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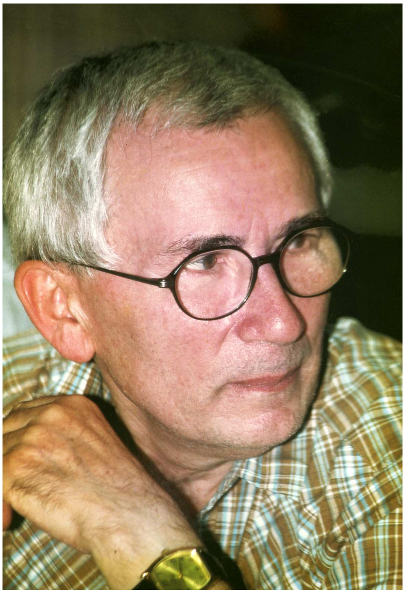
Mikhail Ivanovich Shtogrin (on his 80th birthday)



On 25 September 2018 the prominent Russian geometer, leading researcher in the Department of Geometry and Topology of the Steklov Mathematical Institute of the Russian Academy of Sciences, and doctor of the physical and mathematical sciences Mikhail Ivanovich Shtogrin observed his 80th birthday.

He was born in the village of Verbyatin in the Buchach district of the Ternopol oblast (a part of Poland before 1939). His father Ivan Vasilievich Shtogrin was a weaver, and his mother Teklya Mikhailovna was a peasant. Mikhail (Misha) was the fourth child in the family. By that time, the family lived in a house in the outskirts of the village. Night visits of uninvited guests to the isolated house, both during the war and in the postwar period, alarmed the family, especially considering that in the Soviet period the father was a secretary of the village council. As was customary in peasant families, Misha and his older sisters worked hard helping their parents. In 1945 he enrolled in a rural school, where he was a good student; mathematics was especially easy for him.

In 1955 Shtogrin enrolled in Chernovtsy State University. Upon graduating, he was retained there as an assistant professor in the Department of Algebra and Geometry. In 1964 V. A. Efremovich visited Chernovtsy University to give lectures on Lobachevsky geometry. He was impressed by a young assistant professor who helped him resolve a mathematical difficulty that arose during his lecture, and upon his return to Moscow he told Boris Delauney about the encounter. In 1965 Delauney invited Shtogrin to begin postgraduate studies as his student. Since then, Shtogrin's life was inextricably linked with the Department of Geometry, and since 1983 with the Department of Geometry and Topology (formed by merging two departments) at the Steklov Mathematical Institute.

Photo by Ludwig Danzer.

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In the 1960s, Delauney worked successfully in the general theory of stereohedra. Stereohedra are convex polytopes that can tile Euclidean space with a transitive symmetry group. The theory of stereohedra is a natural generalisation of the classical theory of parallelohedra, which was created in works of E. S. Fedorov, H. Minkowski, G. F. Voronoi, and others. On the other hand, the theory of stereohedra is connected with an arbitrary crystallographic group, not just with a group of translations as in the case of parallelohedra. Therefore, despite certain progress achieved by Delauney in the general theory, the question of finding the combinatorial types of three-dimensional stereohedra turned out to be much more difficult than in the case of three-dimensional parallelohedra. In this connection, Delauney posed the problem of finding all three-dimensional Dirichlet–Voronoi stereohedra for the second triclinic group for his graduate student Misha Shtogrin. On the one hand, this group is second in its simplicity only to a group of translations, since its orbit is always a bi-lattice, that is, it consists of two parallel and congruent lattices. On the other hand, a bi-lattice in three-dimensional space has nine parameters (six for the three-dimensional lattice plus the three coordinates of the translation vector of one lattice relative to the other), and in this sense the second triclinic group is the most complex among all 230 crystallographic groups.

In solving the problem of finding the Dirichlet–Voronoi stereohedra for the second triclinic group, Shtogrin developed a multistage method. In the first stage he used the bi-lattice nature of the problem to investigate, for each of the five three-dimensional parallelohedra, all possible intersections of the grid of the given parallelohedral tiling of the space with a moving copy of the parallelohedron. He called the dissection of the moving parallelohedron by the grid of the tiling a sketch. As a result of a very thorough and painstaking examination, he found all the 102 combinatorial types of such sketches. In this step Shtogrin obtained an important intermediate result: he found all four-dimensional parallelohedra, and showed that in the famous 1929 work of Delauney one parallelohedron was left out.

In the next stage, Shtogrin examined the regions of different sketch types in the nine-dimensional space of parameters, and solved the problem of finding all combinatorial types of the corresponding edge grids of stereohedral tilings within a given region. To do this, he introduced the concept of a '(lightning) bolt', a certain geometric graph associated with the sketch, and he defined 'bolt triggering' that occurs when a point defining a three-dimensional bi-lattice moves inside a nine-dimensional region of a sketch type. He proved that when a point moves in the region of some sketch type, the combinatorial type of the stereohedron changes if and only if a bolt is triggered. It should be noted that implementing this method required the introduction of complex geometric structures. The result of the research was completely unexpected: Shtogrin found all 180 combinatorial types of Dirichlet–Voronoi stereohedra, including 15 general and 165 special ones corresponding to the bi-lattice case. In the case when a point in the orbit is multiple with respect to the group, the two lattices merge into one. The corresponding Dirichlet–Voronoi stereohedron is one of the five known types of three-dimensional parallelohedra.

This work of Shtogrin made a strong impression on Delauney, and because of the unexpectedness of the result (185 types versus 5) and the depth of the method, and, of course, because of the 52nd four-dimensional parallelohedron that he himself had missed. Delauney was very supportive of Shtogrin's work on stereohedra, and it was published in a separate volume of *Trudy Matematicheskogo Instituta im. V. A. Steklova*.[1]

In 1968, after completing his graduate studies, Shtogrin was retained in the Department of Geometry and became a close collaborator with Delauney. He participated in almost all the projects that Delauney was involved in, and his contributions were always significant. The fact that the last ten years of Delauney's life turned out to be so successful with respect to creativity was largely thanks to Shtogrin.

At the very beginning of the 1970s, Shtogrin was occupied with the theory of lattice coverings of space with equal balls. Among the results in this area, obtained together with Delauney, N. P. Dolbilin, and S. S. Ryshkov, we note the convexity theorem for a certain body in the cone of positive quadratic forms. This theorem implies the uniqueness of a locally sparsest covering in each region of Voronoi type, and provides a method for finding all sparsest lattice coverings. This theory of coverings subsequently became important and was implemented in computer-aided algorithms for finding extreme coverings in higher-dimensional spaces.

In the 1970s, Shtogrin continued studying regular space tilings. Together with Delauney he obtained a complete classification of planigons, or two-dimensional stereohedra, and then actively participated in the development of the local theory of regular systems. Here he, together with Delauney, Dolbilin, and R. V. Galiulin obtained a number of important results and, in particular, a local criterion for both regular and multi-regular systems. He also proved a local restriction on the order of the rotational symmetry axis in locally isometric Delauney sets, which was a key point in the subsequent research by Dolbilin on the radius of regularity for these sets in three-dimensional Euclidean space.

Shtogrin made another remarkable discovery in the theory of regular partitions. Obviously, a Delauney set determines a (unique) Voronoi partition, and if the Delauney set is a regular system, then the corresponding Voronoi partition is also regular. However, he discovered that some regular Voronoi partitions may admit an irregular system of centres of the action, along with a regular system of centres.

In the 1980s–1990s, Shtogrin and his colleagues worked on a series of papers on cubic complexes and on conditions for the existence of a map from a cubic complex to the standard higher-dimensional lattice (S. P. Novikov's problem). In particular, the inflexibility of quadrillages of a sphere, a torus, and a pretzel was proved in these works. This research was continued in a series of important papers (joint with Dolbilin, Yu. M. Zinoviev, A. S. Mishchenko, and M. A. Shtan'ko) on combinatorial methods in statistical physics. In particular, the first rigorous proof of the well-known Kac–Ward formula for the Ising partition function was given.

Shtogrin's work on the theory of flexible and inflexible polyhedral and piecewise smooth surfaces also relates to this period. He proposed an exceptionally ingenious construction of a polyhedral surface of arbitrary genus that admits only a one-parameter bending which is non-trivial at all handles.

[1] Translated as *Proceedings of the Steklov Mathematical Institute.*

In yet another cycle of papers of that period Shtogrin studied piecewise smooth curvilinear embeddings of regular polyhedra (a problem of I. Kh. Sabitov) and extensions of a developable surface. There is a well-known construction of an extension of such a surface across a plane boundary curve, where the resulting surface is no longer smooth at the points of the curve, but the local Euclidean metric is preserved. Shtogrin generalized this construction to a quite unexpected theorem on the existence and uniqueness of the extension of a developable surface embedded in space across any (not necessarily plane) smooth curve that is transverse to the system of generatrix lines of the initial surface. Moreover, a surface consisting of two pieces of smooth developable surfaces intersecting in a given curved edge admits a one-parameter bending. The bending parameter can be chosen to be the angle between any two generatrix lines intersecting in a point on the curved edge. These remarkable results in the theory of isometric embeddings of surfaces, as well as other works of his, testify to Shtogrin's unique geometric talent.

Over the years Shtogrin also collaborated with Michel Deza, who drew him to research on chemical graphs and the combinatorics of fullerenes, simple three-dimensional polyhedra with pentagonal and hexagonal faces. In particular, Shtogrin generalised the concept of a fullerene to polytopes of dimension greater than three. Among the results in this direction, we mention sufficient conditions under which a disk divided into pentagons and hexagons can 'grow up' into a fullerene. In general, the problem of extending such a disk to a fullerene remains unsolved so far. The result of Shtogrin's activities in that period was two monographs that he wrote in collaboration with Deza, V. P. Grishukhin, and M. Dutour, which were published by Springer and Imperial College Press.

The deep and remarkable results that Shtogrin obtained in various fields of geometry have given him an international reputation as a geometer of the highest level. His colleagues and friends are well aware of the enormous responsibility with which he embarks on any task. It is no coincidence that Shtogrin has previously found and continues to find serious flaws in well-known works by well-known mathematicians. Everyone who has had the good fortune to collaborate with Shtogrin, work next to him, or just meet him, has been struck by his legendary modesty, friendliness, and decency.

In addition to his research work, Shtogrin has regularly participated for half a century in the organisation of many scientific conferences at the Steklov Mathematical Institute. For several decades he has worked diligently as the scientific secretary of the Department of Geometry and Topology at the institute.

Friends and colleagues wish Mikhail Ivanovich Shtogrin good health, well-being, and further new successes in his work.

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