

Parameters of galactic disks at optical and NIR wavelengths

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We have analyzed the radial scales, central surface brightnesses, and colors of 404 disks of various types of galaxies. The central surface brightness μ_0 and linear disk scale length h vary smoothly along the Hubble sequence of galaxies within a rather narrow interval. The disks of relatively early type galaxies display higher central surface brightnesses in K , higher central surface densities, smaller sizes (relative to the diameter of the galaxy), redder integrated and central colors. The color gradient normalized to the radius of the galaxy and the blue central surface brightness $\mu_{0,i}^0(B)$ of the disk, are both independent of the galaxy type. The radial disk scales in different photometric bands differ less in early-type than in late-type galaxies. The ratio of linear disk scales measured in different photometric bands increases with the isophote ellipticity e of the disk (the inclination of the galaxy); however, the range of the ratio values for each e value exceeds the range of variations of scale lengths ratio over e . The disks in S0 galaxies have more homogeneous parameters than those in spiral galaxies. However, no sharp boundary in the properties of disks in lenticular, spiral, and irregular galaxies has been found; all parameters vary smoothly along the Hubble sequence. A correlation between the central disk surface brightness and the total luminosity of the galaxy is observed. We also consider the influence of dust on the photometric parameters of the disks. We show that the dust concentrated in dust lines towards the spiral arms and bars does not influence to the scale lengths ratio.

Keywords: Galaxies; Disks; Disk scale length

1 Introduction

Knowledge of the photometric parameters of disk components of galaxies is essential for studies of the dynamics and evolution of galaxies, dark matter, and the distribution of dust in galaxies. In the classical case of a thin exponential disk, the disk is described by two parameters in each photometric band: the scale length h and the central surface brightness μ_0 . Various combinations of photometric parameters can be

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used to determine the radial color gradients (which depend on the age and chemical composition of the stellar population of the disk and the distribution of dust), and to estimate the distribution of the stellar mass and the central surface density of the disk.

In numerous studies of the properties of galactic disks (see the brief review in [1]), either galaxies with narrowly specified properties (e.g., only Sb galaxies [2], E-S0 galaxies [3], galaxies observed “face-on” [4], very inclined galaxies [5]) were considered, or a decomposition was carried out using no more than a few (usually, one) photometric bands. This substantially reduces opportunities for analyzing the photometric parameters of the disks (for example, the absence of IR photometric data makes it impossible to determine the influence of dust on the photometric properties of a disk). Note also that, in most studies, 1D decomposition was based on radial profiles of the galaxies, which can in some cases result in incorrect values for the disk and bulge parameters [6].

The photometric properties of disks in the optical and IR are essential for a number of important problems, both fundamental and applied. Examples of the former include the formation and evolution of disks in S0 galaxies. An important applied problem is determining the mass distributions in galaxies as a whole, and in their disks in particular. Decomposition of the mass distribution into spherical and flat components requires knowledge of the distribution of the radiation in the disk and bulge of the galaxy. In general, the surface density is proportional to the surface brightness of the old stellar population corrected for absorption by dust. In a first approximation, it is assumed that the mass distribution corresponds to the radiation distribution in the *K*-band. However, IR observations are currently much more complicated (and rarer) than *BVRI* observations. Unfortunately, the data from the 2MASS Catalog (*JHK* photometry) cannot be used to study the weak outer regions of galaxies. We have attempted to establish a relationship between the radiation distributions (disk scales) in optical bands and in the IR.

Here, we use a sample of 404 galaxies with various morphological types and wide ranges of disk inclinations and galaxy luminosities to study the dependence of the photometric parameters of the disks on the morphological type of the galaxy, radial variations of the disk color indices, the influence of dust on the photometric parameters of the disks, and the dependence of the observed scale for the radial brightness decrease on the disk inclination.

2 The sample of galaxies

For our study of the photometric parameters of disks in galaxies of various types, we used the data from the literature for 392 galaxies (see [1]), together with our previously obtained CCD photometry for 12 galaxies [1, 7]). The integrated parameters of the galaxies (including the ellipticity e of the disk isophotes) were taken or calculated from data in the LEDA Catalog. We used the derived integrated parameters to determine the disk scale lengths h in kpc and the disk central surface brightness $\mu_{0,i}^0$ in mag/arcsec², corrected for the inclination of the galaxy. Thus, by using the available single-source data for the integrated parameters of the galaxies (taken from the LEDA database), we were able to decrease the dissimilarity of the sample objects.

Table 1 Basic data on the galaxies.

NGC	Bands	Type	$M_B^{0,i}$	D , Mpc	R_{25} , kpc	V_{rot} , km/s	e	M_{dust} , $10^6 M_\odot$
524	<i>U...K</i>	-1.2	-21.63	32.4	17.0	300	0.05	0.35
532	<i>U...K</i>	2.0	-19.48	31.5	16.0	191	0.74	3.3
783	<i>U...K</i>	5.1	-21.14	70.5	16.8	46	0.25	26
1138	<i>U...K</i>	-2.1	-19.57	32.9	8.7	25	0.05	–
1589	<i>U...K</i>	1.8	-21.73	49.5	23.8	323	0.63	1.3
2336	<i>U...K</i>	4.0	-22.32	32.2	30.0	256	0.42	9.7
4136	<i>B...K</i>	5.3	-18.41	7.6	4.1	93	0.18	0.17
5351	<i>B...K</i>	3.1	-21.19	48.9	19.6	202	0.53	1.3
5585	<i>U...I</i>	6.9	-18.48	5.7	3.5	79	0.38	0.12
7280	<i>U...K</i>	-1.0	-19.41	25.9	8.1	131	0.36	0.056
7721	<i>U...I</i>	4.9	-21.14	26.3	11.6	142	0.75	–
I1525	<i>U...I</i>	3.1	-21.85	69.6	19.7	186	0.31	–

In addition to the overall sample of 392 galaxies, we considered a separate subsample containing the 144 galaxies studied in [2–5, 8] via 2D decompositions. We believe that the disk parameters determined in these studies are more reliable than those obtained via 1D decompositions [6].

We also used our previous multi-color CCD photometry data of 12 galaxies. A description of the data reduction is given in [1, 6, 7]; the technique used for the 2D decompositions of the galactic radiation into bulge and disk components is presented in [6, 7].

Table 1 presents the basic data for the 12 added-galaxies: bands of the observations, the type, absolute magnitude $M_B^{0,i}$ corrected for absorption in the Galaxy and due to the disk inclination, distance to the galaxy D in Mpc, radius of the galaxy R_{25} in kpc determined from the $25^m/\text{arcsec}^2$ B isophote, maximum rotational velocity V_{rot} in km/s corrected for the inclination, disk isophote ellipticity e , and mass of dust M_{dust} in solar masses. Most part of parameters were taken from LEDA electronic database (see [1]).

The disk photometric parameters h and μ_0 for the 12 galaxies in the various filters are presented in [1, 7].

3 Analysis of the results

3.1 Central surface brightnesses and color indices of the disks

In spite of the fact that the galaxies considered have a large range of sizes ($R_{25} = 2\text{--}40$ kpc) and luminosities ($L_{max}/L_{min} = 100$), the central surface brightnesses of all the galactic disks $\mu_{0,i}^0$ lie in a fairly narrow interval, from $20.2^m/\text{arcsec}^2$ to

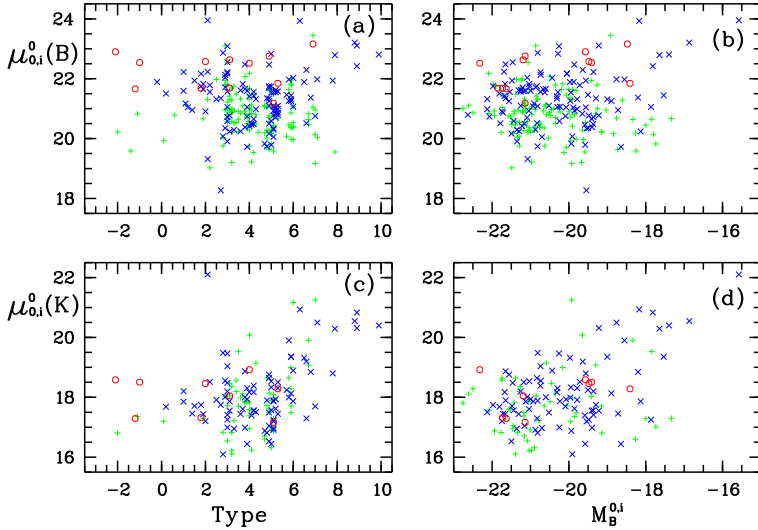


Figure 1 The central disk surface brightnesses $\mu_{0,i}^0$ in B (a, b), and K (c, d) as a function of the type (a, c) and absolute magnitude $M_B^{0,i}$ (b, d) of the galaxy. Circles mark objects from the sample of 12 galaxies, diagonal crosses those from the sub-sample of 144 galaxies, and the vertical crosses the remaining objects from the sample of 392 galaxies.

$22.7^m/\text{arcsec}^2$ in B and from $16.9^m/\text{arcsec}^2$ to $19.3^m/\text{arcsec}^2$ in K . The central disk surface brightnesses of the galaxies considered differ by no more than an order of magnitude. Note, there is no dependence between the central surface brightness $\mu_{0,i}^0$ corrected for the inclination and the disk inclination itself.

Let us consider the dependence between the central disk surface brightnesses in various photometric bands and the luminosity and type of the galaxy (Figs. 1a–1d). In spite of the large scatter in the corresponding plots, a correlation between $\mu_{0,i}^0$ and $M_B^{0,i}$ is observed, as well as a correlation between $\mu_{0,i}^0$ and the galaxy type in long-wavelength bands. The luminosities and types of the galaxies in our sample are weakly correlated: since there are no bright Sd–Irr galaxies, it is difficult to determine which of the parameters (the luminosity or type) is influencing $\mu_{0,i}^0$. Considering the sample of 144 galaxies whose disk parameters were derived from 2D decompositions, we found that the B and K central disk surface brightnesses in the early-type galaxies (S0–Sc) are independent of $M_B^{0,i}$ and the morphological type (the correlation coefficient $|r| < 0.3$). However, the central disk surface brightnesses in the late type galaxies (starting from Sc) increase with the integrated luminosity and depend on the morphological type. Considering the galaxies of all morphological types, we obtained the dependence $\mu_{0,i}(K) \sim (0.16 \pm 0.06)T$ ($r = 0.45$). $\mu_{0,i}(B)$ is independent of the galaxy type (Figs. 1a). Thus, on average, the central red brightness of the disk decreases from earlier to later galaxy type, while the central blue brightness remains constant for galaxies of all morphological types (Figs. 1a, 1c). This conclusion seems controversial, and may be a consequence of our selection of the objects. We can only suggest that the central color indices of the disks depend on the galaxy type.

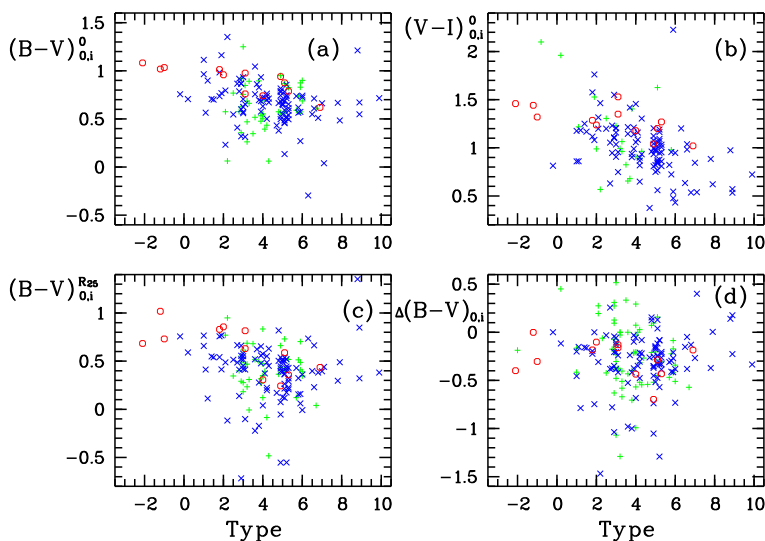


Figure 2 The central disk color indices (a) $(B-V)_{0,i}^0$ and (b) $(V-I)_{0,i}^0$, (c) $(B-V)_{0,i}^{R_{25}}$ color indices at the disk edges, and (d) radial $\Delta(B-V)_{0,i}$ color gradient as functions of the galaxy type. Notation is the same as in Fig. 1.

Figures 2a, 2b present the dependence between the central disk color indices $(B-V)_{0,i}^0$ and $(V-I)_{0,i}^0$ and the galaxy type. The centers of the disks are redder in early type than in late type galaxies for all three samples and for both shown color indices. Similar dependences are derived for the other color indices, with the exception of $J-H$ and $H-K$.

The color indices at the disk edges (at a distance R_{25} from the center of the galaxy), calculated using the formula $(X-Y)_{0,i}^{R_{25}} = (X-Y)_{0,i}^0 + 1.086R_{25}[1/h(X) - 1/h(Y)]$, show the same dependence on the galaxy type as the disk centers (Fig. 2c). Thus, it appears that the radial gradient of the disk color $\Delta(B-V)_{0,i} = (B-V)_{0,i}^{R_{25}} - (B-V)_{0,i}^0$ does not depend on the morphological type of the galaxy (Fig. 2d): most disks in galaxies of any type become bluer towards their periphery.

No correlation was found between the color indices of the disks and the luminosity and inclination of their galaxies.

Two-color diagrams may help us qualitatively estimate the composition of the stellar population of the disks, as well as study the star formation history and the impact of dust in the disks. The interpretation of the results derived from optic two-color diagrams can be ambiguous, since variations of the metallicity/stellar ages and selective absorption by dust both shift the points in the same direction – along the normal color sequence for the integrated colors of galaxies. Two-color diagrams with IR color indices can be used to discriminate between the effects of dust and age or chemical composition variations.

Figure 3 presents the $(B-H)_{0,i} - (J-K)_{0,i}$ diagram for the galaxies. According to the models of [9], variations of the age of the stellar population primarily affect $B-H$, while variations of the metallicity and absorption by dust primarily affect $J-K$. Unfortunately, the photometric parameters of the disks have never been

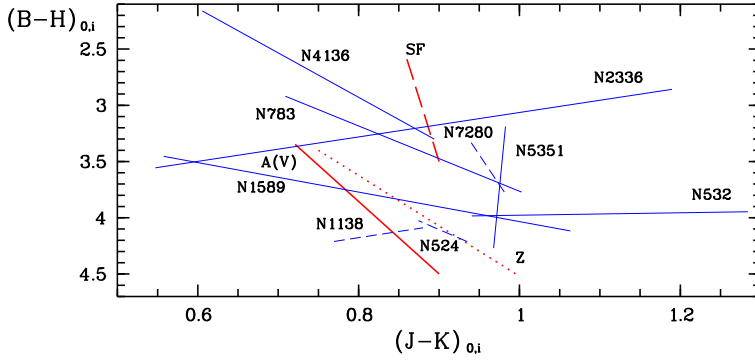


Figure 3 $(B-H)_{0,i} - (J-K)_{0,i}$ two-color diagram for the disks. The bold solid curve indicates the displacement of the points due to absorption by dust, by $A_V = 1.0^m$ up and to the left. The dotted curve shows the metallicity gradient for a stellar system with an age of 10 Gyrs (according to [9]). The bold dashed curve represents the displacement of the points in the case of a burst of star formation [9]. Systems with higher metallicity display higher $J-K$ color indices. The thin solid curves indicate the radial color variations in the disks of spiral galaxies, and the dashed curves the radial color variations in the disks of S0 galaxies (for objects from the sample of 9 galaxies). The NGC numbers of the galaxies are marked in the graph.

studied simultaneously in the $BJHK$ bands, so that we can apply this test only to the 9 galaxies observed by us. Note that the straight-line sections characterizing the radial variations of the disk color indices are appreciably shorter for the S0–Sa galaxies than for the spiral galaxies (Fig. 3). This provides evidence for small radial variations in the stellar population and a weak influence of dust in lenticular galaxies. All the spiral galaxies (with the exception of NGC 5351) display strong radial gradients of their metallicities; it is also possible that absorption by dust increases with distance from the center. In late type (Sbc–Scd) spiral galaxies, the average age of the disk stellar population also typically decreases, and the lower the average age of the disk, the larger the gradient for the decrease in age from the center to the edge of the galaxy.

3.2 Absolute and relative size of the disks

Both the absolute and relative sizes of the galactic disks lie in even narrower intervals than those for the central surface brightness. For the vast majority of galaxies, the absolute length scales for the disks are 2–7 kpc, while their relative scales are $h/R_{25} = 0.20-0.40$. Note that the scatter in h and h/R_{25} substantially exceeds the difference between the absolute and relative disk scales measured in different filters (except for the U and B bands). On average, the measured h and h/R_{25} values decrease from U to K .

The linear disk scale is virtually independent of the morphological type of the galaxy (Fig. 4a). Any variation of $h(I)$ with galaxy type is substantially smaller than the range of $h(I)$ for each given morphological type. Note that the $h(I)$ range in S0 and Sd–Irr galaxies is half that in spiral galaxies: 1–6 kpc vs. 1–12 kpc (Fig. 4a).

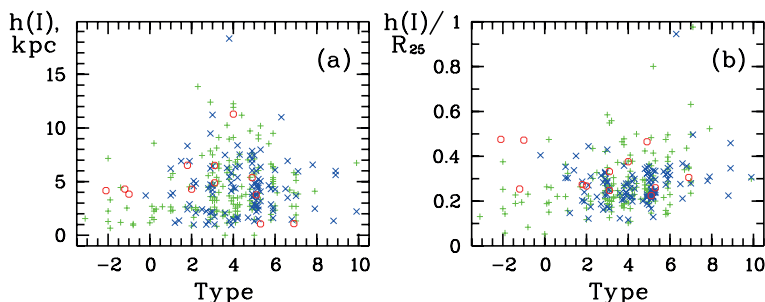


Figure 4 Dependence of the (a) absolute h and (b) relative $h(I)/R_{25}$ sizes of the disk on the galaxy type. The notation is the same as in Fig. 1.

With only one exception, the linear scales for the disk brightness decreases in S0 galaxies do not exceed 5 kpc; note that our sample contains S0 galaxies with both moderate and high (for example, NGC 524) luminosities.

The plot of $h(I)/R_{25}$ as a function of the galaxy type (Fig. 4b) appears more informative than the plot in Fig. 4a. Clearly, the later the morphological type of the galaxy, the larger, on average, the relative size of its disk. For S0 galaxies, $h(I)/R_{25} \approx 0.15\text{--}0.25$, while for Sd–Irr galaxies, $h(I)/R_{25} \approx 0.30\text{--}0.40$. Similar dependences are obtained for the absolute and relative disk sizes measured in the other photometric bands.

3.3 Ratio of linear disk scales in various photometric bands

The parameter $h(X)/h(Y)$, where X and Y are different photometric bands, is most sensitive to the presence of dust, and the dependence of $h(X)/h(Y)$ on the isophote ellipticity e forms the basic observational data for determining the parameters of dust disks in galaxies [2]. It was shown in [1] that the average disk scale ratios obtained by different authors differ strongly, as do the dependences of $h(X)/h(Y)$ on e . Using the total sample, we obtained relatively small average $h(X)/h(Y)$ values. The values for $h(B)/h(I)$ and $h(B)/h(K)$ agree with the estimates of [10] for the case when there are appreciable effects only due to the radial age and metallicity gradients. In our opinion, the large scatter in the average $h(X)/h(Y)$ obtained in earlier studies was due to the broad interval of the observed $h(X)/h(Y)$ values (Figs. 5a–5d).

The dependences between $h(X)/h(Y)$ and the galaxy type in the *BIK* bands were studied previously in [10]. The plot of $h(B)/h(I)$ as a function of the morphological type of the galaxy in Fig. 5d reproduces the results of [10]: $h(B)/h(I) = 1.0\text{--}1.2$ for S0 galaxies, the minimum value $h(B)/h(I) = 1.0$ is characteristic for all types of galaxies, and the maximum ratios increase from 1.2–1.4 for Sa galaxies to 1.6–1.9 for Sc–Sd galaxies.

Figures 5a–5c present $h(B)/h(K)$, $h(B)/h(I)$, and $h(I)/h(K)$ as functions of the ellipticity e . The ranges for the ratios of the linear disk scales are fairly large; the minimum values for $h(B)/h(K)$, $h(B)/h(I)$, and $h(I)/h(K)$ are unity independent of the inclination of the disk, while the maximum ratios increase with increasing inclination. Note that $h(I)/h(K)$ increases with the disk inclination only weakly. For

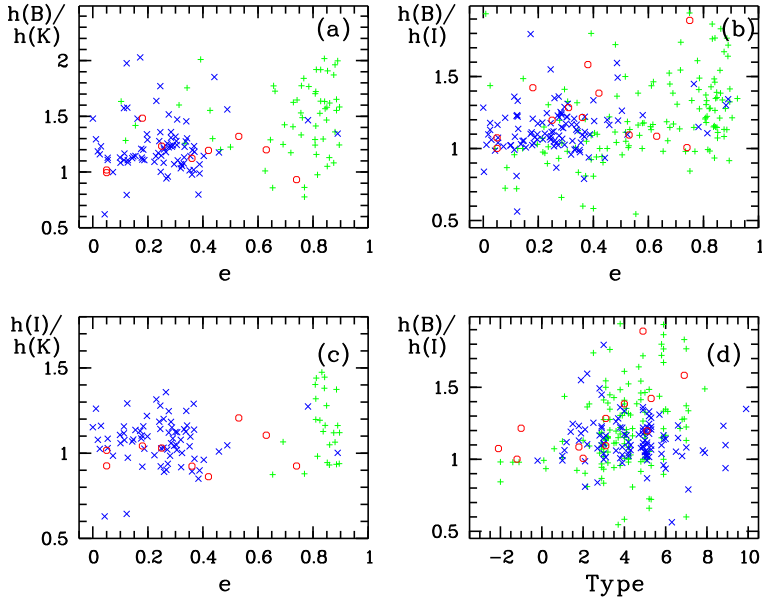


Figure 5 Dependence of the ratios of the radial disk scales (a) $h(B)/h(K)$, (b, d) $h(B)/h(I)$, and (c) $h(I)/h(K)$ on the (a–c) isophote ellipticity e and (d) galaxy type. Notation is the same as in Fig. 1.

the sample of 144 galaxies, we obtained the dependence $h(I)/h(K) = (1.07 \pm 0.03) + (0.02 \pm 0.10)e$. The fit parameters for the dependence of $h(B)/h(K)$ on e have very large errors.

The correlation between $h(B)/h(I)$ and $h(I)$ noted in [2] is generally confirmed; however, the scatter of the data is very large: for galaxies with $h(I) \approx 1$ kpc, $h(B)/h(I) = 0.9$ – 1.4 , while, for galaxies with $h(I) \approx 8$ kpc, $h(B)/h(I) = 1.0$ – 1.7 .

We also considered the dependence of $h(X)/h(Y)$ and on the average surface density of dust $\langle \sigma_{dust} \rangle$. To this end, we used the dust masses derived from the FIR luminosities of 37 out of the 144 galaxies in our sample, as well as the dust masses for 9 galaxies whose disk parameters were determined by us. We calculated the average surface density of dust using the formula $\langle \sigma_{dust} \rangle = M_{dust} / [\pi R_{25}^2 (1 - e)]$. Figure 6 presents the resulting graph for $h(B)/h(K)$ values. No unambiguous dependence between the disk scale ratio and the average surface density of dust can be discerned. The disks of many galaxies with large dust densities display relatively small $h(B)/h(K)$ values (Fig. 6). The data for 7 galaxies from our sample (all except NGC 532 and NGC 783) and for 7 galaxies with modest values from the total sample yield the dependence $h(V)/h(I) = (1.00 \pm 0.04) + (3.2 \pm 1.4) \cdot 10^{-5} \langle \sigma_{dust} \rangle$ ($r = 0.76$), where the units for σ_{dust} are M_{\odot}/kpc^2 . $h(B)/h(K)$ is more weakly correlated with σ_{dust} : using only 5 galaxies from our sample, we can derive the dependence $h(B)/h(K) = (1.02 \pm 0.04) + (11.6 \pm 1.8) \cdot 10^{-5} \langle \sigma_{dust} \rangle$, with $r = 0.96$ (Fig. 6). Why do many galaxies with higher dust abundances display modest $h(V)/h(I)$ and $h(B)/h(K)$? We suggest that, when calculating the integrated dust mass in galaxies from observations, we cannot discriminate between dust that forms the exponential

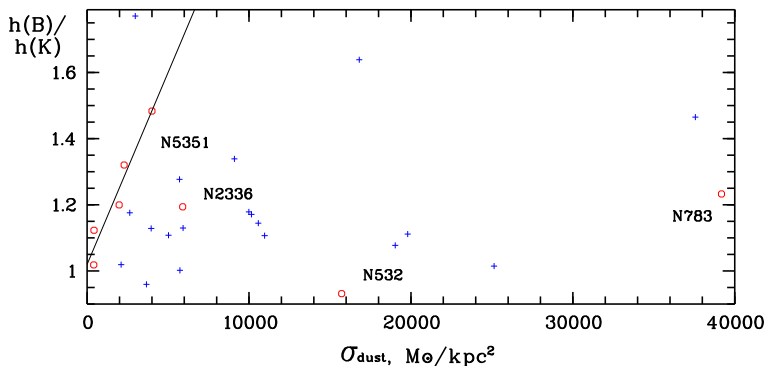


Figure 6 Ratio of the radial disk scales $h(B)/h(K)$ as functions of the average surface density of dust $\langle \sigma_{dust} \rangle$. The circles denote objects from the sample of 9 galaxies, and vertical crosses those from the sample of 37 galaxies. An explanation for the solid line is given in the text. The NGC numbers of the galaxies are indicated.

dust disk and dust that is concentrated towards the spiral arms and bars. For example, the galaxy NGC 5351, whose dust is concentrated in the disk, displays the highest $h(B)/h(K)$. At the same time, NGC 532, NGC 783, and NGC 2336, which are more dust-abundant, display relatively small $h(B)/h(K)$, due to the fact that a large fraction of their dust is concentrated in bands along the inner edges of their spiral arms [6].

3.4 Estimate of the central surface density of the disks

Classical thin exponential disks display a dependence between their central surface density σ_0 , linear scale length h , and maximum disk rotational velocity V_{disk} : $\sigma_0 \approx 0.044V_{disk}^2/h$, where σ_0 is measured in M_{\odot}/pc^2 , V_{disk} in km/s, and h in kpc. Here, $V_{disk} = (0.6 - 0.8)V_{rot}$ (depending on the model for the galaxy), where V_{rot} is the maximum rotational velocity derived from observations. Thus, $\sigma_0 \approx 0.022V_{rot}^2/h$. In most galaxies, the central disk surface density lies in the range 50–500 M_{\odot}/pc^2 . We can see a weak correlation between $\sigma_0(K)$ and the galaxy type: on average, earlier type galaxies display larger σ_0 values (Fig. 7a). This is consistent with the dependence on galaxy type obtained for the central K surface brightness of the disk (Fig. 1c).

The quantities $\mu_{0,i}^0(K)$ and $\sigma_0(K)$ are fairly well correlated in galactic disks (Fig. 7b). $\sigma_0(K)$ can be estimated from $\mu_{0,i}^0(K)$ with an accuracy of $\pm 50\%$. A less clear correlation is observed when $\mu_{0,i}^0$ and σ_0 values measured in other photometric bands are considered.

4 Conclusions

1. In the transition from early to late type galaxies, the central K surface brightness and the central surface density of the galactic disks decrease, the integrated and

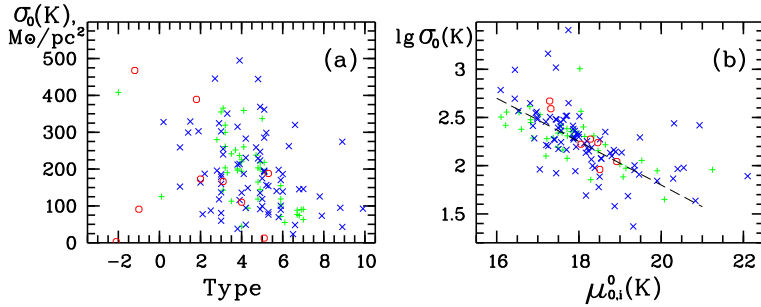


Figure 7 (a) The dependence of the central surface density of the disk σ_0 derived from $h(K)$ on the galaxy type. (b) Logarithm of the central disk surface density $\lg \sigma_0(K)$ as a function of the central surface brightness $\mu_{0,i}^0(K)$. The dotted line in (b) indicates the dependence for the sample of 392 galaxies. Notation is the same as in Fig. 1.

central color indices decrease, and the relative size of the disk h/R_{25} and the ratio $h(X)/h(Y)$ increase (here, X is a shorter wavelength photometric band than Y). The color gradient (normalizing by R_{25}) and the blue central surface brightness $\mu_{0,i}^0(B)$ are independent of the galaxy type. The disks in early type galaxies appear to be denser at the center and shorter than the disks in late type galaxies. The impact of dust on the photometric parameters of the disks and galaxies as a whole increases in the transition to late type galaxies.

2. The disks in S0 galaxies have more homogeneous parameters than those in spiral galaxies. This may be due to the lower linear age and metallicity gradients of their stellar populations, as well as the lower amounts of dust in the disks of S0 galaxies. No sharp boundary in the properties of disks in lenticular, spiral, and irregular galaxies has been found – all parameters vary smoothly along the Hubble sequence.

3. In all photometric bands, the central surface brightnesses of the disks increase with the total luminosity of the parent galaxy.

4. The ratio of linear disk scales measured in different photometric bands $h(X)/h(Y)$ increases with the isophote ellipticity e of the disk (the inclination of the galaxy); however, the range of $h(X)/h(Y)$ values for each e value exceeds the range of variations of $h(X)/h(Y)$ over e . This is due to the fact that very broad intervals are observed for the radial variations of the composition of the stellar population in the disk and the parameters of the dust disks in the galaxies.

5. Assuming that the surface density distribution in the disk corresponds to the K -band surface brightness distribution, the dependence $h(\sigma) = h(I)/[1.07 + 0.02e]$ can be used to determine the linear scale for the decrease of the surface density $h(\sigma)$ with an accuracy of $\pm 15\%$.

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