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On the Relativity and Equivalence Principles in the Gauge Theory of Gravitation.

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One sees the basic ideas of the gauge gravitation theory still not generally accepted, in spite of more than twenty years of its history.

The chief reason lies in the fact that the gauge character of gravity is connected with the whole complex of problems of Einstein general relativity: about the reference system definition, on the (3+1)-splitting, on the presence (or absence) of symmetries in GR, on the necessity (or triviality) of general covariance, on the meaning of equivalence principle, which led EINSTEIN from special to general relativity (1).

The real actuality of this complex of interconnected problems is demonstrated by the well-known work of Fock, who saw no symmetries in general relativity, declared the unnecessary equivalence principle and proposed even to substitute the designation « chronogeometry » instead of « general relativity » (see also HAVAS). Developing this line, BONDI quite recently also expressed doubts about the « relativity » in Einstein theory of gravitation.

All proposed versions of the gauge gravitation theory must clarify the discrepancy between Einstein gravitational field being a pseudo-Riemannian metric field, and the gauge potentials representing connections on some fibre bundles and there exists no group, whose gauging would lead to the purely gravitational part of connection (Christoffel symbols of Fock-Ivanenko-Weyl spinorial coefficients).

We shall not analyse here all attempts to overcome this difficulty (see reviews (2-4)). But we believe that every gauge theory, aiming at describing the gravity, must be in agreement with the basic relativity and equivalence principles of Einstein theory. Moreover, the gauge gravitation theory is to be constructed from this principles.

(1) D. IVANENKO: in *Einstein Centenary* (Jubilee volume of the Nat. Ital. Ac. Sci.) (Firenze, 1979); English version by Johnson preprint Corporation (New York, N. Y., 1979).
(2) A. TRAUTMAN: in *General Relativity and Gravitation*, edited by A. HELD (New York, N. Y., 1979), p. 287.
(3) Y. NE'EMAN: in *General Relativity and Gravitation*, edited by A. HELD (New York, N. Y., 1979), p. 309.
(4) F. HEHL: in *General Relativity and Gravitation*, edited by A. HELD (New York, N. Y., 1979),

To this purpose, let us compare the relativity principle (RP) and the equivalence principle (EP) of gravitation theory with the postulates of gauge theory by using the formalization of both of these theories in terms of fibre bundles.

Without entering in details and comparing here all existing versions, one sees that essentially RP states the equivalence of all reference systems for formulation of fundamental physical laws. So one needs the notion of reference systems for a given theory and must define the class of their transformations. RP requires then the form covariance of physical laws with respect to the group of all transformations of the reference frames. One needs also to indicate some reference system, where the physical laws and equations possess the well-known form. In the case of gravitation theory this is the reference system, corresponding to transition to special relativity, established by EP.

In the fibre bundle approach of the gauge theory the interaction of a field multiplet $\{\varphi\}$ possessing the symmetry group G is described by introducing a geometrical structure (connections) on the fibre bundle λ, whose sections are the matter fields $\{\varphi\}$. The basic principle of the gauge theory—the local invariance—means the covariance of matter field equations under transformations of the fibre bundle λ atlases. These transformations are elements of the local group G_X, which is defined as the group of all sections of the principal fibre bundle associated with λ. In the fibre bundle formalism of the gauge theory a reference system is established by fixing the atlas of the fibre bundle λ. This is expressed in description of the matter fields $\{\varphi\}$ by means of a certain family of functions with values in the space of a typical fibre and with a particular gauge of the gauge fields.

One must distinguish the case in which a special choice of gauging is possible, leading to the reduction of the gauge fields of the group G to the gauge fields of one of its subgroup H (the holonomy group of these fields on the fibre bundle λ is reduced to H), *i.e.* reference systems exist, where the gauge theory of the group G is effectively reduced to the gauge theory of its subgroup H. Then one has the contraction of the structure group G of the fibre bundle λ to the subgroup H. The necessary and sufficient condition for this contraction is the existence of some global section of the quotient bundle associated with λ with the typical fibre G/H. The field σ, arising in such manner in the gauge theory is analogous the Goldstone fields in the well-known models with spontaneous symmetry breaking (5,6).

In the fibre bundle formalism the Einstein gravitational field on the orientable space-time manifold X^4 is (in mathematical language) defined as a section of the fibre bundle Λ of pseudo-Euclidean bilinear forms in the tangent spaces T_x over X^4. Λ is associated with the tangent bundle T_{X^4}, which possesses the structure group $GL^+_{4,R}$. The gravitation fields and connections (Christoffel symbols) expressed by the components of these fields provide a geometry on the tangent bundle T_{X^4}, which is conventionally understood as the geometry of the space-time itself.

Then in analogy with the gauge theory the choice of a reference system in the gravitation theory can be defined as the fixation of an atlas $\Psi = \{U_i, \psi_i\}$ of the tangent bundle T_{X^4}, and the reference frames transformations are the local group $GL^+_{4,R\,X^4}$ transformations of the atlases of this bundle.

This definition of the reference systems is close to that used in the tetrad formulation of general relativity. The fixation of the atlas of T_{X_4} leads to construction at each point of a tetradic reference $\{l_X\}$, which is the image $\{l_X\} = \psi_i^{-1}(x)\{l\}$ of the bases $\{l\}$ of the typical fibre of T_{X^4}, and changes of the atlases are followed by the tetrad trans-

(5) G. SARDANASHVILY: *Abstracts of contributed papers*, in *VIII International Conference on Gravitation* (Canada, 1977), p. 311.

formations. The Einstein (metrical not tetradic) formulation of general relativity corresponds to the case of purely holonomic transformations of the reference frames, when along with a co-ordinate system, defined by the atlas $\Psi_X = \{U_i, \gamma_i\}$ of the manifold X^4, one always chooses the reference system adjusted to it with the atlas $\Psi = \{U_i, \psi_i = d\gamma_i\}$ and this correlation is strictly retained in passing to another co-ordinate frame $\tau: \Psi_X \to \Psi'_X$ accompanied by the corresponding transformation $d\tau: \Psi \to \Psi'$ of the reference system.

Taking all this into account, one can formulate the relativity principle in the gravitation theory as the requirement of the covariance of the equations of matter fields interacting with gravity with respect to local group $GL^+_{4,R}$ and as such it coincides with the basic postulate of local invariance in the gauge theory. This means that the gravitation gauge theory can directly be built from RP as the gauge theory of external symmetrics. Its peculiar character is manifested by the fact, that external symmetries gauge fields act also on the operators of partial derivatives $\{\partial_\mu\}$, as the vectors of tangent spaces. This circumstance leads, for example, to supplementary possibilities of indices contraction and permits, in contrast to the Yang-Mills case, to construct the curvature scalar from the strength tensor of gauge fields and choose it as the Hilbert-Einstein-Lagrangian of the theory. But one must emphasize that the $GL^+_{4,R}$ gauge theory is broader than the theory with pseudo-Riemannian metric gravitational fields, because it possesses metric fields with an undefined signature. The passage to the proper Einstein gravity is connected with the equivalence principle.

In the conventional gauge theory EP is as a rule not explicitely formulated. The representation of gauge fields by 1-forms of connections on fibre bundles leads already to the existence of a reference system, where gauge fields are vanishing in a given point. Hence in the gauge theory of gravitation EP means the indication of the type of gauging only. If one accepts in symmetries language the reduction of external symmetries to the Lorentz group $SO_{3,1}$ as the criterion of transition to special relativity, then EP can be formulated as the requirement of the $GL^+_{4,R}$ gauge theory reduction to the gauge theory of the Lorentz group. This is expressed by the condition of the restriction of the holonomy group on the tangent bundle T_{X^4} to the Lorentz group. Like the general case of the gauge field theory one has then the condition of the reduction of the structure group $GL^+_{4,R}$ of the tangent bundle T_{X^4} to the Lorentz group and hence the important fact of the existence of the global section of the quotient bundle associated with T_{X^4} with the typical fibre $GL^+_{4,R}/SO_{3,1}$ occurs. This fibre bundle is isomorphic to the fibre bundle A of pseudo-Euclidean bilinear forms in the spaces tangent to X^4. One sees that the fulfilment of EP leads to the existence of the metrical gravitational field globally defined on the whole-given space-time manifold X^4.

At the same time this permits to interpret the gravitation field as a Goldstone-type field in the gauge theory of external symmetries (6) (the idea of the possibility to consider the gravity as a Goldstone (Higgs) field was also proposed in (2,3)). But in contrast to the Goldstone fields of internal symmetries, the gravitation field cannot be removed everywhere by any special choice of gauge.

The contraction of the structure group $GL^+_{4,R}$ to the Lorentz $SO_{3,1}$ group and consequently to its maximal compact subgroup SO_3 implies some Klein-Chern geometry of invariants on T_{X^4} (6), and this enables us to interpret the geometrical aspects of gravitation in the spirit of the Erlanger programme. From this point of view one may not agree with the Fack-Havas-Bondi opinion about the absence of any symmetries in general relativity.

Let us shortly remark that the realization of gravitation fields by cross-sections of the quotient bundle defines their topological classification as the classification of the bundles T_{X^4}, whose structure groups can contract to the Lorentz group (4). As $SO_{3,1}$ is the image of the injection of the $SL_{2,C}$-group into $GL^+_{4,R}$, the structure of the $SL_{2,C}$ bundle on T_{X^4} is defined and its classification in Chern $c_{1,2} \in H^{2,4}_{X,Z}$ and Pontryagin $p_1 = c_1^2 - 2c_2$ indices proves to be possible, without passing to Riemannian metrics and SO_4 bundles. For compact manifolds X^4 these indices are represented by Chern differential forms (expressed by curvature) and the integrals $c_2[X^4]$ and $\frac{1}{3}p_1[X^4]$ coincide with Euler number and the homology index (both equal to zero!) of the manifold X^4, respectively.

Finally we can draw the conclusions that the gauge gravitation theory, constructed from relativity and equivalence principles reformulated in terms of fibre bundles, in the $GL^+_{4,R}$ gauge theory of external symmetries, reduced in some reference frames to the Lorentz gauge theory, with the description of the metric gravitational field as a Goldstone-type field, analogous to Goldstone fields in models with spontaneous symmetry breaking.