Abstract—The control system of a plasma unstable vertical position in the tokamak T-15 on the base of a Model Predictive Control (MPC) in discrete time for stationary and non-stationary plant models was synthesized and simulated. A comparison with a system containing a PID controller is presented. The predictive control showed better step response. The adaptive state observer was designed to estimate the unknown plant varying parameter on line which was used in the MPC algorithm to adapt it to the plant model.

I. INTRODUCTION

The MPC approach was used to control plasma current and shape in a tokamak on the quasi-stationary discharge phase when plant parameters were approximately unchanged [1]. The present work continues this direction with the usage of the adaptive MPC system for non-stationary plasma model in the control of the plasma vertical position without an observer in the feedback, when all plant states are available to measure [2].

II. STATEMENT OF THE PROBLEM

The model of the controlled plant is represented by the serial connection of the models of the multiphase thyristor rectifier, the control coil and the plasma in a tokamak, which is unstable in the Z direction (Fig. 1) [2]:

\[
\begin{align*}
T_a \frac{dU}{dt} + U &= K_a V; \\
T_c \frac{dI}{dt} + I &= K_c U; \\
T_p \frac{dz}{dt} - Z &= K_p I.
\end{align*}
\]

(1)

Here: \( Z \) is the vertical plasma position, \( I, U \) are the current and the voltage in the control coil, \( K_p = 0.0000178 \) mA, \( T_p = 0.02088 \) sec, \( K_c = 11.11 \) 1/Ohm, \( T_c = 0.0467 \) sec, \( K_a = 2000 \), \( T_a = 0.0033 \) sec. The \( (A_m, B_m, C_m, D_m) \) realization was built with the help of the transfer functions (1). Since the MPC system works in discrete time, we use the discrete \( (A_d, B_d, C_d, D_d) \) realization with sampling time of 0.001 sec. The goal of this work is to design and simulate a control system of the plasma vertical position with an adaptive MPC algorithm when the plasma behaviour is described by the non-stationary differential equation \( \frac{dz}{dt} = a(t)z + b(t)I \) in (1).

III. CONTROL SYSTEM WITH MPC ALGORITHM

A. MPC algorithm

The MPC algorithm computes a set of increments of the operation input of \( N_c \) on every tact minimising functional

\[
J = (R_s - Y)^T (R_s - Y) + \Delta U \bar{R} \Delta U,
\]

\( N_p \)

where \( R_s = [1 1 \cdots 1]^T r(k_i) = \bar{R} r(k_i), (r(k_i)) \) is the desired set point, \( \bar{R} \) is the weight matrix on the \( N_c \) length interval [3], \( Y \) is the set of outputs on the output horizon of the \( N_p \) length. The optimal set of operation inputs

\[
\Delta U = (\Phi^T \Phi + \bar{R})^{-1} \Phi^T (R_s - F x(k_i))
\]

is found from the necessary extremum condition:

\[
\frac{\partial J}{\partial \Delta U} = 0,
\]

where \( (\Phi^T \Phi + \bar{R})^{-1} \) is the Hessian of the system, in which matrix \( \Phi \) is compiled from matrices \( A, B, C \) of the augmented system. We take the augmented state vector of the system

\[
x = [(\Delta x_m(k_i))^T y(k_i)]^T
\]

to insert an integrator unit into the output \( y \) [3], \( x_m \) is the state vector,

\[
F = [(CA)^T(CA^2)^T \cdots (CA^{N_p})^T]^T.
\]

In the control law we use only the first increment of the input

\[
\Delta u(k_i) = K_g r(k_i) - K x(k_i),
\]

where \( K_g \) is the first element of the vector \( (\Phi^T \Phi + \bar{R})^{-1} \bar{R} \), and \( K \) is the first row of the matrix \( (\Phi^T \Phi + \bar{R})^{-1} \Phi^T F \)(a receding horizon principle). The equation in finite differences
of the closed-loop MPC-system is the following:

\[
\begin{align*}
x(k+1) &= (A - BK)x(k) + BK_ry(k), \\
y(k) &= Cx(k), \quad C = [0_m\ 1].
\end{align*}
\]  

These feedback coefficients can be used both in discrete (Fig.2) and continuous (Fig.3) models. The transient response of continuous system is shown on Fig. 4. When a plant model is in continuous time and a control algorithm is in discrete time the whole control system belongs to the class of hybrid systems [4].

**B. Comparison with a PID controller**

We designed the control law with the PID-controller, that in comparison with the MPC algorithm showed larger overshoot during the same time of the transient response (Fig. 4) at the set point of 0.05 m.

**C. Application of the MPC algorithm to the non-stationary plant model**

Now let in (1) the plasma is described by the differential equation with time-varying parameters instead of the transfer function at the first 0.2 seconds (Fig. 5): 

\[
\dot{Z} = a(t)Z + b(t)I.
\]

Then, on every step of the discretised system the feed-back coefficients $K_x(k)$ and $K_y(k)$ can be computed from $(A, B, C, D)$-matrices of the continuous system in (1) with considering of the unstationary equation (Fig. 6) in line with the principle of the receding horizon in (2). $K_x$ is the vector, computed from the last three elements of $K$. The transient response at a step function input of the closed-loop system (2) with the unstationary MPC algorithm is shown in Fig. 7.

**D. Adaptive parameter estimation**

In case of the unknown $a(t)$ parameter we may design a state estimator that will make possible to create an adaptive MPC algorithm on the base of input and output signals (Fig. 8). Let’s start with observer’s equation for plasma model [5]:

\[
\dot{\hat{Z}} = \hat{\dot{a}}\hat{Z} + bI + k(Z - \hat{Z})
\]

where $\hat{Z}$ is the estimation of the output, $\hat{\dot{a}}$ is the estimation of the unknown parameter, $k$ is the positive coefficient. Let
the function

\[ Q = (\hat{Z} - Z)^2 \]

is the square of the deviation of the estimated output. Then, the anti-gradient descent equation will look like this:

\[ \dot{\hat{a}} = -\lambda_a \nabla_{\hat{a}} Q = 2\lambda_a (Z - \hat{Z})\alpha. \]

Here \( \alpha = \frac{\partial \hat{Z}}{\partial \hat{a}} \) is the sensitivity function, \( \lambda_a \) is the positive coefficient. Finding the derivative of the observer’s equation by the parameter estimation and changing places of derivatives, one gets the differential equation relative to the sensitivity function:

\[ \dot{\alpha} = \hat{Z} + (\hat{a} - k)\alpha. \]

One can get the estimation \( \hat{a} \) of \( a \) parameter by solving this system of 3 differential equations. The process of the estimation of the time-varying parameter \( a \) is shown in Fig. 9. This estimation is inserted into the MPC algorithm instead of the real \( a \) parameter, to calculate the new MPC coefficients \( K_x \) and \( K_y \).

**IV. CONCLUSION**

- The MPC adaptive algorithm was applied to control the non-stationary unstable model of the plasma vertical position of the tokamak T-15.
- The state adaptive observer was designed and applied to the on-line estimation of the unknown plant model parameter.

**REFERENCES**


