STABILIZATION OF HYPERBOLIC PLYKIN ATTRACTOR BY THE PYRAGAS METHOD

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Theory of hyperbolic attractor

The set Λ is called a hyperbolic attractor of the dynamical system if Λ is closed, topologically transitive hyperbolic set and there exists the neighborhood $U \supset \Lambda$ such that $\Lambda = \bigcap_{n \geq 0} f^n U$.

The well-known examples of the hyperbolic attractors are the Smale-Williams' solenoid and Plykin's attractor.

Plykin's sphere is obtained by the transformation of the disc domain $T = S^2$ into itself, where S^2 - a unitary disc in R^2.

Then $f: T \to T$, $f(x,y,z) = (\cos\varphi \sin\phi, \sin\varphi \sin\phi, \cos\phi)$, where k>2 determines the compression "by thickness", sets the disc as a subset $T \subset R^3$.

System of equations that has hyperbolic Plykin attractor

$$\dot{X} = -2\varepsilon Y^2 \Omega_1 \left(\cos(\omega_2 \cos\omega_1 t) - X \sin(\omega_2 \cos\omega_1 t)\right) + kY\Omega_2 \left(\cos(\omega_2 \cos\omega_1 t) - X \sin(\omega_2 \sin\omega_1 t)\right) \sin\omega_1 t,$$

$$\dot{Y} = 2\varepsilon Y \Omega_1 \left(X \cos(\omega_2 \cos\omega_1 t) - \tfrac{1}{2}(1 - X^2 + Y^2) \sin(\omega_2 \cos\omega_1 t)\right) - k\Omega_2 \left(\cos(\omega_2 \sin\omega_1 t) + \tfrac{1}{2}(1 - X^2 + Y^2) \sin(\omega_2 \sin\omega_1 t)\right) \sin\omega_1 t,$$

$$\Omega_1 = \frac{2X \cos(\omega_2 \cos\omega_1 t) + (1 - X^2 - Y^2) \sin(\omega_2 \cos\omega_1 t)}{(1 + X^2 + Y^2)^2},$$

$$\Omega_2 = \frac{-2X \sin(\omega_2 \sin\omega_1 t) + (1 - X^2 - Y^2) \cos(\omega_2 \sin\omega_1 t)}{1 + X^2 + Y^2} + \frac{\sqrt{2}}{2}.$$

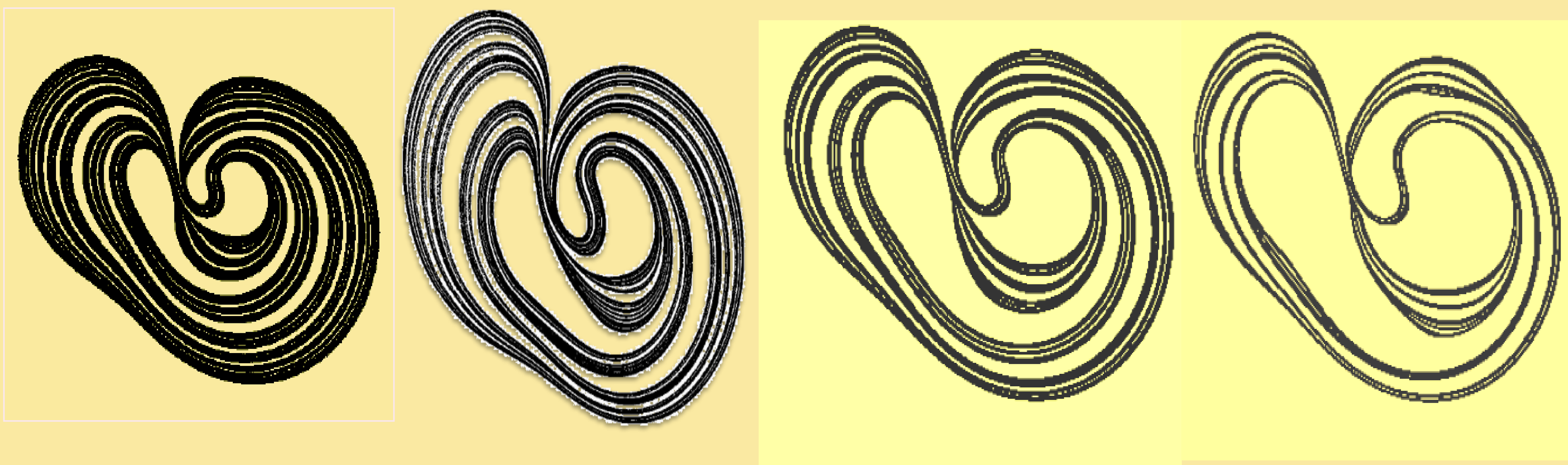
Delayed feedback

$$\dot{X} = -2\varepsilon Y^2 \Omega_1 \left(\cos(\omega_2 \cos\omega_1 t) - X \sin(\omega_2 \cos\omega_1 t)\right) + kY\Omega_2 \left(\cos(\omega_2 \cos\omega_1 t) - X \sin(\omega_2 \sin\omega_1 t)\right) \sin\omega_1 t,$$

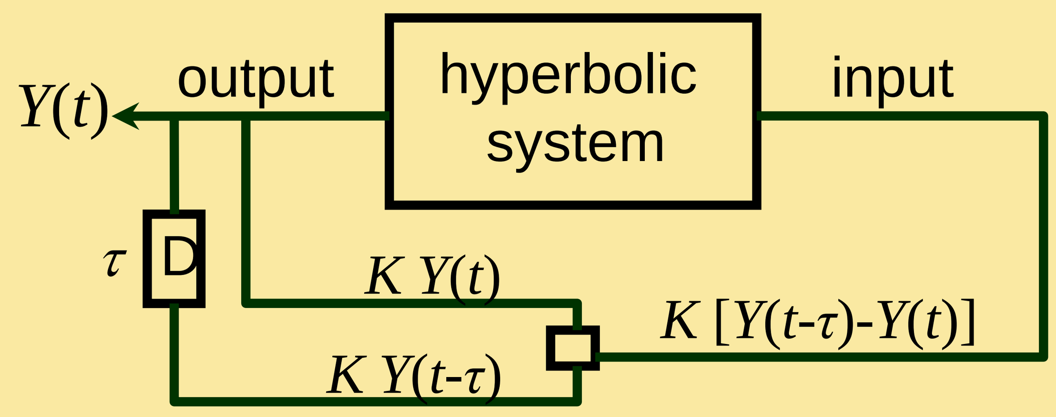
$$\dot{Y} = 2\varepsilon Y \Omega_1 \left(X \cos(\omega_2 \cos\omega_1 t) - \tfrac{1}{2}(1 - X^2 + Y^2) \sin(\omega_2 \cos\omega_1 t)\right) - k\Omega_2 \left(\cos(\omega_2 \sin\omega_1 t) + \tfrac{1}{2}(1 - X^2 + Y^2) \sin(\omega_2 \sin\omega_1 t)\right) \sin\omega_1 t + D_{Y,\tau}.$$

$$D_{Y,\tau} = K[Y(t-\tau) - Y(t)]$$

Evolution of hyperbolic Plykin attractors



Pyragas Method of stabilization of hyperbolic Plykin attractor



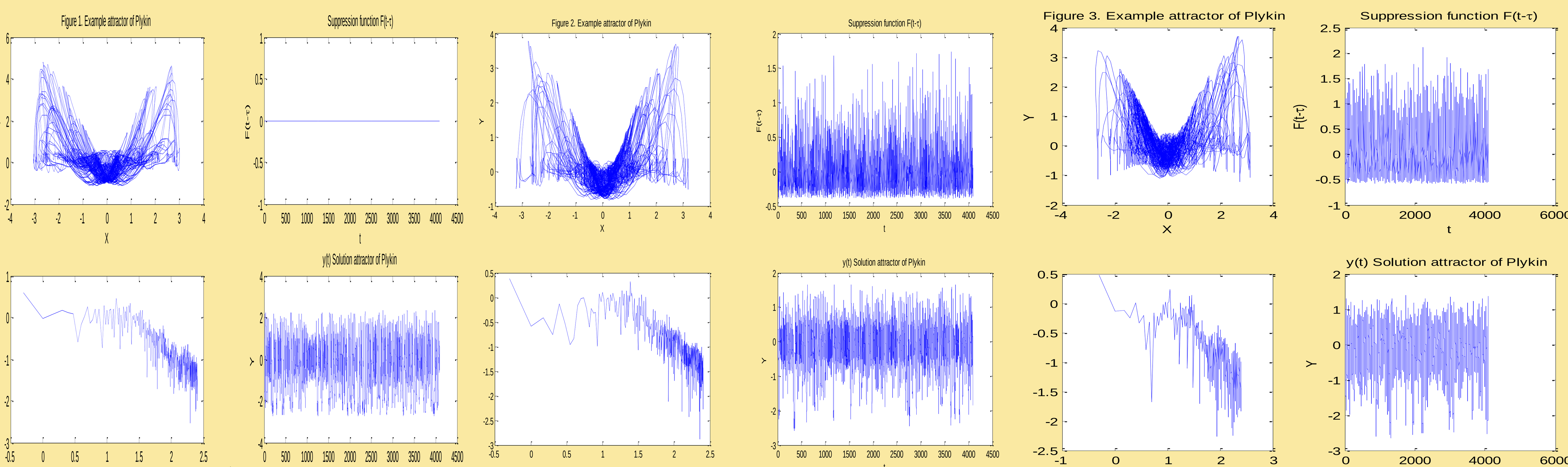
Pyragas Method

The smooth family of the nonlinear systems with control of ODE

$$\dot{x} = F(x, \mu, u)$$

$$x \in M \subset R^m, \mu \in L \subset R^k, u \in U \subset R^n, F \in C^\infty$$

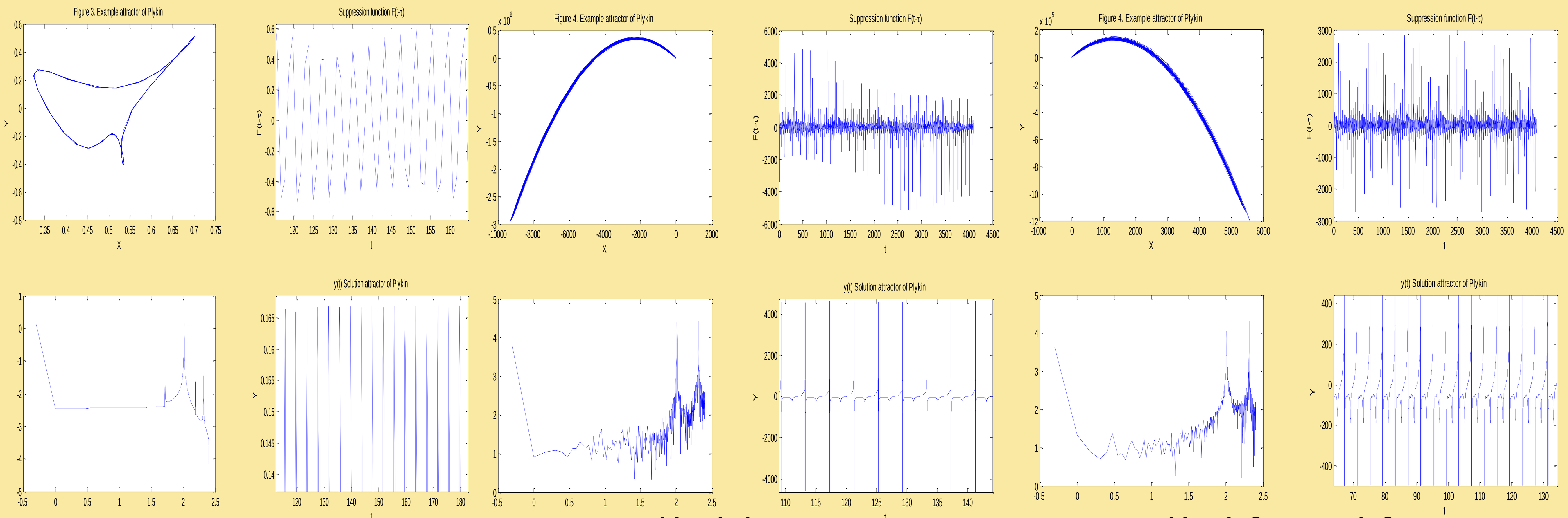
that depends on control parameters vector: u . Let us need to stabilize the unstable limit circle $x^*(t, \mu^*)$ of the period T, which is the solution of the system of the family when $u=0$ and $\mu = \mu^*$. Let, under the same values of the parameters $u=0$ и $\mu = \mu^*$, the family have a regular or singular attractor. Then the stabilization of the circle $x^*(t, \mu^*)$ is found by the feedback law with delay of the form $u(t) = K(x(t) - x(t-T))$, where K — matrix of the coefficients. Initial condition for $x(0)$ we choose in a sufficiently small neighborhood of the circle's orbit. The solution $x(t)$ of the system: $\dot{x} = F(x(t), \mu^*, K(x(t) - x(t-T))$ with the feedback when $\mu = \mu^*$ might converge to the relevant unstable circle $x^*(t, \mu^*)$.



Y/X K=0

K=1.0 , τ =0.6

K=1.0 , τ =1.0

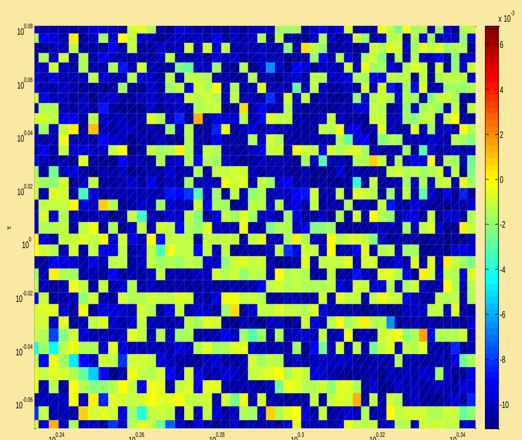


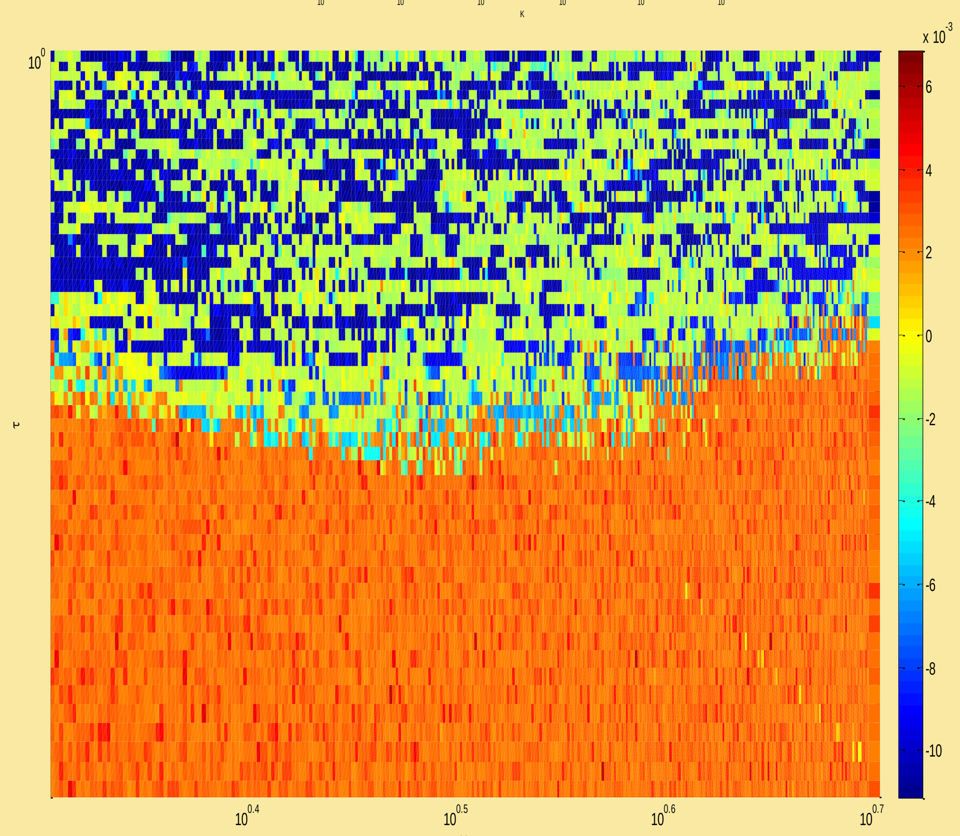
K=1.8 , τ =1.0

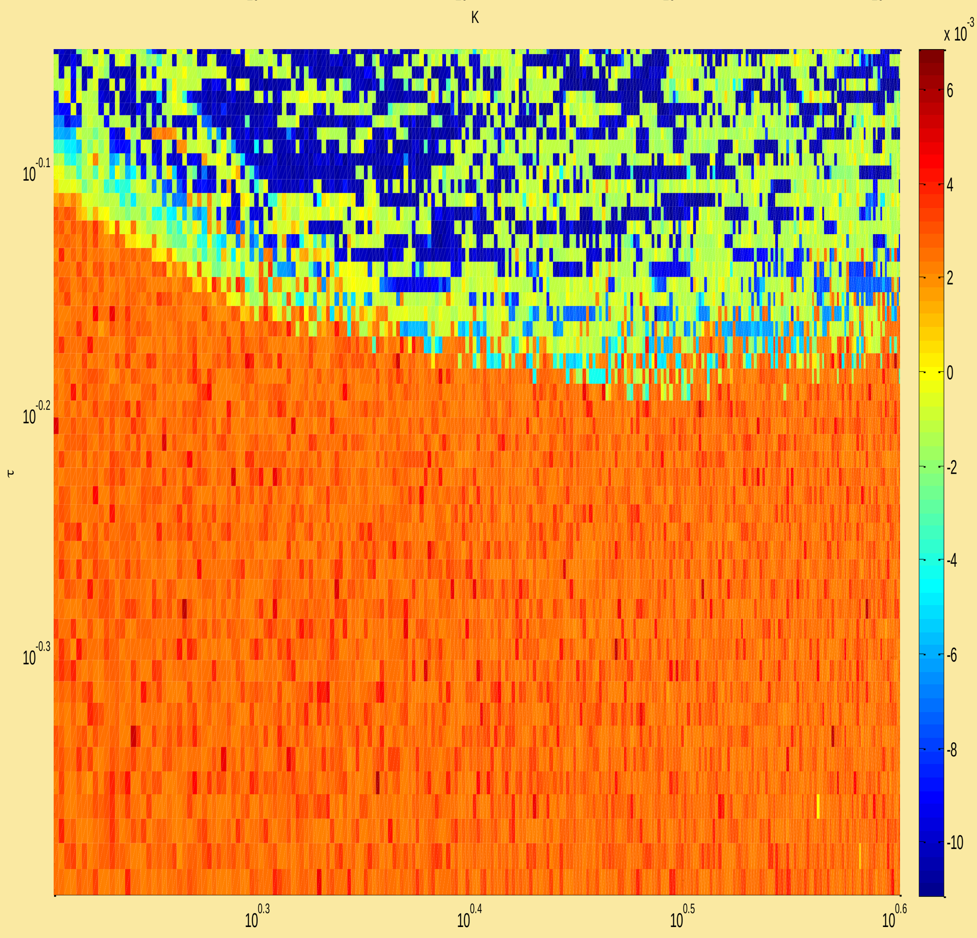
K=-1.1 , τ =1.8

K=-1.0 , τ = 1.9

τ/K







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2. K. Pyragas. Continuous control of chaos by self–controlling feedback. Phys. Lett. A 170 (1992) 421 – 428.