ is a submatrix in the additional columns and additional rows of $\tilde{A}$, $\bar{B}$ is a submatrix in the axial columns and additional rows of $\tilde{A}$, and $\bar{z}$ is a subvector of the computed axial states (see Remark 2).

As nonsingular transformation does not affect invariant zeros, so let us consider the Rosenbrock matrix for the transformed system. Expanding the Rosenbrock determinant at first by the last l rows (marking out the axial columns), then by the last l columns (marking out the terminal rows), then by the additional rows (marking out the additional columns), in the absolute values we shall obtain

$$\begin{aligned}|R(z)| &= \begin{vmatrix} zI - \tilde{A} & -\tilde{B} \\ \tilde{C} & 0 \end{vmatrix} \\ &= \pm|H|\cdot|I|\cdot|zI-\bar{A}|\cdot\begin{vmatrix} -1 & * & \ldots & * \\ 0 & -1 & \ldots & * \\ 0 & 0 & \ldots & * \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & -1 \end{vmatrix} \\ &= \pm|H|\cdot|zI-\bar{A}|.\end{aligned} \tag{15}$$

It follows from here the correctness of the assertion below.

Proposition 6. *The invariant zeros of the considered system coincide with the eigenvalues of the matrix* $\bar{A}$.

Thus, if the system is invertible and its invariant zeros are stable, then the matrix $\bar{A}$ is stable, the states subvector z' is asymptotically or finitely observable (the observer (14) is convergent), and it can be approximated asymptotically or finitely. In this case, the converging approximation of all the non-axial states is also available according to the discussion, pertained earlier for the case $n = |r|$ (implying that additional states are approximated and axial states are computed). This allows to approximate the unknown inputs and solve the inversion problem.

Note that although the argument is carried out for the discrete case, the results remain valid for continuous systems.

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