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Asymptotically one-dimensional dynamics of high-peak-power ultrashort laser pulses

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Abstract

Laser fields with peak powers $P$ well above the critical power of self-focusing $P_{cr}$ are intrinsically unstable with respect to modulation instabilities, breaking up into multiple filaments as a part of a quintessentially three-dimensional nonlinear beam dynamics. Here, however, we show that—even for $P \gg P_{cr}$—the spatiotemporal field evolution can stay effectively one-dimensional. In this regime, observed as an asymptotic case of large diffraction lengths, the laser field can undergo a rich diversity of pulse transformation scenarios, including, most notably, pulse self-compression to subcycle field waveforms with very high peak powers, while remaining decoupled, within a limited propagation length, from beam dynamics.

Keywords: ultrafast optics, laser filamentation, self-focusing

(Some figures may appear in colour only in the online journal)

1. Introduction

Self-focusing is a universal physical effect [1, 2] that sets a fundamental limit for the peak power of electromagnetic field below which the spatiotemporal field dynamics can still be treated in terms of models with a reduced number of dimensions. Specifically, a natural assumption that a beam retains its cylindrical symmetry in the process of spatiotemporal field evolution reduces the number of spatial dimensions to two. Such an assumption makes the problem much more transparent from the physical point of view [3–6] and greatly simplifies calculations, allowing computations to be run on a standard computer or a small computer cluster instead of a supercomputer.

As one of the most impressive examples, two-dimensional models assuming a cylindrical symmetry of the beam have been instrumental in revealing fundamental aspects of filamentation of ultrashort laser pulses [7, 8]. More than two decades past its discovery [9], this phenomenon remains one of the main thrills in ultrafast optical physics, as it continues to attract much attention as an intriguing manifestation of nonlinear wave dynamics [7, 8] and finds growing applications as a pulse compression technology for high-peak-power optical fields [10–13], a method of high-energy beam delivery and high-brightness supercontinuum generation [14–16], as well as a promising approach for the remote sensing of the atmosphere [17, 18].

The rapid progress in solid-state sources of high-peak-power ultrashort pulses, however, calls for the extension of filamentation-based pulse compression technologies to laser peak powers $P$ well above the critical power of self-focusing $P_{cr} = C \lambda^2/(8 \pi n_0 n_2)$, where $\lambda$ is the wavelength, $n_0$ is the field-free refractive index, $n_2$ is the nonlinear refractive index, and $C$ is the beam-shape-sensitive numerical factor (for a Gaussian beam, $C \approx 3.79$; for a Townesian profile, $C \approx 3.72$ [19]). This regime of nonlinear pulse transformation enables the generation of temporally compressible multioctave mid-infrared supercontinua [20, 21] and opens the routes toward efficient all-solid-state sources of single-cycle and even subcycle pulses in the mid-infrared [22–24]. Nonlinear transformation of laser pulses with $P \gg P_{cr}$ encounters fundamental difficulties, as the laser beam at this level of peak power becomes intrinsically unstable with respect to modulation instabilities, breaking up into multiple filaments. Since modulation instabilities are seeded by random fluctuations of optical parameters of a medium or the field intensity across the laser beam, the assumption of a
cylindrically symmetric beam is no longer valid, and the spatiotemporal field evolution has to be generally treated in terms of a full three-dimensional model.

In this work, we show, however, that even for $P \gg P_c$, the spatiotemporal field evolution, within a limited parameter space, can stay effectively one-dimensional. In this regime, observed as an asymptotic case of large diffraction lengths, the laser field can undergo a rich diversity of pulse transformation scenarios—including, most notably, pulse self-compression to subcycle field waveforms with very high peak powers—while remaining decoupled from beam dynamics. These results provide a better understanding of pulse compression regimes attainable for free-beam ultrashort laser pulses in the bulk of transparent solids, which have been recently shown to allow the generation of single and subcycle field waveforms within a broad range of wavelengths and peak powers.

As the main goal of this work, we aim at identifying special regimes of nonlinear-optical spatiotemporal evolution of ultrashort laser pulses with peak powers well above the self-focusing threshold $P_c$, in which nonlinear pulse dynamics in the time domain is well pronounced, enabling meaningful and practically useful methods of pulse compression, but where the beam dynamics is suppressed, allowing temporal transformations to be scaled using the well-known results of purely one-dimensional analysis. As one of the most interesting and practically significant cases, we mainly focus on nonlinear-optical evolution of ultrashort field waveforms in an anomalously dispersive nonlinear medium [25–30], where solitonic transformations of such waveforms are possible. In practical terms, this means that the central wavelength of the input field waveform $\lambda_0$ is longer than the zero group-velocity dispersion wavelength $\lambda_d$ of the nonlinear material. This condition is satisfied for the output of a broad class of sources of ultrashort pulses in the mid-infrared developed in the past few years [21, 23, 24, 31–33], including multistage optical parametric chirped-pulse amplifiers (OPCPAs) [31] enabling the generation of subterawatt pulses in the mid-infrared [32, 33].

2. The model

Our numerical analysis is based on the three-dimensional time-dependent generalized nonlinear Schrödinger equation (GNSE) [7, 8] for the amplitude of the field, which is referred to hereinafter as the $(3 + 1)$-dimensional GNSE model,

$$
\frac{\partial}{\partial z} A(\omega, x, y, z) = \left[ \frac{ic}{2\omega n_0} \Delta_{\omega} + i\mathcal{D}(\omega) \right] A(\omega, x, y, z)
+ \mathcal{F} \left[ \frac{i \omega_0 T}{c} (n_2 I + n_4 I^3) A(t, x, y, z) 
- \frac{U_W(I)}{2I} A(t, x, y, z) 
- \left( \frac{i \omega_0 I}{2\epsilon n_0 \rho_e (\omega^2 + \tau_e^{-2})} + \frac{\sigma(\omega)}{2} \right) 
\right] F(t) A(t, x, y, z).
$$

Here, $A(t, x, y, z)$ is the field envelope, $A(\omega, x, y, z)$ is its Fourier transform, $I \equiv I(t, x, y, z) = |A(t, x, y, z)|^2$ is the field intensity, $x, y$ are the transverse coordinates, $z$ is the coordinate along the propagation axis, $t$ is the retarded time, $\omega = 2\pi c / \lambda$ is the radiation frequency, $\lambda$ is the wavelength, $\mathcal{F}$ is the Fourier transform operator, $\Delta_{\omega} = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$ is the diffraction operator, $\mathcal{D} = k(\omega) - k(\omega_d) + i\partial / \partial \omega \delta(\omega - \omega_0)$, $\omega_0$ is the central frequency, $k(\omega) = \omega n(\omega)/c$, $n(\omega)$ is the refractive index, $n_0 = n(\omega_0)$, $n_2$ and $n_4$ are the Kerr nonlinearity coefficients, $T = 1 + iw_0^{-1}/i\partial$, $I(t)$ is the time-dependent electron density, $\rho_e = \omega_0^2 m_e c^2 / \epsilon^2$ is the critical electron density, $U_W = U_0 + U_{osc}$, $U_0$ is the ionization potential, $U_{osc}$ is the energy of field-induced electron oscillations, $W(I)$ is the photoionization rate, $\sigma(\omega)$ is the impact ionization cross section, and $e$ and $m_e$ are the electron charge and mass, respectively.

Equation (1) extends the nonlinear Schrödinger equation (NSE) to include all the key physical phenomena that have been identified as significant factors behind the spatiotemporal evolution of ultrashort optical pulses in nonlinear media, such as dispersion and absorption of the medium, beam diffraction, Kerr nonlinearities, pulse self-steepening, spatial self-action phenomena, as well as ionization-induced loss, dispersion, and optical nonlinearities. In this model, the field evolution equation is solved jointly with the rate equation for the electron density $\partial \rho_e / \partial t = W + \sigma(\omega) \mu I^4$, which includes impact ionization and photoionization with the photoionization rate calculated using the Keldysh formalism [34]. Simulations are performed for typical parameters of YAG—a band gap of $U_0 = 6.4$ eV, the Kerr effect nonlinear index $n_2 = 4 \cdot 10^{-16}$ cm$^2$W$^{-1}$, and the higher order Kerr effect (HOKE) coefficient $n_4 = -1 \cdot 10^{-29}$ m$^3$W$^{-2}$. Dispersion of YAG crystal was included in the model through a Sellmeier relation [35]. The zero group-velocity dispersion wavelength for YAG is $\lambda_g \approx 1610$ nm. Similar to many other suitable materials, YAG exhibits anomalous dispersion in the long-wavelength part of the near-IR and in the mid-IR range. We therefore choose to work with an input field at a central wavelength $\lambda_0 = 3.9$ nm. Sub-100 fs pulses with peak powers orders of magnitude higher than the self-focusing threshold for YAG ($P_c \approx 30$ MW at $\lambda_0 = 4$ μm) can be delivered at this central wavelength by mid-infrared OPCPA sources [31–33]. Spatial modulation instabilities leading to the formation of multiple filaments are seeded in our model by superimposing a Gaussian-noise modulation on the input beam profile with a standard deviation of 4%. Simulations were performed using an MPI parallel programming interface and the CUDA graphical architecture on the Lomonosov and Lomonosov-2 supercomputer clusters of Moscow State University.

3. Results and discussion

Results of $(3 + 1)$-dimensional GNSE simulations presented in figures 1 and 2 illustrate the key tendencies in pulse envelope evolution and beam dynamics of an optical field in
an anomalously dispersive nonlinear solid. As one of the most important tendencies, readily observed in figures 1(a)–(d) and 2(a)–(d), the joint action of the Kerr nonlinearity and anomalous dispersion induces a self-compression of laser through physical scenarios identified and studied earlier for optical fibers. In the general case of freely propagating beams, however, this self-compression behavior is observed as a part of complex spatiotemporal dynamics (figures 1(e)–(h), 2(e)–(h)), involving diffraction, self-focusing due to the Kerr nonlinearity, defocusing and scattering by the transverse profile of the electron density induced by ultrafast photoionization, as well as beam filamentation, spatial modulation instabilities, ionization-induced blue shift, shock waves, and the high-order Kerr effect. In the general case of $P \gg P_{cr}$, not only the one-dimensional analysis, but any treatment assuming that the beam preserves its radial symmetry fails as the beam becomes intrinsically unstable with respect to spatial modulation instabilities, which tend to amplify random field intensity fluctuations, which feature no symmetry whatsoever. These instabilities give rise to hot spots randomly distributed across the laser beam (figures 1(i)–(l), 2(i)–(l)), induce random, asymmetric sidelobes in the angular spectra (figures 1(m)–(p), 2(m)–(p)), and lead to beam breakup into multiple filaments (figures 1(e)–(h), 2(e)–(h)). As a result, the beam eventually loses its continuity, connectedness, and spatial coherence. In this regime, the temporal evolution of an ultrashort optical waveform cannot be considered independently of its spatial dynamics, leaving no alternative other than analyzing the spatiotemporal dynamics of ultrashort field waveforms using the full $(3 + 1)$-dimensional model.

Remarkably, while any simplified one- or even two-dimensional treatment generally fails to describe this general-case scenario of pulse self-compression for $P \gg P_{cr}$, within a limited area of parameter space, 1D calculations using the

**Figure 1.** Spatiotemporal dynamics of a laser pulse with central wavelength $\lambda_0 = 3.9 \mu m$ and initial pulse width $\tau_0 = 100$ fs in an anomalously dispersive nonlinear medium: (a)–(d) field intensity within the laser pulse as a function of the propagation distance $z$, (e)–(h) beam dynamics, (i)–(l) transverse beam profiles, and (m)–(p) the angular spectra at the point of maximum pulse compression. The input peak power of the laser pulse and its input beam diameter are $P_0 = 5P_{cr}$, $w_0 = 100 \mu m$ (a), (e), (i), (m), $50P_{cr}$, $w_0 = 310 \mu m$ (b), (f), (j), (n), $150P_{cr}$, $w_0 = 534 \mu m$ (c), (g), (k), (o), and $500P_{cr}$, $w_0 = 980 \mu m$ (d), (h), (l), (p).
anomalously dispersive nonlinear medium: dispersion of optical nonlinearity, high-order dispersion, and even one-dimensional non-NSE effects, such as inertia and standard one-dimensional NSE approximation, which neglects is followed by pulse stretching. Within the framework of the GNSE model of equation

\[ \frac{\partial A}{\partial z} = \frac{1}{2} \frac{\partial^2 A}{\partial t^2} - i \beta_2 |A|^2 A - i \beta_3 |A|^4 A + i \gamma |A|^2 A, \]

Figure 2. Spatiotemporal dynamics of a laser pulse with central wavelength \( \lambda_0 = 3.9 \mu m \) and initial pulse width \( \tau_0 = 200 \text{ fs} \) in an anomalously dispersive nonlinear medium: (a)–(d) field intensity within the laser pulse as a function of the propagation distance \( z \), (e)–(h) beam dynamics, (i)–(l) transverse beam profiles and (m)–(p) the angular spectra at the point of maximum pulse compression. The input peak power of the laser pulse and its input beam diameter are \( P_0 = 5P_{cr} \), \( w_0 = 125 \mu m \) (a), (e), (i), (m), \( 50P_{cr} \), \( w_0 = 400 \mu m \) (b), (f), (j), (n), \( 150P_{cr} \), \( w_0 = 680 \mu m \) (c), (g), (k), (o), and \( 500P_{cr} \), \( w_0 = 1250 \mu m \) (d), (h), (l), (p).

\[ N = (l_d/l_0)^{1/2}, \]

\[ l_d = \frac{\tau^2}{\beta_2}, \]

\[ l_0 = \frac{\lambda}{\beta_2}, \]

\[ \tau = \frac{\lambda}{\beta_2}, \]

\[ \beta_2 = \frac{\lambda}{\beta_2}, \]

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treated as one-dimensional. An obvious condition to satisfy to keep one-dimensional treatment applicable is \( l_c \ll l_{uf} \).

Defining a pertinent spatial scale for spatial self-action effects is less trivial. The threshold for spatial self-action effects is defined in terms of the critical power \( P_{cr} \), which sets a fundamental limit for the peak power of a laser beam that can still propagate without undergoing self-focusing. For \( P \gg P_{cr} \), all spatial self-action effects, MI-induced beam breakup into multiple filaments is the most detrimental to beam continuity and spatial coherence. Field hot spots arising across the laser beam due to MIs can cause optical damage of the material. Modulation instabilities need a finite length \( l_m \) to grow from the noise level into noticeable hot spots across the laser beam (figures 1(i)–(l), 2(i)–(l)) and related distortions of the angular spectra (figures 1(m)–(p), 2(m)–(p)). Unlike the self-focusing threshold \( P_{cr} \), the \( l_m \) length is controlled by the field intensity. Indeed, the Bespalov–Talanov (BT) treatment of spatial modulation instabilities [37] gives the following expression for the MI gain: \( G = \exp(c/l_m) \).

Results of \((3 + 1)\)-dimensional simulations presented in figures 1 and 2 show how the MIs tend to build up across the laser beam as the optical field waveform propagates in a nonlinear medium beyond the \( z = l_m \) point. The BT-theory-based definition of the MI length \( l_m \) (shown by the vertical dashed line in figures 1(e)–(h), 2(e)–(h)) is seen to provide an accurate quantitative measure for the spatial scale of MI onset in \((3 + 1)\)-dimensional simulations. Modulation instabilities give rise to field intensity hot spots, clearly seen in the transverse beam profiles (figures 1(i)–(l), 2(i)–(l)) and in the angular spectra (figures 1(m)–(p), 2(m)–(p)), which define the beam patterns in the far field if plotted in the \( \nu_x = x/(\nu_0 D) \) and \( \nu_y = y/(\nu_0 D) \) coordinates, \( D \) being the distance to the observation point in the far field and \( x \) and \( y \) being the transverse coordinates. As the field propagates further along the nonlinear medium, these hot spots seed the generation of multiple filaments (figures 1(e)–(h), 2(e)–(h)).

With the MI-induced multifilamentary structure across the beam building up from the level of noise, whose intensity is typically many orders of magnitude lower than the on-axis field intensity, we set \( l_m = 5l_{uf} \) for a quantitative lower-bound estimate of the MI length and choose to keep the pulse compression length \( l_c \) shorter than \( l_m \) to avoid MI-induced effects within the length needed for the self-compression phase of the breathing soliton dynamics to occur in an anomalously dispersive nonlinear medium. A simple relation \( l_m = \kappa l_{uf} \) (\( \kappa \) being a numerical factor, which is taken equal to 5 here quite arbitrarily) in no way gives a universal formula for the spatial scale of MI buildup. Pulse self-steepening and dispersion effects, as shown by Bergé et al [38], can enhance MI growth, leading a beam to break up into multiple filaments within much smaller distances. However, as a general tendency verified by our \((3 + 1)\)-dimensional simulations, ionization and the high-order Kerr effect reduce the influence of pulse self-steepening and dispersion on MI buildup. For solid materials with parameters of dispersion and nonlinearity similar to those of YAG (i.e. as those specified above), which have been used in many recent experiments on filamentation-assisted pulse compression in the mid-infrared [39–41], full \((3 + 1)\)-dimensional analysis that includes pulse self-steepening, dispersion, ionization, and the high-order Kerr effect verifies \( l_m = 5l_{uf} \) as a reasonable quantitative estimate of the MI buildup length. As a typical example, for \( l_0 = 4.2 \text{TW cm}^{-2} \), \( \lambda_0 = 3.9 \mu m \), and \( \tau_0 = 100 \) fs, we have \( l_m \approx 2 \text{mm} \) and \( l_c \approx 1.2 \text{mm} \). The \( l_c < l_m \) criterion is thus met. For these parameters, \((3 + 1)\)-dimensional simulations with self-steepening, dispersion, ionization, and high-order Kerr effect disabled predict a total MI gain within the self-compression length \( G = \exp(c/l_m) \approx 380 \). When dispersion and self-steepening effects are included, laser pulses tend to shorten as a result of their solitonic dynamics. Within \( l_c \), the pulse width decreases from 100 to 14 fs, leading to an increase in the peak intensity up to approximately 20 TW cm\(^{-2}\) within this length. This results in a dramatic enhancement of MI buildup, in accordance with the predictions of Bergé et al [38], with a total MI gain within \( l_c \) increasing, according to \((3 + 1)\)-dimensional simulations, up to \( G \approx 8300 \). However, ionization, when added to the \((3 + 1)\)-dimensional model, limits the maximum intensity due to plasma refraction, reducing the MI gain within \( l_c \) to \( G \approx 3100 \). Finally, the high-order Kerr effect limits pulse self-compression, further lowering the maximum intensity. The total MI gain within \( l_c \) is then estimated as \( G \approx 230 \), which is even lower than the MI gain predicted by a model without pulse self-steepening, dispersion, ionization, and the high-order Kerr effect. Thus with all the non-NSE effects included in the \((3 + 1)\)-dimensional model, MIs grow slightly slower than they would grow in the absence of all these effects.

It should be also emphasized that, since \( l_{uf} \) (and, hence, \( l_m \)) is a growing function of the wavelength, while \( l_c \), on the contrary, decreases with \( \lambda_0 \), more favorable conditions for the self-compression of \( P \gg P_{cr} \) pulses without multiple filamentation are achieved deeper in the anomalous dispersion region. This is why multiple filamentation of self-compressing \( P \gg P_{cr} \) pulses at \( \lambda_0 = 3.9 \mu m \) (the wavelength provided by the recently developed mid-infrared OPCPA systems [31–33]) is much easier to avoid than multiple filamentation of high-power pulses with a central wavelength of 1550 nm, chosen in Ref. [38]. Indeed, with the high-order Kerr effect disabled, our \((3 + 1)\)-dimensional simulations accurately reproduce the regime examined in Ref. [38], with the compressed pulse undergoing multiple filamentation because of rapidly growing MIs. The high-order Kerr effect, however, was found to noticeably reduce the MI gain.

Obviously, in the general case, when no limitations are imposed on the propagation length, the 1D GNSE model fails to describe the spatiotemporal evolution of ultrashort field waveforms with \( P \gg P_{cr} \). MI-induced beam breakup into multiple filaments is one of the most prominent examples of quasitessentially three-dimensional nonlinear-optical phenomena that play an especially significant role in the \( P \gg P_{cr} \) regime. Another important manifestation of nonlinear non-1D dynamics is the scattering of a laser beam by the transverse profile of the electron density induced in the medium by ultrafast photoionization. As the electron density grows from the leading edge of a laser pulse toward its trailing
edge, this plasma scattering tends to play an especially significant role on the back of the laser pulse, giving rise to signature ring patterns in output beam profiles (figures 1(i), (k), 2(j), (k)).

Despite all the complexity of spatial dynamics involved in the evolution of high-peak-power ultrashort laser pulses, with the inequalities \( I_0 < I_{at} \) and \( I_m < I_{m} \) satisfied, the 1D model not only offers important physical insights into the temporal dynamics that occurs as a part of spatiotemporal evolution of ultrashort high-peak-power optical field waveforms, but can also provide, as calculations presented in figure 3 show, an accurate quantitative description of the key aspects of the temporal evolution of high-peak-power ultrashort pulses even deep into the \( P \gg P_\infty \) regime. Indeed, with the parameters of the input laser field adjusted in such a way as to satisfy the \( I_0 < I_{at} \) and \( I_m < I_{m} \) conditions, so that the beam does not lose its continuity and ionization does not lead to dramatic scattering within \( I_0 \) (figures 1(i) and 2(i)), results of \((3 + 1)\)-dimensional numerical simulations in figure 3 are seen to fully verify the predictions of the 1D GNSE model regarding the key tendencies in the self-compression dynamics of ultrashort high-peak-power laser pulses in an anomalously dispersive nonlinear medium.

As a striking example of the predictive power of the 1D model, the minimum pulse width of a high-peak-power optical waveform in \((3 + 1)\)-dimensional simulations shown in figures 1(a)–(d) and 2(a)–(d) is achieved almost exactly at the point of maximum pulse compression predicted by the one-dimensional GNSE. Moreover, with \( I_0 < I_{at} \) and \( I_m < I_{m} \), the 1D GNSE also accurately predicts the minimum pulse width of the compressed waveform achieved in \((3 + 1)\)-dimensional simulations. This close agreement between \((3 + 1)\)-dimensional simulations and the one-dimensional GNSE model is articulated in figures 3(a) and (b), where the points of maximum pulse compression in \((3 + 1)\)-dimensional simulations (circles, triangles, and diamonds) are shown against the results of 1D GNSE calculations (solid lines). Predictions of these two models for \( I_0 \) are seen to agree very well within a broad range of parameters.

The predictive power of the 1D model is in no way limited to the characteristic spatial scales of the problem and the minimum pulse widths at the point of maximum pulse compression. Comparison of 1D GNSE and full \((3 + 1)\)-dimensional simulations shows (figure 3(c)) that, with the conditions \( I_0 < I_{at} \) and \( I_m < I_{m} \) satisfied, the 1D GNSE model can also provide accurate predictions for the shape of the compressed laser pulse. As an important example, the temporal envelope of the pulse produced at the point D in figures 3(a) and (b) features a central peak with a pulse width \( \tau_c \approx 12 \text{ fs} \). At the central wavelength defined as the wavelength corresponding to the maximum of the spectrum of this pulse (figure 3(d)), \( \lambda_m \approx 4.3 \mu\text{m} \), this pulse width corresponds to 0.8 field cycles. With the central wavelength calculated from the spectrum of the compressed pulse in figure 3(d) as

\[
\tau_c = \frac{1}{2\pi} \sqrt{\frac{\lambda_m}{c}}
\]
the input beam diameter $w_0$ by means of $(3+1)$-dimensional numerical simulations. The diffraction length is shown in the upper abscissa axis. (a) Input pulse width is $\tau_0 = 100\,\text{fs}$ and the field intensity is $I_0 = 1.6\,\text{TW cm}^{-2}$ (diamonds), 2.5 TW cm$^{-2}$ (triangles), and 4.2 TW cm$^{-2}$ (circles). The horizontal lines show the pulse width at the point of maximum pulse compression calculated using the 1D GNSE for $I_0 = 1.0\,\text{TW cm}^{-2}$ (dashed line), 2.5 TW cm$^{-2}$ (dotted line), and 4.2 TW cm$^{-2}$ (dash-dotted line). (b) Input pulse width is $\tau_0 = 200\,\text{fs}$ and the field intensity is $I_0 = 0.9\,\text{TW cm}^{-2}$ (diamonds), 1.6 TW cm$^{-2}$ (triangles), and 2.6 TW cm$^{-2}$ (circles). The horizontal lines show the pulse width at the point of maximum pulse compression calculated using the 1D GNSE for $I_0 = 0.9\,\text{TW cm}^{-2}$ (dashed line), 1.6 TW cm$^{-2}$ (dotted line), and 2.6 TW cm$^{-2}$ (dash-dotted line). The (t, z) and (x, z) field evolution maps for points A—H calculated with the use of the $(3+1)$-dimensional model are shown in figures 4.(a), (c), (i), (m) (A), (b), (f), (j), (n) (B), (c), (g), (k), (o) (C), (d), (h), (l), (p) (D), 2(a), (e), (i), (m) (E), (b), (f), (j), (n) (F), (c), (g), (k), (o) (G), (d), (h), (l), (p) (H).

$$\lambda_c = \int_{-\infty}^{\infty} |A(\omega)|^2 d\omega / \int_{-\infty}^{\infty} |A(\omega)|^2 d\omega \approx 3.9\,\mu\text{m},$$

we arrive at an estimate of 0.9 field cycles.

The main peak in the envelope of the compressed pulse is preceded by a pedestal, which contains less than 25% of the total energy of the compressed pulse and whose peak intensity is an order of magnitude lower than the intensity at the center of the 12 fs main peak. The peak power achieved within this peak is 50 GW, i.e., 3.1 times higher than the input peak power. Results of 1D GNSE calculations for this sub-cycle field waveform (dashed line in figure 3(c)) are seen to agree very well with full $(3+1)$-dimensional simulations (solid line in figure 3(c)).

Figures 4(a) and (b) highlight the role of the diffraction length as a parameter that largely controls, along with the $l_c$ versus $l_0$ relation, the convergence of the full $(3+1)$-dimensional model to the 1D GNSE model; we use both of these models to calculate the minimum pulse width $\tau_c$ at the point of maximum pulse compression, $\tau_c = \tau_0$, as a function of the input beam diameter $w_0$. As long as the central wavelength of the field waveform $\lambda_0$ is fixed, varying $w_0$ is equivalent to tuning the diffraction length, which is also shown in figures 4(a) and (b) along the upper abscissa axis. For each curve in figures 4(a) and (b), the peak intensity $I_0$ in the input beam is kept constant. The input pulse width is taken $\tau_0 = 100\,\text{fs}$ in figure 4(a) and 200 fs in figure 4(b). Predictions of the one-dimensional GNSE model for the pulse width $\tau_c$ for different $I_0$ are shown by dashed horizontal lines in figures 4(a) and (b).

As can be seen from figures 4(a) and (b), for short diffraction lengths, pulse compression in the full $(3+1)$-dimensional model is much less efficient than predicted by the one-dimensional GNSE. This gap between the predictions of the two models is due to the strong diffraction of the laser beam, which, for $l_{\text{eff}} < l_c$, leads to a considerable decrease in the field intensity within the pulse compression length, thus significantly reducing the efficiency of spectral broadening and suppressing pulse compression.

As the input beam diameter is increased to an extent that $l_{\text{eff}}$ is still shorter than $l_c$, but the peak power within the beam,

$$P = \max_{x,y} \int_{-\infty}^{\infty} I(x, y, t) dx dy,$$

where $t$ is time, becomes higher than $P_{\text{cr}}$, spatial self-action effects come into play. Remarkably, in this regime, the full spatiotemporal dynamics of an optical field waveform in an anomalously dispersive nonlinear medium may lead to pulse self-compression to pulse widths even shorter than those attainable in one-dimensional soliton pulse compression (cf solid curves in figures 4(a) and (b)). This enhancement of pulse compression in three-dimensional field evolution is due to the self-focusing of the laser beam, which serves to provide higher field intensities within the pulse compression length, yet keeping the beam continuous as the peak power is not high enough to induce beam breakup into multiple filaments. Specifically, with $\tau_0 = 200\,\text{fs}$, $w_0 = 370\,\mu\text{m}$, and $I_0 = 0.9\,\text{TW cm}^{-2}$, the self-focusing length estimated using the Marburger formula

$$l_{\text{sf}} \approx 0.367 l_0 \sqrt{|\left|I_0/P_{\text{cr}}\right|^2 - 0.852|^2 - 0.0219|^{-1/2}} \approx 5.8\,\text{mm},$$

is comparable with the self-compression length, $l_c \approx 4.7\,\text{mm}$, indicating the significance of self-focusing for self-compression for this set of parameters. As the beam diameter is increased with fixed $\tau_0$ and $I_0$, $l_{\text{sf}}$ grows faster than $l_c$. For $w_0 = 1.49\,\text{mm}$, $\tau_0 = 200\,\text{fs}$, and $I_0 = 0.9\,\text{TW cm}^{-2}$, for example, we have $l_{\text{sf}} \approx 19\,\text{mm}$, while $l_c \approx 5.6\,\text{mm}$. As self-action no longer plays a significant role in pulse self-compression, the compression coefficient (diamonds and
triangles in figures 4(a), (b)) tends to its value defined by 1D dynamics (dashed and dotted lines in figures 4(a), (b)).

Finally, with sufficiently large \( l_{\text{if}} \), the pulse width \( \tau_c \) and the compression ratio \( \eta \) at the point of maximum pulse compression asymptotically converge, as long as \( l_c \) is kept much shorter than \( l_{\text{if}} \) for \( P \gg P_c \), to the predictions of the one-dimensional GNSE model (figures 4(a) and (b)). In this regime, the one-dimensional GNSE model not only offers useful insights into the temporal part of complex spatiotemporal field dynamics—and can serve as a useful guide for much more complicated and computation costly \((3+1)\)-dimensional modeling—but can also provide highly accurate quantitative predictions for the most significant spatial scales of the problem, as well as minimum pulse widths and maximum field intensities attainable as a part of pulse self-compression in an anomalously dispersive nonlinear medium.

4. Conclusion

To summarize, we have shown in this work that, even for optical field waveforms with peak powers many orders of magnitude higher than the self-focusing threshold, the spatiotemporal field evolution can stay effectively one-dimensional, with full \((3+1)\)-dimensional modeling of spatiotemporal field evolution in a nonlinear medium asymptotically converging to the predictions of the one-dimensional GNSE. In this regime, observed in the limiting case of large diffraction lengths, the laser field can undergo a rich diversity of pulse transformation scenarios, including, most notably, pulse self-compression to subcycle field waveforms with very high peak powers, while remaining decoupled, within a limited propagation length, from beam dynamics.

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