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WAVE PROPERTIES OF METAMATERIALS

Pattern Analysis of Fractal Properties in Multilayer Systems with Metamaterials

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Abstract—Fractal pattern analysis is performed for aperiodic multilayer systems with metamaterials. Morphological features of pattern formation in optical characteristics of structures with different geometry are revealed having regard to the manifestation of the phase-compensation effect and presence of metamaterial layers.

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1. INTRODUCTION

Elements and devices with a complex spatial structure, including those containing metamaterials and possessing fractal properties, are widely used in various optical devices based, for example, on modern nanotechnologies [1, 2]. Advance in this field is closely related to development and improvement of diagnostics methods that allow morphological features to be determined and the quality of the system under consideration to be estimated.

The use of metamaterials with unique optical properties [3] in multilayer systems (MSs) results in occurrence of additional forbidden bands stable against the variation in the inclination of the incident light waves. Studying optical properties of metamaterial-based systems of different configurations holds promise for high technologies in terms of both theory and application [4, 5]. For example, MSs are used to make narrow-band filters, broadband absorbers, polarization beam splitters, antennas with new optical properties, and also for other applications in science and technology [6–8].

Though investigations of fractal properties of aperiodic MSs with metamaterials receive attention in the literature [9–11], a number of issues related to formation conditions for fractal properties of probing light beams remain unstudied.

In this work, using numerical simulation, we consider some of the poorly studied issues related to analysis of scaling features of transmission spectra and their connection to manifestation of self-similarity in the geometry of the investigated structures. The emphasis is placed on the stability of fractal properties for optical characteristics of objects of different configurations with identical self-similarity symmetry, and phase effects, in particular phase compensation, are taken into consideration.

These points are investigated using the comparative analysis of traditional dielectric MSs and MSs with metamaterials within the earlier proposed pattern approach based on registration and determination of features of particular self-similar elements (patterns) [12].

2. OPTICAL PROPERTIES OF MULTILAYER SYSTEMS WITH METAMATERIALS

One-dimensional models of quasi-crystals were used to form MSs [13, 14]. It is convenient to represent them as structural blocks made up of elements A and B alternating in compliance with a given law and corresponding to different generation levels. For example, initial levels of the aperiodic Fibonacci system consist of the elements $S_0 = B$, $S_1 = A$, and $S_2 = AB$. To go to a higher generation level, the following replacement rules are used: $A \to AB$ and $B \to A$. The quantities A and B and their sequence order govern alternation of the MS layers with the refractive indices N_A and N_B, respectively.

Let layers A be a metamaterial characterized by a negative refractive index in a particular spectral interval. We assume that there is vacuum between the layers A. The vacuum gaps correspond to the arrangement of the layers made up of elements B.

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The permittivity ε and permeability μ of the metamaterial are given in the most general form that reflects the experimental data and was already used in a number of works for numerical simulation and verification of experimental results [15, 16]:

$$\begin{aligned} \varepsilon_A(f) &= 1 + \frac{5^2}{0.9^2 - f^2} + \frac{10^2}{11.5^2 - f^2}, \\ \mu_A(f) &= 1 + \frac{3^2}{0.902^2 - f^2}, \end{aligned} \tag{1}$$

where f is the quantity numerically equal to the frequency measured in GHz.

In view of (1), the variation in f was discretized using the relation

$$f_k = 1.5(1 + 0.0033k), \tag{2}$$

where $k = 0, ..., N_{\max}$, $N_{\max}$ is the given discrete value limiting the frequency interval.

Figure 1 graphically interprets relation (1) with allowance for (2). The analysis shows that in the region $k < 330$, where $\varepsilon_A < 0$ and $\mu_A < 0$, layers A have a negative refractive index. The region $330 \leqslant k \leqslant 461$, where $\varepsilon_A < 0$ and $\mu_A > 0$, is a transition region. If $k \geqslant 462$, the material of layers A acquires dielectric properties ($\varepsilon_A > 0$, $\mu_A > 0$).

The transmission and reflection spectra of the MSs with metamaterials were calculated using the known matrix method [17]. It was assumed that a wave of unit amplitude is formed at the exit from the systems under consideration. The power transmission coefficient is then defined as

$$T_k = \left| E_0^{\langle k \rangle} \right|_t^{-2}, \tag{3}$$

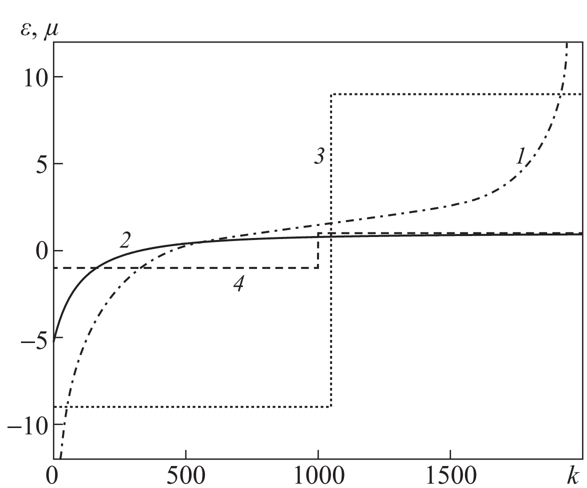

Fig. 1. Frequency dependence of the permittivity ε (curves *1* and *3*) and the permeability μ (curves *2* and *4*). Curves *1* and *2* are the experimental data, and curves *3* and *4* are the theoretical model.

where $(E_0^{\langle k \rangle})_t$ is the strength of the direct wave field, and the index k is defined by (2). As an example, Figs. 2 and 3 show transmission spectra of Fibonacci systems with different geometrical configurations of the constituent layers. The calculated transmission coefficients (3) agree with the matrix representations [10, 11, 15].

The comparative analysis of the spectral dependences was performed using the logarithmic representation [18] $r = -\ln(1-R)$, where R is the reflection coefficient of the MS connected to the transmission coefficient T by the relation $R = 1-T$.

The results of the numerical simulation show that metamaterials appreciably complicate and modify spectral characteristics of multilayer structures. These modifications can be studied using pattern analysis [12] based on monitoring and determination of features of particular self-similar elements (patterns) in the analyzed distributions. Recording the presence and shape of a pattern, we can judge the spatial features of the structures of the systems under consideration and their variation dynamics and identify them.

One of the characteristic fragments in the self-similar spatial spectrum of the analyzed MS formed by taking a Fourier transform of the system of delta functions with the coordinates corresponding to the boundaries of the arising quasicrystalline structure is chosen to be a pattern [12, 18]. Figure 2 shows optical characteristics of the mirror-symmetrical Fibonacci MS. It is evident from the dependence in Fig. 2(c) that it comprises elements similar in shape to the patterns characteristic of the Fibonacci lattice structures (fragments ab, cd). They are formed in the regions corresponding to both the negative and positive refractive indices.

The structural correspondence of the recorded patterns in the spectral characteristics of the systems under investigation was qualitatively estimated by performing the correlation analysis and determining the scaling coefficients. For example, the coefficient of mutual shape correlation between the registered fragments cd and ba of the reduced spectrum is $K \approx 0.87$ (see Fig. 2(e)). In the transition region ($\varepsilon_A < 0$, $\mu_A > 0$), patterns do not manifest themselves.

Dispersion phenomena (1) considerably affect both the position and the shape of the registered patterns. To eliminate, during the analysis, the effect of these phenomena on the transformation of the patterns aimed at revealing the effect of the metamaterials present on their shape, we consider a simplified frequency dependence of the permittivity ε_A and permeability μ_A of layers A (see Fig. 1, curves *3* and *4*). The permittivity of layers A in absolute value

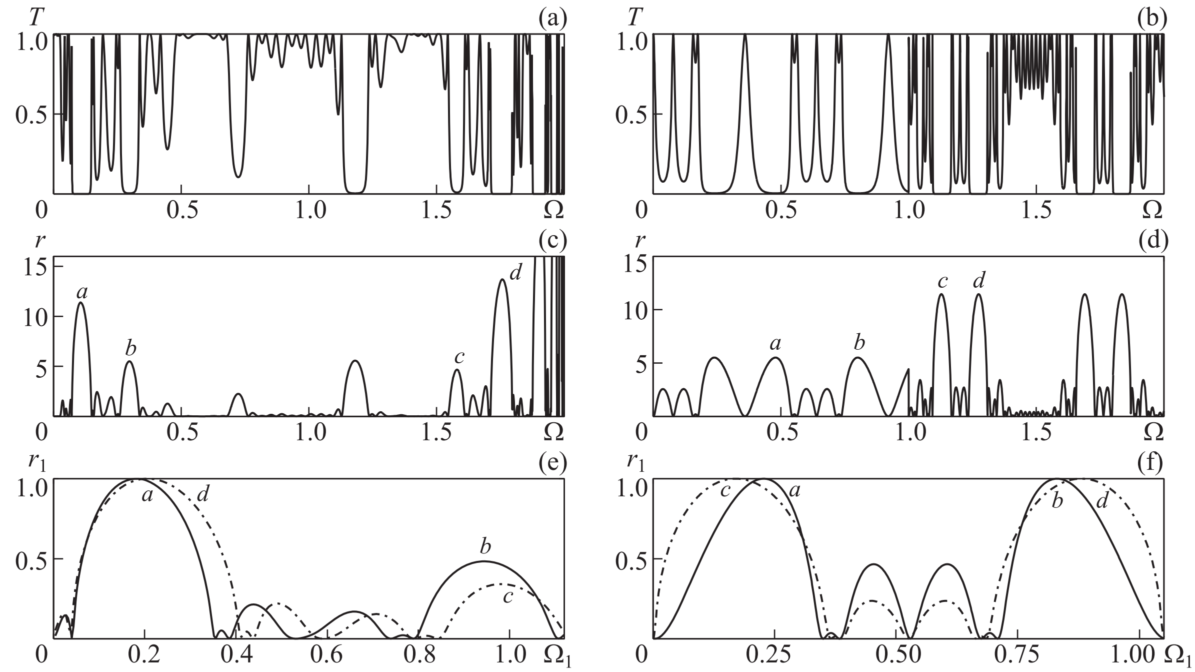

Fig. 2. Optical characteristics of the mirror-symmetrical multilayer Fibonacci system with allowance for dispersion dependences given by formula (1) (a, c, e) and within step approximation (1) (b, d, f) (see Fig. 1). The patterns present in the spectrum are characterized by the peaks ab and cd ($J = 32$ layers, $\Omega = \omega/\omega_0$, where ω is the frequency of the incident wave, and ω_0 corresponds to the frequency at which the permittivity reverses its sign); r_1 and Ω_1 are the reduced values to the field of pattern formation.

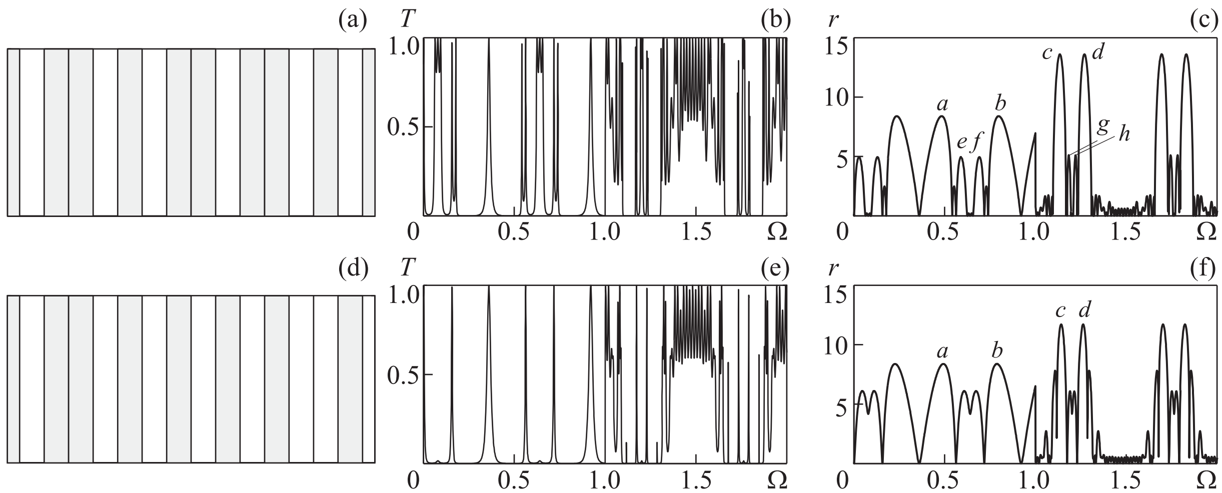

Fig. 3. Optical characteristics of the modified Fibonacci MSs; (a, d) fragments of structures; (b, c, e, f) optical characteristics (model of step approximation of dispersion curves (1)). The patterns present in the spectrum are characterized by the peaks ab and cd ($J = 32$ layers, gray regions correspond to A layers, the frequency Ω is the same as in Fig. 2).

is $|\varepsilon_A| = 9$, and $|\mu_A| = 1$. Layers B are assumed to be of the material with the constant refractive indices $n_B = 1$ and 1.5 in the entire frequency range. The ambient parameters are taken to be $\varepsilon = 1$, $\mu = 1$, and $n = 1$. Spectral dependences of the MSs with the number of layers $J = 32$ for the above parameters are shown at the right of Fig. 2. The plots clearly display patterns that are formed in the regions with negative and positive refractive indices (peaks ab and cd) and have a similar shape. The correlation coefficients K characterizing deviations in the shape of the patterns (ab and cd) varied in the range of 0.8 to 0.9 for different geometrical modifications of the MSs under consideration and different refractive indices of layers B.

However, the local scaling coefficients determined from the ratio of the distances between their peaks are slightly different: $\varsigma = ab/ef \approx 3$ and $\varsigma = cd/gh \approx 3.5$ for different region ab and cd, respectively (see Fig. 3(c)). In this region with the positive refractive index, the peaks obey the golden section rule $cd/ch = ch/hd$, $cd/ch = 1.62$.

This rule suffers changes in the region with the negative refractive index (in particular $ab/af = 1.53$ for the system of peaks $aefb$) due to compensating phase effects. In the transition region, the transmission of the system is close to zero (the highest reflection coefficient).

During additional investigations of the transmission spectra of the MSs with different arrangements of the layers, it was established that formation of patterns at both negative and positive refractive indices is only typical of the systems with an unequal number of layers of different types, which agrees with [11, 15, 16]. For example, in the Fibonacci MS considered above, the number of layers A is about ς times larger than the number of layers B, where $\varsigma \approx 1.62$ is the golden ratio coefficient.

If we take, for example, the Morse–Thue system, whose properties are considered in [11], no patterns will be formed at negative refractive indices. This is because the system has an identical number of layers of different types and therefore the phase-compensation effect typical of transmission spectra of periodic systems with metalayers begins playing an important role [15, 16].

To gain a better insight into the possible phase-compensation effect, various modifications of the Fibonacci MSs were considered. For example, the geometry of the structure in Fig. 3(d) is symmetrically inverted with respect to the classical construction of Fibonacci systems (Fig. 3(a)). In its first half, the layers alternated as usual, and in the second half the layers first alternated in the inverse order and then type A layers were replaced with type B layers. This geometry of an aperiodic MS creates conditions for phase compensation. The role of this effect is illustrated in Fig. 3(e), which shows the transmission spectrum of the modified Fibonacci system. It is seen that while featuring a general decrease in the average transmission, the spectrum in the regions with $\varepsilon_A < 0$, $\mu_A < 0$ and $\varepsilon_A < 0$, $\mu_A > 0$ is qualitatively different from the spectrum in the region with $\varepsilon_A > 0$, $\mu_A > 0$. Nevertheless, the plot of the reduced reflection coefficient (Fig. 3(f)) features, though indistinctly, some signs of patterns characteristic of Fibonacci structures.

The above results are for the case where phase shifts in the layers were identical in absolute value. If the relationship between the geometrical sizes of the layers changed and phase shift disbalance occurred, it was sometimes impossible to identify patterns due to strong distortion of their shape, especially in the region of negative refractive indices.

Thus, the role of dispersion, phase compensation, and phase shift disbalance in layers should be taken into account in the analysis of scaling in optical characteristics of MSs with metalayers. These phenomena can noticeably affect formation of patterns in optical characteristics. Dispersion causes shape distortion of patterns, and phase-compensation effects can almost completely suppress formation of pattern elements in some cases, depending on the ratio between the numbers of layers of different types.

Though the shape of the patterns in the regions with the negative and positive refractive indices is noticeably different, their geometry dictated by the relative positions of the internal peaks is similar. It can therefore be stated that under particular conditions the patterns recorded in the spectral characteristics can be a helpful reference for estimating the scaling properties of the MS under consideration.

Thus, our analysis has shown that rather high stability of pattern formation is typical of both dielectric systems and systems based on metamaterials. This allows the pattern analysis to be extended to estimation of quality of multilayer structures and to their identification.

3. CONCLUSIONS

The analysis performed in this work shows that layers of a metamaterial present in aperiodic MSs can noticeably affect manifestations of scaling in their optical characteristics and sometimes even completely suppress it under the effect of phase compensation. One should bear this in mind when recording patterns in the transmission and reflection spectra of the MSs for estimating features and quality of their structure.

At the same time, the shape of the recorded patterns and the scaling coefficients determined from it are highly stable with respect to the variation in the phase shifts in the layers and the presence of structural defects in the MSs. This allows the pattern analysis to be used for determining types and quality of MSs.

When patterns disappear in the frequency region characterized by the negative refractive index of the metamaterial due to phase compensation, recording of patterns should be performed in the region where the metamaterial acquires dielectric properties.

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