THE GROWTH OF POLYNOMIAL LIE-REINHART ALGEBRAS

D. V. Millionshchikov

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia;

mitia_m@hotmail.com

Consider a commutative algebra A over a commutative unital ring R. The pair $(A, \mathcal{L})$ is called a Lie-Rinehart algebra [1] if

1) $\mathcal{L}$ is a Lie algebra over the ring R which acts on A by left derivations

$$X(ab) = X(a)b + aX(b), \forall a, b \in A, \forall X \in \mathcal{L};$$

2) the Lie algebra $\mathfrak{g}$ is a A-module.

The pair $(A, \mathcal{L})$ must satisfy the compatibility conditions

$$\begin{aligned} [X, aY] = X(a)Y + a[X, Y], \forall X, Y \in \mathcal{L}, \forall a \in A; \\ (aX)(b) = a(X(b)),\ \forall a, b \in A, \forall X \in \mathcal{L}. \end{aligned} \tag{1}$$

Consider an important subclass [2] of graded Lie-Rinehart algebras $(A, \mathcal{L})$, where $A = R[t_1, t_2, \dots, t_p]$ is a graded polynomial algebra over R such that

1) $\mathcal{L}$ is a free left module over $R[t_1, t_2, \dots, t_p]$ of rang N.

2) $\mathcal{L} = \oplus_{i \in \mathbb{Z}} \mathcal{L}_i$ is a $\mathbb{Z}$-graded Lie algebra $[\mathcal{L}_i, \mathcal{L}_j] \subset \mathcal{L}_{i+j},\ i, j \in \mathbb{Z}$, and its grading is compatible with the grading $R[t_1, t_2, \dots, t_p]$.

$$p(t)L \in \mathcal{L}_{i+deg(p(t))},\ deg(L(q(t)) = deg(q(t)) + i, L \in \mathcal{L}_i.$$

where $p(t), q(t)$ are homogeneous polynomials $R[t_1, t_2, \dots, t_p]$ of degree $deg(p(t))$ and $deg(q(t))$ respectively. The algebra grading $R[t_1, t_2, \dots, t_p]$ is defined on generators by the formulas

$$deg(t_1) = m_1, \dots, deg(t_p) = m_p, m_i \in \mathbb{Z}.$$

We will discuss the growth of a Lie algebra (over R), generated by the left free module $\mathcal{L}$ over $R[t_1, t_2, \dots, t_p]$. Its growth rate is related to the integrability of some systems of hyperbolic PDE (Klein-Gordon equation) [3,4].

This work is supported by the Russian Science Foundation under grant 20-11-19998.

REFERENCES

1. *Rinehart G.*, "Differential forms for general commutative algebras", Trans. Amer.Math. Soc. (108), 195–222 (1963).
2. *V.M. Buchstaber, D.V. Leikin* "Polynomial Lie algebras", Funktz. Analys i Prilozh., **36**:4, 18–34 (2002).
3. *Millionshchikov D.V.*, "Lie algebras of slow growth and Klein-Gordon PDE," Algebr. Represent. Theory, **21**:5, 1037–1069 (2018).
4. *Millionshchikov D.V.*, "Polynomial Lie algebras and growth of their finitely generated Lie subalgebras", Proc. Steklov Inst. Math., **302**, 298–314 (2018).