ISSN 2075-1087, Gyroscopy and Navigation, 2021, Vol. 12, No. 2, pp. 186–194. © Pleiades Publishing, Ltd., 2021.
Russian Text © The Author(s), 2021, published in Giroskopiya i Navigatsiya, 2021, Vol. 29, No. 2, pp. 110–125.

INS/Odometer Integration: Positional Approach[1]

A. A. Golovan* (**ORCID:** 0000-0001-5628-248X)

Lomonosov Moscow State University, Moscow Russia

**e-mail: aagolovan@yandex.ru*

Received January 8, 2021; revised April 28, 2021; accepted April 28, 2021

Abstract—The problem of a strapdown inertial navigation system (SINS) integration with an odometer as part of an integrated navigation system is considered. The odometer raw measurement is considered as an increment of the distance traveled along the odometer "measuring" axis. Models of the integration solution components for the case of three-dimensional navigation are presented, among which are the models of inertial autonomous and kinematic odometer dead reckoning (DR), models of relevant error equations, the model of SINS position aiding based on the odometer DR data and using GNSS position and velocity, wherever possible. The models comprise objective components, which do not depend on the type of the inertial sensors used and their accuracy grade, and variable components, which take into account the properties of the navigation sensors used. The integration does not require zero velocity updates, known as ZUPT correction, which are commonly used in navigation application.

Keywords: strapdown inertial navigation system (SINS), odometer, odometer position measurement, SINS/odometer integration

DOI: 10.1134/S2075108721020048

INTRODUCTION

The paper considers the problem of SINS/odometer integration within an integrated navigation system. In this case, the raw measurement of the odometer is an increment of the distance traveled along its "measuring" axis. The solution of this problem is most essential for navigation of oil and gas pipeline inspection robots, wheeled and track-type land vehicles, and railway transport.

In Russia, the following universities carry out research in this field: Saint Petersburg State Electrotechnical University (railway applications) [1–3]; Bauman Moscow State Technical University [4], Moscow Aviation Institute (National Research University) (automotive applications) [5]; Krasnoyarsk State Technical University (pipeline inspection robots) [6, 7]; Lomonosov Moscow State University [8–12].

Russian research institutes and instrument-making companies are represented by All-Russian Research Institute Signal (VNII Signal), Perm Scientific and Production Instrument Making Company, Concern CSRI Elektropribor, JSC [13], Central Research Institute of Automation and Hydraulics, JSC (TsNIIAG), Gyrolab (Perm) [14], and others.

The list of the foreign companies engaged in solving the problem of SINS/odometer integration includes Sagem and SBG Systems (France), iMAR Navigation GmbH (Germany), Advanced Navigation (Australia), Datron Technology (UK), and others.

Among the foreign publications known to the author, of particular interest for us are the works that focus on the algorithms intended to increase the accuracy of the integrated SINS/GNSS/odometer positional solution. The problems of accuracy increase in the situations when GNSS data are degraded or unavailable are considered in [15–17]. Digital maps [16] and additional sensors, such as magnetometers [5] and heading sensors [4, 18], can also be used as sources of aiding data. Some publications describe calibration of the odometer scale factor error in motion using GNSS data [19]. There are two approaches to formation of aiding SINS measurements using odometer readings: the first one is based on velocity [15, 18, 20, 21] and the second, on position information [5, 17, 22, 23].

In the above-mentioned publications, odometer measurements are most often interpreted (formed) as velocity measurements, which are further used to aid SINS, and in the case of kinematic odometer DR, the latter serves as a solution to the navigation problem. The author is unaware of any works in which the results of the odometer DR data are used to aid SINS based on three-dimensional DR and complete models of SINS error equations. It should be noted that the problems of SINS aiding using a vessel's log measure-

[1] The paper is based on presentation made at the 13th Multicoference on Control Problems, Saint Petersburg, 2020.

ments and the measurements of an aircraft air data system are information-related problems.

This paper describes the components of the SINS/odometer integration solution for the case of three-dimensional navigation, specifically for the position interpretation of odometer measurements. At the same time, as is the practice in integrated navigation systems including SINS, other sources of additional navigation data serve as means of SINS aiding. The integration solution comprises objective and variable components. The first one is based on objective models:

- algorithms for autonomous inertial DR, including the SINS vertical channel. Here, DR autonomy means that SINS navigation solutions are generated only using the readings of accelerometers and angular rate sensors (ARS);
- algorithms for three-dimensional kinematic odometer DR;
- SINS error equations;
- odometer DR error equations;
- equations of SINS positional aiding using position data of the odometer DR;
- equations of SINS positional aiding and odometer DR aided with the available GNSS data or known coordinates of marker (reference) points in the problems of pipeline inspection.

The second, variable component, takes into account specific properties of the navigation sensors used in a particular integrated navigation system: accelerometers, ARS, and odometer.

The proposed integration solution does not require ZUPT corrections, which are commonly used in navigation applications mentioned above.

For definiteness, we assume that the SINS/odometer integration solution is based on the Kalman filtering with feedbacks to the SINS and odometer DR.

The paper is a follow-up to the methodical publications [10, 11], which give the classification of possible functional schemes used to solve the problem of SINS/odometer integration. The classification given in these papers correlates with the well-known classification of problems on integration of INS and GNSS, with four levels of integration: separate systems, loosely coupled systems, tightly coupled systems, and deeply integrated systems, see, for example, [28]. Accordingly, loosely coupled odometer/SINS integration is understood as an integration solution wherein an SINS provides attitude information for the odometer DR. The tightly coupled SINS/odometer integration refers to solutions in which the odometer readings (interpreted as position or velocity information) serve as aiding measurements for the SINS. Deeply coupled SINS/odometer integration refers to solutions in which odometer measurements, or the coordinates derived from them, serve as aiding measurements with feedbacks into the SINS DR algorithms.

The paper offered to the reader is methodical in nature, its aim being to describe the objective components of the SINS/odometer integration solution, as well as to present the main components of the variable models. The models presented below partially use the form of differential equations, allowing for their compact representation, and partially the discrete form, where it seems appropriate in the context. Note that in this paper, we do not discuss obvious simplifications of the models based on the specifics of the considered land applications, such as relatively low velocities of the vehicle, especially its vertical component, etc. This is partly due to the fact that some of the models given below are also used in other projects of the Control and Navigation Laboratory of Lomonosov Moscow State University, for example, in the problems of recalibration and realignment [27, 33], and the airborne gravimetry problem [31].

MODEL OF INS AUTONOMOUS DR

In practice, the model of SINS autonomous DR is most often represented by the version wherein a geodetic frame with one or another law of azimuth orientation serves as a reference navigation frame: a geodetic grid, azimuth-free or wander-azimuth frames [24]. To be specific, consider a version of a geodetic grid.

The SINS output is as follows:

- geodetic coordinates: longitude λ', latitude φ', height h';
- components of vector V' of the linear relative velocity: eastern V_E', northern V_N', and vertical V_{UP}' components;
- angles of true heading ψ', pitch υ', roll γ'.

Here, symbol (') is used to indicate the SINS output information; similar parameters without this symbol correspond to their true values.

Mechanization equations of inertial DR can be implemented in different equivalent forms: orientation matrices or quaternions; DR can be carried out directly in the geodetic grid, etc.; numerical methods for integration of ‘fast’ orientation equations can be different. The literature on these issues is rather extensive; see, for example, [24] and [26].

Below we present the differential form of the vertical channel equations, because in the literature, DR models are often given only for horizontal SINS channels. Thus, we have

$$\dot{h}' = V_{UP}',$$
$$\dot{V}_{UP}' = \left(\Omega_N' + 2u\cos\varphi'\right)V_E' - \Omega_E'V_N' + f_{UP}' - g\left(\varphi', h'\right). \quad (1)$$

Here, Ω'_E, Ω'_N are the eastern and northern components of the model vector of the relative angular velocity of the reference geodetic frame; u is the angular rate of the Earth's rotation; f'_{UP} is the 'vertical' accelerometer measurement obtained by the accelerometers' measurement projections on the axes of the geodetic frame; g is the absolute value of the specific force calculated, for example, by the Helmert formula [24].

MODEL OF KINEMATIC ODOMETER DR

Kinematic odometer DR of geodetic coordinates uses the angles of the true heading ψ', pitch υ', and roll γ' provided by the SINS. Odometer measurements represent an increment of the distance traveled along the "measuring" axis of the odometer. In this case, it is assumed that appropriate adjustment procedures were carried out at the vehicle, aimed to ensure the highest possible accuracy in the alignment of the vehicle's longitudinal axis, one of the SINS axes Mz (hereinafter also called the longitudinal axis), and the direction of the odometer's "measuring" axis. The mechanization equations are as follows [11] (here, the equations are given in discrete form; we mean the discrete nature of the odometer positional measurement):

$$\Delta S'_{j+1} = \begin{pmatrix} \Delta s'_{Ej+1} \\ \Delta s'_{Nj+1} \\ \Delta s'_{UPj+1} \end{pmatrix} = A'_j \begin{pmatrix} 0 \\ \Delta s'_{j+1} \\ 0 \end{pmatrix},$$

$$\lambda^0_{j+1} = \lambda^0_j + \frac{\Delta s'_{Ej+1}}{\left(\mathrm{R}_E + h^0_j\right)\cos\varphi^0_j}, \tag{2}$$

$$\varphi^0_{j+1} = \varphi^0_j + \frac{\Delta s'_{Nj+1}}{\left(\mathrm{R}_N + h^0_j\right)},\ h^0_{j+1} = h^0_j + \Delta s'_{UPj+1}.$$

Here, j corresponds to the moment of time t_j; $\Delta s'_{j+1}$ is the odometer measurement in [m] on the odometer measurement interval $[t_j, t_{j+1}]$; $A'_j = A'_j\left(\psi'_j, \vartheta'_j, \gamma'_j\right)$ is the model matrix of orientation of the SINS body frame Mz (axis Mz_1 is directed to starboard, Mz_2 is the longitudinal axis, axis Mz_3 is directed upward, point M is the center of the accelerometer unit) relative to the reference geodetic frame (with the orientation of axes 1, 2, 3 to the east, north, and upward), calculated using the values of the true heading angles ψ'_j, pitch ϑ'_j, and roll γ'_j; $\Delta S'_{j+1} = \left(\Delta s'_{Ej+1}, \Delta s'_{Nj+1}, \Delta s'_{UPj+1}\right)^T$ is the vector of the distance traveled in the eastern, northern, and vertical directions; R_E, R_N are the radii of curvature of the first vertical and meridian sections of the navigation ellipsoid (for land applications, $\mathrm{R_E} + h^0_j$, $\mathrm{R_N} + h^0_j$ can be substituted by the value of a, which is the ellipsoid semimajor axis); $\lambda^0_{j+1}, \varphi^0_{j+1}, h^0_{j+1}$ are the geodetic coordinates derived by odometer DR.

Relations (2) can also be represented as follows [11]:

$$V^0_{j+1} = \begin{pmatrix} V^0_{Ej+1} \\ V^0_{Nj+1} \\ V^0_{UPj+1} \end{pmatrix} = A'_j \begin{pmatrix} 0 \\ V^0_{sj+1} \\ 0 \end{pmatrix},$$

$$V^0_{sj+1} = \frac{\Delta s'_{j+1}}{\Delta t},\quad \Delta t = t_{j+1} - t_j,$$

$$\lambda^0_{j+1} = \lambda^0_j + \frac{V^0_{Ej+1}}{\left(\mathrm{R}_E + h^0_j\right)\cos\varphi^0}\Delta t, \tag{3}$$

$$\varphi^0_{j+1} = \varphi^0_j + \frac{V^0_{Nj+1}}{\left(\mathrm{R}_N + h^0_j\right)}\Delta t,$$

$$h^0_{j+1} = h^0_j + V^0_{UPj+1}\Delta t.$$

Here, $V^0_{j+1} = \left(V^0_{Ej+1}, V^0_{Nj+1}, V^0_{UPj+1}\right)^T$ is the vector of the "odometric" velocity in the axes of the geodetic frame.

Note that the "odometric" height h^0_j can be used to aid the exponentially unstable autonomous vertical channel (1) of the SINS, for example, in the following conventional way:

$$\dot{h}' = V'_{UP} - q_1\left(h' - h^0\right),$$

$$\dot{V}'_{UP} = \left(\Omega'_N + 2u\cos\varphi'\right)V'_E - \Omega'_E V'_N \tag{4}$$

$$+ f'_{UP} - g\left(\varphi^0, h^0\right) - q_2\left(h' - h^0\right),$$

where q_1, q_2 are the feedback coefficients ensuring the stability of the SINS vertical channel [24].

It is also important to note that when forming the absolute value of specific gravity g in (4), we can use the values of both h' (which actually leads to the exponential instability of the SINS autonomous vertical channel, in addition to the instrumental errors of the inertial sensors) and h^0 of the odometric height. The corresponding modification of the SINS vertical channel error equations for this case is described below.

It is quite obvious that the obtained solution for the SINS vertical channel will be "tracking" the behavior of the odometric height without exponential accumulation of its error.

MODEL OF SINS ERROR EQUATIONS

The model given below is widespread in airborne applications; see, for example, [27]. Its distinctive feature is that the errors of relative linear velocity are presented as a sum of dynamic and kinematic components [24], and SINS attitude errors, as two misalignment errors of the instrument vertical and the kinematic azimuth error of the geodetic frame. This model is one of the objective components of the SINS/odometer integration solution. The right parts of the SINS errors equations include the parameters of instrumental errors of the accelerometers and gyroscopes, which refer to the variable component of the model. For simplicity, the instrumental errors models given below take into account only biases and noise components. The designers of a specific integrated navigation system can use their own models in accordance with the specifics of the inertial sensors used.

In this case, the composite state vector of the SINS errors and instrumental errors of the inertial sensors has the 15th order, which includes the following parameters:

$$\begin{gathered}\Delta r_E, \Delta r_N, \Delta h, \delta V_E, \delta V_N, \delta V_{UP}, \alpha_E,\\ \alpha_N, \beta_3, \nu_1^0, \nu_2^0, \nu_3^0, \Delta f_1^0, \Delta f_2^0, \Delta f_3^0.\end{gathered} \tag{5}$$

Here, Δr_E, Δr_N are position errors [m] in the horizontal plane in the east and north directions; Δh is the inertial height error; δV_E, δV_N, δV_{UP} are the dynamic errors in determining the eastern, northern, and vertical components of the relative linear velocity; α_E, α_N are the eastern and northern misalignment errors of the instrument vertical; β_3 is the azimuth kinematic error; $\nu_i^0, \Delta f_i^0$, $(i = 1,2,3)$ are biases of the gyros and accelerometers.

The presentation of the INS errors as a sum of two fractions, dynamic and kinematic, was described in the classic work [29] in the 1960s, where the author called them the errors of the first and second groups, respectively. In [30], they were called dynamic and kinematic components of the INS errors. Briefly, the kinematic error equations reflect the contribution of the ARS (in the case of SINS) to the errors of the navigation system. The dynamic errors are due to accelerometers inaccuracies at the integration stage of the SINS dynamic equations.

In the following models of the SINS error equations, such separation was made for the velocity errors: the corresponding dynamic errors δV_E, δV_N, δV_{UP} are used instead of the total velocity errors ΔV_E, ΔV_N, ΔV_{UP}. The dynamic errors δV_E, δV_N, δV_{UP} are convenient to use, firstly, because the right parts of the equations do not include any accelerometer readings for these variables (derivation of the corresponding error equations is given in detail in [24]) in contrast to the equations for total velocity errors. This is undoubtedly convenient for real-time applications. Secondly, for navigation sensors, such as logs, air data system, odometer as a velocity sensor, etc., measuring the vehicle's velocity along its longitudinal axis, the corresponding velocity aiding measurements represent the measurements of dynamic velocity errors. For example, it is easy to see from the formulas (2.4.11), (3.4.1) in [26] in the section devoted to the models of SINS aiding using log measurements.

The errors equations [24] take the following forms (differential form with insignificant simplifications):

$$\begin{aligned}
\Delta\dot r_E &= \delta V_E + \frac{V'_{UP}}{a}\Delta r_E + \Omega'_{UP}\Delta r_N\\
&\quad - \Omega'_N\Delta h^* - V'_{UP}\alpha_N + V'_N\beta_3,\\
\Delta\dot r_N &= \delta V_N - \Omega'_{UP}\Delta r_E + \frac{V'_{UP}}{a}\Delta r_N\\
&\quad + \Omega'_E\Delta h^* + V'_{UP}\alpha_E - V'_E\beta_3,\\
\Delta\dot h &= \delta V_{UP} - V'_N\alpha_E + V'_E\alpha_N,\\
\delta\dot V_E &= \left(\Omega'_{UP} + 2u\sin\varphi'\right)\delta V_N\\
&\quad - \left(\Omega'_N + 2u\cos\varphi'\right)\delta V_{up} - g_e\alpha_N\\
&\quad + \sum_{i=1}^{3} a_{1i}\left(\Delta f_i^0 + \Delta f_i^s\right),\\
\delta\dot V_N &= -\left(\Omega'_{UP} + 2u\sin\varphi'\right)\delta V_E + \Omega'_E\delta V_{up}\\
&\quad + g_e\alpha_E + \sum_{i=1}^{3} a_{2i}\left(\Delta f_i^0 + \Delta f_i^s\right),\\
\delta\dot V_{UP} &= 2\omega_0^2\Delta h^* + \left(\Omega'_N + 2u\cos\varphi'\right)\delta V_E\\
&\quad - \Omega'_E\delta V_N + \sum_{i=1}^{3} a_{3i}\left(\Delta f_i^0 + \Delta f_i^s\right),\\
\dot\alpha_E &= -\frac{u\sin\varphi'}{a}\Delta r_E - \frac{1}{a}\delta V_N\\
&\quad + \left(\Omega'_{UP} + u\sin\varphi'\right)\alpha_N - u\cos\varphi'\beta_3\\
&\quad + \sum_{i=1}^{3} a_{1i}\left(\nu_i^0 + \nu_i^s\right),\\
\dot\alpha_N &= -\frac{u\sin\varphi'}{a}\Delta r_N + \frac{1}{a}\delta V_E\\
&\quad - \left(\Omega'_{UP} + u\sin\varphi'\right)\alpha_E + \sum_{i=1}^{3} a_{2i}\left(\nu_i^0 + \nu_i^s\right),\\
\dot\beta_3 &= \frac{\Omega'_E}{a}\Delta r_E + \frac{\left(\Omega'_N + u\cos\varphi'\right)}{a}\Delta r_N\\
&\quad + \left(\Omega'_N + u\cos\varphi'\right)\alpha_E - \Omega'_E\alpha_N\\
&\quad + \sum_{i=1}^{3} a_{3i}\left(\nu_i^0 + \nu_i^s\right).
\end{aligned} \tag{6}$$

Here, a is the semimajor axis of the navigation ellipsoid; g_e is the nominal absolute value of specific gravity; ω_0^2 is the square of the Schuler frequency; $\Omega'_{UP} = \Omega'_N \tan\varphi'$ is the vertical component of the relative angular rate vector of the geodetic reference frame; $v_i^s, \Delta f_i^s$, $(i = 1,2,3)$ are the gyro and accelerometer noise error components; $\Delta h^* = \Delta h$ in the case of the autonomous SINS vertical channel; $\Delta h^* = \Delta h^0$, where Δh^0 is the error in the odometric height when the latter is used to damp the SINS vertical channel.

Note. Equations (6) can certainly be simplified, taking into account the specifics of land applications, neglecting low-level terms, for example, those containing the factor $\left(\Omega'_{UP} + u\sin\varphi'\right)$.

In accordance with the adopted errors model of inertial sensors, simplified for the illustration, equations (6) should be added by the forming equations:

$$\dot{v}_i^0 = \Delta\dot{f}_i^0 = 0, \quad (i = 1,2,3). \tag{7}$$

MODEL OF KINEMATIC ODOMETRIC DR ERROR EQUATIONS

Objectively, the model parameters are vector $\Delta r^0 = (\Delta r_E^0, \Delta r_N^0, \Delta h^0)^T$ of the odometric positional error as well as SINS angular errors, since the odometer DR uses angular information provided by the SINS. The errors equation model for kinematic odometric DR also includes the error of the odometer scale factor; two misalignment errors of the odometer 'measuring' axis with respect to the heading and pitch channels. On the one hand, these three parameters also belong to the objective part of the odometer errors equation model, because they are included in the problem as residual errors of the scale factor and misalignment angles after calibration and adjustment of the odometer. On the other hand, these parameters can also be attributed to the variable part, when the developer of the system nuances the models of these errors, for example, as it was done for the odometer scale factor error model in [3].

Now let us take a closer look at the odometer error components. Without going into details of its hardware design, we consider the odometer as a device that measures the number N_j, $(N_0 = 0)$ of partial angles of rotation of the measuring wheel over the time interval $[t_0, t_j]$. Let n be the number of marks uniformly spaced circumferentially on the wheel, then the value of the partial angle of rotation is given as $\Delta a = 2\pi/n$. Let us denote the radius of the wheel by R. Then the distance traveled s with the wheel-surface contact point W on the time interval $[t_0, t_j]$ can be written as

$$s_j = \frac{2\pi R N_j}{n} = K N_j, \quad K = \frac{2\pi R}{n}, \tag{8}$$

where K is the odometer scale factor, which converts measurements N_j into the distance traveled, and the difference $\Delta N_{j+1} = N_{j+1} - N_j$ characterizes the distance traveled Δs_{j+1} on the interval $[t_j, t_{j+1}]$; in this case, $\Delta s_{j+1} = K\Delta N_{j+1}$.

Idealized odometer model

Consider the commonly used a priori idealized odometer model, which assumes that, firstly, the vehicle moves without slipping and secondly, the odometer measuring wheel is in close continuous touch with the road.

This allows us to pass from the scalar interpretation of the odometer measurement to the vector one: the vector of the path increment in the axes of the body-fixed frame Ws is a vector with two zero components (projections onto the lateral Ws_1 and vertical Ws_3 axes) and one nonzero component Δs_{j+1} (when the vehicle is moving) along the longitudinal axis Ws_2:

$$\Delta S_{j+1} = \begin{pmatrix} 0 \\ \Delta s_{j+1} \\ 0 \end{pmatrix}. \tag{9}$$

Realistic odometer model

The realistic odometer measurement model should take into account at least the following factors:

• error k of the scale factor K:

$$K' = K + kK, \tag{10}$$

where K is the true value of the scale factor, K' is the value used in the calculations;

• the odometer "measuring" axis Ws_2 and the SINS longitudinal axis Mz_2 may be misaligned. This case is formalized by introducing a vector of small rotation (if adjustment errors are small) $\delta = (\delta_1, \delta_2, \delta_3)^T$, which characterizes the mutual orientation of the SINS instrumental frame Mz and the body frame Ws:

$$l_z = \left(I + \hat{\delta}\right) l_s, \quad \hat{\delta} = \begin{pmatrix} 0 & \delta_3 & -\delta_2 \\ -\delta_3 & 0 & \delta_1 \\ \delta_2 & -\delta_1 & 0 \end{pmatrix}, \tag{12}$$

where l is the same vector in the projections on the corresponding axes, I is the unit (3×3) matrix, $\hat{\delta}$ is the skew-symmetric matrix assigned to vector δ.

Further, to derive the equations of odometric DR errors, we need a model of the distance traveled increment $\Delta s_z = (\Delta s_{z1}, \Delta s_{z2}, \Delta s_{z3})^T$ in the axes of the SINS instrument frame Mz. Taking into account models (11), (12), (2), we obtain

$$\Delta S'_{j+1} = \begin{pmatrix} 0 \\ \Delta s'_{j+1} \\ 0 \end{pmatrix} = \begin{pmatrix} \Delta s_{z1} \\ \Delta s_{z2} \\ \Delta s_{z3} \end{pmatrix} + \begin{pmatrix} -\delta_3 \\ k \\ \delta_1 \end{pmatrix} \Delta s'_{j+1} + \begin{pmatrix} 0 \\ \Delta s^s_{j+1} \\ 0 \end{pmatrix}, \quad (13)$$

where Δs^s_{j+1} is a random error in the odometer measurement.

It can be shown that for positional errors $\Delta r^0 = (\Delta r^0_E, \Delta r^0_N, \Delta h^0)^T$ of odometric DR in the eastern, northern, and vertical directions, the following model of error equations (differential form) is valid [10–12]:

$$\begin{pmatrix} \Delta \dot r^0_E \\ \Delta \dot r^0_N \\ \Delta \dot h^0 \end{pmatrix} = \begin{pmatrix} 0 & \Omega^0_{UP} & -\Omega^0_N \\ -\Omega^0_{UP} & 0 & \Omega^0_E \\ \Omega^0_N & -\Omega^0_E & 0 \end{pmatrix} \begin{pmatrix} \Delta r^0_E \\ \Delta r^0_N \\ \Delta h^0 \end{pmatrix} + k \begin{pmatrix} V^0_E \\ V^0_N \\ V^0_{UP} \end{pmatrix}$$
$$+ \begin{pmatrix} 0 & \beta_3 & -\beta_N \\ -\beta_3 & 0 & \beta_E \\ \beta_N & -\beta_E & 0 \end{pmatrix} \begin{pmatrix} V^0_E \\ V^0_N \\ V^0_{UP} \end{pmatrix} + A'_{xz} V^0_s \begin{pmatrix} -\delta_3 \\ 0 \\ \delta_1 \end{pmatrix} + \Delta V^s. \quad (14)$$

Here, Ω^0_i, $(i = E, N, UP)$ are the components of the relative angular velocity of the odometric geodetic frame calculated by the known formulas using the values of the odometric coordinates λ^0, φ^0, h^0 and the vector of the relative odometric velocity $V^0 = (V^0_E, V^0_N, V^0_{UP})^T$;

ΔV^s is the random error of the odometric velocity.

Formula (14) includes the SINS kinematic errors β_E, β_N, β_3, which are related to the components of the state vector (5) of the SINS errors as follows [24]:

$$\beta_E = \alpha_E + \frac{\Delta r_N}{a}, \quad \beta_N = \alpha_N - \frac{\Delta r_E}{a}. \quad (15)$$

Below is one of the possible variants of the model (variable part) for the odometer scale factor error and misalignment of its ‘measuring’ axis:

$$k = k^0 + k^s, \quad \dot k^0 = 0,$$
$$\delta_i = \delta^0_i + \delta^s_i, \quad \dot\delta^0_i = 0, \quad i = 1, 2. \quad (16)$$

Model (16) represents two fractions: constant nominal components k^0, δ^0_1, δ^0_3 and variable components k^s, δ^s_1, δ^s_3 modeled by random processes with a priori specified characteristics. The latter are the least formalized elements of the mathematical model, the characteristics of which are selected based on the experience in processing experimental data of a particular integrated system.

The total state vector of the SINS/odometer integration problem consists of two subvectors. The first one, ‘objective’, consists of the following components:

$$\Delta r_E, \Delta r_N, \Delta h, \delta V_E, \delta V_N, \delta V_{UP}, \alpha_E, \alpha_N, \beta_3, \Delta r^0_E, \Delta r^0_N, \Delta h^0. \quad (17)$$

The second subvector, “variable”, consists of the parameters of the illustrative models used in this paper for the instrumental errors of the inertial sensors and odometer:

$$\nu^0_1, \nu^0_2, \nu^0_3, \Delta f^0_1, \Delta f^0_2, \Delta f^0_3, k^0, \delta^0_1, \delta^0_3. \quad (18)$$

MODEL OF AUTONOMOUS SINS AIDING USING ODOMETRIC DR DATA

The difference in the geodetic coordinates (longitude, latitude, altitude) determined by the SINS and odometric DR is used as SINS aiding:

$$z_\lambda = \lambda' - \lambda^0 = (\lambda' - \lambda) - (\lambda^0 - \lambda) = \Delta\lambda - \Delta\lambda^0,$$
$$z_\varphi = \varphi' - \varphi^0 = (\varphi' - \varphi) - (\varphi^0 - \varphi) = \Delta\varphi - \Delta\varphi^0, \quad (19)$$
$$z_h = h' - h^0 = (h' - h) - (h^0 - h) = \Delta h - \Delta h^0.$$

These differences in linear approximation characterize the difference between the positional errors of the SINS and the odometer DR:

$$\Delta\lambda = \frac{\Delta r_E}{a\cos\varphi'}, \ \Delta\varphi = \frac{\Delta r_N}{a},$$
$$\Delta\lambda^0 = \frac{\Delta r^0_E}{a\cos\varphi^0}, \ \Delta\varphi^0 = \frac{\Delta r^0_N}{a}. \quad (20)$$

Based on (19), we derive the objective model of positional aiding z_{pos}:

$$z_{pos} = (z_\lambda, z_\varphi, z_h)^T = Hy + r, \quad (21)$$

$$y = (\Delta r_E, \Delta r_N, \Delta h, \delta V_E, \delta V_N, \delta V_{UP},$$
$$\alpha_E, \alpha_N, \beta_3, \Delta r^0_E, \Delta r^0_N, \Delta h^0, \nu^0_1, \nu^0_2, \nu^0_3, \quad (22)$$
$$\Delta f^0_1, \Delta f^0_2, \Delta f^0_3, k^0, \delta^0_1, \delta^0_3)^T$$

$$H = (H_{pos}, 0_{(3\times6)}, -H_{pos}, 0_{(3\times3)}),$$
$$H_{pos} = \begin{pmatrix} \frac{1}{a\cos\varphi'} & 0 & 0 \\ 0 & \frac{1}{a} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (23)$$

Here, $0_{(n\times m)}$ are zero matrices of the specified dimension; r is the noise component of the measurement error—introduced into the measurement model (20)—due to possible errors in synchronization of the SINS and odometer data, the representative value of the minimum increment in the distance traveled measured by the odometer, etc. Characteristics of noise r are adjusted based on the experience in processing experimental data.

Thus, measurement (19) allows for continuous autonomous SINS aiding using odometric coordinates. In this case, there is no need for a vehicle to make stops for SINS aiding with zero velocity updates. In addition, it becomes possible to recalibrate the instrumental errors of inertial sensors, instrumental (k^0) and geometric (misalignments δ_1, δ_3) odometer errors since these parameters are included in the state vector of the corresponding estimation problem. Estimation of these parameters from the standpoint of their quality and quantity is beyond the scope of this paper as it is supposed to be the subject of a separate study.

USING EXTERNAL INFORMATION FOR AIDING THE SINS/ODOMETER INTEGRATED SYSTEM

It is also possible that we have additional external navigation information on position λ^{GNSS}, φ^{GNSS}, h^{GNSS}, and velocity $V_E^{\text{GNSS}}, V_N^{\text{GNSS}}, V_{UP}^{\text{GNSS}}$ provided by the GNSS receiver; information on the coordinates λ^{ref}, φ^{ref}, h^{ref} of markers, reference points in the problems of oil and gas pipeline inspection; positional cartographic information.

The availability of external positional information makes it possible to carry out only one-step aiding of both SINS and odometer DR, using objective measurement models:

$$\begin{aligned} z_\lambda &= \lambda' - \lambda^{\text{GNSS}} = \Delta\lambda + r_\lambda,\ z_\varphi = \varphi' - \varphi^{\text{GNSS}} \\ &= \Delta\varphi + r_\varphi,\ z_h = h' - h^{\text{GNSS}} = \Delta h + r_h, \\ z_\lambda^0 &= \lambda^0 - \lambda^{\text{GNSS}} = \Delta\lambda^0 + r_\lambda,\ z_\varphi^0 = \varphi^0 - \varphi^{\text{GNSS}} \\ &= \Delta\varphi^0 + r_\varphi,\ z_h^0 = h^0 - h^{\text{GNSS}} = \Delta h^0 + r_h, \end{aligned} \tag{24}$$

where r_λ, r_φ, r_h are the errors in the positional data from the GNSS receiver, for which a discrete white-noise model of a specified intensity is commonly used in SINS/GNSS integration problems.

Obviously, model (24) is also valid when coordinates λ^{ref}, φ^{ref}, h^{ref} of the reference points are used for aiding.

Information on external velocity also makes it possible to form aiding measurements and present the corresponding objective models [24]:

$$\begin{aligned} z_{V_E} &= V_E' - V_E^{\text{GNSS}} \\ &= \delta V_E + (\beta_3 + \Delta\lambda\sin\varphi)V_N' - \alpha_N V_{UP}' + r_{V_E}, \\ z_{V_N} &= V_N' - V_N^{\text{GNSS}} \\ &= \delta V_N - (\beta_3 + \Delta\lambda\sin\varphi)V_E' + \alpha_E V_{UP}' + r_{V_N}, \\ z_{V_{UP}} &= V_{UP}' - V_{UP}^{\text{GNSS}} \\ &= \delta V_{UP} + \alpha_N V_E' - \alpha_E V_E' + r_{V_{UP}}, \end{aligned} \tag{25}$$

where $r_{V_E}, r_{V_N}, r_{V_{UP}}$ are the errors in the velocity data from the GNSS receiver, for which a discrete white-noise model of a specified intensity is also commonly used in SINS/GNSS integration problems.

Note. Equations (25) can be simplified, taking into account the specifics of land applications, neglecting the low-level terms containing the factor V_{UP}'.

NUMERICAL IMPLEMENTATION OF THE SINS/ODOMETER INTEGRATION ALGORITHM

The SINS/odometer integration solution is based on the estimation problem solution for the state vector y (whose components are represented by (22)) of the linear dynamic system of the form (see (6), (7), (14), (15)):

$$\dot{y} = Ay + v, \tag{26}$$

using measurements $z_{pos} = (z_\lambda, z_\varphi, z_h)^T$ (also see (19), (20, (23)) and, if available, positional (24) and velocity (25) GNSS measurements, as well as available information on the coordinates of reference points.

The general model for the above-mentioned measurements can be written as follows:

$$z = Hy + w, \tag{27}$$

where the composition of the measurement vector z and the model of matrix H are determined by the above formulas.

Vector v in (26) is a combined vector of the noise errors of the inertial sensors (see (6)), odometer measurements (11), and its variable parameters (16); w is the vector of the GNSS and odometer measurement noise errors.

To solve problems (26), (27) written in the equivalent discrete form, we use a Kalman filter algorithm (with parameterization of the corresponding hypotheses). The algorithm of the extended Kalman filter is used with the feedbacks at each step of the measurement arrival, as is done in [26], see section 4.1.1.3. Feedback is also based on the estimates of the SINS instrumental error parameters

$$\begin{aligned} \tilde{\omega}_z &= \omega_z' + \tilde{\nu}_z,\ \tilde{f}_z = f_z' - \Delta\tilde{f}_z^0, \\ \tilde{\nu}_z &= (\widetilde{\nu_1^0}, \widetilde{\nu_2^0}, \widetilde{\nu_3^0})^T,\ \Delta\tilde{f}_z^0 = (\widetilde{\Delta f_1^0}, \widetilde{\Delta f_2^0}, \widetilde{\Delta f_3^0})^T \end{aligned} \tag{28}$$

and odometer

$$\Delta\tilde{S}_{j+1} = \begin{pmatrix} \tilde{\delta}_3 \\ \dfrac{1}{1+\tilde{k}} \\ -\tilde{\delta}_1 \end{pmatrix} \Delta s'_{j+1}. \qquad (29)$$

Here, ω'_z, f'_z are measurements of the gyroscopes and accelerometers of the SINS in its instrument reference frame *Mz*; $\tilde{\omega}_z, \tilde{f}_z$ are the measurements corrected by the estimates of biases; $\Delta\tilde{S}_{j+1}$ is the corrected "vector" odometer measurement (13).

A few words about the experimental work on the development and application of the software for the SINS/odometer integration solution. The software was developed within the framework of a research and development contract between the Laboratory of Control and Navigation of Lomonosov Moscow State University with industrial partners (some of them are mentioned in the introduction) and handed over to the contractors [12, 25, 32]. These applications used inertial measurement units that included MEMS sensors [32], fiber optic gyroscopes of different accuracy grades [12, 25], and laser gyroscopes. The minimum increment of the distance traveled measured by the odometer ranged from a few millimeters for oil and gas pipeline inspection robots to a few tens of centimeters for land applications. The navigation facilities were oil and gas pipeline inspection robots, a car-laboratory for diagnostics of a roadbed, a land wheeled vehicle [12], and a railway carrier of the navigation system. Some of the navigation results obtained during the tests on the Soyuz trunk pipeline with the inspection robot developed by Orgenergogaz Saratovorgdiagnostika are considered in [25].

We emphasize that the objective models of inertial and odometric DR, models of errors equations and aiding measurements—the core of the SINS/odometer integration solution—remained unchanged for different applications and various composition of inertial sensors, while the driving noise and measurements were adjusted to the accuracy grade of the inertial sensors used and the odometer resolution. We should also note that these objective models have long been used by the Laboratory of Control and Navigation in other navigation applications, for example, for recalibration and realignment of laser SINS for airborne applications [27, 33].

CONCLUSIONS

This methodical paper describes the main models of the SINS/odometer integration solution for the positional interpretation of odometer measurements. The structure of the integration solution is presented with its objective and variable components.

The objective part includes the well-known model of autonomous inertial three-dimensional SINS DR; the model of kinematic odometric DR; equations of SINS and odometric DR errors; models of SINS aiding using odometric coordinates and, when available, additional navigation information provided by the GNSS receiver and information on the coordinates of the reference points.

The variable part includes models of instrumental errors of inertial sensors, the odometer positional measurement model for a specific navigation integrated system.

The structure of the SINS/odometer integration solution has made it possible to modify the basic software for various applications: navigation of oil and gas pipeline inspection robots, land vehicles, and railway transport.

FUNDING

This work was supported by the Russian Foundation for Basic Research, project no. 19-01-00179 .

REFERENCES

1. Boronakhin, A.M., Inertial Methods and Equipment for Measuring Geometric Parameters of Rail Tracks, *Cand. Tech. Sci. Dissertation*, St. Petersburg, 2002.
2. Boronakhin, A.M., Gupalov, V.I., and Kazantsev, A.V., Method for aiding the sensor to measure the distance traveled: RF Patent No. 2243505, 2004.
3. Boronakhin, A.M., Integrated Inertial Technologies for Dynamic Monitoring of Rail Tracks: *Dr. Tech. Sci. Dissertation*, St. Petersburg, 2013.
4. Gorbachev, A.Yu., Application of odometers to aid integrated navigation systems, *Vestnik MGTU im. N.E. Baumana,* Ser. Priborostroenie, no. 4, pp. 37–53, 2009.
5. Kuznetsov, I.M., Pron'kin, A.N., and Veremeenko, K.K., Navigation complex for an airport vehicle. Electronic journal *Trudy MAI*, no. 47, 2011.
6. Andropov, A.V., Improving positioning accuracy of pipeline inspection pigs by using satellite radio navigation systems: *Dr. Tech. Sci. Dissertation*, Krasnoyarsk, 2006.
7. Andropov, A.V., Improving positioning accuracy of pipeline inspection pigs by using GLONASS/GPS data, *Vestnik SibGAU*, Special Issue, pp. 28–35, 2006.
8. Golovan, A.A., Goritsky, A.Yu., Parusnikov, N.A., and Tikhomirov, V.V., *Algorithms of aided inertial navigation systems solving the topographic location problem, Preprint no. 2*, Moscow: Moscow State University, 1994.
9. Panev, A.A., Navigation problem for an inline diagnostic tool*, Vestnik moskovskogo universiteta, Mathematics. Mechanics*, Moscow: Moscow State University, 2011, pp. 53–56.
10. Golovan, A.A. and Nikitin, I.V., Combined use of strapdown inertial navigation systems and odometers from the standpoint of mechanics of inertial navigation systems. Part 1. *Moscow Univ. Mech. Bull.*, 2015, 70, pp. 46–49. https://doi.org/10.3103/S0027133015020065

11. Golovan, A.A. and Nikitin, I.V., Combined use of strapdown inertial navigation systems and odometers from the standpoint of mechanics of inertial navigation systems. Part 2. *Moscow Univ. Mech. Bull.*, 2015, 70, pp. 101–105.
https://doi.org/10.3103/S0027133015040056

12. Nikitin, I.V., The problem of land vehicle navigation based on strapdown INS/odometer data fusion, *Cand. Tech. Sci. Dissertation*, 2015.

13. Dmitriev, S.P., *Inertsial'nye metody v inzhenernoi geodezii* (Inertial Methods in Engineering Geodesy), St. Petersburg, CSRI Elektropribor, 1997.

14. Mal'gin, N.V., Nesterov, I.I., Kutman, A.B., Yaudinov A.Yu., and Malikov N.Sh., A strapdown inertial navigation system M500. Modern systems of orientation, navigation and georeferencing, *Oboronnaya tekhnika*, Nos. 5–6, 2014, pp. 87–92.

15. Jacques Georgy, Tashfeen Karamat, Umar Iqbal, and Aboelmagd Noureldin, Enhanced MEMS-IMU/odometer/GPS integration using mixture particle filter, *GPS Solutions*, 2011, Vol. 15, No. 3, pp. 239–252.

16. Dragan Obradovic, Henning Lenz, Markus Schupfner, and Kai Heesche, *Multimodal Fusion for Car Navigation Systems. Signal Processing Techniques for Knowledge Extraction and Information Fusion*, part II, pp. 141–158. Springer US, 2008.

17. Wankerl, M. and Trommer, G. F., Evaluation of a Segmented Navigation Filter Approach for Vehicle Self-Localization in Urban Environment, *Gyroscopy and Navigation*, 2014, Vol. 5, No. 2, pp. 98–107.

18. Jianchen Gao, GPS/INS/G Sensors/Yaw Rate Sensor/Wheel Speed Sensors Integrated Vehicular Positioning System, *ION 2006*, Fort Worth TX 26–29 Sep., Session E3, 2006.

19. Libin Zhu and Wei Wang, CDGPS-Based Calibration of Odometer's Scale Factor with Temperature for Vehicle Navigation System, *Proc. 2010 Int. Conf. on Optoelectronics and Image Processing*, Vol. 01, November 2010, pp. 317–320.

20. Elder M. Hemerly and Valter R. Schad, Implementation of a GPS/INS/Odometer navigation system. *ABCM Symposium Series in Mechatronics*, 2008, Vol. 3, pp. 519–524.

21. Jaewon Seo, Hyung Keun Lee, Jang Gyu Lee, and Chan Gook Park. Lever Arm Compensation for GPS/INS/Odometer Integrated System, *Int. J. Control, Automation, and Systems,* 2006, Vol. 4, No. 2, pp. 247–254.

22. Wang Qingzhe, Fu Mengyin, Xiao Xuan, and Deng Zhihong, Automatic calibration and in-motion alignment of an odometer-aided INS, *Control Conference (CCC)*, 2012, 31st Chinese, pp. 2024–2028.

23. Wei Jia, Xuan Xiao, and Zhihong Deng, Self-calibration of INS/Odometer Integrated System via Kalman Filter, *2012 IEEE 5th Int. Conf. on Advanced Computational Intelligence (ICACI)*, pp. 224–228.

24. Vavilova, N.B., Golovan, A.A., and Parusnikov, N.A., *Matematicheskiye osnovy navigatsionnykh system* (Mathematical foundations of navigation systems. Mathematical models of inertial navigation), Moscow: Moscow State University, 2020.

25. Vavilova, N.B., Golovan, A.A., Kozlov, A.V., Nikitin, I.V. et al., A navigation system of a pipeline inspection system for oil and gas pipelines: The results of the development and testing, *22nd St. Petersburg Int. Conf. on Integrated Navigation Systems*, St. Petersburg: Elektropribor, 2015, pp. 318–323.

26. Emel'yantsev, G.I. and Stepanov, A.P. *Integrirovannye inertsial'no-sputnikovye sistemy orientatsii i navigatsii (Integrated INS/GNSS orientation and navigation systems)*, Ed. V.G. Peshekhonov, St.Petersburg: Elektropribor, 2016.

27. Zorina, O.A., Izmailov, E.A., Kukhtevich, S.E., Portnov, B.I., Fomichev, A.V., Vavilovac, N.B., Golovan, A.A., Papusha, I.A., and Parusnikov, N.A., Enhancement of INS/GNSS Integration Capabilities for Aviation-Related Applications, *Gyroscopy and Navigation*, 2017, vol. 8, no. 4, pp. 248–258.

28. Phillips, R.E. and Schmidt, G.T., GPS/INS Integration. In: *System Implications and Innovative Applications of Satellite Navigation. AGARD Lecture Series 207*, 9, 1–18. Canada Communication Group, Qu'ebec, 1996.

29. Andreev, V.D., *Teoriya inertsial'noi navigatsii* (Avtonomnye sistemy) (The Theory of Inertial Navigation. (Autonomous Systems)). Moscow: Nauka, 1966.

30. Parusnikov, N.A., Morozov, V.M., and Borzov, V.I., *Zadacha korrektsii v inertsial'noi navigatsii* (Correction Problem in Inertial Navigation), Moscow: Moscow State University, 1982.

31. *Sovremennye metody i sredstva izmerenya parametrov gravitatsionnogo polya Zemli* (Modern Technologies and Methods for Measuring the Earth's Gravity Field Parameters), Eds., V.G. Peshekhonov, O.A. Stepanov, St. Petersburg, Concern CSRI Elektropribor, JSC, 2017.

32. Vavilova, N.B., Vyaz'min, V.S., and Golovan, A.A., Development of a Low-Cost INS/GNSS/Odometer Integration Algorithm for a Road Surface Testing Laboratory Software, *26th St. Petersburg Int. Conf. on Integrated Navigation Systems*, St. Petersburg: Elektropribor, 2019.

33. Vavilova, N., Golovan, A., and Kozlov, A., et al. Impact of antenna displacement and time delays of GNSS solutions on the INS-GNSS Complex Data Fusion Algorithm, *27th St. Petersburg Int. Conf. on Integrated Navigation Systems*, St. Petersburg: Elektropribor, 2020.