Nonequilibrium laser plasma of noble gases: Prospects for amplification and guiding of the microwave radiation

A. V. Bogatskaya, Hou Bin, A. M. Popov, and I. V. Smetanin

Citation: Physics of Plasmas 23, 093510 (2016); doi: 10.1063/1.4962515
View online: http://dx.doi.org/10.1063/1.4962515
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/23/9?ver=pdfcov

Published by the AIP Publishing

Articles you may be interested in

Different roles of electron beam in two stream instability in an elliptical waveguide for generation and amplification of THz electromagnetic waves
Phys. Plasmas 23, 083110 (2016); 10.1063/1.4960383

Terahertz electromagnetic wave generation and amplification by an electron beam in the elliptical plasma waveguides with dielectric rod
Phys. Plasmas 21, 092122 (2014); 10.1063/1.4896718

Laser induced avalanche ionization in gases or gas mixtures with resonantly enhanced multiphoton ionization or femtosecond laser pulse pre-ionization
Phys. Plasmas 19, 083508 (2012); 10.1063/1.4747344

Numerical model of a laser-sustained argon plasma
J. Laser Appl. 21, 169 (2009); 10.2351/1.3263120

Formation of laser plasma channels in a stationary gas
Phys. Plasmas 13, 043106 (2006); 10.1063/1.2195383
Nonequilibrium laser plasma of noble gases: Prospects for amplification and guiding of the microwave radiation

A. V. Bogatskaya,1,2,3 Hou Bin,2 A. M. Popov,1,2,3 and I. V. Smetanin1,a)

1P.N. Lebedev Physical Institute, Leninski prosp. 53, 119991 Moscow, Russia
2Department of Physics, Moscow State University, 119991 Moscow, Russia
3D.V. Skobeltsyn Institute of Nuclear Physics, Moscow State University, 119991 Moscow, Russia

(Received 29 April 2016; accepted 21 August 2016; published online 20 September 2016)

We developed the analytical model of relaxation of a low-density plasma channel produced in noble gases (Xe, Ar) by a femtosecond KrF laser pulse and investigated the temporal evolution of its dielectric permittivity. It was demonstrated that the strong nonequilibrium of the photoelectron energy spectrum and the presence of Ramsauer minimum in transport scattering cross section make such a plasma channel an optically denser medium in comparison with non-ionized gas in the microwave frequency band and consequently such a channel appears to be a waveguide. In xenon this nonequilibrium state of a plasma leads to both transportation and amplification of the microwave signal during the relaxation of the photoelectron energy spectrum. It was also shown that a circular metal waveguide partially filled with such a nonequilibrium Xe plasma provides efficient amplification of the sub-THz microwave signal. Published by AIP Publishing. [http://dx.doi.org/10.1063/1.4962515]

I. INTRODUCTION

Novel approaches in generation and amplification of coherent microwave radiation in the THz spectral domain are now of particular interest due to a great number of prospective applications.1–3 Various physical principles and mechanisms result in efficient THz sources ranging from relativistic electronics (free-electron lasers and gyrotrons)4 to solid-state microelectronics (quantum cascade lasers)5.

A highly nonequilibrium plasma produced in a medium by ultrashort laser pulses reveals various specific electrodynamic and kinetic properties which can be utilized for lasing and amplification in spectral ranges from X-ray and UV to THz. The use of the filamentation phenomenon6,7 provides formation of long plasma channels of different configurations and plasma densities. In the field of an ultrashort laser pulse, the dominant multiphoton ionization of gas atoms and molecules results in a highly nonequilibrium initial plasma state followed by the complicated relaxation process. Diverse dynamics and kinetics of plasma density, excited states population, and conductivity make available the generation of the extreme ultraviolet attosecond pulses,8,9 triggering of high-voltage discharges and lightning,10–13 remote sensing of the atmosphere,14 lasing via excited nitrogen transitions along the plasma channel in the atmospheric air,15 etc. Under filamentation, microwave radiation of THz and sub-THz frequency band can be effectively generated as a result of the optical pulse rectification.16,17 Microwave transport in gases can be realized using laser produced hollow plasma waveguides of different configurations.18–20 In Ref. 21 the most efficient microwave transport concept was proposed, based on the effect of total internal reflection at the interface of an optically less dense medium (i.e., plasma walls of the waveguide). Such a sliding-mode transportation regime arises when the microwave wavelength is sufficiently smaller in comparison with the inner radius of a tubular plasma waveguide $\lambda \ll R$. In this case the effective angle of incidence for lowest-order modes exceeds the critical angle determined by the ratio of air and plasma refractive indices and therefore high conductivity of the plasma walls is not necessary.22

In this paper, we propose a new approach to the problem of microwave amplification and guiding in a plasma channel produced by an intense UV femtosecond laser pulse. This approach is based on the strong nonequilibrium of the electron energy distribution function (EEDF) in a plasma channel created in the field of an ultrashort laser pulse. Particularly, the energy spectrum of photoelectrons formed in the multiphoton ionization process under conditions when the pulse duration is smaller than the characteristic time between electron-atomic collisions consists of a set of peaks corresponding to the absorption of a certain number of photons. Such an energy distribution function has energy ranges characterized by a population inversion which can be used for the amplification of electromagnetic radiation in a plasma.23,24 In Refs. 25–27 authors analysed the amplification process in a number of atomic and molecular gases. In Ref. 28 it was demonstrated that the population inversion in the EEDF also leads to specific refractive features of a plasma. It was found that plasma produced by a KrF short laser pulse in xenon and argon appears to be an optically denser medium in comparison with nonionized gas. Such a plasma channel can be considered as a dielectric sliding-mode waveguide maintaining both transportation and amplification of radio-frequency (RF) radiation. In the light of future prospects of experimental realization of sub-THz microwave amplification we are interested here in the dispersion characteristics of an ideal circular metal waveguide partially filled by a co-axial channel of nonequilibrium plasma. Using the fact that a low-density nonequilibrium plasma...
inserts a small perturbation into dielectric permittivity, we have calculated the lowest eigen microwave modes and demonstrated that in such partially filled waveguides an amplification coefficient close to the maximum one (peculiar to the case of a plane wave in the infinite plasma volume) can be attained. Estimates show that sufficiently high modal gain coefficients can be attained in experiment rather easily to demonstrate feasibility of the proposed new approach in amplification and guiding sub-THz radiation.

II. EVOLUTION OF THE EEDF IN NOBLE GASES: ANALYTICAL SOLUTION OF THE BOLTZMANN EQUATION

We intend to analyze the evolution of a plasma channel created in noble gases (xenon and argon) by the high-intensity femtosecond UV laser (KrF laser, $h\omega = 5.0$ eV) pulse. We assume the laser pulse to be short enough so that the plasma channel appears only owing to the multiphoton ionization of the gas medium, while all collisional processes during the pulse can be neglected. Then, evolution of the electron energy distribution function (EEDF) proceeds in the direct numerical simulation. The EEDF (1) is normalized as

$$n(e, t = 0) = \frac{1}{\Delta e \sqrt{\pi \Delta e}} \exp\left(-\frac{(e - e_0)^2}{\Delta e^2}\right),$$

where $\Delta e \approx 0.2$ eV is the width of initial peak, $e_0 = N h \omega - I$, ($I_i$ is the ionization potential of Xe or Ar atoms) corresponding to the position of the first peak in the photoelectron spectrum. $N$ is the multiphoton order of photoionization ($N = 3$ for Xe atoms and $N = 4$ for Ar atoms). As it is well known, above-threshold photon absorption becomes pronounced when $U_p/h\omega \geq 1$, where $U_p$ is the ponderomotive energy of an electron in the laser field. Ponderomotive energy scales with the laser wavelength as $U_p \propto \lambda^2$, so the range of KrF laser intensities $10^{12}$-$10^{13}$ W/cm$^2$ corresponds to rather moderate fields and higher above threshold ionization (ATI) peaks can be neglected. This approximation is confirmed by the direct numerical simulation. The EEDF (1) is normalized as

$$\int_0^{+\infty} n(e, t = 0) \sqrt{\Delta e} de = 1.$$  

Temporal evolution of the EEDF (1) in noble gases was analysed using the Boltzmann kinetic equation

$$\frac{\partial n(e, t)}{\partial t} \sqrt{\Delta e} = \frac{\partial}{\partial e} \left( e^2 E_0^2 \nu_T(e) \left( e^{3/2} \frac{\partial n}{\partial e} \right) \right) + 2m \frac{\partial}{M \partial e} \left( \nu_T(e) e^{3/2} n(e, t) + T_g \frac{\partial n(e, t)}{\partial e} \right)$$

$$+ Q_{ee}(n) + Q'(n).$$

Here, $E_0$ is the amplitude of the transported radio-frequency (RF) field, $\nu_T = N \sigma_T v$, $\sigma_T$ are the rate and the transport cross section of electron-atomic collisions, respectively, $T_g$ is the gas temperature (below we assume $T_g = 0.03$ eV), $m$ is the mass of the electron, $M$ is the mass of the gas atom, $Q_{ee}(n)$ and $Q'(n)$ are integrals of electron-electron and inelastic collisions. In noble gases energy-lowest excitation thresholds for electronic states are 8.31 and 11.50 eV for xenon and argon atoms, respectively. This is significantly higher than the energy of peak of photoelectrons produced by the femtosecond laser pulse. Throughout this paper we are going to restrict ourselves by the case of sufficiently low electron density in order to avoid electron-electron collisions and also we will assume amplified and transported microwave radiation being sufficiently weak and not affecting the EEDF. Under the above conditions evolution of the electron energy spectrum in rare gases is guided only by elastic collisions

$$\frac{\partial n(e, t)}{\partial t} \sqrt{\Delta e} = \frac{2m}{M} \frac{\partial}{\partial e} \left( \nu_T(e) e^{3/2} n(e, t) \right).$$

Here we also neglect the term $T(\partial n/\partial e)$ as the EEDF width $\Delta e \gg T_g$.

The analytical solution of Equation (4) for the arbitrary dependence of the transport cross section on energy was found in Ref. 28. This general solution can be concretized for definite dependences of transport cross sections on electron energy $\sigma_T(e)$. For example, for the power dependence

$$\sigma_T(e) = \sigma_0 (e/e_0)^{\beta},$$

the solution has the form

$$f(e, t) = \frac{1}{\sqrt{\pi \Delta e}} \frac{1}{(1 - g(e, t))^{(\beta+1)/\beta}} \exp\left(-\frac{(e - e_0)^2}{\Delta e^2}\right) \left(1 - g(e, t)\right)^{\beta}.$$  

Here, $g(e, t) = \beta(2m/M)\nu_T(e) t$, $\nu = \nu_0 (e/e_0)^\beta$, $\beta = \alpha + 1/2$, and $\nu_0 = N \sigma_0 \sqrt{2/\alpha} / m$.

The analysis of transport cross sections for xenon and argon (their transport cross sections can be found in Refs. 31 and 32) leads to the conclusion that the power dependence can be used for the discussed dependences (see Table I and Fig. 1).

In Ref. 28 it was demonstrated that the above analytical solution is in qualitative agreement with the results of the numerical integration of Eq. (4) at least in the case of Ar and Xe atoms. In further consideration we will assume that the EEDF has the form of a delta-peak, whose position is governed by the equation for the averaged over EEDF energy

$$\frac{d\langle e \rangle}{dt} = -\frac{2m}{M} \langle \nu_T(e) e \rangle.$$
Really, for narrow peak in photoelectron spectrum (i.e.,
the condition \( \langle \nu_T(x) \rangle \approx \langle \nu_T(x) \rangle \) is satisfied) is given by
the expression

\[
\langle e(t) \rangle = \frac{\varepsilon_0}{(1 + 2m/M)\beta \nu'(t = 0)t^{1/2}}.
\] (8)

The dependences \( \langle e(t) \rangle \) obtained from expression (8)
and from numerical integration of Equation (4) are
presented in Fig. 2 and are found to be in good quantitative
agreement.

### III. ANOMALOUS DIELECTRIC PROPERTIES

OF THE NOBLE GAS NONEQUILIBRIUM PLASMA

Knowledge of the temporal evolution of the EEDF
makes it possible to find permittivity \( \varepsilon_{eo} = \text{Re} \varepsilon_{eo} + i \times \text{Im} \varepsilon_{eo} \) of a plasma channel for the transported microwave
radiation frequency \( \omega \),

\[
\text{Re} \varepsilon_{eo} = 1 - \frac{2}{3} \omega_p^2 \left[ \int_0^{\infty} \frac{e^{3/2} \nu_T}{\omega^2 + \nu_T^2} \left( -\frac{\partial n}{\partial \omega} \right) d\omega \right],
\] (9)

\[
\text{Im} \varepsilon_{eo} = \frac{2}{3} \omega_p^2 \left[ \int_0^{\infty} \frac{e^{3/2} \nu_T}{\omega^2 + \nu_T^2} \left( -\frac{\partial n}{\partial \omega} \right) d\omega \right].
\] (10)

Here, \( \omega_p^2 = 4\pi e^2 n_e / m \) is the plasma frequency squared, \( n_e \) is
the electron concentration, and \( \nu_T \) is the transport frequency
of electron-atomic collisions. The real part (9) describes
refraction properties of the plasma channel, while the imagi-
nary part (10) stands for the dissipation of electromagnetic
wave energy in plasma: for the plane electromagnetic wave
with the frequency \( \omega \) we have the following expression for the
absorption coefficient:

\[
\mu_\omega = (\omega/c) \text{Im} \varepsilon_{eo}.
\] (11)

It is known\textsuperscript{25,26,28} that the non-equilibrium spectrum of
electrons which appears in the process of photoionization of
gas media by the femtosecond laser pulse results in energy
ranges with population inversion (or positive value of the
derivative \( \partial n / \partial \omega > 0 \)) that can provide the negative absorp-
tion effect for low frequency radiation \( \omega < \nu_T \), i.e., the possi-
bility of amplification of electromagnetic wave in a plasma.

FIG. 1. Transport scattering cross section (solid line) and its approximation (dashed line) in xenon (a) and argon (b).

FIG. 2. Time dependences of \( \text{Re} (\varepsilon_{eo} - 1) \) ((a) and (c)) and \( \text{Im} \varepsilon_{eo} \) ((b) and
(d)) in argon ((a) and (c)) and xenon ((b) and (d)) obtained by analytical
expressions (13), (14) (dashed line) and by numerical calculations (solid line) at plasma density \( n_e = 10^{12} \text{ cm}^{-3} \).
Microwave frequencies are \( \omega = 10^{11}, 2 \times 10^{11}, 5 \times 10^{11} \text{ c}^{-1} \).
Also in Ref. 28 it was demonstrated that a plasma channel with the population inversion in the EEDF can be optically more dense in comparison with unionized gas. Such a situation is of interest with respect to the creation of the sliding-mode plasma waveguide.

Numerical simulations of Equation (3) for xenon and argon media in the energy range 0–5 eV for different values of electron concentration were performed in Ref. 26. It was demonstrated that within a time interval of 25 ns electron-electron collisions do not influence significantly the evolution of the EEDF for the electron density of \( n_e = 10^{12} \text{ cm}^{-3} \). Therefore, the EEDF represents the Gaussian-like shape peak which gradually shifts towards lower energies (Fig. 2), consistent with Eq. (8). For qualitative analysis of the electrodynamic features of such a nonequilibrium plasma it is convenient to introduce the EEDF as a \( \delta \)-shaped peak,

\[
n(e, t) \sqrt{e} = \delta(e - \langle e(t) \rangle), \tag{12}
\]

where \( \langle e(t) \rangle = \int n(e, t) e^{3/2} d e \) is the spectrum-averaged photoelectron energy in dependence on time. Such an assumption does not take into account the spreading of the EEDF due to energy diffusion. Further calculations show that the \( \delta \)-shaped EEDF approximation provides correct results at times of tens of nanoseconds.

For the \( \delta \)-shaped distribution function and cross sections defined from (5), integrals (9) and (10) can be easily found,

\[
Re \xi_{\omega} = 1 - \frac{2}{3} \frac{\omega_{tr}^2}{\omega^2 + \nu_{tr}^2(t)} \left( \frac{1}{\sqrt{\delta e d e \omega^2 + \nu_{tr}^2(e)}} \right) \tag{13}
\]

\[
Im \xi_{\omega} = \frac{2}{3} \frac{\omega_{tr} \nu(t)}{\omega^2 + \nu^2(t)} \left( x + 2 \omega^2 + (1 - x) \nu^2(t) \right) \tag{14}
\]

where \( \nu(t) = \nu(\langle e(t) \rangle) = N \sigma_p(\langle e \rangle) \sqrt{2 \langle e \rangle}/m \) is the transport frequency corresponding to the spectrum-averaged electron energy. In the low frequency range \( \omega \ll \nu_{tr} \) one can obtain from (13) and (14)

\[
Re \xi_{\omega} \approx 1 + \frac{2}{3} \frac{\omega_{tr}^2}{\nu(t)} \left( 2x - 1/2 \right), \tag{15}
\]

\[
Im \xi_{\omega} \approx - \frac{2}{3} \frac{\omega_{tr} \nu(t)}{\omega^2 + \nu(t)} \left( x \right). \tag{16}
\]

To fulfill the condition \( Re \xi_{\omega} > 1 \), it is necessary that \( x > 1/4 \). This condition is valid for the correct time range in a number of gases, including xenon and argon. The amplification condition \( (x > 1) \) is considerably more strict. For example, the amplification condition is fulfilled for xenon atoms, but not for argon atoms, where \( x = 1 \), which leads to propagation of the electromagnetic radiation in the guiding regime (propagation without absorption or amplification).

Time evolution of the real and imaginary parts of plasma permittivity obtained from expressions (13) and (14) for Xe and Ar atoms for different frequencies of the transported microwave field is shown in Fig. 2. The same dependences extracted from the numerical solution of the Boltzmann equation are also given. First of all, we would like to note the qualitative agreement between analytical and numerical data. It means that the difference between the analytical and numerical solutions of the Boltzmann equation do not contribute significantly to the integrals (9), (10) and the \( \delta \)-like EEDF approximation is still valid.

Thus, as expected, the guiding regime of the radio-frequency field propagation can be realized in the argon plasma channel, and for pulse durations of 10 ns, the damping effect is practically negligible. In the case of RF pulse propagating in a xenon plasma channel, the simultaneous amplification can be achieved. Increasing the RF frequency makes the abnormal plasma properties less pronounced. Both guiding propagation mode and amplification regime (in the case of xenon plasma) are realized for a smaller time interval determined by the EEDF relaxation process. A further increase of the frequency in the region \( \omega > \nu_{tr} \) makes these propagation modes impossible.

Results presented in Fig. 2 are obtained under the assumption that electron-electron collisions do not contribute to the evolution of the EEDF. This is correct only within the time interval when the diffusion spreading of the photoelectron peak due to electron-electron collisions can be neglected. The estimations performed in Ref. 28 lead to the conclusion that for atmospheric pressure electron-electron collisions can be neglected for \( n_e = 3 \times 10^{12} \text{ cm}^{-3} \) for a time interval of tens of nanoseconds.

IV. MICROWAVE SLIDING-MODE PROPAGATION AND AMPLIFICATION IN A WIDE-APERTURE PLASMA CHANNEL

The results presented in Sec. III reveal that within a time interval of about tens of nanoseconds the refractive index of xenon or argon plasma channel appears to be greater than unity. In xenon this effect is accompanied by the amplification of microwave radiation. Therefore below we will restrict ourselves to consideration of xenon plasma channel and analyze both amplification and propagation of microwave radiation in such a dielectric plasma waveguide in the sliding-mode regime (see Fig. 3).

To analyze the electrodynamic properties of a plasma waveguide we will assume a laser pulse producing a well-defined plasma channel of radius \( R \). The role of plasma wall density spreading was studied in detail in Ref. 22. It was shown that sliding mode propagation is not affected by this density spreading until its characteristic size is smaller compared with the wavelength of the microwave signal. Plasma permittivity \( \xi_{\omega} \) is assumed to be uniform inside the channel and can be written in the following form:

\[
\xi_{\omega} = 1 + Re(\xi_{\omega} - 1) + i \times Im \xi_{\omega}, \tag{17}
\]

while outside the channel it equals unity.
First we consider an axial-symmetric TM mode \( E_{01} \) and TE mode \( H_{01} \). The distribution of longitudinal field components \( E_z \) for TM and \( H_z \) for TE modes) has the form \( J_0(\kappa_1 r) \exp[i(\omega t - \kappa_1 x)] \) inside the plasma waveguide and \( H_{01}^{(1)}(\kappa_2 r) \exp[i(\omega t - \kappa_2 x)] \) outside the plasma channel. Here, \( J_0 \), \( H_{01}^{(1)} \) are Bessel and first-kind Hankel functions of zero order; \( \kappa_1 \) is the transverse wave-number inside the plasma waveguide and \( \kappa_2 \) is the transverse wave-number out of the waveguide in nonionized gas, and \( h \) is the longitudinal wave-number.

Similar to Ref. 22, one obtains the dispersion equation for propagation of axial-symmetric sliding modes \( E_{01} \) and \( H_{01} \) in such a plasma waveguide,

\[
\frac{\chi J_1(\kappa_1 R) - \kappa_1 R J_0(\kappa_1 R)}{\kappa_1 R J_0(\kappa_1 R)} = \frac{1}{\kappa_2 R} H_{01}^{(1)}(\kappa_2 R),
\]

\[
\kappa_1 = \sqrt{k^2 + \xi_{\omega}^2} - \frac{h^2}{\kappa_2} \quad \kappa_2 = \sqrt{k^2 + \xi_{\omega}^2}, \quad \omega = \frac{c}{\lambda}.
\]

Here, \( \chi = \xi_{\omega} \) stands for the TM \( E_{01} \) mode and \( \chi = 1 \) for the TE \( H_{01} \) mode, and \( J_1, H_{01}^{(1)} \) are Bessel and first-kind Hankel functions of the first order, respectively. It is important that for the sliding-mode regime that \( R \gg \lambda \), where \( \lambda = 2\pi c/\omega \) is the transported RF radiation wave-length.

We also consider one more important axial-asymmetric lowest mode \( EH_{11} \) since this mode is the main operation mode of dielectric waveguides.\(^{33}\) This mode contains all six components of the electromagnetic field. Then longitudinal components of the electric and magnetic field have the form \( J_1(\kappa_1 r) \exp[i(\omega t - \kappa_1 x)] \) inside the plasma waveguide and \( H_{11}^{(1)}(\kappa_2 r) \exp[i(\omega t - \kappa_2 x)] \) in the nonionized gas. In analogy with Ref. 34 the dispersion equation for this mode can be written in the form

\[
\frac{1}{\kappa_1^2 J_1(\kappa_1 R)} - \frac{1}{\kappa_2^2 J_1(\kappa_2 R)} - \left( \frac{1}{\kappa_1^2 R} + \frac{1}{\kappa_2^2 R} \right)
\]

\[
\times \left[ \left( \frac{\xi_{\omega} J_0(\kappa_1 R)}{\kappa_1 J_1(\kappa_1 R)} - \frac{1}{\kappa_2 J_1(\kappa_2 R)} \right) - \left( \frac{\xi_{\omega} J_0(\kappa_2 R)}{\kappa_2 J_1(\kappa_2 R)} - \frac{1}{\kappa_1 J_1(\kappa_1 R)} \right) \right]
\]

\[
= \frac{\xi_{\omega}}{\kappa_1^2 R} + \frac{1}{\kappa_2^2 R} + \frac{1}{\kappa_1^2 R} \quad \xi_{\omega}.
\]

Here, \( K_0, K_1, K_2, K_3, K_4 \) are Macdonald functions of zero and first order, respectively.

Dispersion equations (18) and (20) were solved numerically for the transported radiation frequency \( \omega = 5 \times 10^{11} \text{c}^{-1} \) and different values of plasma channel radius \( R \). For the above mentioned modes \( Re(\xi_{\omega} - 1) = -Im(\xi_{\omega}) = 4.5 \times 10^{-3} \) (see Fig. 2) and dependences of the amplification length on the waveguide radius \( R \) are presented in Fig. 4. One can see that while increasing the waveguide radius, the distinction between amplification lengths for \( E_{01} \) and \( EH_{11} \) modes decreases. This fact means that at rather large waveguide radius the approach of the plane wave (10) is fulfilled with good precision.

From a practical point of view it may also be relevant to consider the plasma channel created not by multiphoton ionization of the gas, but by single-photon ionization of easily ionized impurities. In this case the laser pulse forming the channel may be of relatively low intensity. However, the ionization potential of an impurity (and laser photon energy) should be selected so that the electron photoionization peak is located in an appropriate energy range.

V. AMPLIFICATION OF THE MICROWAVE MODES IN THE CIRCULAR WAVEGUIDE PARTIALLY FILLED WITH NONEQUILIBRIUM PLASMA

As we have found in Sec. IV, in order to implement efficient amplification of microwave signal one should use an...
ionized channel with the radius greatly exceeding wavelength of a signal. It may be quite difficult to realize this condition in experiment: a large channel aperture requires the use of proportionally large power of an ultrashort laser pulse in order to create plasma density which is sufficient for microwave guiding and amplification. As an alternative, one can exploit a scheme with the conventional metal waveguide which provides efficient coupling between microwave signal and amplifying plasma medium within the restricted (by the waveguide wall) interaction area.

Let us consider an ideal circular waveguide filled with gaseous xenon in which the co-axial circular plasma channel is created by an ultrashort laser pulse. According to the above consideration, within a sufficiently large time interval of a few dozen nanoseconds a plasma channel is found to be in the non-equilibrium excited state and therefore is characterized by dielectric permittivity (17) with small values of $\text{Re} (\xi_0 - 1)$, $\text{Im} (\xi_0) \approx 10^{-2} - 10^{-3}$. The imaginary part of the dielectric permittivity is negative, so one should expect amplification of the correspondent waveguide mode and we intend to calculate the modal gain. Let the radius of the plasma channel be $R_1$, and the radius of the metal waveguide be $R_2$. As far as $|\xi_0 - 1| \ll 1$ we are going to solve the problem using corresponding perturbation theory provided there is a small difference between the real amplified mode in this partially filled waveguide and the conventional mode of the empty cylindrical waveguide. Again, electromagnetic mode is characterized by transverse wave numbers inside $k_1$ and outside $k_2$ of the plasma cylinder which satisfy dispersion relations (19). We will assume that these transverse wave numbers differ slightly from each other as well as from the transverse wave number of the corresponding empty waveguide mode $k_0$, $k_1 = k_0 + \delta k_1$, $k_2 = k_0 + \delta k_2$ so that

$$k_1 - k_2 = \delta k_1 - \delta k_2 \approx (\xi_0 - 1) \frac{k^2}{2k_0} \ll k_0. \quad (21)$$

Below we will calculate amplification increments for the most practically important modes of a circular waveguide, namely, for the lowest axially symmetric modes $E_{01}$ and $H_{01}$, and the lowest asymmetric modes $E_{11}$ and $H_{11}$. It is natural to compare these increments with their maximum values which can be attained for the completely filled with the plasma waveguide. In that case one can easily find

$$\text{Re} h \approx \text{Re} \xi_0 k^2 = \frac{k^2}{2} \approx k^2 - k_0^2, \quad (\text{Im} h)_{\text{max}} \approx \text{Im} \xi_0 \frac{k^2}{2 \sqrt{k^2 - k_0^2}}. \quad (22)$$

So the problem is to determine the optimum filling factor $x = R_1/R_2$ for the plasma cylinder providing modal gain close to the maximum one.

**A. Axially symmetric TM mode $E_{01}$**

Let us start with the axially symmetric TM mode $E_{01}$. For an empty cylindrical waveguide, the longitudinal component of the electric field mode is determined by the Bessel function $J_0(k_0 \rho)$, where the transverse wavenumber is given by its first zero, $k_0 R_2 = J_0(1) \approx 2.40483$. In the waveguide partially filled with plasma we will seek field amplitude components in the following form:

$$E_z = A_0(k_0 r), \quad E_r = \frac{h}{k_1} A_1'(k_1 r), \quad H_\phi = i \frac{k}{k_1} A_0'(k_1 r); \quad (23)$$

inside the plasma cylinder $0 < r < R_1$, $z, r, \phi$ are the cylindrical reference frame coordinates, so

$$E_z = B J_0(k_2 r) + C Y_0(k_2 r), \quad E_r = \frac{h}{k_1} [B J_1'(k_2 r) + C Y_0'(k_2 r)], \quad H_\phi = \frac{k}{k_1} [B J_0'(k_2 r) + C Y_0'(k_2 r)]. \quad (24)$$

for the field amplitude components in the region between the plasma cylinder and waveguide wall. To derive field amplitudes coefficients $A$, $B$, and $C$ we have to apply conventional boundary conditions at $r = R_1$ and $r = R_2$. The boundary condition at the waveguide wall $E_z|_{r=R_2} = 0$ yields, to first order of the perturbation theory,

$$B J_0(k_2 R_2) + C Y_0(k_2 R_2) = B J_0(k_0 R_2) + C Y_0(k_0 R_2) + B J_1'(k_0 R_2) \delta k_2 R_2 + C Y_0'(k_0 R_2) \delta k_2 R_2 = 0. \quad (25)$$

So the relation between coefficients $B$ and $C$ is

$$C = -B \frac{J_0(k_0 R_2)}{Y_0(k_0 R_2)} \delta k_2 R_2 = B \Gamma_0^{TM} \delta k_2 R_2. \quad (26)$$

As far as we are interested in the $E_{01}$ mode, we have to put $k_0 R_2 = J_0(1) \approx 2.40483$ with the value of the characteristic parameter $\Gamma_0^{TM}$ being $\Gamma_0^{TM} = J_1(\xi_0)/Y_0(\xi_0) \approx 1.01809$.

Boundary conditions at the plasma interface $r = R_1$ comprise the continuity of tangential field components, i.e., $E_z$ and $H_\phi$, which can be written as

$$A [J_0(k_0 R_1) + J_0'(k_0 R_1) \delta k_1 R_1] 
\approx B [J_0(k_0 R_1) + J_0'(k_0 R_1) \delta k_2 R_1 + \Gamma_0^{TM} Y_0(k_0 R_1) \delta k_2 R_2] \quad (27)$$

for the longitudinal electric field component and for the azimuthal magnetic field component $H_\phi$ as

$$i \frac{k_0}{\xi_0} \frac{k_0}{k_0} \left( 1 - \frac{\delta k_1}{k_0} \right) A [J_1'(k_0 R_1) + J_0'(k_0 R_1) \delta k_1 R_1] 
\approx i \frac{1}{\xi_0} \frac{k_0}{k_0} \left( 1 - \frac{\delta k_2}{k_0} \right) B [J_1'(k_0 R_1) + J_0'(k_0 R_1) \delta k_2 R_1 
+ \Gamma_0^{TM} Y_0'(k_0 R_1) \delta k_2 R_2]. \quad (28)$$

With the use of relation (21), $\delta k_1 - \delta k_2 \approx (\xi_0 - 1) (k^2/2k_0)$, we find
\[
\Gamma_0^{TM} [J_0(k_0 R_1)Y_1(k_0 R_1) - J_1(k_0 R_1)Y_0(k_0 R_1)] \delta k_2 R_2 \\
\approx (\xi_w - 1) \left\{ J_0(k_0 R_1)J_1(k_0 R_1) + \frac{k^2 R_1^2}{2k_0} \left[ J_2^2(k_0 R_1) \right. \\
\left. + J_1^2(k_0 R_1) - \frac{2}{k_0 R_1^2} J_0(k_0 R_1)J_1(k_0 R_1) \right] \right\}. \tag{29}
\]

Taking into account the relation for the Wronskian of the Bessel functions\textsuperscript{35}
\[
W\{J_\nu(z), Y_\nu(z)\} = J_{\nu+1}(z)Y_{\nu}(z) - J_{\nu}(z)Y_{\nu+1}(z) = 2/(\pi z), \tag{30}
\]
we finally find
\[
\delta k_2 R_2 \approx -\pi \xi_w - 1 \frac{k^2 R_1^2}{2k_0} \left[ J_2^2(k_0 R_1) + J_1^2(k_0 R_1) \right], \tag{31}
\]
As follows from the definition (19) of transverse wavenumber \( k_2 \) the modal gain increment is given by the relation
\[
\text{Im} \ h \approx -\frac{k_0}{\sqrt{k^2 - k_0^2}} \text{Im} \ \delta k_2, \tag{32}
\]
and for the ratio of the modal gain to the maximum one
\[
K^{TM}(x) = \frac{\pi}{18} \left\{ \frac{z_0^2 k_0}{2} \left[ J_0^2(z_0^2 x) + J_1^2(z_0^2 x) \right] \\
- \left[ 1 - \frac{2}{k_0^2 R_2^2} \right] x_0 J_0(z_0^2 x)J_1(z_0^2 x) \right\}, \tag{33}
\]

\section*{B. Axially symmetric TE mode \( H_{01} \)}

In the case of axially symmetric TE mode \( H_{01} \), analysis will be done in the same way. In an empty cylindrical waveguide the longitudinal component of the magnetic field for the unperturbed mode is governed by the Bessel function \( J_0(k_0 r) \), and the eigen value for the transverse wavenumber is given by the dispersion relation \( J''_0(k_0 R_2) = 0 \) so that we have \( k_0 R_2 = x_{11} \approx 3.83171 \). By analogy with the case of the TM mode we can write the solution of the Maxwell equations in the region of the plasma cylinder \( 0 < r < R_1 \) as follows:
\[
\begin{align*}
H_z &= AJ_0(k_1 r), \\
H_r &= i \frac{h}{k_1} AJ'_0(k_1 r), \\
E_\phi &= i \frac{k}{k_1} AJ''_0(k_1 r),
\end{align*}
\tag{34}
\]
and in the region between the plasma and waveguide wall \( R_1 < r < R_2 \) as
\[
H_z = B J_0(k_2 r) + C Y_0(k_2 r),
\]
\[
H_r = i \frac{h}{k_2} B J'_0(k_2 r) + C Y'_0(k_2 r),
\]
\[
E_\phi = i \frac{k}{k_2} B J''_0(k_2 r) + C Y''_0(k_2 r). \tag{35}
\]
Boundary conditions at waveguide wall \( \partial H_z/\partial r |_{r=R_2} = 0 \) and \( E_\phi |_{r=R_2} = 0 \) lead to the same result for the relation between the amplitude coefficients \( B \) and \( C \),
\[
C \approx -B \frac{J''_0(k_0 R_2)}{Y''_0(k_0 R_2)} \delta k_2 R_2 = \Gamma_0^{TE} B \delta k_2 R_2, \tag{36}
\]
where the value of the parameter \( \Gamma_0^{TE} = -J_0(k_0 R_3)/J_1(k_0 R_2) \)
\( \approx 0.976345 \) as far as \( k_0 R_2 = x_{11} \approx 3.83171 \). Boundary conditions at the plasma interface \( r = R_1 \) include the continuity of tangential components \( H_z \) and \( E_\phi \),
\[
A[J_0(k_0 R_1) + J''_0(k_0 R_1) \delta k_1 R_1] \\
\approx B[J_0(k_0 R_1) + J''_0(k_0 R_1) \delta k_2 R_1 + \Gamma_0^{TE} Y_0(k_0 R_1) \delta k_2 R_2], \tag{37}
\]
and
\[
\frac{i A k_0}{k_0} \left[ 1 - \frac{\delta k_1}{k_0} \right] \left[ J_0''(k_0 R_1) + J''_0(k_0 R_1) \delta k_1 R_1 \right] \\
\approx \frac{i B k_0}{k_0} \left[ 1 - \frac{\delta k_2}{k_0} \right] \left[ J_0''(k_0 R_1) + J''_0(k_0 R_1) \delta k_2 R_1 \\
+ \Gamma_0^{TE} Y''_0(k_0 R_1) \delta k_2 R_2 \right]. \tag{38}
\]
Taking relation (21) into account we can find the following first-order amendment to the transverse wavenumber:
\[
\delta k_2 R_2 \approx -(\xi_w - 1) \pi \frac{k^2 R_1^2}{4 \Gamma_0^{TE}} \left[ J_0''(k_0 R_1) + J''_1(k_0 R_1) \\
- \frac{2}{k_0 R_1} J_0(k_0 R_1)J_1(k_0 R_1) \right], \tag{39}
\]
and for the ratio of the modal gain to the maximum one we obtain
\[
K^{TE}(x) = \frac{\pi}{18} x_{11}^2 \left[ J_0''(x_{11}^2 x) + J''_1(x_{11}^2 x) \\
- \frac{2}{x_{11}^2} J_0(x_{11}^2 x)J_1(x_{11}^2 x) \right]. \tag{40}
\]

\section*{C. Non-symmetrical modes \( E_{11} \) and \( H_{11} \)}

The presence of a plasma cylinder representing the non-uniform perturbation leads to hybridization of these modes, i.e., all components of electric and magnetic field should be taken into account. In the region \( 0 < r < R_1 \) field amplitudes of tangential components are
\[ E_z = AJ_1(\kappa_1 r) \cos \varphi, \]
\[ H_z = BJ_1(\kappa_1 r) \sin \varphi, \]
\[ E_\varphi = -i \left\{ \frac{h}{\kappa_1^2} AJ_1(\kappa_1 r) + \kappa_1 B J'_1(\kappa_1 r) \right\} \sin \varphi, \]
\[ H_\varphi = i \left\{ \xi_\alpha \frac{k}{\kappa_1} AJ_1(\kappa_1 r) + \frac{h}{\kappa_1^2} BJ_1(\kappa_1 r) \right\} \cos \varphi, \] (41)

while in the region between the plasma and waveguide wall \( R_1 < r < R_2 \) one should write

\[ E_z = \left[ CJ_1(\kappa_2 r) + C Y_1(\kappa_2 r) \right] \cos \varphi, \]
\[ H_z = \left[ DJ_1(\kappa_2 r) + D Y_1(\kappa_2 r) \right] \sin \varphi, \]
\[ E_\varphi = -i \left\{ \frac{h}{\kappa_2^2} \left[ CJ_1(\kappa_2 r) + C Y_1(\kappa_2 r) \right] \right\} \sin \varphi, \]
\[ H_\varphi = i \left\{ \xi_\alpha k_0 \left[ CJ_1(\kappa_2 r) + C Y_1(\kappa_2 r) \right] \right\} \cos \varphi. \] (42)

Radial components of electromagnetic field can be calculated by means of the Maxwell equations

\[ \frac{i h E_r}{k_0} = \frac{\partial E_z}{\partial r}, \]
\[ \frac{i h H_r}{k_0} = -i \xi_\alpha k_0 E_\varphi - \frac{\partial H_z}{\partial r}, \] (43)

but for our particular purpose we do not need to use their explicit forms in our calculations. It should be noted that we keep in (41) and (42) only azimuthal terms of field components which are coupled via Maxwell equations. One can easily find that the reversed (with respect to the replacement \( \sin \varphi \rightarrow \cos \varphi \) in the longitudinal field components) set of solutions leads to the same dispersion relation.

Let us first consider the perturbed mode \( E_{11} \). The dispersion relation for the unperturbed mode \( E_{11} \) of an empty ideal metal waveguide is determined by vanishing longitudinal electric field \( E_z \) at the waveguide wall, \( J_1(\kappa_0 R_2) = 0 \), which conducts the same result as for the symmetric mode \( H_{01} \) eigen transverse wavenumber \( \kappa_0 R_2 = \lambda_{11} = 3.83171 \). In the presence of the plasma cylinder, boundary conditions at the waveguide wall for the perturbed mode are

\[ E_z |_{r=R_2} = 0, \frac{\partial H_z}{\partial r} |_{r=R_2} = 0, E_\varphi |_{r=R_2} = 0. \] (44)

Using the same procedure as for the derivation of Eqs. (26) and (36) we find the following relations for coefficients \( C \), \( D \), and \( C_1 \), \( D_1 \):

\[ C_1 \approx -C \Gamma_1^E \delta \kappa_2 R_2, \ D_1 \approx -D \Gamma_1^E, \] (45)

with coefficients being

\[ \Gamma_1^E = \frac{J_1'(\kappa_0 R_2)}{Y_1(\kappa_0 R_2)}, \ G_1^E = \frac{J_1'(\kappa_0 R_2)}{Y_1(\kappa_0 R_2)}. \] (46)

Boundary conditions at the plasma interface lie in the continuity of field tangential components at \( r=R_1 \). Namely, continuity of \( E_z \) in the first order of the perturbation theory can be written as

\[ A[J_1(\kappa_0 R_1) + J_1'(\kappa_0 R_1) \delta \kappa_1 R_1] \]
\[ \approx C[J_1(\kappa_0 R_1) + J_1'(\kappa_0 R_1) \delta \kappa_2 R_1 - \Gamma_{11}^E Y_1(\kappa_0 R_1) \delta \kappa_2 R_2], \] (47)

and continuity of \( H_z \) is

\[ BJ_1(\kappa_0 R_1) \approx D[J_1(\kappa_0 R_1) - G_1^E Y_1(\kappa_0 R_1)]. \] (48)

Continuity conditions for azimuthal components are represented as

\[ -i \frac{h}{\kappa_0} A \left[ \left( 1 - \frac{2 \delta \kappa_1}{\kappa_0} \right) J_1(\kappa_0 R_1) + J_1'(\kappa_0 R_1) \delta \kappa_1 R_1 \right] \]
\[ - \frac{k}{\kappa_0} BJ_1' \] (49)

for the \( E_\varphi \) and for the \( H_\varphi \) we have

\[ i \frac{k}{\kappa_0} A \left[ \left( \xi_\alpha - \frac{\delta \kappa_1}{\kappa_0} \right) J_1'(\kappa_0 R_1) + J_1''(\kappa_0 R_1) \delta \kappa_1 R_1 \right] \]
\[ + \frac{h}{\kappa_0} BJ_1 \]
\[ \approx i \frac{k}{\kappa_0} C \left[ \left( 1 - \frac{2 \delta \kappa_2}{\kappa_0} \right) J_1(\kappa_0 R_1) + J_1'(\kappa_0 R_1) \delta \kappa_2 R_1 \right] \]
\[ - \Gamma_{11}^E Y'_1(\kappa_0 R_1) \delta \kappa_2 R_2 \]
\[ + i \frac{h}{\kappa_0} D[J_1(\kappa_0 R_1) - G_1^E Y_1(\kappa_0 R_1)]. \] (50)

After some algebra one can obtain the relation between \( C \) and \( D \):

\[ \frac{2h}{k_0 \kappa_0 R_1} C(\delta \kappa_1 - \delta \kappa_2) J_1'(\kappa_0 R_1) \]
\[ = D G_1^E \left[ J_1(\kappa_0 R_1) Y'_1(\kappa_0 R_1) - J_1'(\kappa_0 R_1) Y_1(\kappa_0 R_1) \right], \] (51)

which means that for the perturbed \( E_{11} \) mode, coefficients \( D \) and \( B \) (in accordance with Eq. (48)) are of the first order of smallness with respect to coefficients \( A \) and \( C \). Combining Eqs. (47)–(50) we find

\[ \left( \xi_\alpha - 1 \right) J_1(\kappa_0 R_1) J_1'(\kappa_0 R_1) + \left( \delta \kappa_1 - \delta \kappa_2 \right) R_1 \left( J_1'(\kappa_0 R_1) \right)^2 \]
\[ - J_1(\kappa_0 R_1) J_1''(\kappa_0 R_1) + \frac{1}{\kappa_0 R_1} J_1(\kappa_0 R_1) J_1'(\kappa_0 R_1) \]
\[ \approx \Gamma_{11}^E \delta \kappa_2 R_2 \left[ Y_1(\kappa_0 R_1) J_1'(\kappa_0 R_1) - J_1(\kappa_0 R_1) Y'_1(\kappa_0 R_1) \right]. \] (52)
Taking Eqs. (21) and the Wronskian for Bessel functions into account we finally derive the correction to the transverse wave-number in the first order of the perturbation theory:

$$\delta k_2 R_2 \approx (\xi_{\omega} - 1) \frac{\pi}{2 \Gamma_{11}} \times \left\{ \frac{k^2 R_1^2}{2} \left( J_0^*(\kappa_0 R_1) + \left( 1 - \frac{1}{\kappa_0^2 R_1^2} \right) J_1^*(\kappa_0 R_1) \right) - \kappa_0 R_1 J_1(\kappa_0 R_1) J'_1(\kappa_0 R_1) \right\}. \tag{53}$$

For the ratio of the $E_{11}$ modal gain to the maximum one we have

$$K_{11}^H(x) \approx -\frac{\pi}{2 \Gamma_{11}} \times \left\{ \frac{J_1(x \xi_{11})}{2} \left( J_0^*(\kappa_0 R_1) + \left( 1 - \frac{1}{\kappa_0^2 R_1^2} \right) J_1^*(\kappa_0 R_1) \right) - \frac{k_0^2}{k^2} J_1(x \xi_{11}) J'_1(x \xi_{11}) \right\}. \tag{54}$$

Here, $\kappa_0 = \xi_{11}/R_2$, $\xi_{11} = \frac{3.83171}{2}$ and $\Gamma_{11}^H = \frac{J'_1(x \xi_{11})}{J_1(x \xi_{11})} \approx -0.9763$.

Let us consider now the mode $H_{11}$. The structure of the electromagnetic field for the mode $H_{11}$ is determined by Equations (41) and (42), the same as for the mode $E_{11}$, but now “electric” amplitude coefficients $A$ and $C$ are of the first order of smallness with respect to “magnetic” amplitude coefficients $B$ and $D$. The dispersion relation for the unperturbed mode $H_{11}$ of an empty waveguide is determined by the first root of equation $J'_1(\kappa_0 R_2) = 0$, so the eigen value is $\kappa_0 R_2 = \xi_{11} \approx 1.84118$.

Boundary conditions (44) at the waveguide wall now lead to the following relations between coefficients $C$, $D$, and $C_1$, $D_1$:

$$C_1 \approx -CG_{11}^H, \quad D_1 \approx -D\Gamma_{11}^H \delta k_2 R_2, \tag{55}$$

with coefficients being written

$$G_{11}^H = \frac{J_1(\kappa_0 R_2)}{Y_1(\kappa_0 R_2)}, \quad \Gamma_{11}^H = \frac{J'_1(\kappa_0 R_2)}{Y'_1(\kappa_0 R_2)}. \tag{56}$$

Continuity conditions at the plasma interface for longitudinal electromagnetic field components are

$$B[J_1(\kappa_0 R_1) + J'_1(\kappa_0 R_1) \delta k_1 R_1] \approx D[J_1(\kappa_0 R_1) + J'_1(\kappa_0 R_1) \delta k_2 R_1 - \Gamma_{11}^H Y_1(\kappa_0 R_1) \delta k_2 R_2], \tag{57}$$

for $H_z$, and for $E_z$,

$$A\overline{J}_1(\kappa_0 R_1) \approx C[J_1(\kappa_0 R_1) - G_{11}^H Y_1(\kappa_0 R_1)]. \tag{58}$$

The continuity condition for the azimuthal electric field component $E_\varphi$ at the plasma interface is written as

$$-i \frac{\hbar}{k_0^2 R_1} A J_1(\kappa_0 R_1) - i \frac{k}{k_0} B \left[ \left( 1 - \frac{\delta k_1}{k_0} \right) J'_1(\kappa_0 R_1) + J''_1(\kappa_0 R_1) \delta k_1 R_1 \right] \approx -i \frac{\hbar}{k_0^2 R_1} C J_1(\kappa_0 R_1) - G_{11}^H Y_1(\kappa_0 R_1)]$$

and for $H_\varphi$,

$$i \frac{\hbar}{k_0^2 R_1} A J'_1(\kappa_0 R_1) + i \frac{k}{k_0} B \left[ \left( 1 - \frac{2 \delta k_2}{k_0} \right) J_1(\kappa_0 R_1) + J'_1(\kappa_0 R_1) \delta k_2 R_1 \right] \approx i \frac{k}{k_0} C J'_1(\kappa_0 R_1) - G_{11}^H Y'_1(\kappa_0 R_1)]. \tag{60}$$

After some calculations we obtain the following equation guiding corrections to transverse wavenumbers:

$$(\delta k_1 - \delta k_2) R_1 \left[ \left( J'_1(\kappa_0 R_1) \right)^2 - J_1(\kappa_0 R_1) J''_1(\kappa_0 R_1) + \frac{1}{k_0 R_1} J_1(\kappa_0 R_1) J'_1(\kappa_0 R_1) \right] \approx \Gamma_{11}^H \delta k_2 R_2 Y_1(\kappa_0 R_1) J'_1(\kappa_0 R_1) - J_1(\kappa_0 R_1) Y'_1(\kappa_0 R_1)]. \tag{61}$$

Finally, we come to the following relation for the correction to the transverse wavenumber of mode $H_{11}$:

$$\delta k_2 R_2 \approx (\xi_{\omega} - 1) \frac{\pi}{2 \Gamma_{11}^H} \times \left\{ \frac{k^2 R_1^2}{2} \left( J_0^*(\kappa_0 R_1) + \left( 1 - \frac{1}{\kappa_0^2 R_1^2} \right) J_1^*(\kappa_0 R_1) \right) \right\}, \tag{62}$$

and the ratio of the $H_{11}$ modal gain to the maximum one is...
where the value of coupling parameter $\Gamma_{11}^H = J_0^2(\kappa_0 R_2)/\gamma_0(\kappa_0 R_2) \approx -0.690332$ for the eigen value $\kappa_0 R_2 = \gamma_1 \approx 1.84118$. In real experiments with a microwave signal of super-terahertz frequency domain the transverse wavenumber $\kappa_0$ should be small with respect to the vacuum wavenumber $k$. So the last term in curly brackets in Eq. (54) can be considered as vanishing and the only difference between Eqs. (54) and (63) is the eigen value of the correspondent dispersion equation for the unperturbed mode: $\gamma_1 = 3.83171$ for the $E_{11}$ mode in Eq. (61) and $\gamma'_{11} \approx 1.84118$ in Eq. (63).

In Fig. 5 dependencies of the modal gain increments on the waveguide filling factor given by Eqs. (33), (40), (54), and (63) are presented. One can see that for the axially symmetric TM mode $E_{01}$ the ratio of increment to the maximum one (22) reveals lowest values, while for the axially symmetric TE mode $H_{01}$ this ratio reaches maximum among all considered modes. At low filling factors $x < 0.5$ the ratio coefficient $K_{11}^T(x)$ of the $E_{11}$ mode depicts high modal gain increments, almost as high as for the mode $H_{01}$. However, with the increase of $x$ the growth rate of $K_{11}^T(x)$ decreases and the modal increment becomes closer to that for the TM mode $E_{01}$. For the $H_{11}$ modal gain increment we have the opposite behaviour. At low filling factors the function $K_{11}^H(x)$ is significantly less than that for $E_{11}$ and $H_{01}$ modes, $K_{11}^T(x)$ and $K_{11}^{TE}(x)$, respectively. However, with the increase of filling factor at $x > 0.5$ the function $K_{11}^H(x)$ becomes closer to $K_{11}^{TE}(x)$ corresponding to the $H_{01}$ mode.

VI. DISCUSSION

Let us consider a possible scheme of experiment in which the above proposed novel amplification mechanism could be demonstrated and make estimates of proper plasma and laser parameters. We will consider the circular waveguide filled with the noble gas (xenon) at the normal pressure of 1 atm. As far as efficient amplification can be attained at signal frequencies less than the characteristic transport collision rate, we will assume that the microwave signal frequency is $\omega = 5 \times 10^{11}\text{s}^{-1}$, which corresponds to the vacuum wavenumber $k_0 \approx 17\text{ cm}^{-1}$ and the signal wavelength is $\lambda \approx 3.77\text{ mm}$. Let the radius of the waveguide be $R = 1\text{ cm}$ and the length be $L = 1\text{ m}$. Assuming the laser plasma density in the ionized channel $n_i = 10^{12}\text{ cm}^{-3}$, one can find from Fig. 2 the peak value of the imaginary part of plasma dielectric permittivity is $\text{Im}(\varepsilon_{\text{pl}}) \approx -2 \times 10^{-3}$ and the temporal interval of negative absorption is about 30 ns. Assuming that the radius of the ionized plasma channel is 3/4 of the waveguide radius, i.e., 7.5 mm, we find from Fig. 5 that the maximum modal gain is attained for the axial symmetric TE $H_{01}$ mode and the $H_{11}$ mode, $K^{TE} \approx K_{11}^H \approx 0.8$ while $K_{11}^T \approx 0.6$ and the lowest modal gain is for the axial symmetric $E_{01}$ mode $K^{TH} \approx 0.4$. Thus, total gain of the signal intensity at the rear end of the waveguide $\exp\{2\text{Im}(\varepsilon_{\text{pl}})K_i L\}$ can be estimated from 11.8 dB for the $E_{01}$ mode up to 17.7 dB for the $E_{11}$ mode and the highest gain is 23.6 dB for $H_{01}$ and $H_{11}$ modes.

A low-density plasma channel in xenon is created with the use of a KrF ultrashort laser pulse through the direct three-photon ionization process with the cross-section $\sigma_3 \approx 5 \times 10^{-22}\text{ cm}^2$. Keeping in mind the GARPUN-MTV multistage hybrid KrF laser system, one can estimate that for 1ps pulse duration the intensity $3.4 \times 10^{10}\text{W/cm}^2$ and the pulse energy 59 mJ are needed to create an ionized channel with the radius 7.5 mm and density $n_i = 10^{12}\text{ cm}^{-3}$ inside the waveguide of 1 m length. Then the energy utilized for ionization of xenon atoms is 0.42 mJ, which is small compared with the total pulse energy, so no pulse depletion will arise which provides homogeneous density profile along the waveguide. The energy per plasma electron converted into the microwave signal energy can be estimated as the difference between the initial kinetic energy of the electron, $3h\omega - I_{\text{c}}$, and the electron energy corresponding to the bottom of the Ramsauer minimum (see Fig. 1(a)), so this results in the total value of 68 $\mu$J. In this estimate, we assume that the inequality $\omega < v_{\text{tr}}(\epsilon)$ holds throughout all the amplification process. Note that this inequality determines the upper limiting frequency for a signal to be amplified: In order to apply the proposed physical mechanism for shorter wavelengths and attain 1 THz-frequency domain one should increase the gas pressure almost by the order of magnitude.

High values of the modal gain coefficients make it possible to suggest a multi-pass amplification scheme. According to Fig. 2, the dynamical interval of negative high-frequency conductivity $\text{Im}(\varepsilon_{\text{pl}}) < 0$ is of a few dozen of nanoseconds. This allows at definite conditions to get the free oscillation regime in which the microwave signal can be generated from noise.

VII. SUMMARY

Thus, the present study demonstrates that a high-intensity KrF laser pulse can create a plasma channel that turns out to be optically denser than a non-ionized gaseous medium on the time scale of relaxation of a strongly non-equilibrium photoelectron spectrum. Such an ionized channel can be used as a waveguide for both transportation and amplification of microwave pulses of a few tens of nanoseconds duration in a sliding-mode regime. A metal waveguide partially filled with such nonequilibrium xenon plasma can provide modal amplification increments close to the maximum value peculiar to the plane wave in the infinite plasma medium on the time scale of relaxation of a strongly non-equilibrium photoelectron spectrum. Such an ionized channel can be used as a waveguide for both transportation and amplification of microwave pulses of a few tens of nanoseconds duration in a sliding-mode regime. A metal waveguide partially filled with such nonequilibrium xenon plasma can provide modal amplification increments close to the maximum value peculiar to the plane wave in the infinite plasma medium on the time scale of relaxation of a strongly non-equilibrium photoelectron spectrum.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research (Project Nos. 15-02-00373 and 16-32-00123).


L. Chin, Femtosecond Laser Filamentation (Springer Verlag, New York, 2010).


