



# Modeling highly-dispersive transparency in planar nonlinear metamaterials



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## ABSTRACT

We consider propagation of light in planar optical metamaterial, which basic element is composed of two silver stripes, and it possesses strong dispersion in optical range. Our method of numerical modeling allows us to take into consideration the nonlinearity of the material and the effects of light self-action without considerable increase of the calculation time. It is shown that plasmonic resonances originating in such a structure result in multiple enhancement of local field and high sensitivity of the transmission coefficient to the intensity of incident monochromatic wave.

## 1. Introduction

The interest to optical metamaterials demonstrating the electromagnetically induced transparency (EIT) is due to the possible application of these structures for group velocity control [1], formation of “slow light” [2], construction of highly sensitive detectors [3,4], new devices in THz range [5–7]. “Classic” media for EIT are alkali metal vapors and some other gaseous media [8]. The resonance width achieved in them could be many orders less than in plasmonic metamaterials. However, metamaterials are easier to use, compact and remain relatively stable under laser radiation, which make them more preferable for some applications. High degree of field localization and high sensitivity of plasmonic excitations to optical properties of the material and to the geometry of the metamaterial basic element stimulate the investigation of its nonlinear optical properties [9] and development of new numerical modeling algorithms and methods. The latter allows to reduce the amount of expensive experimental work and optimize the structure of metamaterial and its basic component in order to obtain a sample with required optical properties. It is specifically important for the visible range in optics, because the amount of the experimental works for this range is small comparing to the infrared, THz and microwave ranges.

There are mainly two difficulties in numerical modeling of light propagation in metamaterial, when the material of the basic element or the surrounding medium possesses nonlinearity. First, one should specifically elaborate the problem of the relation between the experimental results and the results of numerical modeling, while the latter includes the solution of complicated system of nonlinear equations. Second, taking the nonlinearity into account usually makes the

numerical modeling very resource-consuming procedure, while the effect of it is not always worthwhile.

In the frequency domain, the most widely-used numerical technique for solving the heterogeneous medium problem of Maxwell's equations is the finite element method [10]. The reason for its popularity is the ability it gives to solve the Maxwell's equations with high accuracy for arbitrary geometry of a sample [11,12].

The complexity of the solution of Maxwell's equations for plasmonic metamaterials originates due to the drastic difference between the characteristic scales of the electric field variation in dielectric and in metal. One requires the numerical methods with the ability to adjust locally for the peculiarities of the solution in a places, where the characteristics of a medium experience abrupt changes. In such a problems the most prospective methods are the finite element methods with high-order approximation [11]. They allow to use calculation resources efficiently by changing locally characteristic size of the mesh cells and the order of the approximation [12,13]. In the given work we use the advantages of such methods in description of light self-action caused by the cubic nonlinearity of the metal in planar metamaterial with a basic element consisting of two silver stripes.

## 2. Formulation of the problem. Numerical simulation

As in [14], we consider a metamaterial which basic element consists of two silver stripes with thickness  $D=20$  nm, lengths  $l_1 = 150$  nm,  $l_2 = 135$  nm, transversal size  $w=65$  nm, and spacing  $d=70$  nm (see Fig. 1). Silver stripes are considered to be in a vacuum. A period of metamaterial in  $x$  and  $y$  directions is the same,  $L=520$  nm,  $z$ -axis is perpendicular to the Fig. 1 plane. Complex dielectric permittivity of the

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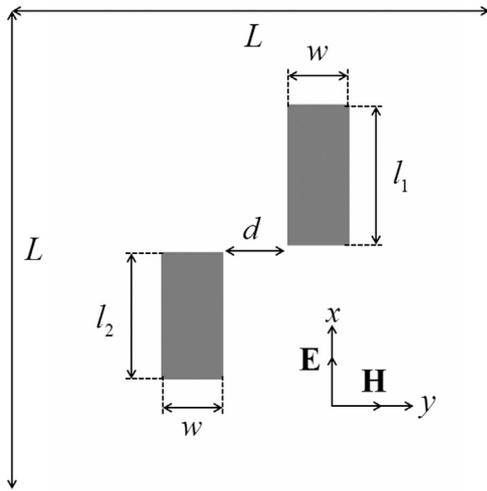


Fig. 1. The scheme of the basic element of the metamaterial.

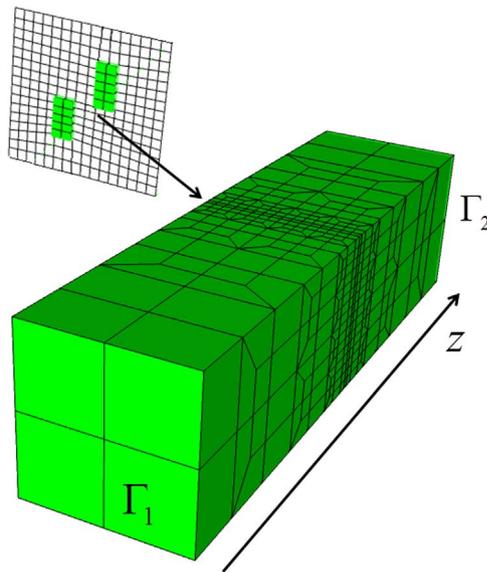


Fig. 2. The computation domain covered by the unstructured hexahedral mesh, which does not have hanging nodes and consists of 2288 cells.  $\Gamma_{1,2}$  are the plane boundaries of the area perpendicular to the  $z$ -direction.

metal is taken as  $\tilde{\epsilon} = 1 - \omega_p^2 / [\omega(\omega - i\gamma)]$ , where  $\omega_p = 1.366 \cdot 10^{16} \text{ rad}\cdot\text{s}^{-1}$  is a plasmonic frequency,  $\gamma = 3.07 \cdot 10^{13} \text{ s}^{-1}$  is a damping constant, and  $\omega$  is a frequency of the propagating electromagnetic wave. Unlike in [14], we take into consideration the nonlinear optical properties of the material of the stripes described by the local cubic susceptibility tensor

$\hat{\chi}^{(3)}(\omega; -\omega, \omega, \omega)$ . All of its non-zero components in isotropic medium can be expressed through  $\chi_1 = \chi_{xxxx}^{(3)}$  and  $\chi_2 = \chi_{xyxy}^{(3)}$ , which specific values can be found in [15,16].

When plane monochromatic wave propagates in a metal with nonlinear optical response, the following equation for the electric field vector  $\mathbf{E} = \{E_x, E_y, E_z\}$  directly follows from the system of Maxwell's equations:

$$\text{rot rot } \mathbf{E} - \frac{\omega^2}{c^2}(\tilde{\epsilon} + 6\pi\chi_2|\mathbf{E}|^2)\mathbf{E} - \frac{\omega^2}{c^2}(3\pi\chi_1|\mathbf{E}|^2)\mathbf{E}^* = 0 \quad (1)$$

We consider linearly polarized (along  $x$ -axis) monochromatic radiation with amplitude  $\mathbf{E}^{\text{inc}}$  normally (along  $z$ -axis) incident on a metamaterial. On a surface of a metal the tangential components of the electric and magnetic field strength vectors are continuous, and normal components have a jump. Finite-element approximation of Maxwell's equations retaining the above mentioned properties after numerical discretization was built in [17] for the first time, and it allowed one to eliminate the divergence of the numerical solution, which often originated when using more coarse approximations [18].

For numerical discretization (1) by finite-element method we used the hierarchic shape functions, which allow us to change locally the approximation order on the unstructured hexahedral mesh [11,12], which does not have hanging nodes. Along with the local mesh refinement this method provides very efficient use of calculation time. Fig. 2 shows the typical example of such a mesh covering the computation domain confined between plane boundaries  $\Gamma_{1,2}$  perpendicular to the  $z$ -direction. The specific scale of electric field variation in metal is much less than the wavelength  $\lambda = 2\pi c/\omega$ , and in order to make the solution more precise, the mesh becomes finer close to the metal elements (see Fig. 2), and the approximation order becomes higher in a places, where the solution is smooth. The position of the  $\Gamma_1$  boundary is chosen in such a way, that the wave reflected from the metallic elements having the wave vector anticollinear to  $z$ -direction could be treated as a flat one at this border. In our calculations we used so called scattering boundary conditions at  $\Gamma_{1,2}$  and periodic boundary conditions at the other boundaries. The main goal of our study was to obtain a dependence of transmission coefficient  $T$  on the wavelength of the incident radiation  $\lambda$ , the former being equal to the ratio of the powers of transmitted and incident wave. Also we acquired electric field distributions in the metamaterial elements and analyzed them.

### 3. The discussion of results

The EIT-effect observed in “classic” media (e.g., alkali metal vapors) can be described with a model of three-level  $\Lambda$ -system. In such a system the transition between two lower energy levels is forbidden, and the transition frequencies to the third, higher energy level, are close to each other. Higher energy level provides the coherent interaction between two frequency resonances close to each other, and their

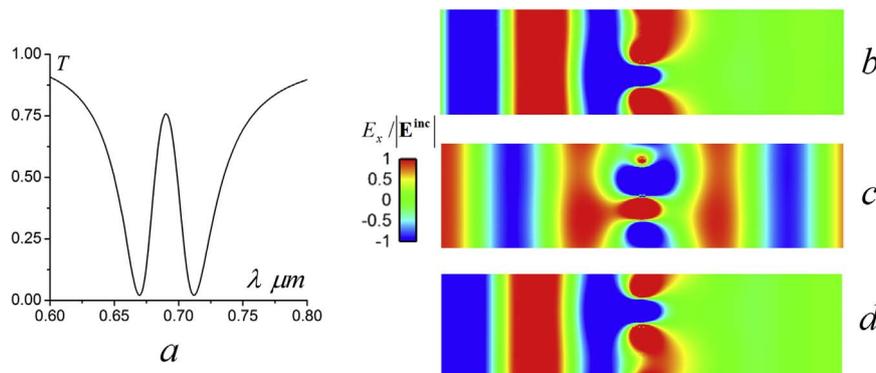
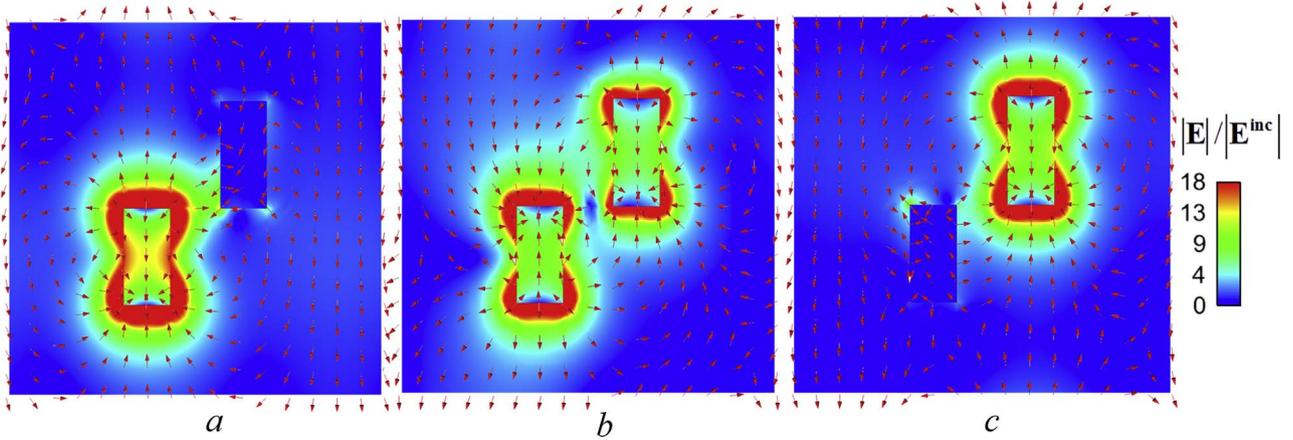
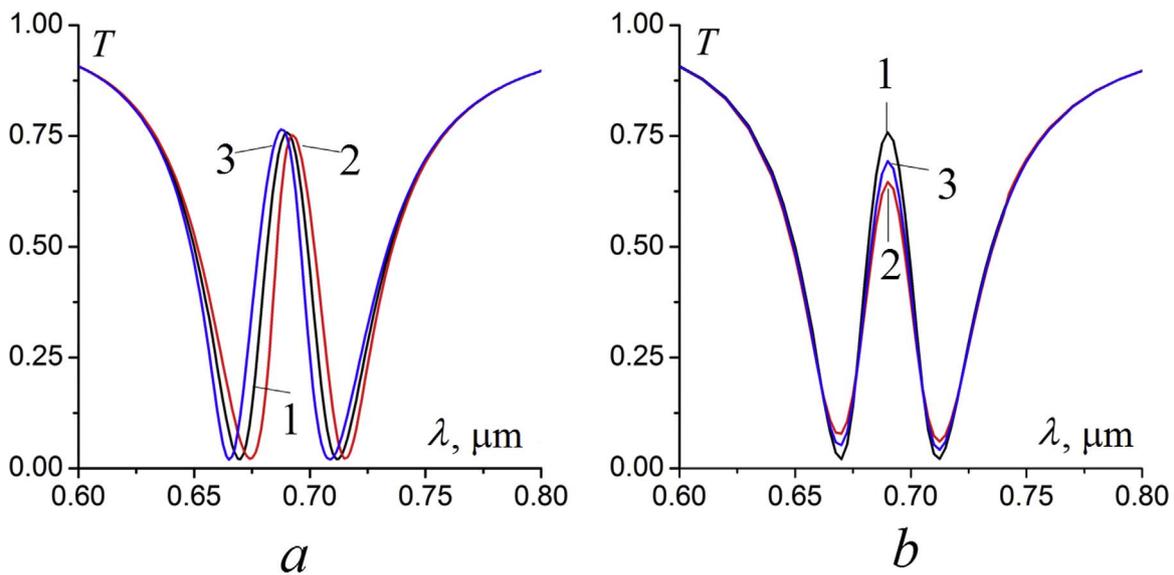


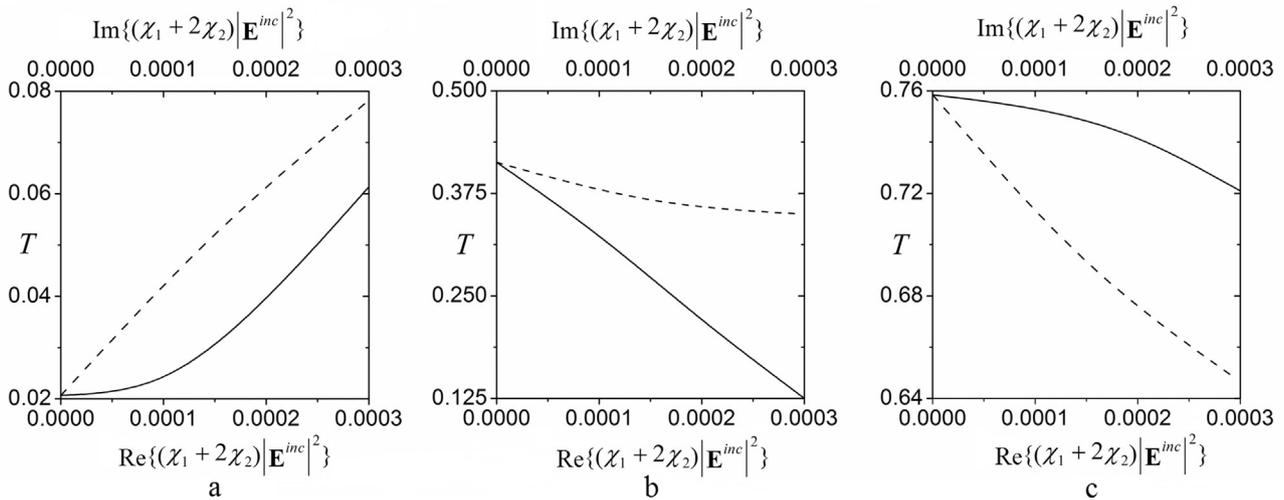
Fig. 3. The dependence of the transmission coefficient  $T$  on the wavelength  $\lambda$  of the incident radiation (a) and the distributions of the  $x$ -component of the electric field vector in a plane  $y=0$  at (b)  $\lambda=670 \text{ nm}$ ; (c)  $\lambda=690 \text{ nm}$ ; (d)  $\lambda=712.5 \text{ nm}$ .



**Fig. 4.** Distributions of the local field factor  $|\mathbf{E}|/|\mathbf{E}^{inc}|$  in a plane  $z=0$  in case of incidence of linearly polarized wave at  $\lambda=670$  nm (a);  $\lambda=690$  nm (b);  $\lambda=712.5$  nm (c). Small red arrows show the projections of the electric field normalized vector on the  $y=0$  plane. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



**Fig. 5.** The dependencies of the transmission coefficient  $T$  on the wavelength  $\lambda$  of the incident radiation. In (a) curves 1–3 correspond to the values  $\text{Re}\{(\chi_1 + 2\chi_2)|\mathbf{E}^{inc}|^2\} = 0; 0.0003; -0.0003$  accordingly, and  $\text{Im}\{\chi_1 + 2\chi_2\} = 0$ . In (b) curves 1–3 correspond to  $\text{Im}\{(\chi_1 + 2\chi_2)|\mathbf{E}^{inc}|^2\} = 0; -0.0003; -0.00015$  accordingly, and  $\text{Re}\{\chi_1 + 2\chi_2\} = 0$ .



**Fig. 6.** Dependencies of the transmission coefficient  $T$  on  $\text{Re}\{(\chi_1 + 2\chi_2)|\mathbf{E}^{inc}|^2\}$ . ( $\text{Im}\{\chi_1 + 2\chi_2\} = 0$ , solid lines) and  $\text{Im}\{(\chi_1 + 2\chi_2)|\mathbf{E}^{inc}|^2\}$  ( $\text{Re}\{\chi_1 + 2\chi_2\} = 0$ , dashed lines) for  $\lambda=670$  nm (a);  $\lambda=680$  nm (b);  $\lambda=690$  nm (c).

destructive interference yields the narrow transmission peak between them [8].

So the EIT takes place if the metallic stripes of the metamaterial acting as resonators, have close resonance frequencies, i.e., are slightly different in size. Our calculations made for the stripes demonstrated in Fig. 1 have shown that there is a maximum of  $T(\lambda)$  dependence between the minima each corresponding to the resonance of each stripe, and this maximum appears due to the interaction of the stripes. This effect takes place even in the simplest case, when the incident plane wave is monochromatic and linearly polarized. Under such conditions the transmitted wave retains linear polarization. For this case the distributions of the  $x$ -component of the electric field corresponding to the resonant frequencies (extrema in Fig. 3a) in a plane  $y=0$  are shown in Fig. 3b, c, d for  $\chi_{1,2}=0$ . It was found that the plasmonic waves are extremely sensitive to the optical properties of the metal, and the electric field is highly localized in such a structure. Fig. 4a, b, c show the distributions of the local field factor  $|\mathbf{E}|/|\mathbf{E}^{\text{inc}}|$  in a cross-section of metallic stripes for the same parameters and at the same frequencies as Fig. 3b, c, d. Here Fig. 4a demonstrates resonant response of the first stripe alone at  $\lambda=670$  nm, Fig. 4b shows the response of both stripes close to their resonances at  $\lambda=690$  nm, and Fig. 4c shows the resonant response of the second stripe alone at  $\lambda=712.5$  nm. In Fig. 4b it can be seen that the directions of the electric field vectors in the middle areas of the stripes are opposite. This behaviour is analogous to those of a “classic” EIT-system, where two resonances with close frequencies demonstrate destructive interference.

In comparison with the results of [14], in our work the electric field distributions in Fig. 4 were acquired in details and have high image quality, including all the issues concerning the localization effects. This is especially important for the analysis of the nonlinear response of a sample. High degree of the localization of the electromagnetic field makes it necessary to take into account the self-action of light even for the moderate values of  $|\mathbf{E}^{\text{inc}}|^2$ .

The experimental data on the values of the components of tensor  $\hat{\chi}^{(3)}(\omega; -\omega, \omega, \omega)$  for metals is inconsistent [16,19]. Particularly, it is due to the necessity to carry out complex polarization measurements in order to acquire complete set of data. For linear polarization of the incident monochromatic light the scenario of its further propagation in a nonlinear medium is determined by the real and imaginary parts of the dimensionless parameter  $(\chi_1 + 2\chi_2)|\mathbf{E}^{\text{inc}}|^2$ . Real part  $\text{Re}\{\hat{\chi}^{(3)}(\omega; -\omega, \omega, \omega)\}$  is responsible for the self-action of light in metal, and, as a consequence, it causes a shift of the resonances of the stripes and the one of the transmission resonance of the structure. Imaginary part  $\text{Im}\{\hat{\chi}^{(3)}(\omega; -\omega, \omega, \omega)\}$  causes nonlinear absorption, and it decreases the amplitude of the resonances in the transmission spectrum of the metamaterial. It can be seen in Fig. 5, where the influence of  $\text{Re}\{(\chi_1 + 2\chi_2)|\mathbf{E}^{\text{inc}}|^2\}$  (a) and  $\text{Im}\{(\chi_1 + 2\chi_2)|\mathbf{E}^{\text{inc}}|^2\}$  (b) is shown.

Fig. 6 shows the dependencies of  $T$  on  $\text{Re}\{(\chi_1 + 2\chi_2)|\mathbf{E}^{\text{inc}}|^2\}$  and  $\text{Im}\{(\chi_1 + 2\chi_2)|\mathbf{E}^{\text{inc}}|^2\}$  for  $\lambda=670$  nm (a);  $\lambda=680$  nm (b);  $\lambda=690$  nm (c). In some cases (dashed line in Fig. 6a and solid line in Fig. 6b)  $T$  is practically proportional to the intensity of the incident wave. This feature of the dependence could be useful for the control of the EIT effect by change of the intensity.

#### 4. Conclusions

Using numerical modeling we have studied optical properties of a metamaterial, which basic element consists of two metallic stripes having slightly different resonant frequencies in visible range. During

the light interaction with the metamaterial there takes place an effect similar to the EIT. This effect can be directly described basing on the linear properties of the material (metal) of the stripes. For our numerical modeling of light-matter interaction we developed and used an algorithm based on the finite-element method with ability to change locally the size of the mesh cell and the approximation order, thus allowing to obtain more accurate solution of the problem and to include into consideration the nonlinear optical response with low resource expense. Using this technique we found that the electric field is highly localized, and it is necessary to account for the effect of light self-action, which, in its turn, leads to the dependence of the transmission coefficient on the intensity of the incident wave. This dependence is especially strong near the resonant frequencies of the metamaterial.

The obtained results are interesting for controlling light with light, particularly, in applications when the controlling beam changes the optical properties of the structure, thus, changing the transmission coefficient for the signal beam.

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