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Transitions of $\nu_i \leftrightarrow \nu_j$ $(i \neq j)$ type in an external field

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Transitions of massive neutrinos with flavour violation induced by the field of a circularly polarized electromagnetic wave are studied within the standard model with lepton mixing. The $\nu_i \to \nu_j$ $(i \neq j)$ amplitude and rate are calculated taking account of the polarization of the initial and final neutrinos. The transition $\nu_i \to \nu_j$ is compared with the process $\nu_i \to \nu_j \gamma$ in the absence of an external field. The branching ratio is shown to be independent of both mixing angles and masses of intermediate charged leptons.

1. Introduction

The standard Glashow–Weinberg–Salam (GWS) theory successfully describes all the known weak and electromagnetic interactions and is consistent with all the experimental data obtained at currently accessible energies. Despite this excellent agreement with experiment, the standard theory still needs extension and generalization since it is neither able to predict fermion masses nor to explain the existence of several fermion generations. Neutrinos are unique for the investigation of weak interactions in that the study of their properties may lead to new physics beyond the standard model. Although significant progress has been achieved in experiments with neutrinos during the last decade, some problems that are important for the theoretical description of elementary particles remain unresolved. One of the most substantial problems is that of the massive neutrino, the attractiveness of which is associated with grand unification models. In the case of non-zero mass, the question inevitably arises whether the neutrino is a pure intrinsic weak state or whether it is a superposition of other neutrino states – the eigenstates of the mass matrix. The latter case is referred to as lepton mixing in the framework of the GWS theory, which may lead to such interesting phenomena as neutrino oscillations [1], rare decays with lepton number violation of the types $\mu \to e\gamma$ [2], $\mu \to e\gamma\gamma$ [3], $\nu_i \to \nu_j\gamma$ [2], $\nu_i \to \nu_j\gamma\gamma$ [4], and other processes "forbidden" by the law of conservation of lepton number. It is worth noting that both oscillations [5] and above-mentioned radiative decays are continuously searched for in experiments [6,7].

On the other hand, due to the development of intensive electromagnetic fields generation techniques, the investigation of electroweak processes in strong external fields may be of special interest. In this case, the method of exact solutions of relativistic wave equations in external electromagnetic fields is quite effective and allows one to go beyond the perturbation theory and to predict phenomena hitherto unobserved experimentally [8]. In particular in the case of the GWS theory with lepton mixing, an external field can induce flavour-changing non-radiative lepton transitions $\ell_i \to \ell_j$ $(i \neq j)$ forbidden by energy–momentum conservation without the field. In the present work we study the effect of a circularly polarized wave on flavour-changing transitions of neutral leptons (massive neutrinos) $\nu_i \to \nu_j$.

2. The transition amplitude and rate

The weak interaction lagrangian describing lepton mixing has, in the Feynman gauge, the following form:

$$\mathscr{L}_{\rm int} = -\frac{g}{\sqrt{2}}\,\bar{l}_f \gamma^\mu \frac{1+\gamma_5}{2} K_{fi} \nu_i W_\mu - \frac{g}{\sqrt{2}}\,\bar{l}_f \left(\frac{m_f}{m_W}\frac{1+\gamma_5}{2} - \frac{m_i}{m_W}\frac{1-\gamma_5}{2}\right) K_{fi} \nu_i \phi^+ + \text{h.c.} \tag{1}$$

Here ν_f are neutrino states participating in weak interactions (the index indicates different lepton flavours: electron, muon, tau-lepton, etc.), which are superpositions of ν_i states with masses m_i:

$$\nu_f = \sum_{i=1}^{N} K_{fi} \nu_i ,$$

where K is the unitary mixing matrix which can be parameterized similarly to the quark mixing Kobayashi–Maskawa matrix: l_f, W_μ, ϕ are the wavefunctions of charged lepton of mass m_f, W-boson of mass m_W and non-physical charged scalar of mass m_W, respectively.

In the lowest order of the perturbation theory, a matrix element of the transition $\nu_i \to \nu_j$ in Feynman gauge is described by the two diagrams, represented in fig. 1, where double lines imply that the influence of the external field in the propagators of intermediate particles is taken into account exactly. The explicit expression for the matrix element S_{ji} in the external field in one-loop approximation is

$$S_{ji} = S_{ji}^{(W)} + S_{ji}^{(\phi)} , \tag{2}$$

where

$$S_{ji}^{(W)} = \tfrac{1}{2} g^2 \sum_f K_{fj}^* K_{fi} \int d^4x \, d^4x' \, \bar{\nu}_j(x') \frac{1-\gamma_5}{2} \gamma_\mu$$
$$\times S_f(x'-x) \gamma_\nu \frac{1+\gamma_5}{2} \nu_i(x) G_{\nu\mu}(x, x') \tag{3}$$

is the contribution from the W-boson (diagram (a) in fig. 1),

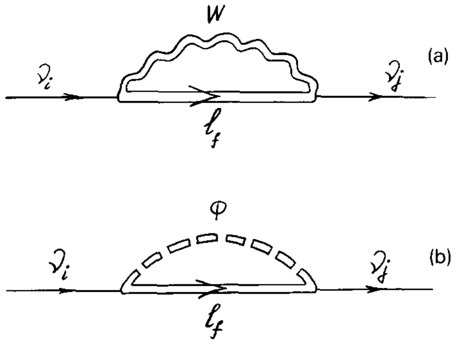

Fig. 1. Transitions of $\nu_e \leftrightarrow \nu_\mu$ type in an external field.

$$S_{ji}^{(\phi)} = \tfrac{1}{2} g^2 \sum_f K_{fj}^* K_{fi} \int d^4x \, dx' \, \bar{\nu}_j(x')$$
$$\times \left(\frac{m_j}{m_W} \frac{1+\gamma_5}{2} - \frac{m_f}{m_W} \frac{1-\gamma_5}{2} \right) S_f(x'-x)$$
$$\times \left(\frac{m_i}{m_W} \frac{1-\gamma_5}{2} - \frac{m_f}{m_W} \frac{1+\gamma_5}{2} \right) \nu_i(x) G(x, x') \tag{4}$$

is the contribution from the charged scalar (diagram (b) in fig. 2). Here m_i, m_j, m_f are the masses of the initial, intermediate and final leptons; $\nu_i(x)$, $\nu_j(x')$ are the wavefunctions of the initial and final neutrino states. For the propagators of the W-boson $G_{\nu\mu}$, charged scalar G and charged lepton S_f, exact solutions are used of the corresponding wave equations in the field of a monochromatic circularly polarized wave with four-potential

$$\mathscr{A}_\mu(x) = a_{1\mu} \cos\varphi + \xi a_{2\mu} \sin\varphi , \quad \varphi = kx , \tag{5}$$

where $k^\mu = (\omega, \boldsymbol{k})$ is the four-wavevector; $k^2 = a_1 k = a_2 k = a_1 a_2 = 0$, $a_1^2 = a_2^2 = a^2$; the parameter $\xi = \pm 1$ indicates the direction of the circular polarization (left- or rightward) [#1].

Calculating the integrals over the space coordinates in (3) and (4), we find

$$S_{ji} = (2p_0 V \cdot 2p_0' V)^{-1/2}$$
$$\times \mathrm{i}(2\pi)^4 \sum_{\sigma=\pm 1} \delta^{(4)}(p - p' + \sigma k) \mathscr{M}_{(\sigma)} , \tag{6}$$

where p_μ ($p^2 = m_i^2$) and p'_μ ($p'^2 = m_j^2$) are the four-momenta of the initial and final neutrinos.

The conservation law $p' = p + \sigma k$ unambiguously determines the kinematics of the process

$$kp = \tfrac{1}{2}\sigma(m_j^2 - m_i^2) > 0 , \tag{7}$$

where $\sigma = \pm 1$ is the difference between the numbers of absorbed and emitted photons of the external field. That only two values of σ are possible is non-trivial and is related not only to the peculiarity of the circularly polarized wave but to the particular decay type as well. The indicated parameter values follow from the conservation of the projection of the total angular momentum on the vector $\boldsymbol{k}$ direction in the process $\nu_i \to \nu_j$.

The invariant amplitude $\mathscr{M}_{(\sigma)}$ in (6) is given by

[#1] Vectors $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ and $\boldsymbol{k}$ form a right-handed coordinate system.

$$\mathcal{M}_{(\sigma)}=\frac{eG_F}{4\pi^2}\bar{U}(p')[(\gamma a_{(\sigma)})(kp)-(\gamma k)(a_{(\sigma)}p)]$$
$$\times\left(\frac{1+\gamma_5}{2}\langle L^{(1)}_{(\sigma)}\rangle+\sqrt{\delta_i\delta_j}\frac{1-\gamma_5}{2}\langle L^{(2)}_{(\sigma)}\rangle\right)U(p), \tag{8}$$

$$\langle L\rangle\equiv\sum_f K^*_{fj}K_{fi}L(\delta_f),$$
$$L^{(r)}_{(\sigma)}=\int_0^\infty\frac{dv}{(1+v)^2}\int_0^\infty dt\exp(-2it\Phi)\,h^{(r)},$$
$$r=1,2 \tag{9}$$

$$\Phi=1+\frac{1}{v}+\delta_f(1+v)-\tfrac{1}{2}(\delta_i+\delta_j)$$
$$+\frac{x^2\delta_i(1+v)^2}{v}[1-j_0^2(\tau)],$$
$$h^{(1)}=i\xi\left(\frac{2}{v}(2+v)+\delta_f(1+2v)\right)j_0(\tau)$$
$$+(2+\delta_f)j_0'(\tau),$$
$$h^{(2)}=j_0'(\tau)+i\xi j_0(\tau),$$
$$\delta_\ell=\frac{m_\ell^2}{m_W^2},\quad \kappa=\frac{2(kp)}{m_W^2}=\sigma(\delta_j-\delta_i),$$
$$a^\mu_{(\sigma)}=\frac{1}{\sqrt{2}}(a_1^\mu+i\sigma\xi a_2^\mu),$$

where $j_0(\tau)=\sin\tau/\tau$, $j_1(\tau)=-j_0'(\tau)$ are spherical Bessel functions, $x^2=-e^2a^2/m_i^2$ is a parameter characterizing the wave intensity.

The square of the matrix element (6) integrated over the final neutrino phase volume determines the probability of the transition involved per unit time,

$$w=\frac{3}{8}\frac{\Gamma_0 x^2 m_i}{|1-\Delta|}F\delta(E-E_\omega), \tag{10}$$

where

$$F=4(1-\Delta)^2\{|\langle L^{(1)}_{(\sigma)}\rangle|^2$$
$$\times[\tfrac{1}{2}(1+\sigma\xi)(1+\lambda)(1-\lambda')$$
$$+\Delta\tfrac{1}{2}(1-\sigma\xi)(1-\lambda)(1+\lambda')]$$
$$-2\,\mathrm{Re}\langle L^{(1)}_{(\sigma)}\rangle\langle L^{(2)}_{(\sigma)}\rangle\delta_i(1+\sigma\xi\lambda)(1-\sigma\xi\lambda')$$
$$+|\langle L^{(2)}_{(\sigma)}\rangle|^2\delta_i\delta_j[\tfrac{1}{2}(1-\sigma\xi)(1-\lambda)(1+\lambda')$$
$$+\Delta\tfrac{1}{2}(1+\sigma\xi)(1+\lambda)(1-\lambda')]\}, \tag{11}$$

$$\Gamma_0=\frac{G_F^2m_i^5}{192\pi^3},\quad E_\omega=\frac{\sigma(m_j^2-m_i^2)}{2\omega(1-v\cos\theta)},\quad \Delta=\frac{m_j^2}{m_i^2}.$$

Here E_ω has the meaning of a "resonance" energy of the initial neutrino, θ is the angle between its velocity v and wavevector $\boldsymbol{k}$. The dependence of the transition rate w on the neutrino polarization is described by the following quantities entering F:

$$\lambda=\frac{m_i(ks)}{(kp)},\quad \lambda'=\frac{m_j(ks')}{(kp)},$$

where s_μ and s'_μ are spin four-vectors characterizing the initial and final neutrino polarizations. λ and λ' can have the values ±1 and have the meaning of twice the spin projection on the wavevector in the massive neutrino rest system.

3. The "weak" field limit

Analysis of the functions $L^{(r)}_{(\sigma)}$ in the expressions for the amplitude (8) and rate (10) shows that nontrivial influence of the wave field on the process $\nu_i\to\nu_j$ is described by the parameter

$$\rho=x^2\delta_i^3(1-\Delta)^2.$$

Let a field for which $\rho\ll1$ be considered as "weak". For the known neutrinos, the parameters δ_i, $\delta_j\ll1$ and, consequently, ρ are small in a very wide range of values of wave intensity parameters.

In the "weak" field limit the function F from the expression for the flavour-changing neutrino transition rate can be substantially reduced:

$$F=|\langle\mathcal{F}\rangle|^2(1-\Delta)^2$$
$$\times\left(\frac{1+\sigma\xi}{2}\frac{1+\lambda}{2}\frac{1-\lambda'}{2}+\Delta\frac{1-\sigma\xi}{2}\frac{1-\lambda}{2}\frac{1+\lambda'}{2}\right), \tag{12}$$

$$\mathcal{F}(\delta_f)=\frac{-3\delta_f}{(1-\delta_f)^2}\left(1+\delta_f+\frac{2\delta_f}{1-\delta_f}\ln\delta_f\right).$$

For δ_i, $\delta_j\ll1$ the dependence of the process rate on the mixing angles and intermediate lepton masses is determined by the factor $|\langle\mathcal{F}\rangle|^2$, the function $\mathcal{F}(\delta_f)$ being identical with the relevant factor in the rate of the radiative neutrino decay $\nu_i\to\nu_j\gamma$ without the field:

$$\omega(\nu_i \to \nu_j \gamma) = \frac{3\alpha}{32\pi} |\langle \mathscr{F} \rangle|^2 (1+\Delta)(1-\Delta)^3 \Gamma_0 . \quad (13)$$

The expectation for transition rate (number of transitions in unit volume per unit time) is obtained as a result of averaging (10) over the initial neutrino spectrum $f(E)$:

$$W(\nu_i \to \nu_j) = \frac{3}{8} \frac{\Gamma_0 x^2 m_i}{|1-\Delta|} F f(E_\omega) . \quad (14)$$

It is curious that, as follows from (12), transitions of left neutrinos $\nu_{iL} \to \nu_j$ are suppressed by the factor m_j^2/m_i^2, provided that $m_i > m_j$. We assume for definiteness that the i-type neutrino is heavier than that of j-type. Thus, transitions turn out to be unsuppressed of only right massive neutrinos which are practically absent in nature because of the V–A structure of weak interaction. However, transitions of a lighter neutrino into heavier ones $\nu_j \to \nu_i$ are possible in external fields. Analysis shows that in this case the rate of transitions of lighter left neutrinos into heavier ones ($\nu_{jL} \to \nu_i$) is exactly equal to that of unsuppressed transitions of heavier right neutrinos into lighter ones $\nu_{iR} \to \nu_j$:

$$W(\nu_{jL} \to \nu_i) = W(\nu_{iR} \to \nu_j) .$$

It is convenient to compare the non-radiative transition $\nu_{jL} \to \nu_i$ in an external field with the radiative decay $\nu_i \to \nu_j \gamma$ without the field. Using the expressions (13) and (14), we obtain for the branching ratio

$$R \equiv \frac{W(\nu_{jL} \to \nu_i)}{W(\nu_i \to \nu_j \gamma)} = \frac{4\pi}{\alpha} x^2 f(E_\omega) \Big/ \int_{m_i}^{\infty} \frac{dE}{E} f(E) . \quad (15)$$

Note that this branching ratio substantially depends on the shape of the spectrum $f(E)$ and, in the limit of "weak" field ($\rho \ll 1$), neither on mixing angles nor on intermediate lepton masses. For a sufficiently narrow energy distribution of the initial neutrinos, we have

$$R \sim 10^3 x^2 \frac{f(E_\omega)}{f(\bar{E})} \frac{\bar{E}}{\Delta E} , \quad (16)$$

where $\bar{E}$ is the neutrino mean energy, ΔE ($\ll \bar{E}$) is the distribution width.

Therefore, substantial amplification of an off-diagonal neutrino transition compared to the corresponding decay $\nu_i \to \nu_j \gamma$ is quite real.

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