Advanced Course in Classical Logic (spring 2021): Program

Lecturer: Evgeny Zolin

1. Set semantics for propositional formulas. Equivalence of the 2-valued and set semantics. The (strong) completeness theorem for the CPC w.r.t. set semantics.

2. Boolean algebra. Algebraic semantics for propositional formulas. Strong Completeness Theorem of the classical propositional calculus w.r.t. the algebraic semantics.

3. Infinitary propositional formulas: syntax, semantics. The lack of compactness. Building the set of all infinitary formulas by transfinite recursion. The cardinality of the set of infinitary formulas. Non-representability of infinitary Boolean functions. The cardinality of the set of subformulas of a formula.

4. Infinitary classical propositional calculus: axioms and rules. Deduction theorem (without proof). Completeness theorem. A saturated set of formulas. Analogue of Lindenbaum lemma: *any consistent set of formulas is contained in some saturated set of formulas* (idea of the proof). Model existence theorem: *any saturated set is satisfiable* (idea of the proof).

5. Decidable and undecidable propositional calculi. Any calculus with a semi-decidable set of axioms is semi-decidable. Linial–Post theorem (formulation). Algorithmic reducibility of one problem (language) to another. Reducibility and decidable / undecidable problems (lemma). Tag system. Undecidability of the halting problem for tag systems (Minski's theorem).

6. Ehrenfeucht game. Elementary equivalence of models. Theorem 1: game characterization of the elementary equivalence (without proof). Quantifier rank of a formula. Game with fixed number of steps. Theorem 2: game characterization of the elementary equivalence up to the quantifier rank n (without proof). Prove that T2 implies T1.

7. Filters: definition, examples, principal filters. Intersection and union of filters. Every centered family of sets is contained in some filter. Ultrafilter: equivalent definitions. Every filter is contained in some ultrafilter.

8. Cartesian product and ultraproduct of sets. Ultraproduct of models. Correctness of the definition. Łoś's theorem: the idea of the proof. Its consequences. The ultrapower of a model is elementarily equivalent to the model.

9. If a class of models is closed under ultraproducts, then it is compact. Malcev's compactness theorem. Any axiomatizable class of models is closed under elementary equivalence and ultraproducts.

10. Keisler's Theorem (criterion of axiomatizability). Criterion of finite axiomatizability.

11. Boolean algebra (BA). Partial order in a BA. Atoms in a BA. Atomic decomposition of elements in a finite BA. Stone's representation theorem for finite BAs. A counterexample to the theorem for infinite atomic BAs.

12. Filters and ultrafilters in Boolean algebras. Stone's representation theorem for arbitrary BAs.

13. Finite Model Theory: Strong categoricity of the theory of a finite model. Axiomatizability of any class of finite models. Criterion of definability (by a single formula) of a class of finite models.

14. Axiomatizable, co-axiomatizable, quasi-axiomatizable classes of models. Criterion for quasi-axiomatizability of a class of models. Criteria for all 4 "species" of classes of models. Examples of classes of each species.