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Parameter symmetry of the interacting boson model

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Abstract

We discuss the symmetry of the parameter space of the interacting boson model (IBM). It is shown that for any set of the IBM Hamiltonian parameters (with the only exception of the U(5) dynamical symmetry limit) one can always find another set that generates the equivalent spectrum. We discuss the origin of the symmetry and its relevance for physical applications. © 1998 Elsevier Science B.V. All rights reserved.

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In this paper we shall consider the simplest version of the interacting boson model (IBM-1) that was proposed by Arima and Iachello [1,2] for the description of spectra of low-lying states of even-even nuclei. IBM is now widely used in nuclear applications. The symmetry properties of IBM that are determined by the structure of the group U(6) have been thoroughly studied (see, e.g., [2,3] and references therein).

Below we shall discuss a symmetry in the space of the parameters of the IBM Hamiltonian. We consider an irreducible set of parameters which because of the structure of the Lie algebra U(6), cannot be reduced to a smaller set [2]. As we shall show, the parameters in the IBM Hamiltonian can be transformed linearly in such a way that the eigenvalue spectrum remains unchanged. We shall refer to this as parameter symmetry (PS). To the best of our knowledge, the parameter symmetry has not been discussed before.

We shall show that PS can be related to ambiguities of the definition of boson operators within IBM or, equivalently, to the possibility of using different realizations of the SU(3) or SO(6) subalgebras of the U(6) algebra. The ambiguities have been discussed by various authors (see, e. g., [2–5]). However, previous studies were mostly restricted to the cases of SU(3) and SO(6) dynamical symmetry limits of IBM. Our approach is

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more general and we emphasize manifestations of the transformations of the boson operators in the space of parameters. We make use of unitary transformations producing the same (formal) effect on a Hamiltonian as transformations of parameters. This was also the basis of Refs. [6,7] devoted to the weak interaction theory. The general principle was carefully discussed in the review [8] where it was called “form invariance”.

The IBM Hamiltonian parameters are obtained by fitting the predictions of the model to nuclear spectra known from the experiment. Because of PS, the fit appears to be essentially ambiguous. We discuss how to resolve the ambiguity of the fit in applications.

Within IBM, nuclear states are labelled by a fixed number N of bosons of two types, s and d , with quantum numbers $J^\pi = 0^+$ and $J^\pi = 2^+$, respectively [1]. The six boson creation operators ($s^+, d_\mu^+, \mu = 0, \pm 1, \pm 2$) and six boson annihilation operators ($s, d_\mu, \mu = 0, \pm 1, \pm 2$) satisfy standard boson commutation relations. The structure of the model is determined by the Lie algebra $U(6)$ generated by 36 bilinear combinations of these boson operators [1–3].

The IBM Hamiltonian H can be expressed as a superposition of the first (C_1) and second (C_2) order Casimir operators of the groups entering the following reduction chains of the $U(6)$ group [1–3]

$$\begin{aligned} & \searrow U(5) \supset SO(5) \supset SO(3) \supset SO(2) \quad \text{I} \\ U(6) \supset & SU(3) \supset SO(3) \supset SO(2) \quad \text{II} \\ & \swarrow SO(6) \supset SO(5) \supset SO(3) \supset SO(2) \quad \text{III} \end{aligned} \quad (1)$$

i. e.

$$\begin{aligned} H(\{k_i\}) = & H_0 + k_1 C_1(U(5)) + k_2 C_2(U(5)) + k_3 C_2(SO(5)) + k_4 C_2(SO(3)) + k_5 C_2(SO(6)) \\ & + k_6 C_2(SU(3)), \end{aligned} \quad (2)$$

where $\{k_i\} \equiv \{H_0, k_1, k_2, k_3, k_4, k_5, k_6\}$ is the set of the Hamiltonian parameters. The parameter set $\{k_i\}$ is irreducible and the number of parameters cannot be reduced [1,2]. We define the Casimir operators in Eq. (2) as in Ref. [3].

The main result of this letter can be stated as follows:

Hamiltonians $H(\{k_i\})$ and $H(\{k'_i\})$ defined by Eq. (2) have identical eigenvalue spectra if the corresponding irreducible parameter sets $\{k_i\}$ and $\{k'_i\}$ are related by

$$\begin{aligned} H'_0 = H_0, \quad k'_1 = k_1 + 2k_6, \quad k'_2 = k_2 + 2k_6, \quad k'_3 = k_3 - 6k_6, \quad k'_4 = k_4 + 2k_6, \quad k'_5 = k_5 + 4k_6, \\ k'_6 = -k_6 \end{aligned} \quad (3)$$

in the case $k_6 \neq 0$, or by

$$\begin{aligned} H'_0 = H_0 + 10k_5 N, \quad k'_1 = k_1 + 4k_5(N+2), \quad k'_2 = k_2 - 4k_5, \quad k'_3 = k_3 + 2k_5, \quad k'_4 = k_4, \quad k'_5 = -k_5, \\ k'_6 = 0 \end{aligned} \quad (4)$$

in the case $k_6 = 0$.

To derive Eqs. (3)–(4) we consider the Hamiltonian matrix in the basis $|Nn\nu_n \Delta LM\rangle$ (see, e.g., [2]) associated with the reduction chain I in Eq. (1). The only quantum number labelling the basis which is essential for the following discussion is the number n of d -bosons. Therefore we introduce the shortened notation $\langle n|H|n'\rangle \equiv \langle Nn\nu_n \Delta LM|H|Nn'\nu' n'_\Delta LM\rangle$. The total number of bosons labelling the totally symmetric representation of the $U(6)$ group $N = n_s + n$ where n_s is the number of s bosons.

Obviously, the matrices of Casimir operators $C_1(\text{U}(5))$, $C_2(\text{U}(5))$, $C_2(\text{SO}(5))$ and $C_2(\text{SO}(3))$ of the groups entering the reduction chain I contribute only to diagonal matrix elements $\langle n|H|n\rangle$. Off-diagonal matrix elements $\langle n|H|n'\rangle$ arise from the operators $C_2(\text{SO}(6))$ and $C_2(\text{SU}(3))$, defined by [3],

$$C_2(\text{SO}(6)) = N(N + 4) - 4(P^+ \cdot P), \tag{5}$$

$$C_2(\text{SU}(3)) = 2(Q \cdot Q) + \frac{3}{4}(L \cdot L), \tag{6}$$

where the multipole operators P , L , and Q are

$$P = \frac{1}{2}((\tilde{d} \cdot \tilde{d}) - (s \cdot s)), \tag{7}$$

$$L = \sqrt{10} [d^+ \times \tilde{d}]^{(1)}, \tag{8}$$

$$Q = [d^+ \times s + s^+ \times \tilde{d}]^{(2)} - \frac{\sqrt{7}}{2} [d^+ \times \tilde{d}]^{(2)}, \tag{9}$$

with $\tilde{d}_\mu = (-1)^\mu d_{-\mu}$, $(t^{(\lambda)} \cdot u^{(\lambda)}) = (-1)^\lambda \sqrt{2\lambda + 1} [t^{(\lambda)} \times u^{(\lambda)}]^{(0)}$, and $[t^{(\lambda_1)} \times u^{(\lambda_2)}]_\mu^{(\lambda)} = \sum_{\mu_1 \mu_2} (\lambda_1 \mu_1 \lambda_2 \mu_2 | \lambda \mu) t_{\mu_1}^{(\lambda_1)} u_{\mu_2}^{(\lambda_2)}$ where $(\lambda_1 \mu_1 \lambda_2 \mu_2 | \lambda \mu)$ is a Clebsch-Gordan coefficient. The operator $C_2(\text{SU}(3))$ contributes to the off-diagonal matrix elements $\langle n|H|n \pm 1\rangle$ and $\langle n|H|n \pm 2\rangle$ while the operator $C_2(\text{SO}(6))$ contributes to off-diagonal matrix elements only of the type $\langle n|H|n \pm 2\rangle$. Both Casimir operators also contribute to the diagonal matrix elements $\langle n|H|n\rangle$. Thus in the general case the matrix of the Hamiltonian (2) is five-diagonal.

Let us now transform the matrix $\langle n|H|n'\rangle$ using the unitary transformation $\langle n|U_1|n'\rangle = (-1)^n \delta_{nn'}$ that produces a transformed $H' = (U_1)^{-1} H U_1$ differing from H only by the sign of the off-diagonal matrix elements $\langle n|H|n \pm 1\rangle$. Since the only Casimir operator which contributes to these matrix elements is $C_2(\text{SU}(3))$, the sign of $\langle n|H|n \pm 1\rangle$ can be restored by setting $k_6 \rightarrow k'_6 = -k_6$. The diagonal matrix elements $\langle n|H|n\rangle$ and the off-diagonal ones of type $\langle n|H|n \pm 2\rangle$ may then be restored by setting $k_i \rightarrow k'_i$ for all the rest of the parameters, where the k'_i were defined in (3). This claim is easily verified using the explicit expressions for the Casimir operators found elsewhere [3].

When $k_6 = 0$, the only non-zero matrix elements are $\langle n|H|n\rangle$ and $\langle n|H|n \pm 2\rangle$, so that the Hamiltonian matrix is tridiagonal. Thus we can use the unitary transformation U_2 that transforms the Hamiltonian into $H' = (U_2)^{-1} H U_2$ with non-zero matrix elements $\langle n|H'|n\rangle = \langle n|H|n\rangle$ and $\langle n|H'|n \pm 2\rangle = -\langle n|H|n \pm 2\rangle$. The sign change of the off-diagonal matrix elements $\langle n|H|n \pm 2\rangle$ can be restored by setting $k_5 \rightarrow k'_5 = -k_5$. To restore the diagonal matrix elements, we redefine the parameters k_i , $1 \leq i \leq 4$ according to (4).

Thus we have shown that for any irreducible set of the IBM Hamiltonian parameters there is another irreducible set which yields the same spectrum. The only exception is the U(5) dynamical symmetry (DS) limit ($k_5 = k_6 = 0$) when the two sets of parameters coincide, as is easily seen from (4).

An intriguing consequence of PS is that it establishes an equivalence between the nuclear spectrum corresponding to a certain DS and a transitional nuclear spectrum. As follows from Eqs. (3), the rotational spectrum of the SU(3) DS limit ($k_1 = k_2 = k_3 = k_5 = 0$) is identical to the spectrum of the transitional Hamiltonian with the parameters

$$\{k'_i\} \equiv \{k'_1 = 2k_6, \quad k'_2 = 2k_6, \quad k'_3 = -6k_6, \quad k'_4 = k_4 + 2k_6, \quad k'_5 = 4k_6, \quad k'_6 = -k_6\}, \tag{10}$$

which does not correspond to any DS. Similarly, it follows from (4) that the γ -unstable spectrum of the SO(6) DS limit ($k_1 = k_2 = k_6 = 0$) is identical to the U(5)–SO(6) transitional nuclear spectrum with the parameters

$$\{k'_i\} \equiv \{k'_1 = 4(N + 2)k_5, \quad k'_2 = -4k_5, \quad k'_3 = k_3 + 2k_5, \quad k'_4 = k_4, \quad k'_5 = -k_5, \quad k'_6 = 0\}. \tag{11}$$

To understand the origin of the PS discussed above, we note that there is an ambiguity in definition of boson operators within IBM [2–4]. For example, one can change the sign of the creation s^+ and annihilation s operators without violating the boson commutation relations. Obviously, the transformation

$$\{s^+ \rightarrow -s^+, \quad s \rightarrow -s\} \tag{12}$$

should not produce any change of the spectra. From Eqs. (5)–(9) it is seen that only the Casimir operator $C_2(\text{SU}(3))$ is changed under the transformation (12) [we note that the Casimir operators $C_1(\text{U}(5))$, $C_2(\text{U}(5))$, $C_2(\text{SO}(5))$ and $C_2(\text{SO}(3))$ can be expressed through bilinear combinations of d boson operators only and are unchanged under any transformation of s boson operators]. The transformation (12) is equivalent to the transformation of the quadrupole operator $Q \rightarrow \bar{Q}$ where

$$\bar{Q} = -[d^+ \times s + s^+ \times \tilde{d}]^{(2)} - \frac{\sqrt{7}}{2}[d^+ \times \tilde{d}]^{(2)}. \quad (13)$$

It is clear from Eqs. (5)–(9) and (13) that the off-diagonal matrix elements $\langle n|H|n \pm 1 \rangle$ change their sign under the transformation (12) while all the remaining matrix elements of the Hamiltonian remain unchanged. Thus, the PS transformation (3) restores the original form of the IBM Hamiltonian H subjected to the transformation (12), or, in other words, the PS transformation (3) is equivalent to the transformation (12).

The quadrupole operators Q and \bar{Q} correspond to different embeddings of the $\text{SU}(3)$ subgroup in the $\text{U}(6)$ group (see also [9] for other realizations of $\text{SU}(3)$). The Casimir operator $C_2(\overline{\text{SU}(3)})$ of the $\overline{\text{SU}(3)}$ algebra associated with the quadrupole operator (13) can be expressed through $C_2(\text{SU}(3))$ and other Casimir operators using (10) with $k_4 = 0$ and $k_6 = 1$:

$$C_2(\overline{\text{SU}(3)}) = 2C_1(\text{U}(5)) + 2C_2(\text{U}(5)) - 6C_2(\text{SO}(5)) + 2C_2(\text{SO}(3)) + 4C_2(\text{SO}(6)) - C_2(\text{SU}(3)), \quad (14)$$

Similarly, in the case $k_6 = 0$ one can change the sign of bilinear combinations $(s^+ \cdot s^+)$ and $(s \cdot s)$ and, consequently, the sign of off-diagonal matrix elements $\langle n|H|n \pm 2 \rangle$, by using the transformation

$$\{s^+ \rightarrow i s^+, s \rightarrow -i s\}. \quad (15)$$

The IBM Hamiltonian H subjected to the transformation (15) can be restored to its original form using the PS transformation (4). On the other hand, in the case $k_6 = 0$ when $C_2(\text{SU}(3))$ is not present in the Hamiltonian, the transformation (15) is equivalent to the transformation of the monopole operator $P \rightarrow \bar{P}$ where

$$\bar{P} = \frac{1}{2}((\tilde{d} \cdot \tilde{d}) + (s \cdot s)). \quad (16)$$

This monopole operator is associated with an alternative embedding of the $\text{SO}(6)$ subgroup in the $\text{U}(6)$ group [3–5]. By using (11) with $k_3 = k_4 = 0$ and $k_5 = 1$, we find the Casimir operator of the $\overline{\text{SO}(6)}$ algebra associated with the monopole operator \bar{P} to be

$$C_2(\overline{\text{SO}(6)}) = 10N + 4(N + 2)C_1(\text{U}(5)) - 4C_2(\text{U}(5)) + 2C_2(\text{SO}(5)) - C_2(\text{SO}(6)). \quad (17)$$

Are the parameter symmetries (3) and (4) the only ones present in IBM? We have shown that these parameter symmetries are associated with the phase ambiguity of the boson operators. The general phase transformation of boson operators consistent with standard boson commutation relations is [4,2]

$$\{b^+ \rightarrow e^{i\varphi} b^+, b \rightarrow e^{-i\varphi} b\}. \quad (18)$$

However time reversal symmetry implies severe restrictions on the values of φ [4,3], namely $\varphi = 0, \pi$ in the case of the general IBM Hamiltonian, and $\varphi = 0, \pm \frac{\pi}{2}, \pi$ in the case of the transitional $\text{SO}(6)$ – $\text{U}(5)$ IBM Hamiltonian with $k_6 = 0$. It is easy to check that we do not obtain new parameter symmetries using all possible phase transformations of s , or d , or both s and d boson operators. Parameter symmetries also cannot be generated by spatial rotations. Thus by using all transformations of boson operators consistent with N and angular momentum conservation and the time reversal symmetry one can obtain only the parameter symmetries (3) and (4). Even the particle-hole boson transformation discussed in Refs. [9,3], which is not N -conserving but is nevertheless isospectral for some particular IBM Hamiltonians, does not generate new parameter symmetries.

However our approach based on the study of the structure of the Hamiltonian matrix and its unitary transformations is more general than that based on the transformation of boson operators. Therefore it is possible to derive an additional PS that cannot be formulated in terms of boson transformations. The Hamiltonian $\overline{H}(\{\overline{k}_i\})$ defined by replacing $C_2(\text{SO}(6))$ in (2) by $C_2(\overline{\text{SO}}(6))$ is identical to and therefore isospectral with $H(\{k_i\})$ if

$$\begin{aligned} \overline{H}_0 &= H_0 + 10k_5N, & \overline{k}_1 &= k_1 + 4k_5(N+2), & \overline{k}_2 &= k_2 - 4k_5, & \overline{k}_3 &= k_3 + 2k_5, & \overline{k}_4 &= k_4, & \overline{k}_5 &= -k_5, \\ \overline{k}_6 &= k_6. \end{aligned} \quad (19)$$

One can combine Eqs. (19) and (3) to get one more PS relation. The quasiclassical approach of Ref. [10] applied to the Hamiltonian (2) does not permit additional parameter symmetries but we do not have a formal proof of the absence of additional parameter symmetries of IBM. However it is very probable that additional parameter symmetries not associated with transformations of boson operators, can be found in some more complicated models than IBM-1, for example in IBM-2.

Let us discuss whether there is a possibility of discriminating between the two sets of IBM parameters which gives rise to equivalent spectra. For this purpose it is natural to consider electromagnetic transitions which were studied within IBM in a number of papers (see [11] and references therein). In the Consistent- Q Formalism (CQF) of Warner and Casten [12], the quadrupole operator

$$Q^\chi = [d^+ \times s + s^+ \times \tilde{d}]^{(2)} + \chi [d^+ \times \tilde{d}]^{(2)} \quad (20)$$

is introduced instead of operators P and Q , and both $(P^+ \cdot P)$ and $(Q \cdot Q)$ terms in the IBM Hamiltonian are replaced by a single term $(Q^\chi \cdot Q^\chi)$. The operator (20) should be used for calculations of $E2$ transition rates. The transformation (12) changes Q^χ into the operator

$$\overline{Q}^\chi = -[d^+ \times s + s^+ \times \tilde{d}]^{(2)} + \chi [d^+ \times \tilde{d}]^{(2)} = -Q^{-\chi}. \quad (21)$$

The consistent transformation of the $E2$ transition operator $Q^\chi \rightarrow \overline{Q}^\chi$ and of the quadrupole-quadrupole interaction $(Q^\chi \cdot Q^\chi) \rightarrow (\overline{Q}^\chi \cdot \overline{Q}^\chi)$ in the Hamiltonian guarantees that the $E2$ transition rates remain unchanged. Note that in the general case, i. e. when $k_6 \neq 0$, the parameter χ is nonzero and the quadrupole-quadrupole interaction $(Q^\chi \cdot Q^\chi)$ is unambiguously determined by the set of the Hamiltonian parameters $\{k_i\}$. Thus PS (3) establishes the isospectrality of the Hamiltonian H with the one in which the quadrupole-quadrupole interaction $(Q^\chi \cdot Q^\chi)$ term is replaced by the term $(\overline{Q}^\chi \cdot \overline{Q}^\chi)$. $E2$ transitions cannot be used to distinguish between the two Hamiltonians within the formalism of CQF.

However the statement that one cannot use the operator \overline{Q}^χ for calculation of $E2$ transitions in the case when the quadrupole-quadrupole interaction entering the Hamiltonian is of the form $(Q^\chi \cdot Q^\chi)$, is somewhat doubtful. The operators $(Q^\chi \cdot Q^\chi)$ and $(\overline{Q}^\chi \cdot \overline{Q}^\chi)$ (and the corresponding Hamiltonians) are isospectral and their matrices differ by the sign of off-diagonal elements only. Therefore it would be interesting to study $E2$ transitions using both operators (20) and (21), and to learn whether the phenomenological transition rates follow the prescriptions of CQF.

The case $k_6 = 0$ is much more interesting and complicated. In this case one should substitute χ by 0 in (20). We note that Q^0 is a generator of $\text{SO}(6)$. Under the transformation (15), Q^0 becomes

$$\overline{Q}^0 = -i[d^+ \times s - s^+ \times \tilde{d}]^{(2)} \quad (22)$$

(we note that (22) presents an alternative form of the $\text{SO}(6)$ generator [3]). In contrast to the case $\chi \neq 0$, the interpretation of the quadrupole-quadrupole interaction in the Hamiltonian in terms of the $(Q^0 \cdot Q^0)$ or $(\overline{Q}^0 \cdot \overline{Q}^0)$ operator is now ambiguous. It is easy to show that

$$(Q^0 \cdot Q^0) = -(\overline{Q}^0 \cdot \overline{Q}^0) + 10N + 4(N+2)C_1(\text{U}(5)) - 4C_2(\text{U}(5)) - 2C_2(\text{SO}(5)). \quad (23)$$

Thus in the case $k_6 = 0$ the IBM Hamiltonian can be expressed either through $(Q^0 \cdot Q^0)$ or alternatively through $(\overline{Q}^0 \cdot \overline{Q}^0)$. As a result, the definition of the $E2$ transition operator appears to be ambiguous. The operators Q^0 and \overline{Q}^0 provide different $E2$ rates for some transitions [4] and identical rates for the remaining ones. Thus it would be interesting to compare carefully the results of calculations with Q^0 and \overline{Q}^0 with phenomenological data on electromagnetic transitions in a number of $SO(6)$ – $U(5)$ transitional nuclei. However, due to the ambiguity in the definition of the quadrupole operator, one cannot make use of $E2$ transitions for the sake of distinguishing between two PS-related Hamiltonians in the case $k_6 = 0$.

One might suggest a resolution of the ambiguity by requiring the quadrupole-quadrupole interaction to be attractive and expressing the interaction either by the operator $(Q^0 \cdot Q^0)$ or by the operator $(\overline{Q}^0 \cdot \overline{Q}^0)$ according to the sign of the term in the Hamiltonian. However this prescription seems dubious. While changing the sign of the monopole-monopole (pairing) interaction results in non-trivial physical issues (see below), the change of the sign of the pure quadrupole-quadrupole interaction is not manifested in observables. The two Hamiltonians related by the PS (3) differ only by the sign of the quadrupole-quadrupole term $(Q \cdot Q)$ and are indistinguishable in physical applications at least within the framework of CQF. However the sign of the $(Q^\chi \cdot Q^\chi)$ term in the Hamiltonian in the general case $\chi \neq 0$ is not arbitrary, and is manifested in physical observables (see below). This is because the CQF quadrupole-quadrupole interaction $(Q^\chi \cdot Q^\chi)$ accounts effectively for the pairing interaction, and the sign of the term $(Q^\chi \cdot Q^\chi)$ is determined mostly by the sign of the pairing term $(P^+ \cdot P)$ and not by the sign of the quadrupole-quadrupole term $(Q \cdot Q)$.

There is another possibility for distinguishing between two PS-related Hamiltonians in the case $k_6 = 0$. An intriguing feature of the PS (4) is its N -dependence. Therefore if it is supposed that the spectra of neighboring even-even nuclei are described by the same set of IBM parameters (see for example Ref. [13]), then one can discriminate between the parameters connected by (4) comparing the spectra of the neighboring nuclei. Since the relations (4) involve the total number of bosons N , the two sets of the parameters can yield identical spectra

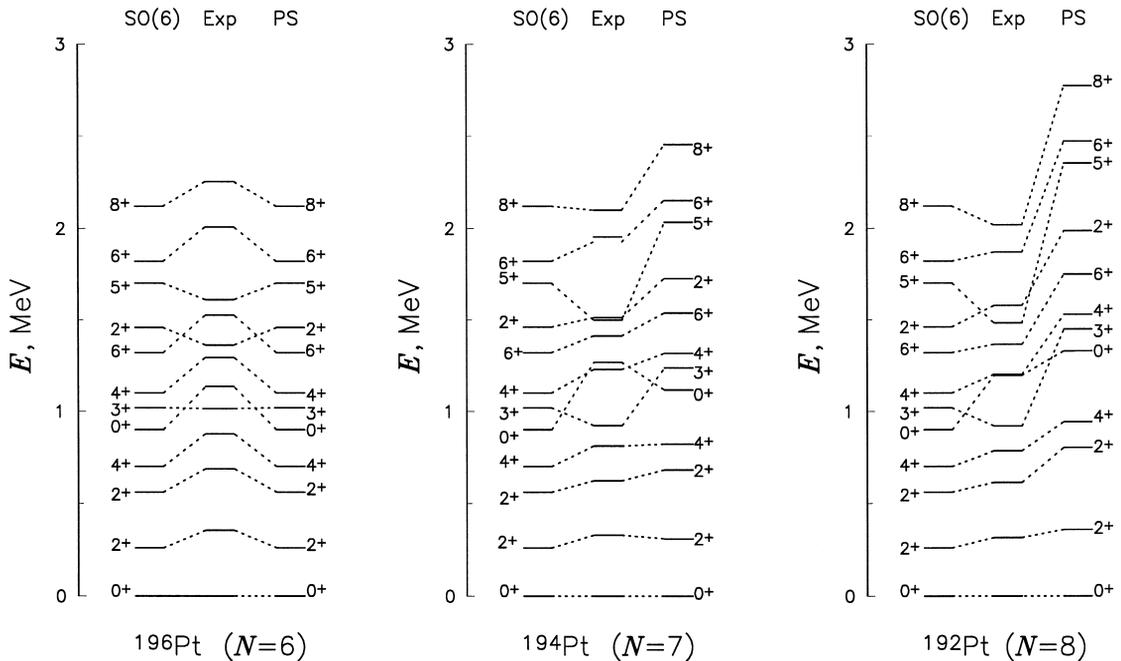


Fig. 1. Few lowest levels of each J^π of Pt isotopes. SO(6): calculations within SO(6) DS limit with the parameters suggested in [14] for the description of ^{196}Pt ($N=6$); PS: calculations with the set of parameters obtained using (4) with $N=6$; Exp: experimental data of Ref. [15].

only for a particular nucleus, and will yield different ones for its isotopes or isotones. In Fig. 1 we present the spectra of few isotopes of Pt. The set of parameters $k_1 = k_2 = k_6 = 0$, $k_3 = 50$ keV, $k_4 = 10$ keV, $k_5 = -42.75$ keV was suggested in [14] for the description of ^{196}Pt ($N=6$) within the SO(6) DS limit of IBM. The corresponding spectra are given in the left columns labelled by SO(6). The set of parameters $k'_1 = -1368$ keV, $k'_2 = 171$ keV, $k'_3 = -35.5$ keV, $k'_4 = 10$ keV, $k'_5 = 42.75$ keV and $k'_6 = 0$ is obtained using (4) with $N=6$. The corresponding spectra are given in the right columns labelled by PS. The SO(6) and PS spectra are, of course, identical in the case of ^{196}Pt but differ for other Pt isotopes. The difference is seen to be essential. The SO(6) DS set of parameters suggested in [14] provides a reasonable description of ^{192}Pt and ^{194}Pt although these isotopes were not involved in the fit. The alternative set of parameters obtained using PS fails to reproduce the spectra of ^{192}Pt and ^{194}Pt .

The spectra of Casimir operators $C_2(\overline{\text{SO}(6)})$ and $C_2(\text{SO}(6))$ are, of course, identical for all values of N . However by expressing $C_2(\overline{\text{SO}(6)})$ in terms of $C_2(\text{SO}(6))$ using (17), we obtain the difference in the N -dependence of the spectra. This N -dependence results from the change in the sign of the pairing interaction: $C_2(\overline{\text{SO}(6)})$ may be expressed in terms of the monopole operator \bar{P} by replacing $(P^+ \cdot P)$ with $(\bar{P}^+ \cdot \bar{P})$ in Eq. (5). Hence Eq. (17) is just the expression of the attractive (repulsive) pairing interaction $(P^+ \cdot P)$ in terms of the alternative repulsive (attractive) pairing interaction $(\bar{P}^+ \cdot \bar{P})$. Equivalently, the PS (4) states the N -dependent isospectrality of two IBM Hamiltonians with opposite signs of the pairing interaction. Thus, in contrast to that of the quadrupole-quadrupole interaction, the sign of the pairing interaction is not arbitrary and is manifested in the N -dependence of the spectrum. As a result, we can exclude one of the PS-related Hamiltonians by the requirement that the pairing interaction in a physically-acceptable Hamiltonian should be attractive. This is illustrated in Fig. 1. The SO(6) DS Hamiltonian with the attractive pairing interaction fitted to the ^{196}Pt spectra in Ref. [14] also reproduces the spectra of ^{192}Pt and ^{194}Pt , while the unphysical Hamiltonian with the repulsive pairing term fails to do so.

The change of sign of the pairing interaction is also manifested in the N -dependence of the PS (19). Thus the PS (19) also demonstrates the N -dependent isospectrality between a physically acceptable IBM Hamiltonian and an unphysical one with a repulsive pairing interaction.

In this paper we have shown that there is a symmetry in the parameter space of IBM which manifests itself by two IBM Hamiltonians defined by different irreducible parameter sets having identical spectra. Some of the PS relations but not others can be associated with the ambiguities in the definition of boson operators. We have shown that PS (3) relates two physically indistinguishable IBM Hamiltonians. In our opinion, this indistinguishability originates in the arbitrariness of the sign of the quadrupole-quadrupole interaction $(Q \cdot Q)$. In contrast, the other parameter symmetries relate IBM Hamiltonians which have pairing interactions of opposite sign. The sign of the pairing interaction is not arbitrary and its change manifests itself in physical observables like the N -dependency of the spectrum. Thus these PS-related Hamiltonians can be distinguished by studying the spectra of neighboring nuclei, and one of the two can be excluded as unphysical. We have shown also that there is an ambiguity in the definition of the quadrupole operator in the case of transitional SO(6)–U(5) nuclei. We believe that these results are of importance for applications, since the IBM parameters are conventionally obtained by fitting experimentally known nuclear spectra.

IBM is not the only boson model to display parameter symmetries. In particular, we have shown in Ref. [16] that PS is present in the vibron model [17] describing the low-lying excitations of diatomic molecules. Various parameter symmetries are surely present in extensions of IBM-1, such as IBM-2 which distinguishes proton and neutron degrees of freedom, *sdg*-IBM, etc. However we have not yet performed a careful study of all possible parameter symmetries of either IBM-2 or *sdg*-IBM. It would be also interesting to study parameter symmetries of fermion models used in nuclear physics.

One non-trivial consequence of parameter symmetries is that the energy spectrum of a DS limit of a boson model can be exactly reproduced by an irreducible set of parameters which does not correspond to any DS limit. From a physical point of view this means, for example, that a typical rotational nuclear spectrum can be reproduced by a combination of rotations and vibrations. As a result, boson Hamiltonians demonstrate the

so-called ‘hidden symmetries’ that seem to be important [18] in the studies of chaos and quantum nonintegrability.

Parameter symmetries originate from the possibility of using different realizations of $SO(6)$ and $SU(3)$ algebras within IBM which correspond to different embeddings of $SO(6)$ and $SU(3)$ subgroups in the $U(6)$ group. Thus we used the parameter symmetries (3) and (4) to derive expressions (14) and (17) of the Casimir operators $C_2(\overline{SU(3)})$ and $C_2(\overline{SO(6)})$ of alternative algebras $\overline{SU(3)}$ and $\overline{SO(6)}$ in terms of Casimir operators $C_2(SU(3))$ and $C_2(SO(6))$ of the original $SU(3)$ and $SO(6)$ algebras. These expressions are valid within the totally symmetric irreducible representation of $U(6)$. This approach seems to be very promising for generating independent algebras in the case of more complicated boson models. For example, we used it to construct the most general IBM-2 Hamiltonian [19].

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