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# MATHEMATICAL MODELLING OF MULTILAYER THIN BODY DEFORMATION

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Abstract. We construct a theory of multilayer thin bodies within the framework of the three-dimensional moment theory by using an efficient parametrization of a multilayer thin domain; in contrast to classic approaches, several base surfaces and an analytic method with Legendre and Chebyshev polynomial systems are used. Geometric characteristics typical for the proposed parametrizations are introduced into consideration. A fundamental theorem for a multilayer thin domain is formulated. Various representations of the equations of motion, the heat influx, and the constitutive relations of physical and heat content are presented for the new body domain parametrization. The definition of the $k$th order moment of a certain quantity with respect to an orthonormal system of second-kind Chebyshev polynomials is given. The expressions of moments of first- and second-order partial derivatives of a certain tensor field are obtained, and this is also done for some important expressions required for constructing different variants of the thin body theory. Various variants of the equations of motion in moments with respect to Legendre and Chebyshev polynomial systems are also obtained. The interlayer conditions are written down under various connections of adjacent layers of a multilayer body.

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An important problem of modern industry is reducing the weight of structures while preserving their operation reliability [58]. In this connection, for a complete study of the real strained-deformed state, it is necessary to consider theories of higher (second, third, etc.) approximations, geometric and physical nonlinearities, moment theories of a deformable rigid body, and also refined methods for reducing three-dimensional problems to two-dimensional ones. Obviously, this new mechanical content leads to new problems requiring mathematical study. At present, a whole number of variants of theories of rods, plates, shells, and multilayer constructions (bodies) have been developed. Analysis of published works shows that the creation of refined theories of these bodies is actively being developed. Moreover,

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nonlinear theories are more widely considered in the literature. The mathematical techniques used are essentially extended both for implementation of already posed problem and in searching for new statements. The experimental way of studying is used in parallel with the theoretical one. Numerical methods and discrete computational models are widely used. Works focused on multilayer bodies allow us to reveal several basic methods for constructing theories of these bodies. They are reduced to the following.

1. Theories constructed on the basis of the Kirchhoff–Love conjecture for the whole layer package.

2. Theories taking into account the transversal shift and rarely transversal normal deformations and stresses in layers (refined theories) on the basis of "integral" assumptions on the character of distribution of tangential transversal stresses or displacements in the layer package as a whole. The order of the equations obtained in this process is independent of the number of layers. Such a construction method is now called a phenomenological or continuous structural method.

3. Theories taking into account the transversal shift (and often transversal and normal deformations and stresses in layers) on the basis of kinematic assumptions for each separate layer. In this case, the order of the system of equations depends on the number of layers. This approach was developed for the first time in the paper of Grigolyuk and Chulkov [33]. It is now called the discrete, or discrete structural, method. A detailed analysis of works of the discrete structural direction is given in [34–36].

4. The theory constructed by independent application of kinematic and static assumptions and the mixed variational principle. If these assumptions are applied for the layer package as a whole, then the order of the system of equations is independent of the number of layers. If displacements and stresses in each separate layer are approximated independently, then the order of the system of equations depends on the number of layers. The disadvantage of the structural method based on kinematic assumptions for each layer is that at the boundaries of layer contact the continuity conditions of transversal stresses do not hold strictly. The complexity of the equations obtained in this process and the relative simplicity and restricted range of applicability of phenomenological models lead to the appearance of works in which independent kinematic and static assumptions and the mixed variational Reissner method are applied simultaneously. Note that such a method can be considered as a further development of the discrete structural method.

5. Theories constructed on the base of the analytic method. In this case a three-dimensional problem of elasticity theory is reduced to a two-dimensional problem of shell theory by a series expansion of the desired functions. The order of the obtained system of equations depends on the number of layers and on the number of terms remaining in the expansions.

6. Theories to which the relations of the three-dimensional theory are applied for analyzing the strained-deformed state and stability of multilayer shells. The application of the spatial approach to calculating layered anisotropic thin-walled constructions was difficult because of the following two circumstances [83]: the absence of sufficiently powerful computational tools and insufficient justification of numerical methods for solving the indicated class of problems. Starting from 1980, the appearance of powerful computers and efficient mathematical software allowed one to eliminate the first circumstance. Research into the convergence and stability of the boundary element method in calculating thin-walled composite constructions allow one to assume that the second problem is solved for this method (including that for geometrically nonlinear problems). The works in this direction showed that the strained-deformed state of multilayer composite shells and plates has an essentially three-dimensional character, so that the transversal stresses and deformations cannot be neglected. The determination and study of exact solutions and also the error estimate of approximate solutions are

essential because intuitive auxiliary assumptions introduced to simplify the equations are not justified and lead to mistakes of not only quantitative but qualitative character.

7. Theories constructed via asymptotic integration of the equations of elasticity theory [27–32]. From the mathematical viewpoint, this method leads to a uniform approximation of solution in all elements of the theory (kinematic and force) because terms of the same order are always considered. All the methods listed above were applied in the construction of theories of one-layer thin bodies. Also, the following two methods for constructing the theory of one-layer thin bodies (rods, plates, and shells) should be mentioned.

8. Theories in the construction of which the method of sequential differentiation of relations of the three-dimensional theory is used [117]. This method proposed by Vekua [117] constructs a non-contradictory moment shell theory. Vekua deduced a system of equations of the tenth order that is in concordance with five physical or kinematic conditions given independently. This method can obviously be used in constructing the rod theory, as well as the multilayer thin body theory.

9. The general nonlinear thin shell theory, which is mainly applied for calculating elastomer shells (rubber-like materials) [17–19], and the general nonlinear theory of thin (Kirchhoff) rods [18], which can be applied for description of large deformations. The basis of these theories consists of the modified geometric Kirchhoff assumptions proposed by Chernykh, the author of these papers. Therefore, these assumptions can be extended to the theory of multilayer thin bodies composed of the corresponding materials. In general, all papers of Chernykh are of special interest. For reviews of works in the theories of one- and multilayer bodies, see [64, 72, 73]. In these papers, there are references to reviews of other authors. In this connection, we do not dwell on a detailed review of the literature here. In principle, any problem of the thin body theory[1] can be considered (solved) in a three-dimensional statement, which is more precise compared to the two-dimensional one. However, it is not always possible to implement this approach in practice because of the complexity of solving three-dimensional problems and the large variety of statements of problems that are necessary. In connection with what was said above and with the wide use of thin bodies (one-, two, three-, and multilayer constructions), there arises a necessity to create new refined thin body theories within the framework of the classic theory, as well as the moment theory and improved methods for their calculation. Therefore, the construction of refined thin body theories and development of efficient methods for their computation are important and urgent problems. It should be noted that the analytic method with the use of the Legendre polynomial system in constructing the one-layer thin body theory [3–8, 20, 24, 38–42, 55, 57, 104, 106, 111–115, 117–123] and multilayer thin body theory [11–16, 25, 88–90] was also applied by other authors. In this direction, the author published the papers [65–68, 72, 74–76, 81, 82] and others with the application of Legendre and Chebyshev polynomial systems. These expansions can be successfully used in constructing any thin body theory. Despite this, the classical theories constructed by this method are far from complete, the more so the moment theories [56].

## 1. Parametrization of a Multilayer Thin Domain of the Three-Dimensional Euclidean Space with Several Base Surfaces

Consider a multilayer thin domain of the Euclidean space consisting of no more than countably many layers. We perform the parametrization of this domain in the same way as in [63, 73]. Let the layers be enumerated in the ascending order, i.e., for example, if $\alpha$ is the serial number of a certain

[1]Three-dimensional bodies where one or two dimensions are less than the others are called thin bodies, and a two-dimensional domain where one dimension is less than the other is called a thin domain.

layer, then the serial number of the previous layer is $\alpha-1$ and the serial number of the next layer is $\alpha+1$. Each layer has two frontal surfaces. The frontal surface of the layer $\alpha$, which lies to the side of the previous layer $\alpha-1$, is called the interior base surface and is denoted by $\overset{(-)}{\underset{\alpha}{\mathrm{S}}}$, whereas the frontal surface of the layer $\alpha$, which lies to the side of the next layer $\alpha+1$, is called the exterior base surface and is denoted by $\overset{(+)}{\underset{\alpha}{\mathrm{S}}}$. We assume that the frontal surfaces of each layer are regular surfaces and its lateral surface is a ruled surface in the case where the layer is bounded and unclosed.

**1.1. Vector parametric equation of the layer $\alpha$ and the system of vector parametric equations of a multilayer thin domain.** The position vector of an arbitrary point $\underset{\alpha}{\mathrm{M}}$ of the layer $\alpha$ is represented in the form

$$\underset{\alpha}{\mathbf{r}}(x^1,x^2,x^3)=\overset{(-)}{\underset{\alpha}{\mathbf{r}}}(x^1,x^2)+x^3\underset{\alpha}{\mathbf{h}}(x^1,x^2)=(1-x^3)\overset{(-)}{\underset{\alpha}{\mathbf{r}}}(x^1,x^2)+x^3\overset{(+)}{\underset{\alpha}{\mathbf{r}}}(x^1,x^2) \tag{1.1}$$

for all $\alpha\in\mathbb{N}$ and $\forall\, x^3\in[0,1]$, where the vector relations

$$\overset{(-)}{\underset{\alpha}{\mathbf{r}}}=\overset{(-)}{\underset{\alpha}{\mathbf{r}}}(x^1,x^2),\quad \overset{(+)}{\underset{\alpha}{\mathbf{r}}}=\overset{(+)}{\underset{\alpha}{\mathbf{r}}}(x^1,x^2),\quad \alpha\in\mathbb{N}, \tag{1.2}$$

are the vector equations of the base surfaces $\overset{(-)}{\underset{\alpha}{\mathrm{S}}}$ and $\overset{(+)}{\underset{\alpha}{\mathrm{S}}}$, respectively, $x^1$ and $x^2$ are curvilinear (Gaussian) coordinates on the interior base surface $\overset{(-)}{\underset{\alpha}{\mathrm{S}}}$, and $\mathbb{N}$ is the set of natural numbers. The vector

$$\underset{\alpha}{\mathbf{h}}(x^1,x^2)=\overset{(+)}{\underset{\alpha}{\mathbf{r}}}(x^1,x^2)-\overset{(-)}{\underset{\alpha}{\mathbf{r}}}(x^1,x^2),$$

which topologically maps the interior base surface $\overset{(-)}{\underset{\alpha}{\mathrm{S}}}$ onto the exterior base surface $\overset{(+)}{\underset{\alpha}{\mathrm{S}}}$, is, in general, not orthogonal to the base surfaces, and, moreover, the endpoint of each $\underset{\alpha}{\mathbf{h}}(x^1,x^2)$ is the initial point of $\underset{\alpha+1}{\mathbf{h}}(x^1,x^2)$, $\forall\alpha$, i.e., the following relation holds:

$$\begin{aligned}\overset{(+)}{\underset{\alpha+\delta}{\mathbf{r}}}(x^1,x^2)&=\overset{(-)}{\underset{\alpha}{\mathbf{r}}}(x^1,x^2)+\sum_{\nu=\alpha}^{\alpha+\delta}\underset{\nu}{\mathbf{h}}=\overset{(+)}{\underset{\alpha}{\mathbf{r}}}(x^1,x^2)+\sum_{\nu=\alpha+1}^{\alpha+\delta}\underset{\nu}{\mathbf{h}}\\&=\overset{(-)}{\underset{\alpha}{\mathbf{r}}}(x^1,x^2)+\sum_{\nu=\alpha}^{\alpha+\delta}\Big[\overset{(+)}{\underset{\nu}{\mathbf{r}}}(x^1,x^2)-\overset{(-)}{\underset{\nu}{\mathbf{r}}}(x^1,x^2)\Big]\\&=\overset{(+)}{\underset{\alpha}{\mathbf{r}}}(x^1,x^2)+\sum_{\nu=\alpha+1}^{\alpha+\delta}\Big[\overset{(+)}{\underset{\nu}{\mathbf{r}}}(x^1,x^2)-\overset{(-)}{\underset{\nu}{\mathbf{r}}}(x^1,x^2)\Big]\quad\forall\alpha,\delta.\end{aligned} \tag{1.3}$$

Let a multilayer domain[2] consist of $K$ layers. Then introducing the notation

$$\mathbf{h}=\sum_{\nu=1}^{K}\underset{\nu}{\mathbf{h}}=\sum_{\nu=1}^{K}\Big[\overset{(+)}{\underset{\nu}{\mathbf{r}}}(x^1,x^2)-\overset{(-)}{\underset{\nu}{\mathbf{r}}}(x^1,x^2)\Big], \tag{1.4}$$

[2]We use the usual rules of tensor calculus [54, 92, 93, 116]. We mainly preserve the notation and conventions of the previous works. Under symbols, we write indices denoting the serial numbers of layers. The Greek indices under symbols assume their values according to circumstances, and capital and small Latin indices assume the values 1, 2 and 1, 2, 3, respectively.

we have

$$\overset{(+)}{\underset{K}{\mathbf{r}}}(x^1,x^2) = \overset{(-)}{\underset{1}{\mathbf{r}}}(x^1,x^2) + \mathbf{h}(x^1,x^2) = \overset{(-)}{\underset{1}{\mathbf{r}}}(x^1,x^2) + \sum_{\nu=1}^{K}\Big[\overset{(+)}{\underset{\nu}{\mathbf{r}}}(x^1,x^2) - \overset{(-)}{\underset{\nu}{\mathbf{r}}}(x^1,x^2)\Big]. \tag{1.5}$$

Note that (1.1) for a fixed $\alpha$ is the vector parametric equation of the layer $\alpha$, and when $\alpha$ varies in the corresponding range and conditions (1.3) hold, it is the system of vector parametric equations of the multilayer thin domain considered. It is easy to see that (1.1) for any $x^1$, $x^2$, and $x^3 = 0$ defines the interior base surface $\overset{(-)}{\underset{\alpha}{\mathrm{S}}}$, and for any $x^1$, $x^2$, and $x^3 = 1$, it defines the exterior lateral surface $\overset{\alpha}{\overset{(+)}{\mathrm{S}}}$, whereas for any $x^1$, $x^2$ and $x^3 = \text{const}$, where $x^3 \in (0,1)$, it defines the equidistance surface for the base surfaces $\overset{(-)}{\underset{\alpha}{\mathrm{S}}}$ and $\overset{(+)}{\underset{\alpha}{\mathrm{S}}}$, which is denoted by $\underset{\alpha}{\mathrm{S}}$.

**1.2. Two-dimensional families of bases and the families of parametrizations of the surface of the layer $\alpha$ generated by them.** For the derivatives of relations (1.1) and (1.2) in $x^P$ at the points $\overset{(\star)}{\underset{\alpha}{\mathrm{M}}} \in \overset{(\star)}{\underset{\alpha}{\mathrm{S}}}$, $\star \in \{-,\emptyset,+\}$ $\forall\alpha$, let us introduce the notation

$$\underset{\alpha P}{\mathbf{r}} \equiv \partial_P \underset{\alpha}{\mathbf{r}} \equiv \frac{\partial_P \underset{\alpha}{\mathbf{r}}}{\partial x^P}, \quad \underset{\alpha P}{\mathbf{r}_{\star}} \equiv \partial_P \overset{(\star)}{\underset{\alpha}{\mathbf{r}}} \equiv \frac{\partial \overset{(\star)}{\underset{\alpha}{\mathbf{r}}}}{\partial x^P}, \quad \star \in \{-,+\},\ \forall\alpha. \tag{1.6}$$

The pair of vectors $\underset{\alpha 1}{\mathbf{r}_{\star}}$ and $\underset{\alpha 2}{\mathbf{r}_{\star}}$, $\star \in \{-,\emptyset,+\}$ $\forall\alpha$, defined at the points $\overset{(\star)}{\underset{\alpha}{\mathrm{M}}} \in \overset{(\star)}{\underset{\alpha}{S}}$, $\star \in \{-,\emptyset,+\}$, $\forall\alpha$, obviously compose two-dimensional covariant surface bases, and $\overset{(\star)}{\underset{\alpha}{\mathrm{M}}}\underset{\alpha 1}{\mathbf{r}_{\star}}, \underset{\alpha 2}{\mathbf{r}_{\star}}$, $\star \in \{-,\emptyset,+\}$ $\forall\alpha$, are two-dimensional covariant surface frames, which, in turn, generate the corresponding parametrizations of the surfaces considered. As is known [54, 92, 93, 116], according to these frames (bases), we can construct the corresponding contravariant frames $\overset{(\star)}{\underset{\alpha}{\mathrm{M}}}\,\underset{\alpha}{\overset{\star}{\mathbf{r}}{}^1}\,\underset{\alpha}{\overset{\star}{\mathbf{r}}{}^2}$ (bases $\underset{\alpha}{\overset{\star}{\mathbf{r}}{}^1}\,\underset{\alpha}{\overset{\star}{\mathbf{r}}{}^2}$), $\star \in \{-,\emptyset,+\}$, $\forall\alpha$. Naturally, the covariant and contravariant bases generate their inherent geometric characteristics. Defining the frames (bases) at each point of the surfaces $\overset{(\star)}{\underset{\alpha}{\mathrm{S}}}$, $\star \in \{-,\emptyset,+\}$, $\forall\alpha$, we obtain the corresponding families of frames (bases), which, in turn, generate the corresponding parametrizations.

**1.3. Three-dimensional families of bases and the families of parametrization of the domain of the layer $\alpha$ generated by them.** Taking into account the expression of the position vector $\underset{\alpha}{\mathbf{r}}$ (1.1) in the first relation in (1.6) and introducing the notation $\underset{\alpha P}{\mathbf{h}} \equiv \partial\mathbf{h}/\partial x^P \equiv \partial_P \underset{\alpha}{\mathbf{h}}$, we obtain

$$\underset{\alpha P}{\mathbf{r}} = \underset{\alpha P}{\mathbf{r}_-} + x^3 \underset{\alpha P}{\mathbf{h}} = (1-x^3)\underset{\alpha P}{\mathbf{r}_-} + x^3 \underset{\alpha P}{\mathbf{r}_+}, \quad \forall\,\alpha. \tag{1.7}$$

Now, differentiating (1.1) in $x^3$, we have

$$\underset{\alpha 3}{\mathbf{r}} \equiv \partial_3 \underset{\alpha}{\mathbf{r}} \equiv \frac{\partial \underset{\alpha}{\mathbf{r}}}{\partial x^3} = \underset{\alpha}{\mathbf{h}}(x^1,x^2), \quad \forall x^3 \in [0,1], \quad \forall\alpha. \tag{1.8}$$

According to (1.8), we can assume that

$$\underset{\alpha 3}{\mathbf{r}_-} \equiv \underset{\alpha 3}{\mathbf{r}} \equiv \underset{\alpha 3}{\mathbf{r}_+} \equiv \partial_3 \underset{\alpha}{\mathbf{r}} = \underset{\alpha}{\mathbf{h}}(x^1,x^2), \quad \forall x^3 \in [0,1], \quad \forall\alpha. \tag{1.9}$$

Relation (1.9) allows us to define the spatial covariant bases $\underset{\alpha p}{\mathbf{r}_{\star}}$, $\star \in \{-,+\}$, $\forall\,\alpha$ at the points $\overset{(\star)}{\underset{\alpha}{M}} \in \overset{(\star)}{\underset{\alpha}{\mathrm{S}}}$, $\star \in \{-,+\}$, $\forall\,\alpha$, respectively. Therefore, the third basis vector of the spatial covariant bases at the points $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}} \in \underset{\alpha}{\overset{(\star)}{\mathrm{S}}}$, $\star \in \{-, \emptyset, +\}$, for each layer $\alpha$ is the same vector $\underset{\alpha}{\mathbf{h}}(x^1, x^2)$. In view of (1.9), we can combine relations (1.7) and (1.8) and represent them as

$$\underset{\alpha}{\mathbf{r}}_p = \underset{\alpha}{\mathbf{r}}_{\bar{p}} + x^3 \underset{\alpha}{\mathbf{h}}_p = (1 - x^3)\underset{\alpha}{\overset{(-)}{\mathbf{r}}}_{\bar{p}} + x^3 \underset{\alpha}{\overset{(+)}{\mathbf{r}}}_{\bar{p}}, \quad \forall \alpha. \tag{1.10}$$

The triples of vectors $\underset{\alpha}{\mathbf{r}}_{\overset{\star}{1}}\, \underset{\alpha}{\mathbf{r}}_{\overset{\star}{2}}\, \underset{\alpha}{\mathbf{r}}_{\overset{\star}{3}}$, $\star \in \{-, \emptyset, +\}$, $\forall \alpha$ defined at the points $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}} \in \underset{\alpha}{\overset{(\star)}{S}}$, $\star \in \{-, \emptyset, +\}$, $\forall \alpha$ obviously compose three-dimensional covariant spatial bases, and $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}}\, \underset{\alpha}{\mathbf{r}}_{\overset{\star}{1}}\, \underset{\alpha}{\mathbf{r}}_{\overset{\star}{2}}\, \underset{\alpha}{\mathbf{r}}_{\overset{\star}{3}}$, $\star \in \{-, \emptyset, +\}$, $\forall \alpha$, compose three-dimensional spatial covariant frames, which, in turn, generate the corresponding parametrizations. As is known [54, 92, 116], according to these frames(bases), we can construct the corresponding contravariant frames $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}}\, \underset{\alpha}{\mathbf{r}}^{\overset{\star}{1}}\, \underset{\alpha}{\mathbf{r}}^{\overset{\star}{2}}\, \underset{\alpha}{\mathbf{r}}_{\overset{\star}{3}}$ (bases $\underset{\alpha}{\mathbf{r}}^{\overset{\star}{1}}\, \underset{\alpha}{\mathbf{r}}^{\overset{\star}{2}}\, \underset{\alpha}{\mathbf{r}}^{\overset{\star}{3}}$), $\star \in \{-, \emptyset, +\}$, $\forall \alpha$.Indeed, by their definition [54, 92, 116], we have

$$\underset{\alpha}{\mathbf{r}}^{\tilde{k}} = \frac{1}{2} \underset{\alpha}{\overset{(\sim)}{\mathrm{C}}}{}^{\tilde{k}\tilde{p}\tilde{q}} \underset{\alpha}{\mathbf{r}}_{\tilde{p}} \times \underset{\alpha}{\mathbf{r}}_{\tilde{q}}, \quad \sim \in \{-, \emptyset, +\},\ \forall \alpha, \tag{1.11}$$

where $\underset{\alpha}{\overset{(\sim)}{\mathrm{C}}}{}^{\tilde{k}\tilde{p}\tilde{q}} = (\underset{\alpha}{\mathbf{r}}^{\tilde{k}} \times \underset{\alpha}{\mathbf{r}}^{\tilde{p}}) \cdot \underset{\alpha}{\mathbf{r}}^{\tilde{q}}$, $\sim \in \{-, \emptyset, +\}$, $\forall \alpha$, are contravariant components of the discriminant tensors [116] of the layer $\alpha$ at the points $\underset{\alpha}{\overset{(\star)}{\mathrm{M}}} \in \underset{\alpha}{\overset{(\star)}{\mathrm{S}}}$, $\star \in \{-, \emptyset, +\}$, $\forall \alpha$. It is easy to see that (1.10) is briefly represented in the form

$$\underset{\alpha}{\mathbf{r}}_p = \underset{\alpha}{g}_p^{\overset{\star}{q}} \underset{\alpha}{\mathbf{r}}_{\overset{\star}{q}} = \underset{\alpha}{g}_{p\overset{\star}{q}} \underset{\alpha}{\mathbf{r}}^{\overset{\star}{q}}, \quad \star \in \{-, +\},\ \forall \alpha, \tag{1.12}$$

where we have introduced the notation

$$\underset{\alpha}{g}_{\breve{p}\tilde{q}} = \underset{\alpha}{\mathbf{r}}_{\breve{p}} \cdot \underset{\alpha}{\mathbf{r}}_{\tilde{q}}, \quad \underset{\alpha}{g}_{\breve{p}}^{\tilde{q}} = \underset{\alpha}{\mathbf{r}}_{\breve{p}} \cdot \underset{\alpha}{\mathbf{r}}^{\tilde{q}}, \quad \smile \in \{-, \emptyset, +\}, \quad \sim \in \{-, +\}, \quad \forall \alpha. \tag{1.13}$$

In view of (1.10) and (1.13), for $\underset{\alpha}{g}_{p\tilde{q}}$ and $\underset{\alpha}{g}_p^{\tilde{q}}$, we have

$$\underset{\alpha}{g}_{p\breve{q}} = (1 - x^3) \underset{\alpha}{g}_{\bar{p}\breve{q}} + x^3 \underset{\alpha}{g}_{\overset{+}{p}\breve{q}}, \quad \underset{\alpha}{g}_p^{\breve{q}} = (1 - x^3) \underset{\alpha}{g}_{\bar{p}}^{\breve{q}} + x^3 \underset{\alpha}{g}_{\overset{+}{p}}^{\breve{q}}, \quad \smile \in \{-, +\}, \quad \forall \alpha. \tag{1.14}$$

Also, it is easy to obtain the expressions for $\underset{\alpha}{g}_{pq}$. Indeed, by (1.12) and (1.14), we have

$$\underset{\alpha}{g}_{pq} = \underset{\alpha}{\mathbf{r}}_p \cdot \underset{\alpha}{\mathbf{r}}_q = \underset{\alpha}{g}_{p\overset{\star}{n}} \underset{\alpha}{g}_q^{\overset{\star}{n}} = (1 - x^3)^2 \underset{\alpha}{g}_{\bar{p}\bar{q}} + x^3(1 - x^3)(\underset{\alpha}{g}_{\bar{p}\overset{+}{q}} + \underset{\alpha}{g}_{\overset{+}{p}\bar{q}}) + (x^3)^2 \underset{\alpha}{g}_{\overset{+}{p}\overset{+}{q}}, \quad \star \in \{-, +\}, \quad \forall \alpha. \tag{1.15}$$

Let us find the expressions for $\sqrt{\underset{\alpha}{g}} = (\underset{\alpha}{\mathbf{r}}_1 \times \underset{\alpha}{\mathbf{r}}_2) \cdot \underset{\alpha}{\mathbf{r}}_3$. By the first relation in (1.12), we obtain

$$\begin{aligned} \sqrt{\underset{\alpha}{g}} = \frac{1}{2}\epsilon^{IJ}(\underset{\alpha}{\mathbf{r}}_I \times \underset{\alpha}{\mathbf{r}}_J) \cdot \underset{\alpha}{\mathbf{r}}_3 = \frac{1}{2}\sqrt{\underset{\alpha}{\overset{(\sim)}{g}}}\,\epsilon^{IJ}\epsilon_{KL} \underset{\alpha}{g}_I^{\tilde{K}} \underset{\alpha}{g}_J^{\tilde{L}} = \\ = \sqrt{\underset{\alpha}{\overset{(\sim)}{g}}}\det(\underset{\alpha}{g}_p^{\tilde{q}}) = \sqrt{\underset{\alpha}{\overset{(\sim)}{g}}}\det(\underset{\alpha}{g}_P^{\tilde{Q}}), \quad \sim \in \{-, +\}, \quad \forall \alpha, \end{aligned} \tag{1.16}$$

where $\epsilon^{IJ}$ and $\epsilon_{KL}$ are the Levi-Civita symbols and

$$\sqrt{\underset{\alpha}{\overset{(\sim)}{g}}} = (\underset{\alpha}{\mathbf{r}}_{\tilde{1}} \times \underset{\alpha}{\mathbf{r}}_{\tilde{2}}) \cdot \underset{\alpha}{\mathbf{r}}_{\tilde{3}}, \quad \sim \in \{-, +\}, \quad \sqrt{\underset{\alpha}{\overset{(-)}{g}}} = \sqrt{\underset{\alpha}{g}}\Big|_{x^3=0}, \quad \sqrt{\underset{\alpha}{\overset{(+)}{g}}} = \sqrt{\underset{\alpha}{g}}\Big|_{x^3=1}, \quad \forall \alpha.$$

In turn, from (1.16), we have

$$\underset{\alpha}{\overset{(\sim)}{\vartheta}} \equiv \sqrt{\underset{\alpha}{g}\, \underset{\alpha}{\overset{(\sim)}{g}}{}^{-1}} = \frac{1}{2}\,\epsilon^{IJ}\epsilon_{KL} \underset{\alpha}{g}_I^{\tilde{K}} \underset{\alpha}{g}_J^{\tilde{L}} = \det(\underset{\alpha}{g}_P^{\tilde{Q}}), \quad \sim \in \{-, +\}, \quad \forall \alpha. \tag{1.17}$$

It should be noted that analogously to (1.16), in a more general case, we have

$$\sqrt{\underset{\alpha}{\overset{(\sim)}{g}}} = \frac{1}{2}\sqrt{\underset{\alpha}{\overset{(\smile)}{g}}}\,\epsilon^{IJ}\epsilon_{KL}\underset{\alpha}{g}{}^{\breve{K}}_{\tilde{I}}\,\underset{\alpha}{g}{}^{\breve{L}}_{\tilde{J}} = \sqrt{\underset{\alpha}{\overset{(\smile)}{g}}}\det(\underset{\alpha}{g}{}^{\breve{Q}}_{\tilde{P}}), \quad \sim, \smile \in \{-, \emptyset, +\}, \quad \forall\alpha. \tag{1.18}$$

It is easy to see from this that

$$\underset{\alpha}{\overset{(\overset{\smile}{\sim})}{\vartheta}} \equiv \sqrt{\underset{\alpha}{\overset{(\sim)}{g}}\,\underset{\alpha}{\overset{(\smile)}{g}}{}^{-1}} = \frac{1}{2}\epsilon^{IJ}\epsilon_{KL}\underset{\alpha}{g}{}^{\breve{K}}_{\tilde{I}}\,\underset{\alpha}{g}{}^{\breve{L}}_{\tilde{J}} = \det(\underset{\alpha}{g}{}^{\breve{Q}}_{\tilde{P}}) = \det(\underset{\alpha}{g}{}^{\breve{q}}_{\tilde{p}}), \quad \smile, \sim \in \{-, \emptyset, +\}, \quad \forall\alpha. \tag{1.19}$$

IT is seen that for $\sim = \emptyset$, $\smile \in \{-, +\}$, from (1.18) we obtain (1.16), and from (1.19), we obtain (1.17). It is easy to verify that by (1.19), we have

$$\underset{\alpha}{\overset{(\overset{\smile}{\sim})}{\vartheta}} = \underset{\alpha}{\overset{(\overset{\sim}{\smile})}{\vartheta}}{}^{-1}, \quad \smile, \sim \in \{-, +\}, \quad \forall\alpha; \qquad \underset{\alpha}{\overset{(\approx)}{\vartheta}} = 1, \quad \sim \in \{-, +\}, \quad \forall\alpha. \tag{1.20}$$

Using (1.19), we can write relations (1.17) in the following more detailed form:

$$\begin{aligned} \underset{\alpha}{\overset{(-)}{\vartheta}} &= \sqrt{\underset{\alpha}{g}\,\underset{\alpha}{\overset{(-)}{g}}{}^{-1}} = (1 - x^3)^2\underset{\alpha}{\overset{(=)}{\vartheta}} + x^3(1 - x^3)\underset{\alpha}{g}{}^{\bar{I}}_{\underset{I}{+}} + (x^3)^2\underset{\alpha}{\overset{(\mp)}{\vartheta}}, \\ \underset{\alpha}{\overset{(+)}{\vartheta}} &= \sqrt{\underset{\alpha}{g}\,\underset{\alpha}{\overset{(+)}{g}}{}^{-1}} = (1 - x^3)^2\underset{\alpha}{\overset{(\pm)}{\vartheta}} + x^3(1 - x^3)\underset{\alpha}{g}{}^{\overset{+}{I}}_{\underset{I}{-}} + (x^3)^2\underset{\alpha}{\overset{(\overset{+}{+})}{\vartheta}}, \quad \forall\alpha. \end{aligned} \tag{1.21}$$

It is easy to express $\underset{\alpha}{\mathbf{r}}{}^{k}$, $\forall\alpha$, through the vectors $\underset{\alpha}{\mathbf{r}}{}_{\tilde{m}}$ or $\underset{\alpha}{\mathbf{r}}{}^{\tilde{m}}$, $\sim \in \{-, +\}$, $\forall\,\alpha$. Indeed, taking into account the first relation in (1.12) in relation (1.11) for $\sim = \emptyset$, we obtain

$$\underset{\alpha}{\mathbf{r}}{}^{k} = \frac{1}{2}\underset{\alpha}{\overset{(\sim)}{\vartheta}}{}^{-1}\epsilon^{kpq}\epsilon_{lmn}\underset{\alpha}{g}{}^{\tilde{m}}_{p}\,\underset{\alpha}{g}{}^{\tilde{n}}_{q}\,\underset{\alpha}{\mathbf{r}}{}^{\tilde{l}}, \quad \sim \in \{-, +\}, \quad \forall\,\alpha, \tag{1.22}$$

where $\epsilon^{kpq}$, $\epsilon_{lmn}$ are the Levi-Civita symbols. By (1.22), we can introduce the notation

$$\underset{\alpha}{g}{}^{k}_{\tilde{l}} = \frac{1}{2}\underset{\alpha}{\overset{(\sim)}{\vartheta}}{}^{-1}\epsilon^{kpq}\epsilon_{lmn}\underset{\alpha}{g}{}^{\tilde{m}}_{p}\,\underset{\alpha}{g}{}^{\tilde{n}}_{q}, \quad \underset{\alpha}{g}{}^{k\tilde{l}} = \frac{1}{2}\underset{\alpha}{\overset{(\sim)}{\vartheta}}{}^{-1}\epsilon^{kpq}\epsilon_{smn}\underset{\alpha}{g}{}^{\tilde{m}}_{p}\,\underset{\alpha}{g}{}^{\tilde{n}}_{q}\,\underset{\alpha}{g}{}^{\tilde{s}\tilde{l}}, \quad \sim \in \{-, +\}, \quad \forall\alpha. \tag{1.23}$$

Using this notation, we represent relation (1.22) in the desired form

$$\underset{\alpha}{\mathbf{r}}{}^{p} = \underset{\alpha}{g}{}^{p}_{\tilde{q}}\,\underset{\alpha}{\mathbf{r}}{}^{\tilde{q}} = \underset{\alpha}{g}{}^{p\tilde{q}}\,\underset{\alpha}{\mathbf{r}}{}_{\tilde{q}}, \quad \sim \in \{-, +\}, \quad \forall\alpha. \tag{1.24}$$

It is easy to see that from the first relation in (1.23), we have

$$\underset{\alpha}{g}{}^{K}_{\tilde{K}} = \underset{\alpha}{\overset{(\sim)}{\vartheta}}{}^{-1}\underset{\alpha}{g}{}^{\tilde{I}}_{I}, \quad \Rightarrow \quad \underset{\alpha}{g}{}^{\breve{K}}_{\tilde{K}} = \underset{\alpha}{\overset{(\underset{\smile}{\sim})}{\vartheta}}{}^{-1}\underset{\alpha}{g}{}^{\tilde{I}}_{\breve{I}} = \underset{\alpha}{\overset{(\overset{\smile}{\sim})}{\vartheta}}\,\underset{\alpha}{g}{}^{\tilde{I}}_{\breve{I}}, \quad \sim \in \{-, +\}, \quad \forall\,\alpha. \tag{1.25}$$

Note that in writing the second relation in (1.25), (1.19) and (1.20) were taken into account. Also, let us introduce into consideration the following objects (matrices):

$$\underset{\alpha\beta}{g}{}_{\breve{p}\cdot}^{\cdot\tilde{q}} = \underset{\alpha}{\mathbf{r}}{}_{\breve{p}} \cdot \underset{\beta}{\mathbf{r}}{}^{\tilde{q}}, \quad \smile, \sim \in \{-, \emptyset, +\}, \quad \forall\alpha, \beta, \tag{1.26}$$

and the objects obtained from (1.26) by alternating the indices. It is easy to calculate that the number of such objects is equal to 36. It is easy to see that for $\alpha = \beta$, (1.26) contains (1.13), (1.15), and (1.23). Indeed, from (1.26), we have

$$\underset{\alpha}{g}{}^{\tilde{q}}_{\breve{p}} = \underset{\alpha\alpha}{g}{}_{\breve{p}\cdot}^{\cdot\tilde{q}} = \underset{\alpha}{\mathbf{r}}{}_{\breve{p}} \cdot \underset{\alpha}{\mathbf{r}}{}^{\tilde{q}}, \quad \smile, \sim \in \{-, \emptyset, +\}, \quad \forall\alpha, \tag{1.27}$$

and alternating the indices, we obviously obtain the objects considered above and also $\underset{\alpha}{g}^{pq} = \underset{\alpha}{\mathbf{r}}^p \cdot \underset{\alpha}{\mathbf{r}}^q$, $\forall\alpha$, i.e., in this case, the number of the introduced quantities is equal to 36. It is easy to see that by (1.26) and (1.27), the connections between the families of bases are represented in the form

$$\underset{\alpha}{\mathbf{r}}_{\tilde{p}} = \underset{\alpha}{g}_{\tilde{p}}^{\breve{n}} \underset{\alpha}{\mathbf{r}}_{\breve{n}} = \underset{\alpha\beta}{g}_{\tilde{p}\cdot}^{\cdot\breve{n}} \underset{\beta}{\mathbf{r}}_{\breve{n}}, \quad \smile, \sim \in \{-, \emptyset, +\}, \quad \forall\alpha, \beta, \tag{1.28}$$

which remains valid under index alternation. By (1.28), it is easy to show that the following relation holds:

$$\underset{\alpha\beta}{g}_{\tilde{p}\cdot}^{\cdot\breve{q}} = \underset{\alpha\delta}{g}_{\tilde{p}\cdot}^{\cdot\overset{\star}{n}} \underset{\beta\beta}{g}_{\overset{\star}{n}\cdot}^{\cdot\breve{q}}, \quad \smile, \sim, \star \in \{-, \emptyset, +\}, \quad \forall\alpha, \beta, \delta. \tag{1.29}$$

Differentiating (1.3)–(1.5) in $x^I$ and taking into account (1.28), we obtain

$$\begin{gathered}
\underset{\alpha+\beta}{\mathbf{r}}_{\overset{+}{I}} = \underset{\alpha}{\mathbf{r}}_{\overset{-}{I}} + \sum_{\nu=\alpha}^{\alpha+\beta} \Big[\underset{\nu}{g}_{\overset{+}{I}}^{\bar{k}} - \underset{\nu}{g}_{\overset{-}{I}}^{\bar{k}}\Big] \underset{\nu}{\mathbf{r}}_{\bar{k}} = \underset{\alpha}{\mathbf{r}}_{\overset{+}{I}} + \sum_{\nu=\alpha+1}^{\alpha+\beta} \Big[\underset{\nu}{g}_{\overset{+}{I}}^{\bar{k}} - \underset{\nu}{g}_{\overset{-}{I}}^{\bar{k}}\Big] \underset{\nu}{\mathbf{r}}_{\bar{k}}, \\
\partial_I \mathbf{h}(x^1, x^2) = \sum_{\nu=1}^{N} \partial_I \underset{\nu}{\mathbf{h}}(x^1, x^2) = \sum_{\nu=1}^{N} \Big[\underset{\nu}{\mathbf{r}}_{\overset{+}{I}}(x^1, x^2) - \underset{\nu}{\mathbf{r}}_{\overset{-}{I}}(x^1, x^2)\Big] = \\
= \sum_{\nu=1}^{N} \Big[\underset{\nu}{g}_{\overset{+}{I}}^{\bar{k}}(x^1, x^2) - \underset{\nu}{g}_{\overset{-}{I}}^{\bar{k}}(x^1, x^2)\Big] \underset{\nu}{\mathbf{r}}_{\bar{k}}(x^1, x^2), \\
\underset{N}{\mathbf{r}}_{\overset{+}{I}}(x^1, x^2) = \underset{1}{\mathbf{r}}_{\overset{-}{I}}(x^1, x^2) + \sum_{\nu=1}^{N} \Big[\underset{\nu}{g}_{\overset{+}{I}}^{\bar{k}}(x^1, x^2) - \underset{\nu}{g}_{\overset{-}{I}}^{\bar{k}}(x^1, x^2)\Big] \underset{\nu}{\mathbf{r}}_{\bar{k}}(x^1, x^2).
\end{gathered} \tag{1.30}$$

Naturally, all spatial covariant and contravariant bases constructed above have geometric characteristics specific for the parametrizations generated by them. Defining the spatial frames (bases) at each point of the surfaces $\underset{\alpha}{\overset{(\star)}{S}}$, $\star \in \{-, \emptyset, +\}$, $\forall\,\alpha$, we obtain the corresponding families of spatial frames (bases), which, in turn generate the corresponding families of parametrizations. Once more, we note that the structure of $\underset{\alpha}{\overset{(\sim)}{S}}_{\underset{\alpha}{\overset{(\sim)}{g}}}$-family frames (bases), $\sim \in \{-, \emptyset, +\}$, $\forall\alpha$, is such that the third basis vectors $\underset{\alpha}{\mathbf{r}}_{\tilde{3}} = \underset{\alpha}{\mathbf{h}}(x^1, x^2)$, $\sim \in \{-, \emptyset, +\}$, $\forall\,\alpha$, are not perpendicular to the corresponding base surface $\underset{\alpha}{\overset{(\sim)}{S}}$, $\sim \in \{-, \emptyset, +\}$, $\forall\,\alpha$, in general. However, in a particular case, they can be perpendicular, and in a more particular case, they can be unit normal vectors to the surfaces $\underset{\alpha}{\overset{(\sim)}{S}}$, $\sim \in \{-, \emptyset, +\}$, $\forall\,\alpha$, which are denoted by $\underset{\alpha}{\overset{(\sim)}{n}}$, $\sim \in \{-, \emptyset, +\}$, $\forall\,\alpha$, respectively. It is seen from the material presented above that in the parametrization of the multilayer domain considered, for each layer all the relations of the first chapter in [72] or the second chapter in [70] hold under the condition that the root letters of quantities entering these relations must be equipped with the bottom index, which denotes the number of the layer considered. In this connection, we do not consider the problems on the parametrization of a multilayer domain in detail. In what follows, if necessary, we write the necessary formulas from the corresponding relations of the works mentioned in this paragraph by the above method (equipping the root letters of quantities with the bottom index of the layer considered) and obtain some relations, which do not enter into the above works.

**1.4. Representation of the unit tensor of the second rank.** It is easy to find this representation. Indeed, starting from the usual representation of this tensor [54, 92], by (1.28) and (1.29), we obtain the relation

$$\underset{\sim}{\mathbf{E}} = \underset{\widetilde{\alpha}}{\underline{\mathbf{E}}} = \underset{\alpha}{g}{}^{\breve{n}}_{\tilde{p}}\,\underset{\alpha}{\mathbf{r}}{}^{\tilde{p}}\,\underset{\alpha}{\mathbf{r}}{}_{\breve{n}} = \underset{\widetilde{\beta}}{\underline{\mathbf{E}}} = \underset{\beta}{g}{}^{\breve{n}}_{\tilde{p}}\,\underset{\beta}{\mathbf{r}}{}^{\tilde{p}}\,\underset{\beta}{\mathbf{r}}{}_{\breve{n}} = \underset{\alpha\beta}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\breve{n}}\,\underset{\alpha}{\mathbf{r}}{}^{\tilde{p}}\,\underset{\beta}{\mathbf{r}}{}_{\breve{n}}, \quad \sim, \quad \smile\, \in \{-,\emptyset,+\}, \quad \forall\alpha,\beta, \tag{1.31}$$

which remains valid under index alternating. As is seen from (1.31), the quantities (1.26) and (1.27) introduced above represent the components of the unit tensor of the second rank (UTSR) for a multilayer thin domain of the three-dimensional Euclidean space. Now let us introduce the following definitions.

**Definition 1.1.** The parametrization considered above, which is characterized by assigning the radius-vector of an arbitrary point of any layer $\alpha$ in the form (1.1) and by the fulfillment of relation (1.3), is called the new parametrization of a multilayer thin domain.

**Definition 1.2.** The components $\underset{\alpha\beta}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\breve{n}}$, for $\tilde{p} \in \{\overset{-}{p},\, p,\, \overset{+}{p}\}$, $\breve{n} \in \{\overset{-}{n},\, n,\, \overset{+}{n}\}$, $\forall\,\alpha \neq \beta$, and also the components $\underset{\alpha}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\breve{n}}$, $\forall\,\alpha$, for $\sim\, \neq \smile$, where $\sim, \smile\, \in \{+,\emptyset,-\}$, and the images obtained from them by index alternating are called the components of the unit second-rank tensor translation under the new parametrization of a multilayer thin domain.

**Definition 1.3.** The components $\underset{\alpha\beta}{g}{}_{\tilde{p}\tilde{q}}^{\,\cdot\,\cdot}$, $\underset{\alpha\beta}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\tilde{q}}$, $\underset{\alpha\beta}{g}{}^{\tilde{p}\tilde{q}}_{\cdot\,\cdot}$ for $\sim\, = -$ $(\sim\, = +)$, $\forall\,\alpha,\,\beta$, and the components of the translation $\underset{\alpha\beta}{g}{}_{\tilde{p}\breve{q}}^{\,\cdot\,\cdot}$, $\underset{\alpha\beta}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\breve{q}}$, for $\sim\, = +, \smile\, = -$ $(\sim\, = -, \smile\, = +)$, $\forall\,\alpha,\,\beta$, are called the basic components of the second-rank unit tensor under the new parametrization of a multilayer thin domain if, as the base surface, the inner (exterior) base surface of layers are taken.

It is easy to find the expressions for $\underset{\alpha\beta}{g}{}_{pq}$ via basic translation components. Indeed, by (1.14), (1.26), and (1.29), we have

$$\underset{\alpha\beta}{g}{}_{pq} = \underset{\alpha}{g}{}_{p}^{\breve{m}}\,\underset{\beta}{g}{}_{q}^{\tilde{n}}\,\underset{\alpha\beta}{g}{}_{\breve{m}\tilde{n}} = (1-x^3)^2\,\underset{\alpha\beta}{g}{}_{\overset{-}{p}\,\overset{-}{q}} + x^3(1-x^3)\big(\underset{\alpha\beta}{g}{}_{\overset{-}{p}\,\overset{+}{q}} + \underset{\alpha\beta}{g}{}_{\overset{+}{p}\,\overset{-}{q}}\big) + (x^3)^2\,\underset{\alpha\beta}{g}{}_{\overset{+}{p}\,\overset{+}{q}}, \tag{1.32}$$

where $\sim, \smile\, \in \{-,+\}$, $\forall\alpha,\beta$. Whence, for $\alpha = \beta$, we obtain (1.15). It is easy to prove that by UTSR components, the components of the second tensors of surfaces of multilayer constructions [73] are defined. In turn, this implies the following theorem.

**Theorem** (fundamental theorem for a multilayer thin domain in $\mathbb{R}^3$ under its new parametrization). *The existence of the unit second-rank tensor represented in the form*

$$\underset{\sim}{\mathbf{E}} = \underset{\alpha}{g}{}^{\breve{n}}_{\tilde{p}}\,\underset{\alpha}{\mathbf{r}}{}^{\tilde{p}}\,\underset{\alpha}{\mathbf{r}}{}_{\breve{n}} = \underset{\beta}{g}{}^{\breve{n}}_{\tilde{p}}\,\underset{\beta}{\mathbf{r}}{}^{\tilde{p}}\,\underset{\beta}{\mathbf{r}}{}_{\breve{n}} = \underset{\alpha\beta}{g}{}_{\tilde{p}\,\cdot}^{\,\cdot\,\breve{n}}\,\underset{\alpha}{\mathbf{r}}{}^{\tilde{p}}\,\underset{\beta}{\mathbf{r}}{}_{\breve{n}}, \quad \sim, \smile\, \in \{-,\emptyset,+\}, \quad \forall\alpha,\beta,$$

*whose translation components satisfy the equations of* [70]*, which are analogous to the Gauss and Peterson–Codazzi equations, is necessary and sufficient for the existence and for the uniqueness up to a motion in* $\mathbb{R}^3$ *of a certain regular multilayer thin domain under its new parametrization. Moreover, the number of independent basis components of UTSR depends on the type of parametrization and the number of layers.*

**1.5. Representation of isotropic tensors of the fourth rank.** These tensors play a special role in deformable rigid body mechanics. In particular, they are used, e.g., in writing the constitutive relations (CR) of linear isotropic elasticity theory. Therefore, it is appropriate to have their representation in the proposed variant of the theory. It is easy to write these representations. Indeed, as is easily seen, under a complete contraction of indices to multiplicative bases composed of an even number of basis vectors, under the condition that each pair of linked indices belongs, e.g. to one of the $\underset{\alpha}{\overset{(\sim)}{g}}$-families of indices, $\sim\,\in\{-,\emptyset,+\}$, $\forall\,\alpha$, the multiplicative bases become isotropic tensors. Having revealed such a nature of multiplicative bases, we can expect that under a complete contraction of indices to a multiplicative basis consisting of four basis vectors, all isotropic tensors of the fourth rank are obtained. Since the number of isomers of a multiplicative basis of four vectors is equal to 24, it is easy to show that under a complete contraction of indices with respect to isomers, only the following three tensors are not reducible to each other:

$$\begin{gathered}
\underset{\overset{\approx}{\alpha\beta}}{\mathbf{C}}{}_{I}=\underset{\alpha\beta}{\mathbf{R}}{}_{\tilde m}{}^{\cdot\,\tilde m\,\breve n\,\cdot}{}_{\cdot\,\cdot\,\breve n}=\underset{\alpha}{\mathbf{r}}{}_{\tilde m}\underset{\alpha}{\mathbf{r}}{}^{\tilde m}\underset{\beta}{\mathbf{r}}{}^{\breve n}\underset{\beta}{\mathbf{r}}{}_{\breve n}=\underset{\overset{\sim}{\alpha}}{\mathbf{E}}\underset{\overset{\smile}{\beta}}{\mathbf{E}}=\underset{\sim}{\mathbf{E}}\underset{\smile}{\mathbf{E}},\quad \underset{\overset{\approx}{\alpha\beta}}{\mathbf{C}}{}_{II}=\underset{\alpha\beta}{\mathbf{R}}{}_{\tilde m\,\breve n}{}^{\cdot\,\cdot\,\tilde m\,\breve n}{}_{\cdot\,\cdot}=\underset{\alpha}{\mathbf{r}}{}_{\tilde m}\underset{\beta}{\mathbf{r}}{}_{\breve n}\underset{\alpha}{\mathbf{r}}{}^{\tilde m}\underset{\beta}{\mathbf{r}}{}^{\breve n},\\
\underset{\overset{\approx}{\alpha\beta}}{\mathbf{C}}{}_{III}=\underset{\alpha\beta}{\mathbf{R}}{}_{\breve n\,\tilde m}{}^{\cdot\,\cdot\,\tilde m\,\breve n}{}_{\cdot\,\cdot}=\underset{\alpha}{\mathbf{r}}{}_{\tilde m}\underset{\overset{\smile}{\beta}}{\mathbf{E}}\underset{\alpha}{\mathbf{r}}{}^{\tilde m}=\underset{\alpha}{\mathbf{r}}{}_{\tilde m}\underset{\smile}{\mathbf{E}}\underset{\alpha}{\mathbf{r}}{}^{\tilde m},\quad \sim,\smile\,\in\{-,\emptyset,+\},\quad \forall\alpha,\beta.
\end{gathered}\tag{1.33}$$

It is easy to see that for example, by (1.31), the relation $\underset{\alpha}{\mathbf{r}}{}^{\breve n}\underset{\alpha}{\mathbf{r}}{}_{\breve n}=\underset{\beta}{\mathbf{r}}{}^{\tilde p}\underset{\beta}{\mathbf{r}}{}_{\tilde p}$, holds, and using it, we obtain

$$\underset{\overset{\approx}{\alpha}}{\mathbf{C}}{}_{I}=\underset{\overset{\approx}{\alpha\beta}}{\mathbf{C}}{}_{I},\quad \underset{\overset{\approx}{\alpha}}{\mathbf{C}}{}_{II}=\underset{\overset{\approx}{\alpha\beta}}{\mathbf{C}}{}_{II},\quad \underset{\overset{\approx}{\alpha}}{\mathbf{C}}{}_{III}=\underset{\overset{\approx}{\alpha\beta}}{\mathbf{C}}{}_{III},\quad \forall\alpha,\beta.$$

If we omit the indices $\alpha$, $\beta$, and $\sim$, $\smile$, then tensors (1.33) evidently coincide with the fourth-rank isotropic tensors considered in [54].

**1.6. On components of UTSR.** As was said above, we do not write all relations of this section, since for a given layer, they can be obtained from the corresponding relations of the first chapter in [72] or second chapter in [70] if the root letters in these relations are equipped with the bottom index denoting the number of layer. Therefore, assuming that these relations are known, and taking the base interior surface $\underset{\alpha}{\overset{(-)}{\mathrm{S}}}$ of the layer $\alpha$ as the base surface, by Definition 3, for $\beta=\alpha$, we obtain that the basic components of UTSR of the layer $\alpha$ are the components $\underset{\alpha}{g}{}_{\underset{p}{-}\underset{q}{-}}$, $\underset{\alpha}{g}{}_{\underset{p}{-}}^{\bar q}$, $\underset{\alpha}{g}{}^{\bar p\,\bar q}$ and the translation components $\underset{\alpha}{g}{}_{\underset{p}{+}\underset{q}{-}}$, $\underset{\alpha}{g}{}_{\underset{p}{+}}^{\bar q}$, which play an important role in the sense that the other components and the most geometric characteristics are expressed through them. Also, the components $\underset{\alpha\beta}{g}{}_{\underset{p}{+}\,\cdot}^{\,\cdot\,\bar q}=\underset{\alpha}{\mathbf{r}}{}_{\underset{p}{+}}\cdot\underset{\beta}{\mathbf{r}}{}^{\bar q}$ for $\beta=\alpha+1$ ($\underset{\alpha\alpha+1}{g}{}_{\underset{p}{+}\,\cdot}^{\,\cdot\,\bar q}=\underset{\alpha}{\mathbf{r}}{}_{\underset{p}{+}}\cdot\underset{\alpha+1}{\mathbf{r}}{}^{\bar q}$) are of interest; they characterize the connections between surfaces in contact; owing to this, they can be called the contact components of UTSR. Note that a complete presentation of parametrization problems for one- and multilayer three-dimensional domains with one small dimension is contained in [59, 67, 70, 72, 75] (see also [61, 62]) and [63, 73], for a plane domain in [60, 76], and for a three-dimensional domain with two small dimensions, in [74]. To obtain a certain relation (system of equation, CR, boundary and initial conditions) in moments for multilayer thin bodies under the considered parametrization of the thin body domain, it suffices to equip the root letters of quantities with the bottom index $\alpha$, which denotes the number of the layer $\alpha$ in the corresponding relation, and vary this index in the range from 1 up to $K$, where $K$ is the number of layers. Hence for a correct statement of problems, to the equations of motion and boundary and initial conditions in moments, we need to add the inter-layer contact conditions. In what follows, using this rule, we write certain systems of equations in moments for multilayer thin bodies, and also consider the inter-layer contact conditions for various conditions of neighboring layer connection.

## 2. Representation of Equations of Motion and Heat Influx and Constitutive Relations of Physical and Heat Contents of Micropolar Theory of Multilayer Elastic Thin Bodies with One Small Dimension

In what follows, for brevity, we present certain representations of equations of motion and constitutive relations in the case of a one-layer thin body, and then we show how one can obtain the desired relations using the rule presented above and write certain relations.

**2.1. Representation of equations of motion and constitutive relations of physical and heat contents of the micropolar theory of one-layertic thin bodies with one small dimension.** The new parametrization of a one-layer thin domain [67, 70, 72] is performed by the relation obtained from (1.1) under the absence of index $\alpha$ under the symbols. To obtain the representations of equations of motion and constitutive relations, we need the representations of the gradient and the divergence under the parametrization considered. Let us obtain the representations of these operators. Omitting the index $\alpha$, from (1.12) and (1.24), we find that

$$\mathbf{r}_p = g_p^{\bar{m}}\mathbf{r}_{\bar{m}} = g_{p\overset{+}{m}}\mathbf{r}^{\overset{+}{m}}, \quad \mathbf{r}^p = g^p_{\bar{m}}\mathbf{r}^{\bar{m}} = g^p_{\overset{+}{m}}\mathbf{r}^{\overset{+}{m}}, \tag{2.1}$$

and also from (1.23), we have

$$\begin{gathered} g^P_{\bar{M}} = \overset{(-)}{\vartheta}{}^{-1}A^P_{\bar{M}}, \quad \overset{(-)}{\vartheta} = \det(g_I^{\bar{J}}), \quad g^3_{\bar{M}} = -g_P^{\bar{3}}g^P_{\bar{M}}, \quad g_P^{\bar{3}} = x^3 g^{\bar{3}}_{\overset{+}{P}}, \\ g^{\bar{3}}_{\overset{+}{P}} = \partial_P \ln h, \quad h = |\mathbf{h}|, \quad A^P_{\bar{M}} \equiv g^{\bar{P}}_{\bar{M}} + x^3 a^P_{\overset{+}{M}}, \quad a^P_{\overset{+}{M}} \equiv (g^{\bar{I}}_{\overset{+}{I}} - 1)g^{\bar{P}}_{\bar{M}} - g^{\bar{P}}_{\overset{+}{M}}. \end{gathered} \tag{2.2}$$

Moreover, it is easy to note that the following relations hold [67, 70, 72]:

$$g^P_{\bar{M}} = \sum_{s=0}^{\infty} \underset{(s)}{A}{}^{\bar{P}}_{\overset{+}{M}}(x^3)^s, \quad \underset{(s)}{A}{}^{\bar{P}}_{\overset{+}{M}} = (g^{\bar{P}}_{\bar{N}_1} - g^{\bar{P}}_{\overset{+}{N}_1}) \cdot (g^{\bar{N}_1}_{\bar{N}_2} - g^{\bar{N}_1}_{\overset{+}{N}_2}) \cdot \ldots \cdot (g^{\bar{N}_{s-1}}_{\bar{M}} - g^{\bar{N}_{s-1}}_{\overset{+}{M}}), \quad \underset{(0)}{A}{}^{\bar{P}}_{\overset{+}{M}} = g^{\bar{P}}_{\bar{M}}. \tag{2.3}$$

By the first and third relations in (2.2) and the second relation in (2.1), we find that

$$\mathbf{r}^P = g^P_{\bar{M}}\mathbf{r}^{\bar{M}}, \quad \mathbf{r}^3 = g^3_{\bar{M}}\mathbf{r}^{\bar{M}} + \mathbf{r}^{\bar{3}} = \mathbf{r}^{\bar{3}} - g_P^{\bar{3}}\mathbf{r}^P = \mathbf{r}^{\bar{3}} - g_P^{\bar{3}}g^P_{\bar{M}}\mathbf{r}^{\bar{M}}. \tag{2.4}$$

The gradient operator can be applied to any tensor. Therefore, denoting a certain tensor quantity by $\mathbb{F}(x', x^3)$, by the definition of the gradient [54, 92, 116] and by (2.4), we have

$$\mathrm{grad}\mathbb{F} = \nabla\mathbb{F} = \mathbf{r}^p\partial_p\mathbb{F} = \mathbf{r}^P\partial_P\mathbb{F} + \mathbf{r}^3\partial_3\mathbb{F} = \mathbf{r}^{\bar{M}}g^P_{\bar{M}}(\partial_P - g_P^{\bar{3}}\partial_3)\mathbb{F} + \mathbf{r}^{\bar{3}}\partial_3\mathbb{F}.$$

Whence, introducing the differential operator

$$N_p = \partial_p - g_p^{\bar{3}}\partial_3, \quad \mathbf{N} = \mathbf{r}^p N_p = \mathbf{r}^P N_P = \mathbf{r}^{\bar{M}}g^P_{\bar{M}}N_P, \quad N_3 = 0, \tag{2.5}$$

we obtain the desired representation of the gradient in the form

$$\mathrm{grad}\mathbb{F} = \nabla\mathbb{F} = \mathbf{N}\mathbb{F} + \mathbf{r}^{\bar{3}}\partial_3\mathbb{F} = \mathbf{r}^P N_P\mathbb{F} + \mathbf{r}^{\bar{3}}\partial_3\mathbb{F} = \mathbf{r}^{\bar{M}}g^P_{\bar{M}}N_P\mathbb{F} + \mathbf{r}^{\bar{3}}\partial_3\mathbb{F}. \tag{2.6}$$

The divergence operator is applied to a tensor whose rank is no less than 1. Applying this operator, e.g. to a second-rank tensor $\underset{\sim}{\mathbf{P}}$, by the definition, the third relation in (2.2), and (2.5), we obtain

$$\mathrm{div}\,\underset{\sim}{\mathbf{P}} = \nabla\cdot\underset{\sim}{\mathbf{P}} = g^P_{\bar{M}}N_P\mathbf{P}^{\bar{M}} + \partial_3\mathbf{P}^{\bar{3}} \quad (\mathbf{P}^{\bar{m}} = \mathbf{r}^{\bar{m}}\cdot\underset{\sim}{\mathbf{P}}). \tag{2.7}$$

Note that (2.7) can also be easily obtained from (2.6) if in this relation, we replace the sign of the tensor product, which is omitted, by the sign of the inner product.

*2.1.1. Representations of equations of motion.* As is known [1, 2, 49, 84, 85], the equations of motion of moment deformable rigid body mechanics in stress tensors and moment stress tensors are represented in the form

$$\nabla \cdot \underset{\sim}{\mathbf{P}} + \rho \mathbf{F} = \rho \partial_t^2 \mathbf{u}, \quad \nabla \cdot \underset{\sim}{\boldsymbol{\mu}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \rho \mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2 \boldsymbol{\varphi}. \tag{2.8}$$

Here, $\underset{\sim}{\mathbf{P}}$ and $\underset{\sim}{\boldsymbol{\mu}}$ are tensors of true stresses and moment stresses, $\underset{\approx}{\mathbf{C}}$ is the discriminant tensor (third-rank tensor) [116], $\mathbf{u}$ is the vector of displacements, $\boldsymbol{\varphi}$ is the vector of (inner) rotation, $\rho$ is the material density, $\mathbf{F}$ is the mass force density, $\mathbf{m}$ is the mass moment density, and $\overset{2}{\otimes}$ is the inner 2-product (for example, $\underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} = \mathbf{r}^i C_{ijk} P^{jk}$). The definition of inner $r$-product and the problems related to it are considered in [69, 77, 78, 116]. Proceeding analogously to [117], for the equations of the classical deformable rigid body mechanics (DRBM) under the classical parametrization of the thin body domain, in the case considered, from (2.8), we find the following form of representation of equations of moment DRMB:

$$\begin{gathered}
\big(1/\sqrt{\overset{(-)}{g}}\big)\partial_P\big(\sqrt{\overset{(-)}{g}}\,\overset{(-)}{\vartheta}\,\mathbf{P}^P\big) + \partial_3\big(\overset{(-)}{\vartheta}\,\mathbf{P}^3\big) + \rho\,\overset{(-)}{\vartheta}\,\mathbf{F} = \rho\,\overset{(-)}{\vartheta}\,\partial_t^2 \mathbf{u}, \\
\big(1/\sqrt{\overset{(-)}{g}}\big)\partial_P\big(\sqrt{\overset{(-)}{g}}\,\overset{(-)}{\vartheta}\,\boldsymbol{\mu}^P\big) + \partial_3\big(\overset{(-)}{\vartheta}\,\boldsymbol{\mu}^3\big) + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \big(\overset{(-)}{\vartheta}\,\underset{\sim}{\mathbf{P}}\big) + \rho\,\overset{(-)}{\vartheta}\,\mathbf{m} = \overset{(-)}{\vartheta}\,\underset{\sim}{\mathbf{J}} \cdot \partial_t^2 \boldsymbol{\varphi}, \\
\overset{(-)}{g} = \det\big(g_{\bar{m}\bar{n}}\big), \quad g_{\bar{m}\bar{n}} = \mathbf{r}_{\bar{m}} \cdot \mathbf{r}_{\bar{n}}.
\end{gathered} \tag{2.9}$$

It is easy to see that by (2.7), Eqs. (2.8) can be rewritten in the form

$$g^P_{\bar{M}} N_P \mathbf{P}^{\bar{M}} + \partial_3 \mathbf{P}^{\bar{3}} + \rho \mathbf{F} = \rho \partial_t^2 \mathbf{u}, \quad g^P_{\bar{M}} N_P \boldsymbol{\mu}^{\bar{M}} + \partial_3 \boldsymbol{\mu}^{\bar{3}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \rho \mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2 \boldsymbol{\varphi}. \tag{2.10}$$

It is easy to note that the following relations hold:

$$g^P_{\bar{M}} N_P \mathbf{P}^{\bar{M}} = g^P_{\bar{m}} N_P \mathbf{P}^{\bar{m}} = N_P\big(g^P_{\bar{m}} \mathbf{P}^{\bar{m}}\big) = N_P\big(g^P_{\bar{M}} \mathbf{P}^{\bar{M}}\big) = N_P \mathbf{P}^P;$$

using them, we can represent Eqs. (2.10) in the form

$$\begin{gathered}
N_P \mathbf{P}^P + \partial_3 \mathbf{P}^{\bar{3}} + \rho \mathbf{F} = \rho \partial_t^2 \mathbf{u}, \quad N_P \boldsymbol{\mu}^P + \partial_3 \boldsymbol{\mu}^{\bar{3}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \rho \mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2 \boldsymbol{\varphi}, \\
N_P\big(g^P_{\bar{M}} \mathbf{P}^{\bar{M}}\big) + \partial_3 \mathbf{P}^{\bar{3}} + \rho \mathbf{F} = \rho \partial_t^2 \mathbf{u}, \quad N_P\big(g^P_{\bar{M}} \boldsymbol{\mu}^{\bar{M}}\big) + \partial_3 \boldsymbol{\mu}^{\bar{3}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \rho \mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2 \boldsymbol{\varphi}.
\end{gathered} \tag{2.11}$$

Multiplying each relation in (2.10) by $\overset{(-)}{\vartheta}$ and taking into account the first relation in (2.2), we have

$$\begin{gathered}
A^P_{\bar{M}} N_P \mathbf{P}^{\bar{M}} + \overset{(-)}{\vartheta}\,\partial_3 \mathbf{P}^{\bar{3}} + \rho\,\overset{(-)}{\vartheta}\,\mathbf{F} = \rho\,\overset{(-)}{\vartheta}\,\partial_t^2 \mathbf{u}, \\
A^P_{\bar{M}} N_P \boldsymbol{\mu}^{\bar{M}} + \overset{(-)}{\vartheta}\,\partial_3 \boldsymbol{\mu}^{\bar{3}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \big(\overset{(-)}{\vartheta}\,\underset{\sim}{\mathbf{P}}\big) + \rho\,\overset{(-)}{\vartheta}\,\mathbf{m} = \overset{(-)}{\vartheta}\,\underset{\sim}{\mathbf{J}} \cdot \partial_t^2 \boldsymbol{\varphi}.
\end{gathered} \tag{2.12}$$

Note that (2.9) – (2.12) are different forms of representation of the equations of moment DRBM (2.8) for the parametrization of the thin body domain considered. They are called the different forms of equations of moment deformable rigid thin body mechanic (DRTBM) under the new parametrization of thin body domain. Taking into account the first relation in (2.3), we can write Eqs. (2.10) in the form

$$\begin{aligned}
&\sum_{s=0}^{\infty} A^{\bar{P}}_{\overset{+}{M}}(x^3)^s N_P \mathbf{P}^{\bar{M}} + \partial_3 \mathbf{P}^{\bar{3}} + \rho \mathbf{F} = \rho \partial_t^2 \mathbf{u}, \\
&\sum_{s=0}^{\infty} A^{\bar{P}}_{\overset{+}{M}}(x^3)^s N_P \boldsymbol{\mu}^{\bar{M}} + \partial_3 \boldsymbol{\mu}^{\bar{3}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \rho \mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2 \boldsymbol{\varphi}.
\end{aligned} \tag{2.13}$$

It is seen that Eqs. (2.13) contain infinitely many summands. Therefore, they cannot be used in practice. Naturally, we need to consider approximate equations with finitely many summands. In this connection, let us introduce the following definition.

**Definition 2.1.** The equations, which are obtained from (2.10) if, in the expansion of $g^{P}_{\overset{-}{M}}$ (see the first formula in (2.3)), we preserve the first $s+1$ terms, are called the equations of approximation of order $s$.

Obviously, the equation of approximation of order $s$ are represented in the form

$$\underset{(s)}{g}{}^{P}_{\overset{-}{M}} N_P \mathbf{P}^{\bar{M}} + \partial_3 \mathbf{P}^{\bar{3}} + \rho \mathbf{F} = \rho \partial_t^2 \mathbf{u}, \quad \underset{(s)}{g}{}^{P}_{\overset{-}{M}} N_P \boldsymbol{\mu}^{\bar{M}} + \partial_3 \boldsymbol{\mu}^{\bar{3}} + \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \rho \mathbf{m} = \underset{\sim}{\mathbf{J}} \cdot \partial_t^2 \boldsymbol{\varphi}, \tag{2.14}$$

where

$$\underset{(s)}{g}{}^{P}_{\overset{-}{M}} = \sum_{m=0}^{s} A^{\bar{P}}_{\overset{+}{M}}(x^3)^m, \tag{2.15}$$

From (2.14), for $s = 0$, we obtain the equations of zero approximation, for $s = 1$, the equations of first approximation, etc.

*2.1.2. Representation of the equation of heat influx in moment DRTBM.* In the general case, the heat influx equation in moment DRBM can be represented in the form [95]

$$-\nabla \cdot \mathbf{q} + \rho q - T \frac{d}{dt}(\underset{\sim}{\mathbf{a}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \underset{\sim}{\mathbf{d}} \overset{2}{\otimes} \underset{\sim}{\boldsymbol{\mu}}) + W^* = \rho c_p \partial_t T, \tag{2.16}$$

where $\mathbf{q}$ is the vector of exterior heat influx, $q$ is the mass heat influx, $T$ is the temperature, $\underset{\sim}{\mathbf{a}}$, $\underset{\sim}{\mathbf{d}}$ are the tensors of heat extension, $\underset{\sim}{\mathbf{P}} \neq \underset{\sim}{\mathbf{P}}^T$ is the stress tensor, $\underset{\sim}{\boldsymbol{\mu}} \neq \underset{\sim}{\boldsymbol{\mu}}^T$ is the moment stress tensor, $W^*$ is the scattering function, $\rho$ is the medium density, and $c_p$ is the heat capacity under constant pressure. If we consider the physically linear medium, then the nonlinearity in (2.16) is in the third summand of the left-hand side. A similar situation holds in a particular variant of this equation, which is obtained from (2.16) for $\underset{\sim}{\mathbf{d}} = 0$ (see [93]). In the latter case, since both heat capacities $c_p$ and $c_v$ (heat capacity under constant volume) cannot be constant simultaneously (independent of the temperature), very often one assumes that in this summand the temperature $T$ is replaced by the temperature $T_0 = \text{const}$. Taking into account this assumption, we see that the desired representation of the heat influx equation has the following form similar to (2.10):

$$-g^{P}_{\overset{-}{M}} N_P q^{\bar{M}} - \partial_3 q^{\bar{3}} + \rho q - T_0 \frac{d}{dt}(\underset{\sim}{\mathbf{a}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \underset{\sim}{\mathbf{d}} \overset{2}{\otimes} \underset{\sim}{\boldsymbol{\mu}}) + W^* = \rho c_p \partial_t T. \tag{2.17}$$

If necessary, it is easy to write the relations similar to (2.9) and (2.12). Therefore, for brevity, we do not dwell on this. Note that by (2.17), analogously to (2.14), the heat influx equation of approximation of order $s$ is represented in the form

$$-\underset{(s)}{g}{}^{P}_{\overset{-}{M}} N_P q^{\bar{M}} - \partial_3 q^{\bar{3}} + \rho q - T_0 \frac{d}{dt}(\underset{\sim}{\mathbf{a}} \overset{2}{\otimes} \underset{\sim}{\mathbf{P}} + \underset{\sim}{\mathbf{d}} \overset{2}{\otimes} \underset{\sim}{\boldsymbol{\mu}}) + W^* = \rho c_p \partial_t T. \tag{2.18}$$

*2.1.3. Representations of constitutive relations of physical and heat content.* In linear moment elasticity theory, the constitutive relations of physical content under nonisothermal processes can be represented in the following form by the generalized Duhamel–Neumann principle [93, 95]:

$$\underset{\sim}{\mathbf{P}} = \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} (\underset{\sim}{\boldsymbol{\gamma}} - \underset{\sim}{\mathbf{a}}\vartheta) + \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} (\underset{\sim}{\boldsymbol{\varkappa}} - \underset{\sim}{\mathbf{d}}\vartheta), \quad \underset{\sim}{\boldsymbol{\mu}} = \underset{\approx}{\mathbf{D}} \overset{2}{\otimes} (\underset{\sim}{\boldsymbol{\varkappa}} - \underset{\sim}{\mathbf{d}}\vartheta) + \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} (\underset{\sim}{\boldsymbol{\gamma}} - \underset{\sim}{\mathbf{a}}\vartheta), \tag{2.19}$$

where $\underset{\sim}{\boldsymbol{\gamma}} = \nabla \mathbf{u} - \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi}$ is the deformation tensor in moment theory (see [49]), $\underset{\sim}{\boldsymbol{\varkappa}} = \nabla \boldsymbol{\varphi}$ is the bend-torsion tensor, $\underset{\approx}{\mathbf{C}}$, $\underset{\approx}{\mathbf{A}}$, $\underset{\approx}{\mathbf{D}}$, $\underset{\approx}{\mathbf{B}}$ are material tensors of the fourth rank, and $\vartheta$ is the temperature drop. Taking into account the expression for $\underset{\sim}{\boldsymbol{\gamma}}$, we can write (2.19) in the form

$$\underset{\sim}{\mathbf{P}} = \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \nabla \mathbf{u} + \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} \nabla \boldsymbol{\varphi} - \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\mathbf{b}}\vartheta, \quad \underset{\sim}{\boldsymbol{\mu}} = \underset{\approx}{\mathbf{D}} \overset{2}{\otimes} \nabla \boldsymbol{\varphi} + \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} \nabla \mathbf{u} - \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\boldsymbol{\beta}}\vartheta, \tag{2.20}$$

where

$$\underset{\sim}{\mathbf{b}} = \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\sim}{\mathbf{a}} + \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} \underset{\sim}{\mathbf{d}}, \quad \underset{\sim}{\boldsymbol{\beta}} = \underset{\approx}{\mathbf{D}} \overset{2}{\otimes} \underset{\sim}{\mathbf{d}} + \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} \underset{\sim}{\mathbf{a}},$$

which are called the tensors of thermomechanical properties. Note that a particular case of law (2.20) was considered in [49, 84], and more general relations were presented in [94, 95]. Now it is easy to find the desired representations of the Hooke's law (2.20) under the new parametrization of thin body domain. Indeed, taking into account the representation of the gradient operator (2.6), after simple transformations, from (2.20), we have

$$\begin{aligned} \underset{\sim}{\mathbf{P}} &= \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} (g^{P}_{\bar M} \mathbf{r}^{\bar M} N_P \mathbf{u} + \mathbf{r}^{\bar 3} \partial_3 \mathbf{u}) + \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} (g^{P}_{\bar M} \mathbf{r}^{\bar M} N_P \boldsymbol{\varphi} + \mathbf{r}^{\bar 3} \partial_3 \boldsymbol{\varphi}) - \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\mathbf{b}}\vartheta, \\ \underset{\sim}{\boldsymbol{\mu}} &= \underset{\approx}{\mathbf{D}} \overset{2}{\otimes} (g^{P}_{\bar M} \mathbf{r}^{\bar M} N_P \boldsymbol{\varphi} + \mathbf{r}^{\bar 3} \partial_3 \boldsymbol{\varphi}) + \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} (g^{P}_{\bar M} \mathbf{r}^{\bar M} N_P \mathbf{u} + \mathbf{r}^{\bar 3} \partial_3 \mathbf{u}) - \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\boldsymbol{\beta}}\vartheta. \end{aligned} \tag{2.21}$$

Taking into account the first relation in (2.3), it is easy to note that relations (2.21) contain infinitely many summands. Therefore, they cannot be used in such a form. In applications, one uses approximate constitutive relations (CR), i.e., relations represented by finitely many summands. In this connection, we introduce the following definition.

**Definition 2.2.** The relations obtained from (2.21) under the condition that in the expansion of $g^{P}_{\bar M}$ (see the first formula in (2.3)), the first $s+1$ terms are preserved are called the CR of approximation of order $s$.

It is easy to see that analogously to Eqs. (2.14) and (2.18), CR of approximation of order $s$ are represented in the form

$$\begin{aligned} \underset{\sim}{\mathbf{P}}_{(s)} &= \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} (\underset{(s)}{g}{}^{P}_{\bar M} \mathbf{r}^{\bar M} N_P \mathbf{u} + \mathbf{r}^{\bar 3} \partial_3 \mathbf{u}) + \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} (\underset{(s)}{g}{}^{P}_{\bar M} \mathbf{r}^{\bar M} N_P \boldsymbol{\varphi} + \mathbf{r}^{\bar 3} \partial_3 \boldsymbol{\varphi}) - \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\mathbf{b}}\vartheta, \\ \underset{\sim}{\boldsymbol{\mu}}_{(s)} &= \underset{\approx}{\mathbf{D}} \overset{2}{\otimes} (\underset{(s)}{g}{}^{P}_{\bar M} \mathbf{r}^{\bar M} N_P \boldsymbol{\varphi} + \mathbf{r}^{\bar 3} \partial_3 \boldsymbol{\varphi}) + \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} (\underset{(s)}{g}{}^{P}_{\bar M} \mathbf{r}^{\bar M} N_P \mathbf{u} + \mathbf{r}^{\bar 3} \partial_3 \mathbf{u}) - \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\boldsymbol{\beta}}\vartheta. \end{aligned} \tag{2.22}$$

**Definition 2.3.** The relations obtained from (2.22) for $s = 0$ are called CR of zero approximation, and for $s = 1$, they are called CR of the first approximation.

It is easy to see that CR of zero approximation have the form

$$\underset{\sim}{\mathbf{P}}_{(0)} = \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} (\mathbf{r}^{\bar M} N_P \mathbf{u} + \mathbf{r}^{\bar 3} \partial_3 \mathbf{u}) + \underset{\approx}{\mathbf{A}} \overset{2}{\otimes} (\mathbf{r}^{\bar M} N_P \boldsymbol{\varphi} + \mathbf{r}^{\bar 3} \partial_3 \boldsymbol{\varphi}) - \underset{\approx}{\mathbf{C}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\mathbf{b}}\vartheta,$$

$$\underset{\sim}{\boldsymbol{\mu}}_{(0)} = \underset{\approx}{\mathbf{D}} \overset{2}{\otimes} (\mathbf{r}^{\bar M} N_P \boldsymbol{\varphi} + \mathbf{r}^{\bar 3} \partial_3 \boldsymbol{\varphi}) + \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} (\mathbf{r}^{\bar M} N_P \mathbf{u} + \mathbf{r}^{\bar 3} \partial_3 \mathbf{u}) - \underset{\approx}{\mathbf{B}} \overset{2}{\otimes} \underset{\simeq}{\mathbf{C}} \cdot \boldsymbol{\varphi} - \underset{\sim}{\boldsymbol{\beta}}\vartheta.$$

Note that if we consider a body with center of symmetry [49, 84], then $\underset{\approx}{\mathbf{A}} = 0$, $\underset{\approx}{\mathbf{B}} = 0$, and in this case, the CR presented above simplify. Let us find the corresponding representation for the Fourier heat conduction law (which defines the relations of heat content) under the new parametrization of the thin body domain. Since the Fourier heat conduction law [84, 93] has the form $\mathbf{q}=-\underset{\sim}{\mathbf{\Lambda}}\cdot\nabla T$, where the second-rank positive-definite tensor $\underset{\sim}{\mathbf{\Lambda}}$ is called the heat conduction tensor, by (2.6), the Fourier heat conduction law of zero approximation and approximation of order $s$ is represented in the form

$$\mathbf{q}_{(0)} = -\mathbf{\Lambda}^{\bar{M}} N_{P} T - \mathbf{\Lambda}^{\bar{3}} \partial_3 T, \quad \mathbf{q}_{(s)} = -\mathbf{\Lambda}^{\bar{M}} \underset{(s)M}{\overset{P}{\bar{g}}} N_{P} T - \mathbf{\Lambda}^{\bar{3}} \partial_3 T, \quad \mathbf{\Lambda}^{\bar{m}} = \underset{\sim}{\mathbf{\Lambda}} \cdot \mathbf{r}^{\bar{m}}. \tag{2.23}$$

## 3. Moment Theory with Respect to System of Orthonormal Chebyshev Polynomials of Second Kind

To construct the moment theory with respect to a certain system of orthogonal polynomials (Legendre, Chebyshev, etc.), we need recursive relations for these polynomials. For example, for the shifted Chebyshev polynomials, the main recursive relations on the orthogonality closed interval [0,1] are represented in the following form [67, 72]:

$$\begin{aligned} 4tU_n^*(t) &= U_{n-1}^*(t) + 2U_n^*(t) + U_{n+1}^*(t), && n \geq 1, \\ 2tU_n^{*'}(t) &= 2nU_n^*(t) + U_{n-1}^{*'}(t) + U_n^{*'}(t), && n \geq 1, \\ U_n^{*'}(t) &= 4nU_{n-1}^*(t) + U_{n-2}^{*'}(t), && n \geq 2, \quad 0 \leq t \leq 1. \end{aligned} \tag{3.1}$$

Note that formulas (3.1) are obtained in the same way as analogous formulas for Legendre polynomials on the orthogonality closed interval $[-1, 1]$, e.g, in [105] (see also [65]). Using the main recursive relations (3.1), it is easy to obtain the following relations, which are necessary for constructing thin body theories [67, 72]:

$$2^{2s} t^s U_n^*(t) = \sum_{p=0}^{2s} C_{2s}^p U_{n-s+p}^*(t), \quad s, n \in \mathbb{N}_0; \tag{3.2}$$

$$2^{2s} t^s U_m^*(t) U_n^*(t) = \sum_{p=0}^{m} \sum_{q=0}^{2s} C_{2s}^q U_{n-m-s+2p+q}^*(t), \quad n, m, s \in \mathbb{N}_0; \tag{3.3}$$

$$U_n^{*'}(t) = 4 \sum_{k=0}^{[(n-1)/2]} (n-2k) U_{n-(2k+1)}^*(t) = 4 \sum_{k=0}^{[(n-1)/2]} (2k+1+a)\, U_{2k+a}^*(t), \quad n \geq 1; \tag{3.4}$$

$$\begin{aligned} 2^{2s} t^s U_n^{*'}(t) &= 4 \sum_{k=0}^{[(n-1)/2]} \sum_{p=0}^{2s} (n-2k) C_{2s}^p U_{n-(s+2k+1)+p}^*(t) = \\ &= 4 \sum_{k=0}^{[(n-1)/2]} (2k+1+a) U_{2k+a-s+p}^*, \quad n \geq 1, \ s \geq 0; \end{aligned} \tag{3.5}$$

$$\begin{aligned} U_n^{*''}(t) &= 2^4 \sum_{k=0}^{[(n-2)/2]} (k+1)(n-k)[n-(2k+1)] U_{n-(2k+2)}^*(t) = \\ &= 2^2 \sum_{k=0}^{[(n-2)/2]} (2k+2-a)[(n+1)^2 - (2k+2-a)^2] U_{2k+1-a}^*(t), \quad n \geq 2; \end{aligned} \tag{3.6}$$

$$2^{2s}t^s U_n^{*''}(t) = 2^4 \sum_{k=0}^{[(n-2)/2]} \sum_{p=0}^{2s} (k+1)(n-k)[n-(2k+1)] C_{2s}^p U_{n-(s+2k+2)+p}^* =$$

$$= 2^2 \sum_{k=0}^{[(n-2)/2]} (2k+2-a)[(n+1)^2-(2k+2-a)^2] C_{2s}^p U_{2k+1-a-s+p}^*, \quad n \geq 2, \ s \geq 0. \quad (3.7)$$

Here, $a = n-1-2\big[(n-1)/2\big]$, $[x]$ is the integral part of $x$, and $C_m^n$ are the binomial coefficients. Note that all relations (3.2)–(3.7), which also hold for the system of orthonormal Chebyshev polynomials of the second kind $\{\hat{U}_k^*\}_{k=0}^{\infty}$, except for (3.3), can be proved by induction. For a system of orthonormal polynomials, (3.3) is represented in the form

$$2^{2s}t^s \hat{U}_m^*(t)\hat{U}_n^*(t) = \hat{U}_0^* \sum_{p=0}^{m} \sum_{q=0}^{2s} C_{2s}^q \hat{U}_{n-m-s+2p+q}^*(t), \quad n, m, s \in \mathbb{N}_0. \quad (3.8)$$

Note that extending the definition of the system of Chebyshev polynomials of the second kind to the set of negative numbers, we obtain the relation $U_{-n}^* = -U_{n-2}^*$, $n \in \mathbb{N}_0$, under which (3.2)–(3.7) were obtained.

Consider a certain tensor field $\mathbb{F}(x^1, x^2, x^3)$, which depends on the coordinates $x^1, x^2, x^3$ of the thin body domain under its new parametrization. For brevity, instead of $\mathbb{F}(x^1, x^2, x^3)$, we write $\mathbb{F}(x', x^3)$, where $x' = (x^1, x^2)$, $x^3 \in [0,1]$. Moreover, we assume that the tensor fields considered are sufficiently smooth. For example, $\mathbb{F}(x', x^3) \in C_m(V \cup \partial V)$, $m \geq 1$; $V$ is the domain occupied by the thin body considered and $\partial V$ is its boundary. Then the tensor field $\mathbb{F}(x', x^3)$ can be expanded in a series with respect to the system of shifted Chebyshev polynomials of the second kind $\{\hat{U}_k^*\}_{k=0}^{\infty}$ with respect to the coordinate $x^3 \in [0,1]$ for each fixed point $x' \in \overset{(-)}{S}$ (of the inner base surface) [105]. This expansion is represented in the form

$$\mathbb{F}(x', x^3) = \sum_{k=0}^{\infty} \overset{(k)}{\mathbb{F}}(x')\hat{U}_k^*(x^3), \quad x' \in \overset{(-)}{S}, \quad x^3 \in [0,1], \quad (3.9)$$

where $\overset{(k)}{\mathbb{F}}(x')$ is called the coefficient with number $k$ in the expansion of $\mathbb{F}(x', x^3)$ in the series with respect to the polynomial system $\{\hat{U}_k^*\}_{k=0}^{\infty}$.

**Definition 3.1.** The moment of the $k$th order of a certain tensor field $\mathbb{F}(x', x^3)$ with respect to the polynomial system $\{\hat{U}_k^*\}_{k=0}^{\infty}$, which is denoted by $\overset{(k)}{\mathbb{M}}(\mathbb{F})$, is the integral

$$\overset{(k)}{\mathbb{M}}(\mathbb{F}) = \int_0^1 \mathbb{F}(x', x^3)\hat{U}_k^*(x^3)h^*(x^3)dx^3, \quad k \in \mathbb{N}_0. \quad (3.10)$$

It is easy to prove that the following assertions hold.

**Assertion 3.1.** *For any tensor fields $\mathbb{F}(x', x^3)$ and $\mathbb{G}(x', x^3)$ and any functions $\alpha(x')$ and $\beta(x')$, the following relation holds*:

$$\overset{(k)}{\mathbb{M}}[\alpha(x')\mathbb{F} + \beta(x')\mathbb{G}] = \alpha(x')\overset{(k)}{\mathbb{M}}(\mathbb{F}) + \beta(x')\overset{(k)}{\mathbb{M}}(\mathbb{G}), \quad k \in \mathbb{N}_0. \quad (3.11)$$

This implies that the moment operator is a linear operator.

**Assertion 3.2.** *The $k$th-order moment of a tensor field $\mathbb{F}(x', x^3)$ with respect to the polynomial system $\{\hat{U}^*_k\}_{k=0}^\infty$ is equal to the coefficient with number $k$ in the expansion of $\mathbb{F}(x', x^3)$ with respect to $x^3$ in this polynomial system, i.e.,*

$$\overset{(k)}{\mathbb{M}}(\mathbb{F}) = \int\limits_0^1 \mathbb{F}(x', x^3)\hat{U}^*_k(x^3)h^*(x^3)dx^3 = \overset{(k)}{\mathbb{F}}(x'), \quad k \in \mathbb{N}_0. \tag{3.12}$$

Relation (3.11) follows from definition (3.10), whereas relation (3.12) is proved by using (3.9) and (3.10) and taking into account the fact that the system $\{\hat{U}^*_k\}_{k=0}^\infty$ is orthonormal. It is easy to prove that the following relations hold:

$$\overset{(k)}{\mathbb{M}}(\partial_i\mathbb{F}) = \begin{cases} \partial_I \overset{(k)}{\mathbb{F}}(x'), & i = I, \\ \overset{(k)}{\mathbb{F}}{}'(x'), & i = 3, \end{cases} \qquad \overset{(k)}{\mathbb{M}}(\partial_i\partial_j\mathbb{F}) = \begin{cases} \partial_I\partial_J \overset{(k)}{\mathbb{F}}(x'), & i = I,\ j = J, \\ \partial_I \overset{(k)}{\mathbb{F}}{}'(x'), & i = I,\ j = 3, \\ \overset{(k)}{\mathbb{F}}{}''(x'), & i = j = 3, \end{cases} \tag{3.13}$$

where we have introduced the following notation:

$$\begin{aligned} \overset{(k)}{\mathbb{F}}{}'(x') &= 2^2(k+1)\sum_{p=0}^\infty \overset{(k+2p+1)}{\mathbb{F}}(x') = 2(k+1)\sum_{p=k}^\infty \left[1-(-1)^{k+p}\right]\overset{(p)}{\mathbb{F}}(x') \\ &= 2(k+1)\left[\sum_{p=k}^N \left(1-(-1)^{k+p}\right)\overset{(p)}{\mathbb{F}}(x') + \overset{(+)}{\mathbb{F}}{}'(x') - (-1)^k \overset{(-)}{\mathbb{F}}{}'(x')\right], \\ \overset{(k)}{\mathbb{F}}{}''(x') &= \big(\overset{(k)}{\mathbb{F}}{}'\big)' = 2^4(k+1)\sum_{p=0}^\infty (p+1)(k+p+2)\overset{(k+2p+2)}{\mathbb{F}}(x') \\ &= 2(k+1)\left[\sum_{p=k}^N (p-k)(k+p+2)\left(1+(-1)^{k+p}\right)\overset{(p)}{\mathbb{F}}(x') + \overset{(+)}{\mathbb{F}}{}''(x') + (-1)^k \overset{(-)}{\mathbb{F}}{}''(x')\right], \end{aligned} \tag{3.14}$$

$$\begin{aligned} \overset{(+)}{\mathbb{F}}{}'(x') &= \sum_{p=N+1}^\infty \overset{(p)}{\mathbb{F}}(x'), & \overset{(+)}{\mathbb{F}}{}''(x') &= \sum_{p=N+1}^\infty (p-k)(k+p+2)\overset{(p)}{\mathbb{F}}(x'), \\ \overset{(-)}{\mathbb{F}}{}'(x') &= \sum_{p=N+1}^\infty (-1)^p \overset{(p)}{\mathbb{F}}(x'), & \overset{(-)}{\mathbb{F}}{}''(x') &= \sum_{p=N+1}^\infty (-1)^p (p-k)(k+p+2)\overset{(p)}{\mathbb{F}}(x'). \end{aligned} \tag{3.15}$$

Note that the first relation in (3.14) can be taken as the definition of the "prime" operator, and the second can be obtained by applying the "prime" operator to $\overset{(k)}{\mathbb{F}}$ two times. The following relations are generalizations of (3.13):

$$\overset{(k)}{\mathbb{M}}[P_N(x^3)\partial_i^p\partial_j^q\mathbb{F}] = \left[\begin{array}{ll} \partial_I^p\partial_J^q \overset{(k)}{\mathbb{M}}\big[P_N(x^3)\mathbb{F}\big], & i = I,\ j = J, \\ \partial_I \big\{\overset{(k)}{\mathbb{M}}\big[P_N(x^3)\mathbb{F}\big]\big\}^{(q)}, & i = I,\ j = 3, \\ \big\{\overset{(k)}{\mathbb{M}}\big[P_N(x^3)\mathbb{F}\big]\big\}^{(p+q)}, & i = j = 3, \end{array}\right. \tag{3.16}$$

where $P_N(x^3)$ is a polynomial of degree $N$, $k$, $N$, $p$, $q \in \mathbb{N}_0$, and $\big\{\overset{(k)}{\mathbb{M}}\big[P_N(x^3)\mathbb{F}\big]\big\}^{(m)}$, $m \in \mathbb{N}_0$, means that the "prime" operator is applied $m$ times. To prove the first lines of (3.13) and (3.16), we use definition (3.10). The second and third lines of (3.13) are proved by using (3.4) and (3.6), respectively, and the second and third lines of (3.16) are proved by induction. Using (3.2) and the last relation in (3.16), we can prove the relations

$$
\begin{aligned}
&\overset{(n)}{\mathbb{M}}\big[(x^3)^s\partial_3^m\mathbb{F}\big]=\sum_{p=0}^{2s}2^{-2s}C_{2s}^{p}\overset{(n-s+p)}{\mathbb{F}}{}^{(m)}(x'), && n-s\geq 0,\ s,\ m\in\mathbb{N}_0,\\
&\overset{(n)}{\mathbb{M}}\big[(x^3)^s\partial_3^m\mathbb{F}\big]=\sum_{p=1}^{2n+2}2^{-2(n+1)}C_{2n+2}^{p}\overset{(p-1)}{\mathbb{F}}{}^{(m)}(x'), && s=n+1,\ n,m\in\mathbb{N}_0,\\
&\overset{(n)}{\mathbb{M}}\big[(x^3)^s\partial_3^m\mathbb{F}\big]=-\sum_{q=2}^{s-n}2^{-2s}C_{2s}^{q-2}\overset{(s-n-q)}{\mathbb{F}}{}^{(m)}(x')\\
&\qquad+\sum_{p=s-n}^{2s}2^{-2s}C_{2s}^{p}\overset{(n-s+p)}{\mathbb{F}}{}^{(m)}(x'), && s\geq n+2,\ n,m\in\mathbb{N}_0.
\end{aligned}
\tag{3.17}
$$

Let us represent (3.17) for $m=1$ in another form. Using easy transformations, the first relation in (3.14), and the first two relations in (3.15), from (3.17), for $m=1$, we obtain

$$
\begin{aligned}
&\overset{(k)}{\mathbb{M}}\big[(x^3)^{s+1}\partial_3\mathbb{F}\big]=\overset{(k)}{\mathbb{M}}{}'\big[(x^3)^{s+1}\mathbb{F}\big]=\sum_{p=0}^{2s+2}\sum_{q=l-1}^{N}2^{-(2s+1)}C_{2s+2}^{p}l\big[1+(-1)^{l+q}\big]\overset{(q)}{\mathbb{F}}(x')+\\
&\qquad+(2k-s+1)\overset{(+)}{\mathbb{F}}{}',\quad l\equiv k-s+p,\quad k\geq s+1,\quad N\geq k+s+1,\quad s\geq 0,\\
&\overset{(k)}{\mathbb{M}}\big[(x^3)^{k+1}\partial_3\mathbb{F}\big]=\overset{(k)}{\mathbb{M}}{}'\big[(x^3)^{k+1}\mathbb{F}\big]=\sum_{p=1}^{2k+2}\sum_{q=p-1}^{N}2^{-(2k+1)}C_{2k+2}^{p}p\big[1+(-1)^{p+q}\big]\overset{(q)}{\mathbb{F}}(x')+\\
&\qquad+(k+1)\overset{(+)}{\mathbb{F}}{}',\quad N\geq 2k+1,\quad k\geq 0,\\
&\overset{(k)}{\mathbb{M}}\big[(x^3)^{s+1}\partial_3\mathbb{F}\big]=\overset{(k)}{\mathbb{M}}{}'\big[(x^3)^{s+1}\mathbb{F}\big]=-\sum_{p=0}^{s-k-1}\sum_{q=p}^{N}2^{-(2s+1)}C_{2s+2}^{s-k-1-p}(p+1)\big[1-(-1)^{p+q}\big]\overset{(q)}{\mathbb{F}}+\\
&\qquad+\sum_{p=s+1-k}^{s+k+1}\sum_{q=p}^{N}2^{-(2s+1)}C_{2s+2}^{s+1-k+p}(p+1)\big[1-(-1)^{p+q}\big]\overset{(q)}{\mathbb{F}}+a_{(s,k)}\overset{(-)}{\mathbb{F}}{}'+b_{(s,k)}\overset{(+)}{\mathbb{F}}{}',\\
&\qquad s\geq k+1,\quad N\geq s+k+1,\quad k\geq 0,
\end{aligned}
\tag{3.18}
$$

where we have introduced the notation

$$
\begin{aligned}
a_{(s,k)}&=2^{-(2s+1)}\left[-\sum_{p=0}^{s-k-1}C_{2s+2}^{s-k-1-p}(p+1)(-1)^{p+1}+\sum_{p=s+1-k}^{s+k+1}C_{2s+2}^{s+1-k+p}(p+1)(-1)^{p+1}\right],\\
b_{(s,k)}&=2^{-(2s+1)}\left[-\sum_{p=0}^{s-k-1}C_{2s+2}^{s-k-1-p}(p+1)+\sum_{p=s+1-k}^{s+k+1}C_{2s+2}^{s+1-k+p}(p+1)\right],\quad s\geq k+1.
\end{aligned}
$$

Let us find the expression for $\overset{(k)}{\mathbb{M}}(\underset{(s)M}{g}{}^{P}_{-}\mathrm{N}_P\mathbb{F})$. By (2.5) and (3.11), we obtain

$$
\overset{(k)}{\mathbb{M}}(\underset{(s)M}{g}{}^{P}_{-}\mathrm{N}_P\mathbb{F})=\overset{(k)}{\mathbb{M}}(\underset{(s)M}{g}{}^{P}_{-}\partial_P\mathbb{F})-\underset{+}{\overset{-}{g}}{}^{3}_{P}\overset{(k)}{\mathbb{M}}(x^3\underset{(s)M}{g}{}^{P}_{-}\partial_3\mathbb{F}).
\tag{3.19}
$$

Furthermore, by (2.15), (3.11), (3.16), and (3.17), for $m = 1$, we find that

$$
\begin{aligned}
&\overset{(k)}{\mathbb{M}}\big(\underset{(s)M}{g^{P}_{-}}\partial_P\mathbb{F}\big) = \sum_{m=0}^{s}\underset{(m)}{A}{}^{\bar{P}}_{\underset{M}{+}}\partial_P\overset{(k)}{\mathbb{M}}[(x^3)^m\mathbb{F}] = \sum_{m=0}^{k+1}\sum_{p=0}^{2m}\underset{(m)}{A}{}^{\bar{P}}_{\underset{M}{+}}2^{-2m}C^{p}_{2m}\partial_P\overset{(k-m+p)}{\mathbb{F}} \\
&\quad+\sum_{m=k+2}^{s}\underset{(m)}{A}{}^{\bar{P}}_{\underset{M}{+}}\Big(-\sum_{p=2}^{m-k}2^{-2m}C^{q-2}_{2m}\partial_P\overset{(m-k-q)}{\mathbb{F}}+\sum_{p=m-k}^{2m}2^{-2m}C^{p}_{2m}\partial_P\overset{(k-m+p)}{\mathbb{F}}\Big), \quad k\ge 0, \quad s\ge 0. \quad (3.20)
\end{aligned}
$$

Whence, for $s = 0$ and $s = 1$, we obtain

$$
\begin{aligned}
&\overset{(k)}{\mathbb{M}}\big(\underset{(0)M}{g^{P}_{-}}\partial_P\mathbb{F}\big) = \overset{(k)}{\mathbb{M}}(\partial_M\mathbb{F}), \quad k\ge 0, \\
&\overset{(k)}{\mathbb{M}}\big(\underset{(1)M}{g^{P}_{-}}\partial_P\mathbb{F}\big) = \overset{(k)}{\mathbb{M}}[(g^{\bar{P}}_{\underset{M}{-}}+x^3A^{\bar{P}}_{\underset{M}{+}})\partial_P\mathbb{F}] = \partial_P\overset{(k)}{\mathbb{F}}+\frac{1}{4}A^{\bar{P}}_{\underset{M}{+}}\partial_M\big(\overset{(k-1)}{\mathbb{F}}+2\overset{(k)}{\mathbb{F}}+\overset{(k+1)}{\mathbb{F}}\big), \quad k\ge 0.
\end{aligned} \quad (3.21)
$$

Here, we have introduced the notation

$$
A^{\bar{P}}_{\underset{M}{+}} \equiv \underset{(1)}{A}{}^{\bar{P}}_{\underset{M}{+}} = g^{\bar{P}}_{\underset{M}{-}} - g^{\bar{P}}_{\underset{M}{+}}.
$$

Moreover, we assume that $\overset{(m)}{\mathbb{F}} = 0$ if $m < 0$. In what follows, we assume that this condition holds. Similarly, by (2.15), (3.11), (3.16), and (3.18), we have

$$
\begin{aligned}
\overset{(k)}{\mathbb{M}}\big(x^3\underset{(s)M}{g^{P}_{-}}\partial_3\mathbb{F}\big) &= \sum_{m=0}^{s}\underset{(m)}{A}{}^{\bar{P}}_{\underset{M}{+}}\overset{(k)}{\mathbb{M}}'[(x^3)^{m+1}\mathbb{F}] \\
&= \sum_{m=0}^{k}\underset{(m)}{A}{}^{\bar{P}}_{\underset{M}{+}}\sum_{p=0}^{2m+2}\sum_{q=l-1}^{N}2^{-(2m+1)}C^{p}_{2m+2}l\Big(1+(-1)^{l+q}\Big)\overset{(q)}{\mathbb{F}}(x') \\
&\quad+\sum_{m=0}^{s}\underset{(m)}{A}{}^{\bar{P}}_{\underset{M}{+}}\Bigg[-\sum_{p=0}^{m-k-1}\sum_{q=p}^{N}2^{-(2m+1)}C^{m-k-1-p}_{2m+2}(p+1)\Big(1-(-1)^{p+q}\Big)\overset{(q)}{\mathbb{F}}(x') \\
&\qquad+\sum_{p=m+1-k}^{m+k+1}\sum_{q=p}^{N}2^{-(2m+1)}C^{m+1-k+p}_{2m+2}(p+1)\Big(1-(-1)^{p+q}\Big)\overset{(q)}{\mathbb{F}}(x')\Bigg] \\
&\quad+\Big(\sum_{m=k+1}^{s}\underset{(m)}{A}{}^{\bar{P}}_{\underset{M}{+}}a_{(s,k)}\Big)\overset{(-)}{\mathbb{F}}{}'+\Bigg[\sum_{m=0}^{k}(2k-m+1)\underset{(m)}{A}{}^{\bar{P}}_{\underset{M}{+}}+\sum_{m=k+1}^{s}\underset{(m)}{A}{}^{\bar{P}}_{\underset{M}{+}}b_{(s,k)}\Bigg]\overset{(+)}{\mathbb{F}}{}', \\
&\qquad l\equiv k-m+p, \quad N\ge s+k+1, \quad k\ge 0, \quad s\ge 0. \quad (3.22)
\end{aligned}
$$

Whence, for $s = 0$ and $s = 1$, we find that

$$
\begin{aligned}
\overset{(k)}{\mathbb{M}}\big(x^3\underset{(0)M}{g^{P}_{-}}\partial_3\mathbb{F}\big) &= g^{\bar{P}}_{\underset{M}{-}}\overset{(k)}{\mathbb{M}}'(x^3\mathbb{F}) = \frac{1}{4}g^{\bar{P}}_{\underset{M}{-}}\big(\overset{(k-1)}{\mathbb{F}}{}'+2\overset{(k)}{\mathbb{F}}{}'+\overset{(k+1)}{\mathbb{F}}{}'\big) \\
&= g^{\bar{P}}_{\underset{M}{-}}\left[k\overset{(k)}{\mathbb{F}}+2(k+1)\left(\sum_{p=k}^{N}\overset{(p)}{\mathbb{F}}-\overset{(k)}{\mathbb{F}}+\overset{(+)}{\mathbb{F}}{}'\right)\right], \quad k\ge 0, \quad (3.23)
\end{aligned}
$$

$$\overset{(k)}{\mathbb{M}}(x^3 \underset{(1)}{g}{}^{P}_{-M} \partial_3 \mathbb{F}) = \overset{(k)}{\mathbb{M}} \left[ (g^{\bar{P}}_{-M} x^3 + A^{\bar{P}}_{+M} (x^3)^2) \partial_3 \mathbb{F} \right]$$

$$= g^{\bar{P}}_{-M} \overset{(k)}{\mathbb{M}}'(x^3 \mathbb{F}) + A^{\bar{P}}_{+M} \overset{(k)}{\mathbb{M}}'[(x^3)^2 \mathbb{F}] = g^{\bar{P}}_{-M} \left[ k \overset{(k)}{\mathbb{F}} + 2(k+1) \left( \sum_{p=k}^{N} \overset{(p)}{\mathbb{F}} - \overset{(k)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}' \right) \right]$$

$$+ \frac{1}{4} A^{\bar{P}}_{+M} \left[ (k-1) \overset{(k-1)}{\mathbb{F}} - 4(k+2) \overset{(k)}{\mathbb{F}} - (k+3) \overset{(k+1)}{\mathbb{F}} + 8(k+1) \left( \sum_{p=k}^{N} \overset{(p)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}' \right) \right], \quad k \geq 0. \quad (3.24)$$

Taking into account (3.20) and (3.22), from (3.19), we obtain the desired relation in the form

$$\overset{(k)}{\mathbb{M}}( \underset{(s)}{g}{}^{P}_{-M} N_P \mathbb{F}) = \sum_{m=0}^{k+1} \sum_{p=0}^{2m} \underset{(m)}{A}{}^{\bar{P}}_{+M} 2^{-2m} C^{p}_{2m} \partial_P \overset{(k-m+p)}{\mathbb{F}}$$

$$+ \sum_{m=k+2}^{s} \underset{(m)}{A}{}^{\bar{P}}_{+M} \left( - \sum_{p=2}^{m-k} 2^{-2m} C^{p-2}_{2m} \partial_P \overset{(m-k-p)}{\mathbb{F}} + \sum_{p=m-k}^{2m} 2^{-2m} C^{p}_{2m} \partial_P \overset{(k-m+p)}{\mathbb{F}} \right)$$

$$- g^{\bar{3}}_{+P} \left\{ \sum_{m=0}^{k} \underset{(m)}{A}{}^{\bar{P}}_{+M} \sum_{p=0}^{2m+2} \sum_{q=l-1}^{N} 2^{-(2m+1)} C^{p}_{2m+2} l \Big( 1 + (-1)^{l+q} \Big) \overset{(q)}{\mathbb{F}}(x') \right.$$

$$+ \sum_{m=k+1}^{s} \underset{(m)}{A}{}^{\bar{P}}_{+M} \left[ - \sum_{p=0}^{m-k-1} \sum_{q=p}^{N} 2^{-(2m+1)} C^{m-k-1-p}_{2m+2} (p+1) \Big( 1 - (-1)^{p+q} \Big) \overset{(q)}{\mathbb{F}}(x') \right.$$

$$\left. \left. + \sum_{p=m+1-k}^{m+k+1} \sum_{q=p}^{N} 2^{-(2m+1)} C^{m+1-k+p}_{2m+2} (p+1) \Big( 1 - (-1)^{p+q} \Big) \overset{(q)}{\mathbb{F}}(x') \right] \right\}$$

$$- g^{\bar{3}}_{+P} \left\{ \left( \sum_{m=k+1}^{s} \underset{(m)}{A}{}^{\bar{P}}_{+M} a_{(m,k)} \right) \overset{(-)}{\mathbb{F}}{}' + \left[ \sum_{m=0}^{k} (2k-m+1) \underset{(m)}{A}{}^{\bar{P}}_{+M} + \sum_{m=k+1}^{s} \underset{(m)}{A}{}^{\bar{P}}_{+M} b_{(m,k)} \right] \overset{(+)}{\mathbb{F}}{}' \right\},$$

$$l \equiv k - m + p, \quad N \geq s + k + 1, \quad k \geq 0, \quad s \geq 0. \quad (3.25)$$

Whence, for $s = 0$ and $s = 1$, by (3.21), (3.23), and (3.24), we find that

$$\overset{(k)}{\mathbb{M}}( \underset{(0)}{g}{}^{J}_{-I} N_J \mathbb{F}) = \overset{(k)}{\mathbb{M}}(N_I \mathbb{F}) = \nabla_I \overset{(k)}{\mathbb{F}} - g^{\bar{3}}_{+I} \left[ k \overset{(k)}{\mathbb{F}} + 2(k+1) \left( \sum_{p=k}^{N} \overset{(p)}{\mathbb{F}} - \overset{(k)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}' \right) \right], \quad k \geq 0, \quad (3.26)$$

$$\overset{(k)}{\mathbb{M}}( \underset{(1)}{g}{}^{J}_{-I} N_J \mathbb{F}) = \overset{(k)}{\mathbb{M}}[(g^{\bar{J}}_{-I} + x^3 A^{\bar{J}}_{+I}) N_J \mathbb{F}]$$

$$= \nabla_I \overset{(k)}{\mathbb{F}} + \frac{1}{4} A^{\bar{J}}_{+I} \nabla_J \Big( \overset{(k-1)}{\mathbb{F}} + 2 \overset{(k)}{\mathbb{F}} + \overset{(k+1)}{\mathbb{F}} \Big) - g^{\bar{3}}_{+J} \left\{ g^{\bar{J}}_{-I} \left[ k \overset{(k)}{\mathbb{F}} + 2(k+1) \left( \sum_{p=k}^{N} \overset{(p)}{\mathbb{F}} - \overset{(k)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}' \right) \right] \right.$$

$$\left. + \frac{1}{4} A^{\bar{J}}_{+I} \left[ (k-1) \overset{(k-1)}{\mathbb{F}} - 4(k+2) \overset{(k)}{\mathbb{F}} - (k+3) \overset{(k+1)}{\mathbb{F}} + 8(k+1) \left( \sum_{p=k}^{N} \overset{(p)}{\mathbb{F}} + \overset{(+)}{\mathbb{F}}{}' \right) \right] \right\}, \quad k \geq 0. \quad (3.27)$$

Therefore, we have deduced the main relation in the form (3.25); using this relation from the equations of motion (2.14), the heat influx equation (2.18), CR of physical content (2.22), and CR of heat content

(2.23) (second formula) of approximation of order $s$, we obtain the corresponding relations in moments; in turn by using the above rule, from them, we obtain the corresponding relations for multilayer thin bodies. Analogously, we obtain the boundary conditions of physical and heat contents in moments. Formulas (3.26) and (3.27) are applied in deducing the above relations from the corresponding relations of zero and first approximations. Formulas analogous to (3.26) and (3.27) certainly hold for the Legendre polynomial system. The relation for Legendre polynomial system analogous to (3.25) is very cumbersome,and so we do not write it.

## 4. Systems of Equations of Motion in Moments for Multilayer Thin Bodies with One Small Dimension

**4.1. Systems of equations of motion in moments of contravariant components of stress tensors and moment stresses with respect with respect to Chebyshev polynomial systems for multilayer thin bodies with one small dimension.** We restrict ourselves to obtaining the systems of equations of motion of approximations $(0, N)$ and $(1, N)$ in moments. Using the rule presented above, by analogous systems of equations from [72],we represent the desired systems of equations in the form

$$\begin{gathered}\Big\{\nabla_I \underset{\alpha}{\overset{(k)}{\mathbf P}}{}^{\bar I} - \underset{\alpha P}{g_{+}^{\bar 3}}\Big[k\underset{\alpha}{\overset{(k)}{\mathbf P}}{}^{\bar I} + 2(k+1)\Big(\sum_{p=k}^{N}\underset{\alpha}{\overset{(p)}{\mathbf P}}{}^{\bar I} - \underset{\alpha}{\overset{(k)}{\mathbf P}}{}^{\bar I}\Big)\Big] \\ + 2(k+1)\sum_{p=k}^{N}\Big[1-(-1)^{k+p}\Big]\underset{\alpha}{\overset{(p)}{\mathbf P}}{}^{\bar 3}\Big\} + \underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf F}} = \underset{\alpha}{\rho}\partial_t^2\underset{\alpha}{\overset{(k)}{\mathbf u}}, \qquad (4.1)\end{gathered}$$

$$\{\mathbf P \Rightarrow \boldsymbol\mu\} + \underset{\alpha}{\underset{\approx}{\mathbf C}} \overset{2}{\otimes} \underset{\alpha}{\underset{\sim}{\overset{(k)}{\mathbf P}}} + \underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf m}} = \underset{\alpha}{\underset{\sim}{\mathbf J}}\cdot\partial_t^2\underset{\alpha}{\overset{(k)}{\boldsymbol\varphi}}, \quad k=\overline{0,N}, \quad \alpha=\overline{0,K};$$

$$\begin{gathered}\Big\{\nabla_I \underset{\alpha}{\overset{(k)}{\mathbf P}}{}^{\bar I} + \frac{1}{4}\Big(\underset{\alpha I}{g_{-}^{\bar J}} - \underset{\alpha I}{g_{+}^{\bar J}}\Big)\nabla\nabla_J\Big(\underset{\alpha}{\overset{(k-1)}{\mathbf P}}{}^{\bar I} + 2\underset{\alpha}{\overset{(k)}{\mathbf P}}{}^{\bar I} + \underset{\alpha}{\overset{(k+1)}{\mathbf P}}{}^{\bar I}\Big) \\ - \underset{\alpha I}{g_{+}^{\bar 3}}\Big\{\underset{\alpha J}{g_{-}^{\bar I}}\Big[k\underset{\alpha}{\overset{(k)}{\mathbf P}}{}^{\bar J} + 2(k+1)\Big(\sum_{p=k}^{N}\underset{\alpha}{\overset{(p)}{\mathbf P}}{}^{\bar J} - \underset{\alpha}{\overset{(k)}{\mathbf P}}{}^{\bar J}\Big)\Big] \\ + \frac{1}{4}\Big(\underset{\alpha J}{g_{-}^{\bar I}} - \underset{\alpha J}{g_{+}^{\bar I}}\Big)\Big[(k-1)\underset{\alpha}{\overset{(k-1)}{\mathbf P}}{}^{\bar J} - 4(k+2)\underset{\alpha}{\overset{(k)}{\mathbf P}}{}^{\bar J} - (k+3)\underset{\alpha}{\overset{(k+1)}{\mathbf P}}{}^{\bar J} + 8(k+1)\Big(\sum_{p=k}^{N}\underset{\alpha}{\overset{(p)}{\mathbf P}}{}^{\bar J}\Big)\Big]\Big\} + \\ + 2(k+1)\Big[\sum_{p=k}^{N}(1-(-1)^{k+p})\underset{\alpha}{\overset{(p)}{\mathbf P}}{}^{\bar 3}\Big]\Big\} + \underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf F}} = \underset{\alpha}{\rho}\,\partial_t^2\underset{\alpha}{\overset{(k)}{\mathbf u}}, \qquad (4.2)\end{gathered}$$

$$\{\mathbf P \Rightarrow \boldsymbol\mu\} + \underset{\alpha}{\underset{\approx}{\mathbf C}} \overset{2}{\otimes} \underset{\alpha}{\underset{\sim}{\overset{(k)}{\mathbf P}}} + \underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf m}} = \underset{\alpha}{\underset{\sim}{\mathbf J}}\cdot\partial_t^2\underset{\alpha}{\overset{(k)}{\boldsymbol\varphi}}, \quad k=\overline{0,N}, \quad \alpha=\overline{0,K}.$$

Here, the notation $\{\mathbf P \Rightarrow \boldsymbol\mu\}$ means that the expression in brackets is obtained from the expression in brackets of the previous relation if the letter $\mathbf P$is replaced by $\boldsymbol\mu$; this notation is also used in what follows. Note that Eqs. (4.1) and (4.2) are also obtained by using formulas (3.26) and (3.27).

**4.2. Systems of equations of motion in moments of contravariant components of stress tensors and moment stresses with respect to Legendre polynomial systems for multilayer thin bodies with one small dimension.** Let us write the systems of equations of motion of approximations $(0,N)$ and $(1,N)$ in moments taking into account only boundary conditions of physical content on frontal surface, since the systems of equations without boundary conditions on frontal surfaces, which can be obtained by using the corresponding systems of equations from [72], have a form analogous to (4.1) and (4.2). It is easy to prove that similarly to the system of equations for a one-layer classical elastic body [117], the desired systems of equations are represented in the form

$$\begin{cases} \nabla_I \underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar I} - \underset{\alpha I}{\overset{+}{g}}{}^{\bar 3}\Big[k \underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar I} - (2k+1)\sum_{p=0}^{k} \underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar I}\Big] - (2k+1)\sum_{p=0}^{k}\big[1-(-1)^{k+p}\big]\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar 3} \\ \qquad + (2k+1)\left[\sqrt{\underset{\alpha}{\overset{++}{g}}{}_{33}}\,\underset{\alpha}{\overset{(+)}{\mathbf{P}}} + (-1)^k\sqrt{\underset{\alpha}{\overset{--}{g}}{}_{33}}\,\underset{\alpha}{\overset{(-)}{\mathbf{P}}}\right]\Bigg\} + \rho \underset{\alpha}{\overset{(k)}{\mathbf{F}}} = \underset{\alpha}{\rho}\,\partial_t^2 \underset{\alpha}{\overset{(k)}{\mathbf{u}}}, \quad (4.3) \\ \{\mathbf{P}\Rightarrow\boldsymbol{\mu}\} + \underset{\alpha}{\underset{\approx}{\mathbf{C}}} \overset{2}{\otimes} \underset{\alpha}{\underset{\sim}{\overset{(k)}{\mathbf{P}}}} + \underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf{m}}} = \underset{\alpha}{\underset{\sim}{\mathbf{J}}}\cdot\partial_t^2 \underset{\alpha}{\overset{(k)}{\boldsymbol{\varphi}}}, \quad k=\overline{0,N}, \quad \alpha=\overline{0,K}; \end{cases}$$

$$\begin{cases} \nabla_I \underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar I} + \dfrac{1}{2}\Big(\underset{\alpha M}{\overset{-}{g}}{}^{\bar P} - \underset{\alpha M}{\overset{+}{g}}{}^{\bar P}\Big)\Big(\dfrac{k}{2k-1}\nabla_P \underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar M} + \nabla_P \underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar M} + \dfrac{k+1}{2k+3}\nabla_P \underset{\alpha}{\overset{(k+1)}{\mathbf{P}}}{}^{\bar M}\Big) \\ \qquad - (2k+1)\sum_{p=0}^{k}\big[1-(-1)^{k+p}\big]\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar 3} - \underset{\alpha P}{\overset{+}{g}}{}^{\bar 3}\Big[\underset{\alpha M}{\overset{-}{g}}{}^{\bar P}\Big[k\underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar M} - (2k+1)\sum_{p=0}^{k}\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar M}\Big] \\ \qquad + \big(\underset{\alpha M}{\overset{-}{g}}{}^{\bar P} - \underset{\alpha M}{\overset{+}{g}}{}^{\bar P}\big)\Big[\dfrac{(k-1)k}{2(2k-1)}\underset{\alpha}{\overset{(k-1)}{\mathbf{P}}}{}^{\bar M} + k\underset{\alpha}{\overset{(k)}{\mathbf{P}}}{}^{\bar M} - \dfrac{(k+1)(k+2)}{2(2k+3)}\underset{\alpha}{\overset{(k+1)}{\mathbf{P}}}{}^{\bar M} - (2k+1)\sum_{p=0}^{k}\underset{\alpha}{\overset{(p)}{\mathbf{P}}}{}^{\bar M}\Big]\Big] \\ \qquad + (2k+1)\left[\sqrt{\underset{\alpha}{\overset{++}{g}}{}_{33}}\,\underset{\alpha}{\overset{(+)}{\mathbf{P}}} + (-1)^k\sqrt{\underset{\alpha}{\overset{--}{g}}{}_{33}}\,\underset{\alpha}{\overset{(-)}{\mathbf{P}}}\right]\Bigg\} + \rho \underset{\alpha}{\overset{(k)}{\mathbf{F}}} = \underset{\alpha}{\rho}\,\partial_t^2 \underset{\alpha}{\overset{(k)}{\mathbf{u}}}, \quad (4.4) \\ \{\mathbf{P}\Rightarrow\boldsymbol{\mu}\} + \underset{\alpha}{\underset{\approx}{\mathbf{C}}} \overset{2}{\otimes} \underset{\alpha}{\underset{\sim}{\overset{(k)}{\mathbf{P}}}} + \underset{\alpha}{\rho}\underset{\alpha}{\overset{(k)}{\mathbf{m}}} = \underset{\alpha}{\underset{\sim}{\mathbf{J}}}\cdot\partial_t^2 \underset{\alpha}{\overset{(k)}{\boldsymbol{\varphi}}}, \quad k=\overline{0,N}, \quad \alpha=\overline{0,K}. \end{cases}$$

Note that Eqs. (4.3) and (4.4) are deduced by using formulas for Legendre polynomials that are similar to (3.26) and (3.27). Also, note that $\underset{\alpha}{\overset{(+)}{\mathbf{P}}}$ $(\underset{\alpha}{\overset{(+)}{\boldsymbol{\mu}}})$ and $\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}$ $(\underset{\alpha+1}{\overset{(-)}{\boldsymbol{\mu}}})$, where $\alpha=\overline{1,K-1}$, are stress vectors (moment stresses) of interaction between the layers $\alpha$ and $\alpha+1$, which act on the surfaces $\underset{\alpha}{\overset{(+)}{S}}$ and $\underset{\alpha+1}{\overset{(-)}{S}}$, respectively, and $\underset{1}{\overset{(+)}{\mathbf{P}}}$ $(\underset{1}{\overset{(+)}{\boldsymbol{\mu}}})$ and $\underset{K}{\overset{(-)}{\mathbf{P}}}$ $(\underset{K}{\overset{(-)}{\boldsymbol{\mu}}})$ are given stress vectors (moment stresses) on the frontal surfaces $\underset{1}{\overset{(+)}{S}}$ and $\underset{K}{\overset{(-)}{S}}$, respectively. The systems of equations of heat influx of approximations $(0,N)$ and $(1,N)$, and also CR of heat content for multilayer thin bodies are obtained similarly to (4.1)–(4.4). Therefore, for brevity, we do not dwell on them. To help the reader understand this work (if the large work [72] is not available for and what was said is not sufficient to completely understand this work), we refer to [67], where for the theory of one-layer thin body with one small size with the use of the Chebyshev polynomial system of the second kind, similar problems are presented in detail. In particular, using the deduced recursive relations for the Chebyshev polynomial system, Nikabadze obtained the moments of derivatives of the first and second order of a scalar function, tensors of the first and second ranks and their components, and also some differential operators of these quantities.Also, this work obtained constitutive relations of physical and heat contents, equations of motion and heat influx, boundary conditions of various kind in moments with respect to the Chebyshev polynomial system of the second kind, and also initial conditions of kinematic and heat contents. Moreover, the constitutive relations were also obtained for an inhomogeneous material. This work presented the statements of related and nonrelated dynamical problems in moments of approximation $(r, N)$ of moment thermomechanics of a deformable rigid thin body and also the statement of nonstationary temperature problem in moments of approximation $(r, N)$, where $r$ and $N$ are arbitrary nonnegative integers. All relations for one-layer thin body presented in this paragraph are automatically transferred to the case of multilayer thin body theory by using the rule presented above .

**4.3. Systems of equations in moments of the displacement vector with respect to Legendre and Chebyshev polynomial systems for multilayer thin bodies with one small dimension.** Let us write systems of equations of zero and first approximation in moments for the displacement vector. The system of equations of zero approximation in moments with respect to Legendre and Chebyshev polynomials has the form

$$
\begin{gathered}
\underset{\alpha}{\underset{\sim}{\mathbf{C}}}^{\bar I\cdot\bar J\cdot}\cdot\nabla_I\nabla_J\overset{(k)}{\underset{\alpha}{\mathbf{u}}}+\Big(\underset{\alpha}{\underset{\sim}{\mathbf{C}}}^{\bar 3\cdot\bar I\cdot}+\underset{\alpha}{\underset{\sim}{\mathbf{C}}}^{\bar I\cdot\bar 3\cdot}\Big)\cdot\nabla_I\overset{(k)}{\underset{\alpha}{\mathbf{u}}}{}'+\underset{\alpha}{\underset{\sim}{\mathbf{C}}}^{\bar 3\cdot\bar 3\cdot}\cdot\overset{(k)}{\underset{\alpha}{\mathbf{u}}}{}''\\
-\Big(\underset{\alpha}{\mathbf{b}}^{\bar I}\nabla_I\overset{(k)}{\underset{\alpha}{\vartheta}}+\underset{\alpha}{\mathbf{b}}^{\bar 3}\overset{(k)}{\underset{\alpha}{\vartheta}}{}'\Big)+\underset{\alpha}{\rho}\overset{(k)}{\underset{\alpha}{\mathbf{F}}}=\underset{\alpha}{\rho}\,\partial_t^2\overset{(k)}{\underset{\alpha}{\mathbf{u}}},\quad k\in\mathbb{N}_0,\quad \alpha=\overline{0,K}.
\end{gathered}
\tag{4.5}
$$

Taking into account the formulas for moments of the $k$th order of first and second derivatives of a vector(vector components, a scalar function) with respect to these polynomial systems [72], from (4.5), we obtain the desired systems of equations of zero approximation in moments. For brevity, we do not write them. Similarly to (4.5), the system of equations of the first approximation is represented in the form

$$
\begin{gathered}
\underset{\alpha}{\underset{\sim}{\mathbf{C}}}^{\bar M\cdot\bar N\cdot}\cdot\Bigg\{g^{\bar P}_{\underset{\alpha M}{-}}g^{\bar Q}_{\underset{\alpha N}{-}}\nabla_P\nabla_Q\overset{(k)}{\underset{\alpha}{\mathbf{u}}}+\Big[\underset{(1\alpha)}{B}{}^{\bar P\bar Q}_{\underset{MN}{++}}\nabla_P\nabla_Q\overset{(k)}{\mathbf{M}}(x^3\underset{\alpha}{\mathbf{u}})+\underset{(2\alpha)}{B}{}^{\bar P\bar Q}_{\underset{MN}{++}}\nabla_P\nabla_Q\overset{(k)}{\mathbf{M}}\big((x^3)^2\underset{\alpha}{\mathbf{u}}\big)\Big]\Bigg\}\\
+\Big(\underset{\alpha}{\underset{\sim}{\mathbf{C}}}^{\bar 3\cdot\bar M\cdot}+\underset{\alpha}{\underset{\sim}{\mathbf{C}}}^{\bar M\cdot\bar 3\cdot}\Big)\cdot\Big[g^{\bar P}_{\underset{M}{-}}\nabla_P\overset{(k)}{\underset{\alpha}{\mathbf{u}}}{}'+(g^{\bar P}_{\underset{\alpha M}{-}}-g^{\bar P}_{\underset{\alpha M}{+}})\nabla_P\overset{(k)}{\mathbf{M}}{}'\Big(x^3\underset{\alpha}{\mathbf{u}}\Big)\Big]\\
+\underset{\alpha}{\underset{\sim}{\mathbf{C}}}^{\bar 3\cdot\bar 3\cdot}\cdot\overset{(k)}{\underset{\alpha}{\mathbf{u}}}{}''-\underset{\alpha}{\mathbf{b}}^{\bar M}\Big[g^{\bar P}_{\underset{\alpha M}{-}}\nabla_P\overset{(k)}{\underset{\alpha}{\vartheta}}+\Big(g^{\bar P}_{\underset{\alpha M}{-}}-g^{\bar P}_{\underset{\alpha M}{+}}\Big)\nabla_P\overset{(k)}{\mathbf{M}}(x^3\underset{\alpha}{\vartheta})\Big]\\
-\underset{\alpha}{\mathbf{b}}^{\bar 3}\overset{(k)}{\underset{\alpha}{\vartheta}}{}'+\underset{\alpha}{\rho}\overset{(k)}{\underset{\alpha}{\mathbf{F}}}=\underset{\alpha}{\rho}\,\partial_t^2\overset{(k)}{\underset{\alpha}{\mathbf{u}}},\quad k\in\mathbb{N}_0,\quad \alpha=\overline{0,K},
\end{gathered}
\tag{4.6}
$$

where we have introduced the following notation:

$$
\begin{gathered}
\underset{(1\alpha)}{B}{}^{\bar P\bar Q}_{\underset{MN}{++}}=\Big(g^{\bar P}_{\underset{\alpha M}{-}}-g^{\bar P}_{\underset{\alpha M}{+}}\Big)g^{\bar Q}_{\underset{\alpha N}{-}}+g^{\bar P}_{\underset{\alpha M}{-}}\Big(g^{\bar Q}_{\underset{\alpha N}{-}}-g^{\bar Q}_{\underset{\alpha N}{+}}\Big),\\
\underset{(2\alpha)}{B}{}^{\bar P\bar Q}_{\underset{MN}{++}}=\underset{(2\alpha)}{A}{}^{\bar P}_{\underset{M}{+}}g^{\bar Q}_{\underset{\alpha N}{-}}+\underset{(1\alpha)}{A}{}^{\bar P}_{\underset{M}{+}}\underset{(1\alpha)}{A}{}^{\bar Q}_{\underset{N}{+}}+g^{\bar P}_{\underset{\alpha M}{-}}\underset{(2\alpha)}{A}{}^{\bar Q}_{\underset{N}{+}},\\
\underset{(1\alpha)}{A}{}^{\bar P}_{\underset{M}{+}}=g^{\bar P}_{\underset{\alpha M}{-}}-g^{\bar P}_{\underset{\alpha M}{+}},\quad \underset{(2\alpha)}{A}{}^{\bar P}_{\underset{M}{+}}=\Big(g^{\bar P}_{\underset{\alpha N}{-}}-g^{\bar P}_{\underset{\alpha N}{+}}\Big)\Big(g^{\bar N}_{\underset{\alpha M}{-}}-g^{\bar N}_{\underset{\alpha M}{+}}\Big).
\end{gathered}
$$

Taking into account the expressions for the $k$th-order moments entering (4.6) and using the corresponding formulas for Legendre and Chebyshev polynomial systems of the second kind [72], from (4.6), we obtain various representations of the equations of first approximation of the displacement vector in moments with respect to these polynomial systems. Also, it is easy to deduce the equations of motion of first approximation in moments for the displacement vector with respect to the systems of Chebyshev polynomials of the first kind. For brevity, we do not write the equations in moments mentioned in this paragraph. It should be noted that to close systems (4.1)–(4.4), we need to add to them the system of equations of heat influx, CR, boundary and initial conditions of physical and heat contents in moments of the corresponding approximations, and also inter-layer contact conditions depending on the connections of neighboring surfaces. Hence to close systems (4.5) and (4.6), we need to add to them all relations of the previous proposition, except for CR in the case of first boundary-value problem where kinematic boundary conditions are given on the whole surface . Of course, it is easy to write all missed relations and formulate the statement of problems analogous to those presented in [67, 72, 74–76] (CR; see also above) for one-layer domains, except for inter-layer contact conditions, by using the rule presented above. Otherwise, we need to repeat almost all presentations in [67] applied to multilayer thin body theory. Owing to this, we do not dwell on them and consider inter-layer contact conditions below.

## 5. Inter-Layer Contact Conditions

In studying strained-deformed states of multilayer constructions and composite media, as a rule, one assumes that component layers (elements, phases) work jointly, without sliding. Obviously, such a model does not cover the variety of connection methods used in technology and does not take into account the existence of interphase defects, which manifest themselves in nonperfect connection of phases in contact. Defects of such a type are often inevitable because of peculiarities of technological character (see [9, 46, 101]). Therefore, the deformation of multilayer thin bodies can be without violation or with violation of complete layer contact owing to their separation in normal or tangential direction. Between the layers, there can arise contact domain and contact-free domain. Moreover, the boundaries of these domains can vary in the deformation process, the layer can slide with respect to each other, the sliding can be with friction, etc. All these phenomena can essentially influence the mechanical behavior of a thin body, its strained-deformed state. Of course, account of these phenomena is necessary in studying strained-deformed state of multilayer bodies. In contrast to other parametrizations, the use of frontal surfaces as base surfaces in the parametrization of a multilayer thin body domain allows one to easily take into account these phenomena. In considering the phenomena occurring on frontal surfaces, the main problem is the problem of modelling the interface. In this direction, there exist two approaches. The first approach is physical, which takes into account thin adhesion layers via the generalized weld condition of the elements in contact. For the first time, such an approach was proposed for heat conduction problems in [96]. Later on, it was generalized to mechanical problems [97]. The second approach is phenomenological; it is based on the assumption that, a priori, discontinuity zones of displacements exist. To study these problems, we assume that a multilayer thin construction consists of $K$ layers. Denote by $\underset{\alpha}{\overset{(+)}{S}}$ and $\underset{\alpha}{\overset{(-)}{S}}$ $(\alpha = \overline{1, K})$ the exterior and inner surfaces of the layer $\alpha$ $(\alpha = \overline{1, K})$, respectively, and consider several cases of mutual relation of neighboring surfaces $\underset{\alpha}{\overset{(+)}{S}}\ \underset{\alpha+1}{\overset{(-)}{S}}$ $(\alpha = \overline{1, K-1})$, which are important in practice.

**5.1. Weld conditions (complete ideal contact conditions).** In this case the forces and moments of interaction between the layers $\alpha$ and $\alpha+1$ $(\alpha = \overline{1, K-1})$ are unknown. These forces and moments are evidently equal and have opposite directions. Therefore, there additionally arise six unknown functions. However, in the case considered, we have six additional conditions, which express the continuity of displacement vectors and the rotation of welded surface points. In other words, displacement vectors and rotation vectors of contacting surfaces are equal. Denoting the forces and moments of interaction of the contacted surfaces $\underset{\alpha}{\overset{(+)}{S}}$ and $\underset{\alpha+1}{\overset{(-)}{S}}$ $(\alpha = \overline{1, K-1})$ by $\underset{\alpha}{\overset{(+)}{\mathbf{P}}}$, $\underset{\alpha}{\overset{(+)}{\boldsymbol{\mu}}}$ and $\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}$, $\underset{\alpha+1}{\overset{(-)}{\boldsymbol{\mu}}}$ $(\alpha = \overline{1, K-1})$, respectively, and the displacement and rotation vectors of points of these surfaces by $\underset{\alpha}{\overset{(+)}{\mathbf{u}}}$, $\underset{\alpha}{\overset{(+)}{\boldsymbol{\varphi}}}$ and $\underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}$, $\underset{\alpha+1}{\overset{(-)}{\boldsymbol{\varphi}}}$ $(\alpha = \overline{1, K-1})$, we can represent the complete contact conditions in the moment theory of multilayer thin bodies in the form

$$\underset{\alpha}{\overset{(+)}{\mathbf{P}}} = -\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}, \quad \underset{\alpha}{\overset{(+)}{\boldsymbol{\mu}}} = -\underset{\alpha+1}{\overset{(-)}{\boldsymbol{\mu}}}, \quad \underset{\alpha}{\overset{(+)}{\mathbf{u}}} = \underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}, \quad \underset{\alpha}{\overset{(+)}{\boldsymbol{\varphi}}} = \underset{\alpha+1}{\overset{(-)}{\boldsymbol{\varphi}}}, \quad \alpha = \overline{1, K-1}. \tag{5.1}$$

Neglecting the characteristics of moment theory (the second and fourth relations) in (5.1), we obtain the ideal contact conditions for the classical theory (the first and third relations).

**5.2. Conditions under relative displacement of contacted layer surfaces.** As was said above, in the process of deformation of a multilayer construction, relative displacements of points of the surfaces $\underset{\alpha}{\overset{(+)}{S}}$ and $\underset{\alpha+1}{\overset{(-)}{S}}$ with the same Gaussian coordinates $(x^1, x^2)$ are possible. Let us consider various variants. First of all, we note that there exist bounded limit intensities of coupling forces of the layers $\alpha$ and $\alpha + 1$ $(\alpha = \overline{1, K-1})$ in normal and tangential direction. Denote the normal and tangent components of the limit force of action of the layer $\alpha$ on the layer $\alpha + 1$ by

$$\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}{}^{*}_{(n)} = \underset{\alpha+1}{\overset{(-)}{P}}{}^{*}_{(n)}(x^1, x^2)\underset{\alpha+1}{\overset{(-)}{\mathbf{n}}}, \quad \underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}{}^{*}_{(s)} = \underset{\alpha+1}{\overset{(-)}{P}}{}^{*}_{(s)}(x^1, x^2, \underset{\alpha+1}{\overset{(-)}{\mathbf{s}}})\underset{\alpha+1}{\overset{(-)}{\mathbf{s}}}, \quad \alpha = \overline{1, K-1},$$

respectively. Here, $\underset{\alpha+1}{\overset{(-)}{\mathbf{n}}}$ and $\underset{\alpha+1}{\overset{(-)}{\mathbf{s}}}$ are unit exterior normal and tangent vectors to the surface $\underset{\alpha+1}{\overset{(-)}{S}}$. Note that we take into account the possibility of dependence of the limit tangent force preventing the mutual sliding of layers on the direction in the tangent plane (anisotropy of the limit tangent force).

**5.3. Conditions under relative displacement of points of ideal (smooth) contacted layer surfaces.** In this case, a free slipping of layers with respect to each other can take place in the process of deformation of a multilayer thin body. The parametrization preserves the validity of all relations of the theory of thin bodies in the case considered here; only the required and given functions are changed. Obviously, if the layers are joined, then the following equalities hold:

$$\begin{gathered}\underset{\alpha}{\overset{(+)}{\mathbf{r}}}(x^1, x^2) = \underset{\alpha}{\overset{\circ}{\overset{(+)}{\mathbf{r}}}}(x^1, x^2) + \underset{\alpha}{\overset{(+)}{\mathbf{u}}}(x^1, x^2), \quad \underset{\alpha}{\overset{(-)}{\mathbf{r}}}(x^1, x^2) = \underset{\alpha}{\overset{\circ}{\overset{(-)}{\mathbf{r}}}}(x^1, x^2) + \underset{\alpha}{\overset{(-)}{\mathbf{u}}}(x^1, x^2), \quad \alpha = \overline{1, K}, \\ \underset{\alpha}{\overset{\circ}{\overset{(+)}{\mathbf{r}}}}(x^1, x^2) = \underset{\alpha+1}{\overset{\circ}{\overset{(-)}{\mathbf{r}}}}(x^1, x^2), \quad \underset{\alpha}{\overset{(+)}{\mathbf{r}}}(x^1, x^2) = \underset{\alpha+1}{\overset{(-)}{\mathbf{r}}}(x^1, x^2), \quad (\underset{\alpha}{\overset{(+)}{\mathbf{u}}} = \underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}), \quad \alpha = \overline{1, K-1}.\end{gathered} \tag{5.2}$$

where $\underset{\alpha}{\overset{(+)}{\mathbf{r}}}$ $(\underset{\alpha}{\overset{\circ}{\overset{(+)}{\mathbf{r}}}})$ and $\underset{\alpha}{\overset{(-)}{\mathbf{r}}}$ $(\underset{\alpha}{\overset{\circ}{\overset{(-)}{\mathbf{r}}}})$ are the radius-vectors of the surfaces $\underset{\alpha}{\overset{(+)}{S}}$ $(\underset{\alpha}{\overset{\circ}{\overset{(+)}{S}}})$ and $\underset{\alpha}{\overset{(-)}{S}}$ $(\underset{\alpha}{\overset{\circ}{\overset{(-)}{S}}})$, respectively, in the deformed (nondeformed) state of the multilayer thin body. It is not difficult to see that in this case (under slipping of absolutely smooth contacting surfaces) we have the following relations instead of (5.2):

$$\begin{gathered}
\underset{\alpha}{\overset{(+)}{\mathbf{r}}}(x^1,x^2)=\underset{\alpha}{\overset{\circ}{\overset{(+)}{\mathbf{r}}}}(x^1,x^2)+\underset{\alpha}{\overset{(+)}{\mathbf{u}}}(x^1,x^2),\quad \underset{\alpha}{\overset{(-)}{\mathbf{r}}}(x^1,x^2)=\underset{\alpha}{\overset{\circ}{\overset{(-)}{\mathbf{r}}}}(x^1,x^2)+\underset{\alpha}{\overset{(-)}{\mathbf{u}}}(x^1,x^2),\quad \alpha=\overline{1,K},\\
\underset{\alpha}{\overset{\circ}{\overset{(+)}{\mathbf{r}}}}(x^1,x^2)=\underset{\alpha+1}{\overset{\circ}{\overset{(-)}{\mathbf{r}}}}(x^1,x^2),\quad \underset{\alpha}{\overset{(+)}{\mathbf{r}}}(x^1,x^2)\neq\underset{\alpha+1}{\overset{(-)}{\mathbf{r}}}(x^1,x^2),\quad \underset{\alpha}{\overset{(+)}{\mathbf{u}}}(x^1,x^2)\neq\underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}(x^1,x^2),\\
\underset{\alpha}{\mathbf{v}}(x^1,x^2)=\underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}(x^1,x^2)-\underset{\alpha}{\overset{(+)}{\mathbf{u}}}(x^1,x^2),\quad \alpha=\overline{1,K-1}.
\end{gathered}\tag{5.3}$$

Obviously, $\underset{\alpha}{\mathbf{v}}(x^1,x^2)$ is the vector of the relative displacement of the corresponding points of the contacted surfaces $\underset{\alpha}{\overset{(+)}{S}}$ and $\underset{\alpha}{\overset{(-)}{S}}$ ($\alpha=\overline{1,K-1}$), which is an unknown in the considered case. The absence of friction between the layers allows us to write the following additional relations:

$$\begin{gathered}
\underset{\alpha+1}{\overset{(-)}{u}}{}_{(n)}=\underset{\alpha}{\overset{(+)}{u}}{}_{(n)}\quad (\underset{\alpha}{v}{}_{(n)}=0),\quad \underset{\alpha}{\overset{(+)}{\mathbf{P}}}{}_{(s)}=0,\quad \underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}{}_{(s)}=0,\quad \underset{\alpha}{\overset{(+)}{\mathbf{P}}}{}_{(n)}=-\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}{}_{(n)},\\
\alpha=\overline{1,K-1},\quad x'\in\underset{\alpha}{\overset{(+)}{S}}{}^0\subset\underset{\alpha}{\overset{(+)}{S}},
\end{gathered}\tag{5.4}$$

where $\underset{\alpha}{\overset{(+)}{\mathbf{P}}}{}_{(s)}$ ($\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}{}_{(s)}$) and $\underset{\alpha}{\overset{(+)}{\mathbf{P}}}{}_{(n)}$ ($\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}{}_{(n)}$) are the tangent and normal components of the stress vector (the interaction force intensity) $\underset{\alpha}{\overset{(+)}{\mathbf{P}}}$ ($\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}$), i.e.,

$$\underset{\alpha}{\overset{(+)}{\mathbf{P}}}=\underset{\alpha}{\overset{(+)}{\mathbf{P}}}{}_{(s)}+\underset{\alpha}{\overset{(+)}{\mathbf{P}}}{}_{(n)},\quad \underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}=\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}{}_{(s)}+\underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}{}_{(n)},\quad \alpha=\overline{1,K-1}.$$

It is not difficult to see that (5.4) implies the following relations:

$$\begin{gathered}
\underset{\alpha+1}{\overset{(-)}{u}}{}_{(n)}=\underset{\alpha}{\overset{(+)}{u}}{}_{(n)},\quad \underset{\alpha}{v}{}_{(n)}=0),\quad \underset{\alpha+1}{\overset{(-)}{\mathbf{n}}}\cdot\underset{\alpha+1}{\underset{\sim}{\overset{(-)}{\mathbf{P}}}}(\underset{\alpha+1}{\mathbf{u}},\underset{\alpha+1}{\vartheta})\cdot\underset{\alpha+1}{\overset{(-)}{\mathbf{s}}}=0,\quad \underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\alpha}{\underset{\sim}{\overset{(+)}{\mathbf{P}}}}(\underset{\alpha}{\mathbf{u}},\underset{\alpha}{\vartheta})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{s}}}=0,\\
\underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\alpha}{\underset{\sim}{\overset{(+)}{\mathbf{P}}}}(\underset{\alpha}{\mathbf{u}},\underset{\alpha}{\vartheta})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{n}}}=\underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\alpha+1}{\underset{\sim}{\overset{(-)}{\mathbf{P}}}}(\underset{\alpha+1}{\mathbf{u}},\underset{\alpha+1}{\vartheta})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{n}}},\quad \alpha=\overline{1,K-1},\quad x'\in\underset{\alpha}{\overset{(+)}{S}}{}^0\subset\underset{\alpha}{\overset{(+)}{S}}.
\end{gathered}\tag{5.5}$$

Here the notation $\underset{\alpha}{\underset{\sim}{\overset{(\sim)}{\mathbf{P}}}}(\underset{\alpha}{\mathbf{u}},\underset{\alpha}{\vartheta})$, $\sim\in\{-,+\}$ means the dependence of $\underset{\alpha}{\underset{\sim}{\overset{(\sim)}{\mathbf{P}}}}$ on $\underset{\alpha}{\mathbf{u}}$ and $\underset{\alpha}{\vartheta}$, and $\underset{\alpha}{\underset{\sim}{\overset{(-)}{\mathbf{P}}}}=\underset{\alpha}{\underset{\sim}{\mathbf{P}}}|_{x^3=0}$, $\underset{\alpha}{\underset{\sim}{\overset{(+)}{\mathbf{P}}}}=\underset{\alpha}{\underset{\sim}{\mathbf{P}}}|_{x^3=1}$ (this notation is also further used). The corresponding relations are obtained from the first equality of (2.21) with $\underset{\alpha}{\underset{\approx}{\mathbf{A}}}=0$, $\underset{\alpha}{\boldsymbol{\varphi}}=0$. In the case considered here, relations (5.5) close the system of equations of the classical theory of multilayer thin bodies. In the case of the moment theory of multilayer thin bodies, equalities (5.5) should be replaced by the following ones:

$$\begin{gathered}
\underset{\alpha+1}{\overset{(-)}{u}}{}_{(n)}=\underset{\alpha}{\overset{(+)}{u}}{}_{(n)},\quad \underset{\alpha+1}{\overset{(-)}{\varphi}}{}_{(n)}=\underset{\alpha}{\overset{(+)}{\varphi}}{}_{(n)},\quad \underset{\alpha+1}{\overset{(-)}{\mathbf{n}}}\cdot\underset{\alpha+1}{\underset{\sim}{\overset{(-)}{\mathbf{P}}}}(\underset{\alpha+1}{\mathbf{u}},\underset{\alpha+1}{\boldsymbol{\varphi}},\underset{\alpha+1}{\vartheta})\cdot\underset{\alpha+1}{\overset{(-)}{\mathbf{s}}}=0,\\
\underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\alpha}{\underset{\sim}{\overset{(+)}{\mathbf{P}}}}(\underset{\alpha}{\mathbf{u}},\underset{\alpha}{\boldsymbol{\varphi}},\underset{\alpha}{\vartheta})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{s}}}=0,\quad \underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\alpha}{\underset{\sim}{\overset{(+)}{\mathbf{P}}}}(\underset{\alpha}{\mathbf{u}},\underset{\alpha}{\boldsymbol{\varphi}},\underset{\alpha}{\vartheta})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{n}}}=\underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\alpha+1}{\underset{\sim}{\overset{(-)}{\mathbf{P}}}}(\underset{\alpha+1}{\mathbf{u}},\underset{\alpha+1}{\boldsymbol{\varphi}},\underset{\alpha}{\vartheta})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{n}}},\\
\underset{\alpha+1}{\overset{(-)}{\mathbf{n}}}\cdot\underset{\alpha+1}{\underset{\sim}{\overset{(-)}{\boldsymbol{\mu}}}}(\underset{\alpha+1}{\boldsymbol{\varphi}})\cdot\underset{\alpha+1}{\overset{(-)}{\mathbf{n}}}=0,\quad \underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\alpha}{\underset{\sim}{\overset{(+)}{\boldsymbol{\mu}}}}(\underset{\alpha}{\boldsymbol{\varphi}})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{n}}}=0,\\
\underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\alpha}{\underset{\sim}{\overset{(+)}{\boldsymbol{\mu}}}}(\underset{\alpha}{\boldsymbol{\varphi}})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{s}}}=\underset{\alpha}{\overset{(+)}{\mathbf{n}}}\cdot\underset{\alpha+1}{\underset{\sim}{\overset{(-)}{\boldsymbol{\mu}}}}(\underset{\alpha+1}{\boldsymbol{\varphi}})\cdot\underset{\alpha}{\overset{(+)}{\mathbf{s}}},\quad \alpha=\overline{1,K-1},\quad x'\in\underset{\alpha}{\overset{(+)}{S}}{}^0\subset\underset{\alpha}{\overset{(+)}{S}}.
\end{gathered}\tag{5.6}$$

In this case, along with the vector of relative displacement, the vector of relative rotation $\underset{\alpha}{\boldsymbol{\psi}} = \overset{(-)}{\underset{\alpha+1}{\boldsymbol{\varphi}}} - \overset{(+)}{\underset{\alpha}{\boldsymbol{\varphi}}}$ of the corresponding points of the contacted surfaces is introduced into consideration; $\overset{(-)}{\underset{\alpha+1}{\varphi}}{}_{(n)}$ and $\overset{(+)}{\underset{\alpha}{\varphi}}{}_{(n)}$ are the normal components of the vectors $\overset{(-)}{\underset{\alpha+1}{\boldsymbol{\varphi}}}$ and $\overset{(+)}{\underset{\alpha}{\boldsymbol{\varphi}}}$, respectively. Note that relations (5.6) are written subject to the fact that each layer has a center of symmetry, i.e., the tensors $\underset{\underset{\alpha}{\approx}}{\mathbf{A}} = 0$ and $\underset{\underset{\alpha}{\approx}}{\mathbf{B}} = 0$ in (2.21) for all $\alpha$. If this is not the case, then these relations should be replaced by other ones depending on the considered governing relations. For example, if relations (2.21) are considered as governing ones for a material having no center of symmetry (see [72]), then instead of (5.6) we have

$$
\begin{gathered}
\overset{(-)}{\underset{\alpha+1}{u}}{}_{(n)} = \overset{(+)}{\underset{\alpha}{u}}{}_{(n)}, \quad \overset{(-)}{\underset{\alpha+1}{\varphi}}{}_{(n)} = \overset{(+)}{\underset{\alpha}{\varphi}}{}_{(n)}, \quad \overset{(-)}{\underset{\alpha+1}{\mathbf{n}}} \cdot \overset{(-)}{\underset{\underset{\alpha+1}{\sim}}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha+1}{\vartheta}) \cdot \overset{(-)}{\underset{\alpha+1}{\mathbf{s}}} = 0, \\
\overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(+)}{\underset{\underset{\alpha}{\sim}}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{s}}} = 0, \quad \overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(+)}{\underset{\underset{\alpha}{\sim}}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{n}}} = \overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(-)}{\underset{\underset{\alpha+1}{\sim}}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{n}}}, \\
\overset{(-)}{\underset{\alpha+1}{\mathbf{n}}} \cdot \overset{(-)}{\underset{\underset{\alpha+1}{\sim}}{\boldsymbol{\mu}}}(\underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\vartheta}) \cdot \overset{(-)}{\underset{\alpha+1}{\mathbf{n}}} = 0, \quad \overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(+)}{\underset{\underset{\alpha}{\sim}}{\boldsymbol{\mu}}}(\underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{n}}} = 0, \\
\overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(+)}{\underset{\underset{\alpha}{\sim}}{\boldsymbol{\mu}}}(\underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{s}}} = \overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(-)}{\underset{\underset{\alpha+1}{\sim}}{\boldsymbol{\mu}}}(\underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha+1}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{s}}}, \quad \alpha = \overline{1, K-1}, \quad x' \in \overset{(+)}{\underset{\alpha}{S}}{}^{0} \subset \overset{(+)}{\underset{\alpha}{S}}.
\end{gathered}
\tag{5.7}
$$

Note also that the contact conditions should be supplied with the conditions of heat content on the contacted surfaces, which is not difficult. Therefore, in order to shorten the presentation, we do not consider them here.

**5.4. Conditions under relative displacement of points of uneven contacted surfaces of layers.** In the case considered here, slipping with friction of the layers with respect to each other can take place in the process of deformation of the multilayer thin body. The relative slipping does not occur until the magnitude of the tangent component of the interaction force $\overset{(+)}{\underset{\alpha}{\mathbf{P}}}{}_{(s)}$ $(\overset{(-)}{\underset{\alpha+1}{\mathbf{P}}}{}_{(s)})$ (force of friction) between the contacted surfaces reaches its limit (maximal possible) value $|\overset{(+)}{\underset{\alpha}{\mathbf{P}}}{}^{*}|$ $(|\overset{(-)}{\underset{\alpha+1}{\mathbf{P}}}{}^{*}|)$, therefore,

$$
\underset{\alpha}{\mathbf{v}}(x^1, x^2) = 0, \quad \alpha = \overline{1, K-1}. \tag{5.8}
$$

When the force of friction reaches its limit value, slipping begins, and the relations presented above should be replaced by other ones. First of all, note that for the case of the classical theory of multilayer thin bodies instead of (5.5) we have

$$
\begin{gathered}
\overset{(-)}{\underset{\alpha+1}{u}}{}_{(n)} = \overset{(+)}{\underset{\alpha}{u}}{}_{(n)}, \quad \overset{(-)}{\underset{\alpha+1}{\mathbf{n}}} \cdot \overset{(-)}{\underset{\underset{\alpha+1}{\sim}}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\vartheta}) \cdot \overset{(-)}{\underset{\alpha+1}{\mathbf{s}}} = \overset{(-)}{\underset{\alpha+1}{P}}{}^{*}_{(s)}, \quad \overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(+)}{\underset{\underset{\alpha}{\sim}}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{s}}} = \overset{(+)}{\underset{\alpha}{P}}{}^{*}_{(s)}, \\
\overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(+)}{\underset{\underset{\alpha}{\sim}}{\mathbf{P}}}(\underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{n}}} = \overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(-)}{\underset{\underset{\alpha+1}{\sim}}{\mathbf{P}}}(\underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{n}}}, \quad \alpha = \overline{1, K-1}, \quad x' \in \overset{(+)}{\underset{\alpha}{S}}{}^{0} \subset \overset{(+)}{\underset{\alpha}{S}},
\end{gathered}
\tag{5.9}
$$

and in the case of the moment theory of multilayer thin bodies whose layers do not have a center of symmetry, we assume the following relations instead of (5.7):

$$
\begin{gathered}
\overset{(-)}{\underset{\alpha+1}{u}}{}_{(n)} = \overset{(+)}{\underset{\alpha}{u}}{}_{(n)}, \quad \overset{(-)}{\underset{\alpha+1}{\varphi}}{}_{(n)} = \overset{(+)}{\underset{\alpha}{\varphi}}{}_{(n)}, \quad \overset{(-)}{\underset{\alpha+1}{\mathbf{n}}} \cdot \overset{(-)}{\underset{\alpha+1}{\underset{\sim}{\mathbf{P}}}}(\underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha+1}{\vartheta}) \cdot \overset{(-)}{\underset{\alpha+1}{\mathbf{s}}} = \overset{(-)}{\underset{\alpha+1}{P}}{}^{*}_{(s)}, \\
\overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(+)}{\underset{\alpha}{\underset{\sim}{\mathbf{P}}}}(\underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{s}}} = \overset{(+)}{\underset{\alpha}{P}}{}^{*}_{(s)}, \quad \overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(+)}{\underset{\alpha}{\underset{\sim}{\mathbf{P}}}}(\underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{n}}} = \overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(-)}{\underset{\alpha+1}{\underset{\sim}{\mathbf{P}}}}(\underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha+1}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{n}}}, \\
\overset{(-)}{\underset{\alpha+1}{\mathbf{n}}} \cdot \overset{(-)}{\underset{\alpha+1}{\underset{\sim}{\boldsymbol{\mu}}}}(\underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha+1}{\mathbf{u}}, \underset{\alpha+1}{\vartheta}) \cdot \overset{(-)}{\underset{\alpha+1}{\mathbf{n}}} = \overset{(-)}{\underset{\alpha+1}{\mu}}{}^{*}_{(n)}, \quad \overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(+)}{\underset{\alpha}{\underset{\sim}{\boldsymbol{\mu}}}}(\underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{n}}} = \overset{(+)}{\underset{\alpha}{\mu}}{}^{*}_{(n)}, \\
\overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(+)}{\underset{\alpha}{\underset{\sim}{\boldsymbol{\mu}}}}(\underset{\alpha}{\boldsymbol{\varphi}}, \underset{\alpha}{\mathbf{u}}, \underset{\alpha}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{s}}} = \overset{(+)}{\underset{\alpha}{\mathbf{n}}} \cdot \overset{(-)}{\underset{\alpha+1}{\underset{\sim}{\boldsymbol{\mu}}}}(\underset{\alpha+1}{\boldsymbol{\varphi}}, \underset{\alpha}{\mathbf{u}}, \underset{\alpha+1}{\vartheta}) \cdot \overset{(+)}{\underset{\alpha}{\mathbf{s}}}, \quad \alpha = \overline{1, K-1}, \quad x' \in \overset{(+)}{\underset{\alpha}{S}}{}^{0} \subset \overset{(+)}{\underset{\alpha}{S}}.
\end{gathered}
\tag{5.10}
$$

Here, naturally,

$$
\overset{(+)}{\underset{\alpha}{\mu}}{}^{*}_{(n)} = \overset{(+)}{\underset{\alpha}{\boldsymbol{\mu}}}{}^{*} \cdot \overset{(+)}{\underset{\alpha}{\mathbf{n}}}, \quad \overset{(-)}{\underset{\alpha+1}{\mu}}{}^{*}_{(n)} = \overset{(-)}{\underset{\alpha+1}{\boldsymbol{\mu}}}{}^{*} \cdot \overset{(+)}{\underset{\alpha+1}{\mathbf{n}}},
$$

where $\overset{(+)}{\underset{\alpha}{\boldsymbol{\mu}}}{}^{*}$ $(\overset{(-)}{\underset{\alpha+1}{\boldsymbol{\mu}}}{}^{*})$ is the intensity of the limit momentum. Therefore, $\overset{(+)}{\underset{\alpha}{P}}{}^{*}_{(s)}$, $\overset{(-)}{\underset{\alpha+1}{P}}{}^{*}_{(s)}$, $\overset{(+)}{\underset{\alpha}{\mu}}{}^{*}_{(n)}$, and $\overset{(-)}{\underset{\alpha+1}{\mu}}{}^{*}_{(n)}$ are unknown values in relations (5.9) and (5.10) determined from some a priori dependences, conditions of slipping with friction, which, generally speaking, must depend on the geometric and physical-mechanical properties of the contacting bodies. In the classic case we may assume that the following relations hold:

$$
\mathbf{L}(x^1, x^2, \mathbf{v}_s, \dot{\mathbf{v}}_s, [T], \mathbf{P}^{(l)*}, \dots) = 0, \tag{5.11}
$$

where $\mathbf{v}_s$ and $\dot{\mathbf{v}}_s$ are the tangent components of the vectors of the relative displacement and relative velocity, $[T]$ is the temperature jump, $\mathbf{P}^{(l)*}$ is the limit stress vector on a plane element with the normal $\mathbf{l}$, and the ellipsis denotes dependence on some other parameters. Based on (5.11), we can accept that the generalized model of Coulomb friction is valid:

$$
\mathbf{P}^{*}_{(s)} = \underset{\sim}{\mathbf{f}}(x^1, x^2, [T], \mathbf{P}^{*}_{(n)}) \cdot \dot{\mathbf{v}}_s, \tag{5.12}
$$

which takes into account the anisotropy of the friction. Here $\mathbf{P}^{*}_{(s)}$ and $\mathbf{P}^{*}_{(n)}$ are the limit tangent and normal components of the stress vector $\mathbf{P}^{(l)*}$. The second-rank tensor $\underset{\sim}{\mathbf{f}}(x^1, x^2, [T], \mathbf{P}^{*}_{(n)})$ is called the tensor of friction coefficients. Obviously, in the isotropic case we have $\underset{\sim}{\mathbf{f}} = f\underset{\sim}{\mathbf{E}}$, where $\underset{\sim}{\mathbf{E}}$ is the unit second-rank tensor. Representing (5.12) for contacting surfaces of a multilayer thin body, we obtain the missing required relations. Based on similar arguments in the case of the moment theory, we can assert that the following a priori relations are valid:

$$
\mathbf{L}(x^1, x^2, \mathbf{v}_s, \dot{\mathbf{v}}_s, \boldsymbol{\psi}_n, \dot{\boldsymbol{\psi}}_n, [T], \mathbf{P}^{(l)*}, \dots) = 0, \quad \mathbf{M}(x^1, x^2, \mathbf{v}_s, \dot{\mathbf{v}}_s, \boldsymbol{\psi}_n, \dot{\boldsymbol{\psi}}_n, [T], \boldsymbol{\mu}^{(l)*}, \dots) = 0, \tag{5.13}
$$

where $\boldsymbol{\psi}_n$ and $\dot{\boldsymbol{\psi}}_n$ are the normal components of the vectors of the relative internal rotation and relative internal rotation velocity of adjacent layers, $\boldsymbol{\mu}^{(l)*}$ is the limit vector of the moment stress on a plane element with the normal $\mathbf{l}$, and the other parameters are the same as in (5.11). Based on (5.13) and similar to (5.12), for the moment theory we can assume that the following relations are valid:

$$
\begin{gathered}
\mathbf{P}^{*}_{(s)} = \underset{\sim}{\mathbf{f}}(x^1, x^2, [T], \mathbf{P}^{*}_{(n)}) \cdot \dot{\mathbf{v}}_s + \underset{\sim}{\mathbf{h}}(x^1, x^2, [T], \mathbf{P}^{*}_{(n)}) \cdot \dot{\boldsymbol{\psi}}_n, \\
\boldsymbol{\mu}^{*}_{(n)} = \underset{\sim}{\mathbf{g}}(x^1, x^2, [T], \boldsymbol{\mu}^{*}_{(s)}) \cdot \dot{\boldsymbol{\psi}}_n + \underset{\sim}{\mathbf{l}}(x^1, x^2, [T], \boldsymbol{\mu}^{*}_{(s)}) \cdot \dot{\mathbf{v}}_s,
\end{gathered}
\tag{5.14}
$$

which take into account the anisotropy of the friction. Here $\underset{\sim}{\mathbf{f}}$, $\underset{\sim}{\mathbf{h}}$, $\underset{\sim}{\mathbf{g}}$, and $\underset{\sim}{\mathbf{l}}$ are the second-rank tensors, called the tensors of friction coefficients. Therefore, in the case of isotropic friction we have $\underset{\sim}{\mathbf{f}} = f\underset{\sim}{\mathbf{E}}$, $\underset{\sim}{\mathbf{h}} = h\underset{\sim}{\mathbf{E}}$, $\underset{\sim}{\mathbf{g}} = g\underset{\sim}{\mathbf{E}}$, and $\underset{\sim}{\mathbf{l}} = l\underset{\sim}{\mathbf{E}}$, where $\underset{\sim}{\mathbf{E}}$ is the unit second-rank tensor. It should be noted here that the coefficients of friction are determined by experiments and are given in tables. The author knows little in this direction for the moment theory, but for the classical theory these coefficients can be obtained, e.g., from [10, 44, 45]. Representing (5.14) for the contacting surfaces of a multilayer thin body, we get the missing required relations in the case of the moment theory.

**5.5. Conditions under a partial exfoliation of contacted surfaces of layers.** For the classical theory of multilayer thin bodies in this case we have the conditions

$$\begin{gathered}\underset{\alpha}{\mathbf{v}}(x^1,x^2)=\underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}(x^1,x^2)-\underset{\alpha}{\overset{(+)}{\mathbf{u}}}(x^1,x^2)\neq 0,\quad \underset{\alpha}{\overset{(+)}{\mathbf{P}}}(x^1,x^2)=0,\quad \underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}(x^1,x^2)=0,\\ (x^1,x^2)\subset \underset{\alpha}{\overset{(+)}{S}}{}^0\subset \underset{\alpha}{\overset{(+)}{S}},\quad \alpha=\overline{1,K-1},\end{gathered}\tag{5.15}$$

and for the moment theory of multilayer thin bodies we get the conditions

$$\begin{gathered}\underset{\alpha}{\mathbf{v}}(x^1,x^2)=\underset{\alpha+1}{\overset{(-)}{\mathbf{u}}}(x^1,x^2)-\underset{\alpha}{\overset{(+)}{\mathbf{u}}}(x^1,x^2)\neq 0,\quad \underset{\alpha}{\overset{(+)}{\mathbf{P}}}(x^1,x^2)=0,\quad \underset{\alpha+1}{\overset{(-)}{\mathbf{P}}}(x^1,x^2)=0,\\ \underset{\alpha}{\boldsymbol{\psi}}(x^1,x^2)=\underset{\alpha+1}{\overset{(-)}{\boldsymbol{\varphi}}}(x^1,x^2)-\underset{\alpha}{\overset{(+)}{\boldsymbol{\varphi}}}(x^1,x^2)\neq 0,\quad \underset{\alpha}{\overset{(+)}{\boldsymbol{\mu}}}(x^1,x^2)=0,\quad \underset{\alpha+1}{\overset{(-)}{\boldsymbol{\mu}}}(x^1,x^2)=0,\\ (x^1,x^2)\subset \underset{\alpha}{\overset{(+)}{S}}{}^0\subset \underset{\alpha}{\overset{(+)}{S}},\quad \alpha=\overline{1,K-1}.\end{gathered}\tag{5.16}$$

Note also that if $\underset{\alpha}{\overset{(+)}{S}}{}^0=\underset{\alpha}{\overset{(+)}{S}}$, then we have complete exfoliation of contacting layers. Other conditions posed on deformed and force states of exterior surfaces of multilayer thin bodies are also possible: contact with rigid or elastic bodies, forced displacement of points, etc. It should be noted that before the presentation of the problems concerning the conditions on contacting surfaces of thin bodies the author had acquainted himself with papers [10, 21, 43, 47, 51–53, 86, 87, 91, 96–100, 103, 107–110]. Based on [74, 76] and quite similarly to this paper, one can construct moment theories of multilayer thin bodies with two small dimensions and those of plane domains with one small dimension, respectively (it remains to write down the corresponding relations). We do not pay attention to this in the present paper. In conclusion we note that the classic theory of elasticity predicts sufficiently well the behavior of actual rigid bodies under different loads in all the cases where the "granularity" of the structure of the considered real bodies is not typical. However, the classical theory of elasticity is unable to explain satisfactorily the mechanism of some phenomena observed in actual elastic bodies, not to mention for bodies with other rheology. For example, from the viewpoint of theoretical solutions of the classical theory of elasticity, one cannot explain and predict the laws of propagation of short acoustic waves in crystalline rigid bodies, polycrystalline metals, and high polymers. The classical theory also gives no satisfactory concordance of its results with experimental data for bodies with clearly expressed polycrystalline structure under the conditions of a stressed state with a large stress gradient. In particular, this theory cannot give any intelligible explanation of the influence of the stress gradient on the fatigue characteristics of polycrystalline materials. The reason for such unconformity of the theory and experience should evidently be sought in the fact that a continuous elastic model of a rigid body lying based on the classical theory of elasticity is unable to represent the elastic properties of real bodies determined by their discrete structure. Therefore, the explanation of these phenomena requires a new model of a rigid body in continuum mechanics so that the properties stemming from the discrete structure of real bodies can be explicitly represented [102]. A series of observed effects (phenomena) that cannot be explained on the base of the classic theory was indicated in [58]. As is known, all real bodies have a "granular" structure, and hence they can be considered as a collection of spatial material compounds consisted of particular "grains," material particles positioned relative to each other at distances comparable to their sizes and linked with each other by a complicated system of interconnections. Such “grains” can be molecules of the substance, separate crystals and crystal blocks in polycrystalline materials, etc. It should also be noted that the dispersion of elastic surface Rayleigh waves cannot be explained within the classical model of continuous media [23, 48]. However, this effect receives an explanation in a Cosserat medium (or a more generalized medium). In this case the attenuation rate of the Rayleigh wave amplitude with the depth and also the ellipticity of the wave depend on material constants of the medium, including the parameters describing the moment properties. This fact allows us to hope for an efficient application of such waves in possible experimental studies directed to the determination of moment behavior of materials and further to the determination of material parameters. Thus, the practical application of the theories constructed by the author requires the determination of the material functions entering the systems of equations and determining relations. A review of the corresponding papers [22, 23, 26, 48, 50] and others in this direction indicates that there exist several experimental methods [26, 50] for their determination and an active work is undertaken for the determination of material constants of various media (for some material they had been determined, see [22, 23, 26, 48, 50]). Therefore, we can assume that in the nearest future they will be obtained for most required materials, and the moment theories of thin bodies constructed by the author will have practical applications. Staying within the classical three-dimensional theory, we can get different improved theories from the moment ones under neglect of the moment characteristics, and these theories can be used in practice. In addition, it is necessary to point out, similarly to [37], some problems requiring further development and remaining urgent at present. The further development of mathematical methods reducing three-dimensional problems of the mechanics of a deformable rigid body to two- and one-dimensional ones is an urgent problem. This includes analytic and asymptotic methods and also the method of sequential differentiation of relations of the three-dimensional theory. Such methods should be developed not only for bodies with one small dimension, but also for bodies with two small dimensions. Obviously, the latter problem is more laborious. The main attention should be paid to dynamic theories of such bodies. Reducing three-dimensional theories to two-dimensional ones, we find variational methods very efficient for the determination of internally consistent and mathematically well-posed models. It is necessary to carry out further studies in the direction of mathematical justification of reduction methods, i.e., the study of convergence problems, error estimates, boundary conditions, convergence acceleration, etc. The necessity to compare the results of approximate theories with the results of analytic and numerical solutions to problems of the three-dimensional theory remains urgent. It is reasonable to compare the results of approximate theories with exact solutions for bodies with one small dimension of different cross-sections. The available comparisons in the case of the classical theory based on the plane deformation equations, or on those of the generalized plane stressed state, are unconvincing because these equations are approximate themselves. An important direction is the reduction of three-dimensional models to two-dimensional ones in the case of various rheological properties of the material, of geometrically and physically nonlinear bodies, and also materials subject to the influence of temperature, electromagnetic, and other fields. The other important direction is the determination of eigenvalues and eigentensors for the elasticity modulus tensor (elastic compliances) for different cases of material anisotropy and the representation of the relations of the deformable rigid body mechanics (determining relations, equations, problem statements, etc.) in these terms. The problem of determination of eigenvalues and eigentensors for a tensor of arbitrary even rank and also some problems of tensor calculus were considered in [69, 71, 77–79]. Finally, we note the considerable gap existing now in the field of experimental studies. Note also that the major portion of the material presented here was published in [80].

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