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Theoretical investigation of atomic nitrogen adsorption on Si (111) surface in the framework of molecular dynamics approach

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For many years nitridation process of silicon surface was studied \cite{1}. Today this process is applied at the initial stage of semiconductor heterostructures growth on a substrate of silicon (111) surface by the MBE (molecular beam epitaxy) method. The nitridation process model was proposed. That model contained chain of interdependent processes: adsorption of ammonia on a silicon surface, the ammonia dissociation which is followed by a hydrogen desorption (in the form of molecular hydrogen or molecules of water) and by adsorption of atomic nitrogen on a surface, diffusion of atomic nitrogen through Si layer to the reaction front and into the Si substrate, etc. The complete theoretical research of nitridation process requires research at the atomic level of all listed stages.

In this work theoretical research of one of the initial stages of nitridation process was conducted. Adsorption energy of atomic nitrogen in various high-symmetric positions has been calculated on surface (111) of silicon using molecular dynamics approach. These calculations were used to verify the results of molecular dynamics (MD) modeling of the Si-N system. Tersoff potential was applied with the sets of parameters received as a result of the solution of a problem of parametrical identification \cite{2,3}. Results of ab initio calculations were used as the reference values at the stage of the fitting of interatomic potential. Molecular dynamics calculations of nitrogen atom adsorption at the same positions of Si(111) surface were performed using obtained potential. The results of these calculations were in a good agreement with first-principles calculations. The most energetically favorable position for nitrogen atom adsorption was found to be the same via both approaches \cite{4}. Thus, it was concluded that obtained potential is suitable for use in further calculations for the purpose of modeling of silicon surface nitridation process.

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Approaches to the solution of the optimization problem of interatomic potential fitting

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For the purpose of carrying out practical researches in the field of mathematical simulation of crystalline structures with the given properties as well as for simulation of dynamic processes, such as processes of growth of multi-layer semiconductor nanostructures, it is necessary to apply the modern potentials of interatomic interaction (Tersoff, Brenner-Tersoff, etc.). To solve the task of parametric identification for specific material with the given chemical formula [1], target function of the following form is used:

$$F(\xi) = \sum_{i=1}^{m} \omega_i \left( f_i(\xi) - \hat{f}_i \right)^2 \to \min, \quad \xi \in X,$$

where \( \hat{f}_i \) – reference value of the property \( i \), \( f_i(\xi) \) – value of the same property obtained as a result of a calculation for the given set of basis atoms, \( \xi \in R^n \) – vector of the selected parameters, \( \omega_i \) – weight factor. The admissible set \( X \subseteq R^n \) is a parallelepiped with boundaries chosen so that it will certainly contain possible range of parameters. It is required to define a set of the parameters \( \xi \in R^n \) which minimizes function \( F(\xi) \). Such a set will provide the minimum deviation of the calculated characteristics of material from reference values, thereby it will allow to describe its properties most precisely. This task becomes significantly complicated upon transition from single-component (Al, Si, Fe, N, Ge, etc.) to two-component and multicomponent materials. This complication results from the increase in number of optimizable parameters. Therefore the search time for parameter sets on which the function minimum is reached increases.

It is necessary to point out that due to complication of target function in the case of introduction of additional atomic components, it is possible to increase uniqueness of identification of potential parameter values if the number of atoms in a chemical formula remains the same. In this work parameter \( m \) in (1) takes on values from one to three. The list of characteristics used in (1) includes cohesive energy, lattice parameter, volume
elastic modulus. Single-component and two-component materials were considered. Methods of a zero order (a method of the Nelder-Ministry of Foreign Affairs [2], Granular Radial Search [3]) were applied. Besides, the technology of the fast automatic differentiation (FAD), described in [4] was used. It allowed us to apply the method of conjugate gradients to the solution of the optimization task and to calculate values of the first and second derivative composite functions with a great accuracy. The developed algorithms were realized in the form of the software in language C++. Multisequencing of computation according to input data was applied. It was shown that methods of a zero order gave us a chance to find sets of parameters leading to the minimum of function (1) with the given accuracy keeping the same computing expenses. The method of conjugate gradients was tested for a case when parameters of Tersoff potential for single-component crystalline materials were optimized and the dietary supplement technology [3, 4] was applied. Calculations showed that in spite of the fact that the dietary supplement technology requires additional arithmetic operations, however at the expense of high speed of convergence of the method of conjugate gradients it is possible to pick up sets of parameters for Tersoff potential with computing expenses comparable with methods of a zero order. Further it is planned to apply this technology for the calculation of sets of parameters of Tersoff and Brenner-Tersoff potentials for multicomponent systems on high-performance program complexes.

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On the Cyclicity in Controllable Systems

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We consider a dynamic process with discrete time in the form

\[ x_i(t) = k_i(t)x_i(t - 1), \quad t = 1, 2, \ldots, \]

where \( x(t) = [x_1(t), \ldots, x_n(t)] \) and \( k(t) = [k_1(t), \ldots, k_n(t)] \) are positive vectors in \( R^n \), that is, real \( n \)-dimensional space. The coefficient \( k_i(t) \) is called an expansion rate of the variable \( x_i(t) \) at the time period \( t \). This coefficient is known by the beginning of the time period. The initial vector \( x(0) \) is given.

Assume that the expansion rates are controlled by selecting the values of some regulating parameters. Denote by \( R(t) = [R_1(t), \ldots, R_N(t)] \) the vector of these parameters at the time period \( t \).

We assume that the value of \( R_j(t), j = 1, \ldots, N, \) defines the coefficient \( r_{ji}(t) \) influencing the growth rate \( k_i(t), i = 1, \ldots, n, \) such that

\[ k_i(t) = \prod_{j=1}^{N} r_{ji}(t). \]

In our case, the task consists in holding the proportions \( x_i(t)/x_j(t), i, j = 1, \ldots, n; i \neq j, \) in certain limits. This means that the following inequalities must be satisfied

\[ \frac{x_i(t)}{x_j(t)} \leqslant p_{ij}, \quad i, j = 1, \ldots, n; \quad t = 1, 2, \ldots, \quad (1) \]

where \( p_{ij}, i, j = 1, \ldots, n; i \neq j, \) are some positive constants.

Recall that a dynamic system is called controllable if there are control actions such that the system reaches the goal. In our case, the system is controllable if inequalities (1) are hold for all \( t \).

**Theorem.** Let the system under consideration be controllable. Suppose the spectrum of regulating parameter \( R_j, j = 1, \ldots, M, \) is finite. Then it is possible to use cyclically some finite sequence \( \bar{R}(1), \ldots, \bar{R}(s) \) of regulating parameters such that inequalities (1) are hold for all \( t \). This sequence must be used starting from a certain time period \( \tau_0 \).
Preventive maintenance based on parameters estimation and prediction

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For complex engineering systems under heavy-duty service the failure of which leads to heavy losses or disastrous consequences the main problem of system monitoring and diagnostics becomes not the identification and isolation of failure, but prevention of them. The solution of this task can be based on individual preventive maintenance. Predicting and estimating the state of an engineering system forms an information base for individual (condition-based) maintenance.

The difficulty in solving the problem of individual status prediction is largely caused by the lack or shortage of statistic information on field variation of system parameters. In this case the application of classical methods of mathematical statistics to the solution of status estimation and prediction problem may cause serious errors.

The paper states and solves a problem of adopting optimal estimation and prediction strategies when the stochastic properties of measurement errors and errors of status model are unavailable. We use a technique of individual robust prediction which is based on the extremely properties of Karlin polynomials [1] and the ideas of minimax estimation. This technique makes a prediction even if the number of test measurements is small.

It does not need any stochastic properties of measurement errors and other noises (it is only necessary to know their limits), obtains not only a simple average, but also secures bounds in which an actual value of measurement parameter would lie in future. This technique has adaptive properties improving the prediction accuracy in an instable situation [2].

The approach under discussion meets general requirements to any prediction procedure. Estimates found are unique, optimal and unbiased.

In addition to measurement errors, the approach allows one to take into account some other mistakes caused by the difference of real processes of parameter variation from a mathematical model adopted. But if the basic
model contains an error, then a special-purpose adaptation algorithm is proposed to improve prediction accuracy.

The algorithm is based on the ideas used in the technique of moving average or exponential smoothing and consists in weighing measurement data. By using prediction data we can, in optimal way, solve the problem of specifying the time of next inspection or preventive maintenance [3].

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Construction of the minimal sets of differential equations with polynomial right-hand side in MathCloud system

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Let us take a look at the system of common differential equations with the following vector notation

\[ \dot{x} = f(x), \]  

where \( x = (x_1, \ldots, x_n) \) be a real vector function of a real variable \( t \), and \( f = (f_1, \ldots, f_n) \) be a real vector function, whose every element \( f_i \) be a multidimensional polynomial of variables \( x_1, \ldots, x_n \).

Type (1) systems have long held a great deal interest for applications because numerous process models of various physical, economical and other natures are described by similar systems. The systems in question have become increasingly important as of late, since many well-known systems which presumably contain strange attractors belong to this particular type.

Any attractor by definition is a compact invariant set. Any compact invariant set in its turn contains a compact minimal set, described by recurrent solution. That said, the vital problem is the construction of the whole set of minimal sets, not separate recurrent solutions.

As of today, the question of generic minimal sets construction has been raised only in publications [1,2]. However, the method of modelling minimal sets with the help of numerical techniques offers additional opportunities for research into the complex behavior of dynamic systems. The subject of this publication is the problem of construction and exploration of minimal sets and attractors of system (1) in the distributed computer
environment. This construction and exploration will be based on a special method of building local solutions for the given system taking into account that function $f$, in particular, is a multidimensional polynomial (see [2]).

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**References**

Horner’s generic scheme as applied to multidimensional polynomials in MathCloud system

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Recently the problem of multidimensional polynomial evaluation has become of great importance for a wide range of applications. This in particular concerns the applications of optimal control, mathematical programming and qualitative theory of differential equations (e.g., see [1]).

This publication covers the development of a scheme for quick evaluation of multidimensional polynomials. The approach is based on the Horner’s one-dimensional scheme, which allows for a relatively quick calculation of set values with fixed precision. The calculations are carried out with the help of services provided by MathCloud system.

Horner’s scheme for one-dimensional polynomials is well-known and has a lot of research dedicated to it (e.g., see [2]). Here is a short overview of this method.

Let us consider the real polynomial

\[ P_1(x) = \sum_{i=0}^{n} a_i x^i, \]  

(1)

where \( a_i \) be an arbitrary real numbers and \( x \) be a real variable. According to [2] Horner’s Scheme for polynomials (1) has the following appearance:

\[ P_1(x) = \ldots ((a_n x + a_{n-1})x + a_{n-2})x + \ldots) x + a_0. \]  

(2)

It is easy to see that the scheme (2) allows for reducing the number of calculations in comparison with the direct scheme (1). Let us instantiate it is adaptation for multidimensional polynomials calculation as exemplified
by a two-dimensional polynomial. Specifically, let us take a look at the following polynomial

\[ P_2(x) = \sum_{i=0}^{n} \sum_{j=0}^{m} a_{ij} x^i y^j, \tag{3} \]

where \( a_{ij} \) be an arbitrary real numbers and \( x \) and \( y \) be a real variables. Horner's generic scheme for polynomials (3) can be displayed as follows

\[ P_2(x) = (\ldots ((b_n(y)x + a_{n-1})x + b(y)_{n-2})x + \ldots)x + b_0(y), \tag{4} \]

where

\[ b_i(y) = (\ldots ((a_{im}(y+a_{i,m-1})y+a_{i,m-2})y+\ldots)y+a_{i0}, \quad i = 0, \ldots, n. \tag{5} \]

It is obvious that the scheme (4), (5) converts to parallels quite well. Modeling exercises has shown that using this scheme in comparison to direct scheme (3) makes it possible to reduce the number of calculations by two or three times even in the simplest of cases still preserving the same level of precision.

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References

Zonal Feedback Control for a Heating Problem with Delay in Boundary Conditions

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We study the feedback control problem for objects with distributed parameters on the basis of continuous observation of the phase state of the object at its certain points. Consider a synthesis problem of the feedback law for a tubular heat exchanger with a steam jacket. The mathematical model of the process can be described by the following equation [1]:

$$u_t(x,t) + a \cdot u_x(x,t) = -\alpha \cdot [u(x,t) - \vartheta(t)], (x,t) \in \Omega = [0,l] \times (0,T). \quad (1)$$

Here $a = const$ is the fluid velocity; $l$ the length of the part of the heat exchanger located inside the steam jacket; $\vartheta(t)$ the temperature of the steam jacket, which is a piecewise continuous function of time; $\alpha = const$ the experimentally determined heat transfer coefficient; $u(x,t)$ the fluid temperature at the point $(x,t) \in \Omega$ from the class of functions continuously-differentiable with respect to $x$ and continuous with respect to $t$. Let the initial and boundary conditions be given, for example, in the following form:

$$u(x,0) = u_0(x,\tau) \in G_0, \ x \in [0,l], \ \tau \in [-\Delta,0], \quad (2)$$

$$u(0,t) = (1-\gamma) \cdot u(l,t-\Delta), t \in (0,T]. \quad (3)$$

Here $\gamma = \gamma(t) \in G_1 = (0,\delta), \ 0 < \delta < 1$, is the parameter characterizing the magnitude of heat loss in the process of fluid passage through the heating system; $\Delta$ the transportation delay; the continuous function $u_0(x,\tau)$ and the parameter $\gamma$ are given inaccurately, but their values belong to some given admissible sets $G_0$ and $G_1$, with known distribution functions $\rho_{G_0}(u_0(x,\tau))$ and $\rho_{G_1}(\gamma)$, which characterize the distribution of possible values of the initial and boundary conditions. Assume that inside the furnace, along the heat exchanger, there are installed thermocouples (sensors) measuring the fluid temperature at the points $x_j \in [0,l], \ j = 1,2,\ldots,N$. These measurements are used for correcting
the required temperature of the steam jacket \( \vartheta(t) \). The sensors implement operational observation and input of information about the heating process state at these points into the control system, as defined by the vector:

\[
\varpi(t) = (u(\bar{x}_1, t), \ldots, u(\bar{x}_N, t))^*, \ t \in (0, T].
\] (4)

For the fluid heating process, it is required to synthesize the regulator that, based on the measurements of the temperature at the points \( \bar{x}_j \in [0, l], \ j = 1, 2, \ldots, N \) of the heat exchanger, would ensure the maintenance of the output fluid temperature \( u(l, t) \) at the desired level by manipulating the temperature \( \vartheta(t) \) of the steam jacket. The considered feedback control problem consists in constructing the dependence of the furnace’s temperature values from the measured state values at the observational points:

\[
\vartheta(t) = w(\varpi(t)) \in V, \ t \in (0, T],
\] (5)

minimizing the specified control quality criteria, given, for example, in the form of the following functional:

\[
J(w) = \int_{G_0} \int_{G_1} \int_0^T [u(l, t; w, u_0, \gamma) - \tilde{u}(t)]^2 dt d\rho_{G_1}(\gamma) d\rho_{G_0}(u_0).
\] (6)

Here \( \tilde{u}(t) \) is the function characterizing the desired values of the fluid temperature at the right end (output) of the heat exchanger during the fluid heating process. For numerical solution of the feedback optimal control problem (1)–(6), we propose to use the approach described in [2, 3].

References

On methods of minimizing a sensitivity function under constraints

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A system of two optimization problems is considered, the first of which is a parametric convex programming problem with a parameter \( y \in \mathbb{R}^m_+ \), and the second one is the problem to minimize the sensitivity of function \( \varphi(y), y \in Y \subseteq \mathbb{R}^m_+ \), generated by the first task

\[
\varphi(y) = f(x^*) = \min\{ f(x) \mid g(x) \leq y, \ x \in X \subseteq \mathbb{R}^n \}, \ y \in \mathbb{R}^m_+,
\]

\[
y^* \in \text{Argmin}\{ \varphi(y) \mid y \in Y \subseteq \mathbb{R}^m_+ \}.
\]

Where \( f(x) \) is scalar, and \( g(x) \) is vector function, the scalar function and each component of the vector function are also convex, \( y \geq 0 \), \( \mathbb{R}^m_+ \) is positive orthant, \( X \subset \mathbb{R}^n, Y \subset \mathbb{R}^m_+ \) is convex, closed sets. Here \( y \) is vector right side of the functional constraints. The first problem of the system (1),(2) generates the sensitivity function. It is as follows: when the parameter \( y \in \mathbb{R}^m_+ \) runs positive orthant, then domestic tasks in variable \( x \in X \) generates optimum value \( f(x^*) \), which is issued on function \( \varphi(y) \) calculated at \( y \in \mathbb{R}^m_+ \). The system (1),(2) is required find the minimum of the sensitivity functions on the set \( y \in Y \), while the objective function is defined implicitly.

The report proposes saddle-point methods for solving of system (1),(2). The convergence of computing methods is proved.

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REFERENCES

Damping of oscillations of a rectangular membrane by using multiple point dampers

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The oscillations of rectangular membrane are described by equation

\[ u_{tt} = a^2(u_{xx} + u_{yy}) + g(t, x, y), \quad t \geq 0, \quad 0 \leq x \leq l_1, \quad 0 \leq y \leq l_2, \quad a = \text{const}. \]  \hspace{1cm} (1)

The initial conditions: deviation and velocity – are known:

\[ u(0, x, y) = h_0(x, y), \quad u_t(0, x, y) = h_1(x, y). \]  \hspace{1cm} (2)

On the boundary of a rectangular membrane imposes fixing conditions

\[ u(t, 0, y) = u(t, l_1, y) = u(t, x, 0) = u(t, x, l_2) = 0. \]

The problem of damping is: to find the control function \( g(t, x, y) \in L^2 \) (0 < \( t < T \), 0 < \( x < l_1 \), 0 < \( y < l_2 \)), which allows to get the state of a membrane from initial state (2) to final state

\[ u(0, x, y) = 0, \quad u_t(0, x, y) = 0. \]  \hspace{1cm} (3)

In this report we consider damping of rectangular membrane oscillations by using multiple point dampers. The function \( g(t, x, y) \) is considered as

\[ g(t, x, y) = \sum_{i=1}^{n} w_i(t) \delta(x - x_i) \delta(y - y_i), \]  \hspace{1cm} (4)

where \( w_i(t) \) — control functions and \( \delta \) — the Dirac delta-functions. To find the required control function numerically let us use decomposition in the finite difference scheme of (1) and coordinate descent method for minimizing of the membrane’s energy integral

\[ E(T) = \int_0^{l_1} \int_0^{l_2} (u_{tt}^2 + a^2 u_x^2 + a^2 u_y^2) \, dx \, dy. \]
Example 1. Let us consider (1) with \( l_1 = l_2 = 1, \ a = 1. \) The initial conditions will be \( u(0, x, y) = \sin(\pi x) \sin(\pi y), \ u_t(0, x, y) = 0, \) with steps \( h_x = 0.05, \ h_y = 0.05, \ h_t = 0.0353. \) The minimum of energy integral will be searched with precision \( \varepsilon = 0.001. \) As a function (4) we consider only one control function \( w(t) \) with restraint \( \max|w(t)| = 160 \) placed in the point \((x_1, y_1) = (0.5, 0.5).\) The problem was solved by \( T = 5.657. \) Figures 1 and 2 show process of damping for \( u(t, x, 0.5) \) and the control functions \( w(t) \) correspondingly.

![Figure 1. The process of damping.](image1)

![Figure 2. The control function.](image2)

Example 2. Let us take all conditions used in example 1. As a function (5) we consider 5 equal control functions \( w_i(t) = w(t), \ i = 1, \ldots, 5: \) the one will be placed in the point as above \((x_1, y_1) = (0.5, 0.5), \) while other four will be placed in the points: \((x_2, y_2) = (0.25, 0.25), \ (x_3, y_3) = (0.25, 0.75), \ (x_4, y_4) = (0.75, 0.25), \ (x_5, y_5) = (0.75, 0.75). \) The problem was solved by \( T = 2.121. \) Figures 3 and 4 show process of damping of rectangular membrane oscillations of \( u(t, x, 0.5) \) and the control functions \( w(t) \) correspondingly.

![Figure 3. The process of damping.](image3)

![Figure 4. The control functions.](image4)

By using multiple dampers approach we managed to decrease more than two times required minimal time for damping of rectangular membrane oscillations.

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A benchmark of heuristic algorithms for the double integrator traveling salesman problem

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As the first approximation, dynamics of some objects can be considered as a linear system. Namely, it is possible to obtain a good approximation of spacecraft dynamics (in deep space) by using the double integrator model. In this regard, dynamic versions of the traveling salesman problem (TSP) for controlled objects described by linear differential equations is important. A special case of such problem is the TSP for the double integrator [1].

We consider a double integrator visiting a set of given stationary points at a minimum travel time. Control constraints are defined in terms of a convex compact set. We obtain an upper bound for the minimum travel time, by developing the method of transformation of the original problem into a generalised traveling salesman problem. This transformation is based on a discretisation of sets of admissible visiting velocities. To solve time-optimal two-point problems, we use the duality of optimal control problems and convex programming [2].

In [1] STOP-GO-STOP heuristic algorithm was proposed. It was shown that in the worst case scenario STOP-GO-STOP provides solution with total time \( T\sqrt{2n} \), where \( n \) is a number of nodes to visit and \( T \) is the total time provided by the discretisation algorithm. Next we provide a comparison of both heuristics.

**Experiment setup.** We generate 100 random instances of routing problems. In these instances start and finish points coincide with \((0,0,0,0)\) ∈ \( \mathbb{R}^4 \). Geometric coordinates of all visiting points are drawn uniformly from rectangle with coordinates \((10,10)\) and \((110,85)\). In each instance there are exactly 14 ‘cities’ to visit. Each ‘city’ has 13 vectors of admissible visiting speed: \((0,0)\) and \((4\sin \alpha, \cos \alpha)\) where \( \alpha = \frac{\pi}{6}, \frac{2\pi}{6}, ..., \frac{12\pi}{6} \). We fix some control constraint \( p \) in \( \{0.01 \cdot 2^n : n = 0, 1, 2, ..., 12\} \subset [0.01, 40.96] \).
Then we apply heuristics to calculate routing time for all instances. These values are used to obtain the corresponding binary logarithm of average calculated time for the heuristic under constraint $p$. The resulting data are depicted in Fig. 1. Note that the average calculated time is quite similar for $p \leq 0.16$.

![Comparison of heuristics](image)

Fig. 1  Dependence of binary logarithm of average calculated time on control constraint $p$ in \{0.01 $\cdot$ 2$^n$ : $n = 0, 1, 2, \ldots, 12$\} $\subset$ [0.01, 40.96] for the proposed heuristic and STOP-GO-STOP.

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REFERENCES


Experiments with Hybrid Methods of Edgeworth-Pareto Hull Approximation in Nonlinear Multiobjective Problems with Many Objectives

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The talk is devoted to the experience of approximating the Edgeworth-Pareto Hull, i.e. the maximal set, which Pareto frontier coincides with the Pareto frontier of the feasible objective set, in nonlinear multiobjective problems with many objectives (number of objectives is larger than 3). Approximating the Edgeworth-Pareto Hull is the most complicated step of the Interactive Decision Maps technique, which is aimed at interactive visualization of the Pareto frontier and is an efficient tool for decision support in decision problems with many objectives [1]. Graphic information on the Pareto frontier helps the decision makers or the negotiators to specify the preferred non-dominated objective point (feasible goal) consciously. Then, the associated decision is provided by the computer automatically.

In the 2000s, effective hybrid methods for nonlinear problems with non-convex Edgeworth-Pareto Hull were developed. The hybrid methods, which integrate random search techniques and classic gradient-based optimization methods with evolutionary multiobjective techniques, turned out to be an efficient tool for approximating the Edgeworth-Pareto Hull in non-linear problems with hundreds of decision variables and up to 9 objectives. The talk is based on the paper [2].

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REFERENCES

Parallel implementation of genetic algorithm to search for ballistic installations optimal parameters

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The use of genetic algorithms in search for optimal parameters of ballistic installations is becoming more widely used [1]. Optimization of the parameters of such systems places high demands on computing power due to the fact that the solution of the direct problem is accompanied by the integration of a two-phase non-stationary equations of gas dynamics describing intrachamber processes [2]. The solution of the inverse problem of finding the optimal parameters of ballistic systems, for example, by the criterion of maximum output speed, requires multiple solutions of the direct problem, since to determine the numerical value of the fitness function is required every time to solve a system of partial differential equations, it is necessary to evaluate the quality of each chromosome. Experience in the implementation of genetic algorithm on personal computers shows the limitations of this approach because of the large computation time, which does not allow the calculation of large populations. Fast convergence for small populations can not guarantee finding the global extremum. At the same time the structure of genetic algorithm allows to make it fairly simple parallelization. In this work we consider the parallel implementation of the genetic algorithm.

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REFERENCES

Application of Random forest method to estimate the incurred but not reported claims reserve of an insurance company

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The purpose of this report is to explore the applicability of Random forest method to assess the incurred but not reported claims reserve (IBNR) of a non-life insurance company. The research is based on the statistical method of Random forest, presented in [1, 2], and relies on the results in [3, 4, 5]. The actual data on the direct hull insurance of the real company for the period 2009-2014 were used. The following dependence was estimated by Random forest:

\[
paid_{edited} \sim \text{crisis}_year_{of}_\text{ins}_ev + \text{ins}_sum + \text{term}_\text{end} + \text{start}_\text{quar} + \text{region} + \text{claim}_\text{delay}, \quad (1)
\]

where

paid_{edited} — sum of paid losses announced in the year next to the contract start year, RUR,

start_year — year of the policy entrance into force,

crisis_year_{of}_ins_ev — flag of "crisis year of the insurance event",

ins_sum — sum insured, RUR.

term_end — the term of the policy, in days,

start_quar — quarter of the contract beginning date,

region — sales region of the policy,

claim_delay — delay in the receipt of the loss application, calculated from the contract start date, in days.

The value of the IBNR valuated on 31.12.2014 by Random forest was compared with the results of standard calculation methods (chain ladder and Bornhuetter Ferguson on paid triangles). In general, we can say...
that the Random forest method can be applied to assess the IBNR as an alternative algorithm.

REFERENCES

Accelerated Primal-Dual Gradient Method for Linearly Constrained Minimization Problems

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In this work, we consider a constrained convex optimization problem of the following form

\[
(P_1) \min_{x \in Q \subseteq E} \{f(x) : A_1x = b_1, A_2x \leq b_2\},
\]

where \(E\) is a finite-dimensional real vector space, \(Q\) is a simple closed convex set, \(A_1, A_2\) are given linear operators from \(E\) to some finite-dimensional real vector spaces \(H_1\) and \(H_2\) respectively, \(b_1 \in H_1, b_2 \in H_2\) are given, \(f(x)\) is a \(\nu\)-strongly convex function on \(Q\) with respect to some chosen norm \(\| \cdot \|_E\) on \(E\). The last means that for any \(x, y \in Q\)

\[
f(y) \geq f(x) + \langle \nabla f(x), y - x \rangle + \frac{\nu}{2}\|x - y\|_E^2,
\]

where \(\nabla f(x)\) is any subgradient of \(f(x)\) at \(x\) and hence is an element of the dual space \(E^*\). Also we denote the value of a linear function \(g \in E^*\) at \(x \in E\) by \(\langle g, x \rangle\).

Problem \((P_1)\) captures a broad set of optimization problems arising in applications. The first example is classical entropy-linear programming (ELP) problem \([1]\), which arises in many applications such as econometrics, modeling in science and engineering, especially in the modeling of traffic flows \([2]\) and the IP traffic matrix estimation. Other examples are ridge regression and elastic net approaches, which are used in machine learning. Finally, the problem class \((P_1)\) covers problems of regularized optimal transport (ROT) \([3]\) and regularized optimal partial transport (ROPT) \([4]\), which recently have become popular in application to the image analysis.

We extend the Fast Gradient Method \([5,6]\) applied to the dual problem in order to make it primal-dual so that it allows not only to solve the
dual problem, but also to construct nearly optimal and nearly feasible solution to the primal problem ($P_1$). We also equip our method with a stopping criterion which allows an online control of the quality of the approximate primal-dual solution. Unlike [3,4,7] we provide the estimates for the rate of convergence in terms of the error in the solution of the primal problem $|f(x_k) - \text{Opt}[P_1]|$ and the linear constraints infeasibility $\|A_1x_k - b_1\|_{H_1}, \|(A_2x_k - b_2)\|_{H_2}$. In the contrast to the estimates in [5], our estimates do not rely on the assumption that the feasible set of the dual problem is bounded. At the same time our approach is applicable for the wider class of problems defined by ($P_1$) than approaches in [3,7]. In the computational experiments we show that our approach allows to solve ROT problems more efficiently than the algorithms of [3,4,7] when the regularization parameter is small.

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REFERENCES

Time dilatation principle in evolutionary games of approach

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Presentation is devoted to the time dilatation principle for solving the games of approach, for which Pontryagin’s condition, lying at the heart of all direct methods, fails. Deep insight to this condition by M.S. Nikolskij resulted in the modified condition providing construction of the pursuer’s control on the basis of the evader’s one in the past. Establishment of the close relation of the modified condition with the passage from the original game with perfect information to an auxiliary game with delayed information led to the development of an efficient approach to solving complicated games of approach.

We consider the game, whose dynamics is described by an evolutionary system of general form, encompassing a wide range of the functional-differential systems. The gist of the approach consists in artificial worsening the availability of information on the current evader’s control to the pursuer. We pass from the original game with complete information to the game with the same dynamics and the terminal set with special kind information delay, decreasing as the game trajectory approaches the terminal set and vanishing as the game trajectory hits the target. Then the obtained game with delayed information is analyzed on the basis of its equivalence to the perfect-information game with the changed dynamics and terminal set. The central idea of investigation is the introduction of the time dilatation function, through which the time delay function is expressed. The time dilatation principle proves its efficiency for solving the problems of soft meeting for a number of second-order linear differential games for which formula of the time dilatation function is deduced in explicit form. We study in detail the geometric-descriptive situation of finding ‘tracks’ of the evader which provides realization of the time dilatation principle by the way of following the evader’s trajectory by the pursuer with delay in time. Sufficient conditions on the game parameters are derived insuring feasibility of the game termination both under the geometric and the integral constraints on the players’ controls.
On the choice of the best forms of reinsurance

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The problem of choosing the form of reinsurance is a known risk of dividing the task and put in the form of the optimization problem. Standard notation and concepts are as follows:

- $S$ - the total loss on the portfolio of contracts,
- $b$ - The total premium of the portfolio without commission,
- $R$ - The transmitted part of the loss $S$ to the reinsurer,
- $b(R)$ - Award reinsurer, including its administrative costs,
- $Q = S - R$ - the total loss of the insurer,
- $b(Q) = b - b(R)$ - insurer’s premium.

In the framework of this problem is first dispersion model that values $R$ and $b(R)$ should be selected such that the dispersion does not exceed the self-retention of the desired level, and the expected results $b(Q) - E(Q)$ from the net retention was as high as possible.

For the case of one insurer and one reinsurer it is formulated as a problem

\[
\begin{align*}
\text{minimize} & \quad b(Q) - E(Q), \\
\text{subject to} & \quad D[Q] \leq d, \\
& \quad 0 \leq Q \leq S
\end{align*}
\]

The dependence structure $Q$ of $S$ in inequalities other than those subordinated to additional conditions, for example, $b(R) = f(E[R])$ or $b(R) = f(\sigma_R)$.

Additional conditions determine the structure of the solution $Q(S)$. It should be noted that in addressing the problem of determining the function additional solve the problem, or the second dispersion model:

\[
\begin{align*}
\text{minimize} & \quad D[Q] \leq d, \\
\text{subject to} & \quad b(Q) - E(Q), \\
& \quad 0 \leq Q \leq S
\end{align*}
\]

with the same additional conditions.
In this paper, the task set is solved in the original formulation, the solutions obtained are compared with the second model.

REFERENCES

The optimal marketing strategy of the firm of a special type

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We present the mathematical model of a firm selling certain product. The feature of this firm is the structure: the firm is divided into several distribution units (for example, department stores) each of which aims to achieve the best sales performance in comparison with other units. Each point has its own marketing budget, approved by the head office, which can not exceed the total marketing budget. The overall aim of the company is fair development of all units. Hence, there is the following problem of the budget allocation for all units $i$ in the set $A$:

\[
\begin{align*}
&\begin{cases}
  pD_i(c) - c_i \to \max_{c_i} \\
  \sum_{i \in A} c_i \leq C_0
\end{cases}
\end{align*}
\]

where $p$ is the price of product, $D_i(c)$ and $c_i$ are the demand for product and commercial expenses for unit $i$ respectively, $C_0$ is the budget. Thus, there is a kind of competition between units for share of the budget.

The main results of this paper are 1) the proof that there is the unique special solution of described problem and 2) the proof that the problem of fair marketing budget allocation is equivalent to the problem of maximizing the total profit:

\[
\begin{align*}
&\begin{cases}
  \sum_{i \in A} (pD_i(c) - c_i) \to \max_{c=(c_i)_{i \in A}} \\
  \sum_{i \in A} c_i \leq C_0
\end{cases}
\end{align*}
\]

\textbf{References}

The complexities of predicting the individual processes of a technical object parameters changing at the analysis of its condition

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The increasing number of emergencies of technogenic character at the present time has led to necessity of the decision of such problems as maintenance of reliability and efficiency of difficult technical objects of responsible appointment. Basically such objects are created by a small number, are operated under different conditions and realize extreme technologies. It is obvious that their refusals are connected with the big material losses or catastrophic consequences, and it is necessary to solve a problem of reliability of concrete object, instead of reliability of type of objects. Besides, at the analysis of its condition is important not the number of failures, and the ability to prevent them. But the problems of optimizing the reliability, safety and effectiveness can reasonably be considered only at appropriate formalization of such categories as the purpose, usefulness, losses, uncertainty, and decision-making for each concrete object. It is necessary to consider gravity of consequences of the erroneous decisions accepted at all stages of creation and system operation.

The prevention of failures of technical objects for critical applications largely depends on the availability of monitoring and prediction of their technical condition and residual resource \cite{1}. Given their uniqueness, the strategy of technical condition control of object on the basis of individual predicting processes of object parameters change is considered as a priority.

Sense of individual prediction is that the conclusion about potential reliability and possibility of use within a specified period of a certain specific device of their total population based on observations or the value of some chosen informative parameter \cite{2}. The main difficulties in the individual prediction lie in the fact that we have to work with small volumes of the initial data (a small set of testing results) and in the presence
of noise (error control), whose the statistical properties reliably are not known. In such conditions classical methods of mathematical statistics and the theory of random processes lose their attractive properties, and their use for predicting can lead to essential errors and low of the prediction reliability. Dominating in reliability theory probabilistic-statistical approach, which is based on statistics of failure, is also few suitable, since failures are associated with large material losses or catastrophic consequences. And in a considered situation speech should go not about fixing of refusals, and about their prevention.

As more productive, the functional-parametrical (functional-parametric) approach was offered in [3]. Within the frames of this approach failure is a consequence of operational deviations of the parameters from their initial (nominal, design) values, and the form of manifestation of failure is the output of parameters beyond the region of possible values (area of serviceability). But it has the complexities. They, in particular, are connected by that the majority systems are poorly formalized, that do not allow to use functional models for modeling of parameters change processes, and also with deficiency of the information about the parametric perturbations patterns. In the report it is analyzed, as it is possible to overcome the difficulties arising at reduction of the considered problem to a problem of decision-making in the conditions of uncertainty for realization a rational way of providing of guaranteed result. For acceleration of labour-intensive processes of calculations it is offered to use multiple technologies.

REFERENCES

Optimal control problem with state constraints

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A problem from financial mathematics is considered. The investor takes credit in bank and chooses suitable securities to get profit. The problem is to find a maximum return for given securities to meet the cash requirements and liabilities.

Consider on a given compact interval \([0, T]\) a system of differential equations:

\begin{align*}
\dot{x}_1 &= u_1, \\
\dot{x}_2 &= u_2, \\
\dot{x}_3 &= \rho(t)x_1 - \rho_0(t)x_2
\end{align*}  

and state constraints

\begin{align*}
x_1 &\leq M; \quad x_1, x_2, x_4 \geq 0; \\
x_4 &= x_3 + x_2 - (1 + d(t))x_1,
\end{align*}  

where

\[
x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} \text{ is a state vector, } \quad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \text{ is a control,}
\]

\(x_1(t)\) is volume of securities, \(x_2(t)\) is volume of credit, \(x_3(t)\) is profit (return), \(x_4(t)\) is cash position; \(u_1(t)\) is volume rate of securities, \(u_2(t)\) is volume rate of credit; \(\rho_0(t)\) is rate of credit; \(\rho(t)\) is coupon rate; \(d(t)\) is estimated risk.

The maximized value function \(x_3(T)\) could be expressed as

\[
x_3(T) = \int_0^T [\rho(t)x_1 - \rho_0(t)x_2]dt.
\]
The control vector $u(t)$ involves inequality constraints

$$u_{10} \leq u_1(t) \leq u_{11}, \quad u_{20} \leq u_2(t) \leq u_{21} \quad (4)$$

For system (1) initial conditions are given by relations:

$$x_1(0) = x_2(0) = x_3(0) = x_4(0) = 0 \quad (5)$$

The desired final state is:

$$x_1(T) = x_2(T) = 0. \quad (6)$$

Denote

$$R(t) = \begin{bmatrix} \rho_0(t) \\ \rho(t) \\ d(t) \end{bmatrix}. \quad (7)$$

As input forecasted value of vector $R(t)$ is needed; $\rho_0(t) > 0$, $\rho(t) > 0$, $0 < d(t) \leq 1$.

An approach to solution of described optimization problem is proposed. This approach utilizes the system dynamics and adjoint system of differential equations. The resulting boundary value problem is characterized by Jacobi matrices which are small (in the sense of dimension) and ill-conditioned. The proposed analytical solution for special cases serves as first approximation in homotopy techniques.

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REFERENCES

Development of automated scientific information system taking into account query optimization

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The problem of the organization of high-quality access to scientific and bibliographic information can be solved on the basis of use of the modern technologies of the databases (DB) allowing to structure data so that to provide their effective collection, processing, storage and extraction [1, 2].

Some aspects of development of automated system of collection, storage and search of scientific information are provided in [3]. The specified system is developed on the basis of use of relational databases and annual scientific bibliographic Indexes of Dorodnicyn Computing Center of FRC Computer Science and Control. The DB conceptual model Bibliographic Indexes is realized on the basis of Access DB. In an analysis result of data domain the following objects are selected: issuings, articles, authors, editors, reviewers, personnel, keywords, subject links, UDC. The model consists of a set of two-dimensional tables. For identification of entry in the table unique primary keys are used. For communication of objects the device of foreign keys is used. To input personnel data we use the divided form which simultaneously displays the data in Form mode and in Table mode. These two modes are associated with the same data source and synchronized with each other.

In process of query optimization in the DB search of the optimal execution plan of requests from all possible for the given request is carried out. Query optimization is connected to change of structure of a DB for the purpose of reduction of use of computing resources in case of execution of request. The same result can be obtained in DB by different methods which can significantly differ both on expenses of resources, and on runtime.

In relational DB the optimal execution plan of request is sequence of application of operators of relational algebra to the initial and intermediate relations which for a specific current status of a DB (its structure and filling) can be executed with minimum use of computing resources.
Now the following strategy of search of the optimal plan are used: 1) by an assessment of all swaps of the connected tables, the input methods in tables and connection types (i.e. complete search of options); 2) on the basis of the genetic algorithm by an assessment of limited number of swaps [4, 5].

The developed conceptual model Bibliographic Indexes can be used for development of systems of collection, storage and search of scientific and bibliographic information, for structuring bibliographic indexes of publications of research teams. Use of the electronic index including bibliographic resources and links to full-text sources will allow to receive quickly information that will promote increase of level and quality of activities of scientific institution.

REFERENCES

Separation of Trivial Parts from Control Systems

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We consider nonlinear control systems that are linear in controls

$$ \dot{y} = f(y)u, \quad y \in M \subseteq \mathbb{R}^n, \quad u \in \mathbb{R}^r. $$

(1)

Here, $y$ are the phase variables; $u$ are the controls; and $M$ is the phase space that is a domain. We assume that $f$ is an $n$-by-$r$ matrix in which the columns $f_\alpha, \alpha = 1, 2, \ldots, r$, are smooth vector fields; and $\text{rank} f(y) = \text{const} = r$. A solution (or phase trajectory) of system (1) is defined as a continuous piecewise smooth function $y(t)$ for which there exists a piecewise continuous control $u(t)$ such that $y(t)$ and $u(t)$ satisfy (1).

Introduce two types of so-called trivial systems. The first is

$$ \dot{y} = 0. $$

For this system there is only one solution (which is constant) for given initial point $y_0$. The second is

$$ \dot{y} = u. $$

For this system every continuous piecewise smooth function $y(t)$ is solution.

By separation of trivial part of the first type we mean reduction of system (1) by substitution of variables to the system

$$ \dot{z}_1 = 0, $$

(2)

$$ \dot{z}_2 = h(z_1, z_2)v, $$

(3)

where $z_1, z_2$ — new phase variables, $v$ — new controls.

By separation of trivial part of the second type we mean reduction of system (1) by substitution of variables to the system

$$ \dot{z}_1 = v_1, $$

(4)

$$ \dot{z}_2 = h(z_2)v_2, $$

(5)
where $z_1, z_2$ — new phase variables, $v_1, v_2$ — new controls.

The decomposition (2) (3) separates the trivial part (2). By analogy, the decomposition (4) (5) separates the trivial part (4). This is helpful in decomposing any control problem related to system (1) into two problems—a trivial problem related to trivial system (2) or (4) and, in general, a nontrivial problem related to system (3) or (5).

For reduction to (2), (3) we must construct minimal algebra Lie of vector fields containing vector fields $f_\alpha, \alpha = 1, 2, \ldots, r$ [1, p. 33]. If the rank of distribution generated by this algebra is equal to $s < n$ then the dimension of $z_2$ is equal to $s$ and the dimension of $z_1$ is equal to $n - s$. If $s = n$ then reduction to (2), (3) is impossible.

For reduction to (4), (5) we must pass from (1) to dual Pfaffian system of equations

$$\sum_{i=1}^{n} \omega_i^k(y)dy^i = 0, \quad k = 1, \ldots, q = n - r,$$

which can be found from (1) by eliminating the variables $u$ and then multiplying by $dt$. If the class of Pfaffian system of equations (6) (i.e. minimal number of variables on which Pfaffian system of equations equivalent to (6) may depend [1, p. 72]) is equal to $s < n$ then the dimension of $z_2$ is equal to $s$ and the dimension of $z_1$ is equal to $n - s$. If $s = n$ then reduction to (4), (5) is impossible.

**References**

NP-hardness of Minimum Length of Vectors Sum Problems

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We consider the following problems of finding a subset of Euclidean points (vectors) of minimum sum lengths.

**Problem 1** (*Subset of minimum sum length, given cardinality*). Given a set \(Y = \{y_1, \ldots, y_N\}\) of points from \(\mathbb{R}^q\), and a positive integer \(M > 1\). Find: a subset \(C \subseteq Y\) of cardinality \(M\) such that

\[
\left\| \sum_{y \in C} y \right\| \rightarrow \min .
\]

**Problem 2** (*Subset of minimum sum length, arbitrary cardinality*). Given a set \(Y = \{y_1, \ldots, y_N\}\) of points from \(\mathbb{R}^q\). Find: a non-empty subset \(C \subseteq Y\) minimizing the same objective function.

These problems can be interpreted, in particular, as finding a balanced subset of powers from a given set. Note that similar problems with the opposite optimization criterium (maximum) are well studied [1,2].

We prove that these problems are strongly NP-hard if the dimension \(q\) of the space is a part of the input, and NP-hard in the ordinary sense otherwise. Moreover, the problems are NP-hard even in the case of dimension 2 (on a plane).

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**References**

Cost-effective covering of the strip with identical directional sensors

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Sensor networks are often used to monitor the roads, pipelines, perimeter facilities, etc. All these objects can be modeled as a strip. When optimizing the functioning of the sensor networks it is necessary to find an optimal cover of the strip with various figures. To cover a strip the circles [1], ellipses [3] and sectors [2,4,5] are used [6]. If the sensor is equipped with a video camera, then its observation area can be considered as a sector [2,5]. Modern sensors can adjust the observation area [6], i.e. the parameters of the sector (angle and radius).

We study the problem of constructing a cost-effective regular cover of a strip with identical sectors. Three effective coverage models, which were proposed first in [5], are considered and their comparative analysis is performed which allows to obtain an upper bound for the minimum number of the sectors per unit length of the strip.

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REFERENCES

New approach to the theorems of alternatives

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The theorems of alternatives are not only of theoretical but also of considerable computational importance. They provide the opportunity of obtaining solutions to the systems of linear equalities and inequalities, to find the steepest descent direction in convex programming problems, to construct separating hyperplanes, to perform improper problems correction, to create new algorithms for linear programming and so on.

The theorems of alternatives always contain two equality and/or inequality systems which can be expressed through \(I\) and \(II\). The theorems of alternatives state that any of these two systems, either \(I\) or \(II\) is always soluble, but they are never soluble simultaneously. The input data for alternative systems \(I\) and \(II\) are the elements of matrix \(A\) and the components of vector \(b\). Nevertheless alternative systems can be obtained in another way by using different matrices of various dimensions [1].

For a given matrix \(A \in \mathbb{R}^{m \times n}\) of rank \(m\), let us take into consideration matrix \(K \in \mathbb{R}^{\nu \times n}\), where \(\nu = n - m\) is the defect of matrix \(A\). Any matrix can be used as \(K\), if \(\nu\) of its rows make up the null-space basis for matrix \(A\). Therefore \(AK^T = 0_{m\nu}\), where \(0_{ij}\) stand for \((i \times j)\)-matrix with null elements. In selecting matrix \(K\) a certain arbitrariness is observed, i.e. it can be obtained in different ways. If matrix \(A\) is presented in the block form \(A = [B \mid N]\), where \(B\) is non-degenerate, then matrix \(K\) can be put as follows: \(K = [-N^T(B^{-1})^T \mid I_\nu]\). If, using Gauss-Jordan transform, matrix \(A\) is reduced to \(A = [I_m \mid N]\), then matrix \(K\) can be expressed as \(K = [-N^T \mid I_\nu]\). Matrix \(A\) has \(m\) rank, hence system \(Ax = b\) is always soluble, but its solutions may fail to contain non-negative ones. Let us express the arbitrary solution to system \(Ax = b\) as \(\bar{x}\).

**Theorem.** Let us assume that matrix \(A \in \mathbb{R}^{m \times n}\) has rank \(m\), matrix \(K \in \mathbb{R}^{\nu \times n}\) has rank \(\nu = n - m\) and \(AK^T = 0_{m\nu}\), vector \(b \neq 0_m\), \(\rho\) is an arbitrary positive number. Then:

1) either system

\[
Ax = b, \quad x \geq 0_n, \quad (I)
\]
or system
\[ Kv = 0_v, \quad v \geq 0_n, \quad -\bar{x}^\top v = \rho > 0; \quad (II_v) \]
is always consistent;

2) either system
\[ K^\top y \leq \bar{x}, \quad (I_y) \]
or system
\[ A^\top u \leq 0_n, \quad b^\top u = \rho > 0. \quad (II) \]
is always consistent.

Alternative systems (I) and (II_v) are similar to the corresponding alternative systems in Farkas theorem, and (I_y) and (II) — to those in Gale theorem.

Note that systems (I) and (I_y) are equivalent in the sense that they are either simultaneously soluble or simultaneously insoluble. Likewise, systems (II) and (II_v) are equivalent in the sense of their simultaneous solubility or insolvency.

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REFERENCES

Technology for a set of solutions search in one global optimization problem

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The task of finding a global solution of nonconvex finite-dimensional optimization problems is one of the fundamental goals of modern optimization. Most of the developed methods focus on organizing the search in admissible region in a such way to expend fewer resources in unpromising regions. In this case, the desired result is the best value of the function, but not all possible values of the argument, delivering extreme value.

The paper considers the problem of searching the entire set of solutions in a nonlinear programming problem. The solution is a Nash equilibrium for the two-dimensional four-person game, so a global extremum value is zero. Statement of the problem contains four pairs of variables corresponding to the strategies of the players and four auxiliary variables that allow one to convert game statement to a mathematical programming problem.

At first, a series of calculations using the MSBH method and different starting values allowed us to identify the fact that problem has multiple solutions. These solutions have three pairs of strategy-variables, acceptable values for the other pair of variables, auxiliary variables were also different. Using this information as initial, the computational experiments when narrowing the search area was carried out. During calculations we found out that despite the fact that the solution is not unique and belongs to a certain curve, local methods stop at the point of admissible region boundary. The use of the sequential discretization method of the search region allowed to determine the interval, which includes the values of the variables-strategies delivering function maximum, and identify a linear dependence of the auxiliary variables and define the functional form of this dependence. For the stated problem we got the solution set description in general form.

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Numerical study of the problem of spherical mobile robot optimal control

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The problem of spherical robot control is sufficiently modern and relevant, the main application of such robots is a research area, i.e. it could be security, study of the surface or even a space. Prototypes developed by researchers in this field can be divided into two types according to the principle of motion. The first group includes robots, which have inside a mechanism imparting a rotational moment (see eg. \cite{1}). The second type of mechanisms described, e.g., in \cite{2} controls the position of the center of mass.

The paper considers a model describing the robot, which designed as a spherical shell with the rotors (engines) inside. The robot dynamics reduced to contact coordinate is described by the following system of ordinary differential equations:

\begin{equation}
\dot{x} = G(x)J^{-1}(x)J_r \sum_{k=1}^{n} n_k(x)u_k,
\end{equation}

where state and control vectors are $x \triangleq [u_a, v_a, u_o, v_o, \psi]^T$, $u \triangleq [\dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_3]^T$, and $\phi_i$, $i = 1, 3$ denote the rotation angles of the engines.

The position of the contact point on the plane is given by the coordinates $u_a$, $v_a$, and its coordinates on the sphere are specified by the angles $u_o$, $v_o$, $\psi$ is the auxiliary coordinate. Matrices $G$, $J$ and the vectors $n$ as a result of sphericity of the source object contain trigonometric expressions and determine the strongly non-linear right parts of the system.

Consider optimal control problem of transfer the robot from point $x(0)$ to point $x(T)$ under the condition of minimizing the control source

$$J = \int_0^T u^T u dt.$$ 

The problem becomes significant more complicated, if we exclude one of the controls ($u \triangleq [\dot{\phi}_1, \dot{\phi}_2]^T$), i.e. leave only two rotors, which are in
the same plane with the axes of rotation and perpendicular to each other. Then, if the contact point falls on the equator \( u_o = \pm (2k + 1) \frac{\pi}{2}, k = 0, 1, 2, \ldots \) the plane of the rotors placement becomes perpendicular to the plane of rolling. The system becomes locally uncontrollable, it comes to a physical singularity and requires additional settings in the calculations.

![Diagram](image)

The resulting complexity is well illustrated in Fig. 1, where the solid line denotes the trajectory of the contact point on the plane for robot with three engines and dash line is for robot with two rotors. Starting and end points are set as \( x(0) = [0, 0, 0, 0, 0] \) and \( x(10) = [0, 2, 0, 3, 0, 0, \pi/6] \). We simulate horizontal position of the two rotors in the zero point, therefore, to satisfy the terminal equality \( \psi(5) = \pi/6 \) required loop in the line of motion of the plane.

Computational experiments for various end points allowed us to obtain controls that are physically realized, and the trajectories corresponding to the behavior of the prototype robot. Obtained controls satisfy the optimality condition for a considered class of systems, and such criteria (minimum square control), the sum of control squares in any point of time a constant value.

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An exact pseudopolynomial-time algorithm for a NP-hard problem of searching a family of disjoint subsets

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We consider the following intractable problem [1-2].

**Problem.** Given a set \( \mathcal{Y} = \{y_1, \ldots, y_N\} \) of points from \( \mathbb{R}^q \) and some positive integers \( M_1, \ldots, M_J \). Find a family \( \{C_1, \ldots, C_J\} \) of disjoint subsets of \( \mathcal{Y} \) such that

\[
\sum_{j=1}^{J} \sum_{y \in C_j} ||y - \overline{y}(C_j)||^2 \rightarrow \min,
\]

where \( \overline{y}(C_j) = \frac{1}{|C_j|} \sum_{y \in C_j} y \) is the centroid (geometrical center) of the subset \( C_j \), under constraints \( |C_j| = M_j, j = 1, \ldots, J \), and \( \sum_{j=1}^{J} M_j \leq N \), on the cardinalities of the required subsets.

The strong NP-hardness of the problem is implied from the results obtained in [1], since in the cited work it was proved that the special case of the problem when \( J = 1 \) is strongly NP-hard. In [2], a 2-approximation algorithm which time complexity is equal to \( \mathcal{O}(N^2(N^{J+1} + q)) \) was proposed. For the case of problem when the number \( J \) of required subsets is fixed, this algorithm is polynomial. Currently, there are no other algorithmic results for the problem and the known results [3-7] were obtained only for its special case when \( J = 1 \). These results are described below.

It follows from [3] that one can find an exact solution in \( \mathcal{O}(qN^{q+1}) \) time. In [4], a 2-approximation polynomial-time algorithm of complexity \( \mathcal{O}(qN^2) \) was constructed. For the variation of the problem with an additional restriction that the coordinates of the input points are integer and for the case of fixed space dimension, in [5] an exact pseudopolynomial algorithm was presented, which time complexity is equal to \( \mathcal{O}(N(MB)^q) \),
where $B$ is the maximum absolute value of the coordinates of the input points. Furthermore, for the case of the fixed space dimension in [6] an FPTAS was proposed. This scheme for a given relative error $\varepsilon$ finds $(1 + \varepsilon)$-approximate solution in $O(N^2(M/\varepsilon)^q)$ time, that is polynomial in the size of input and $1/\varepsilon$. A PTAS of complexity $O(qN^2/\varepsilon+1(9/\varepsilon)^{3/\varepsilon})$, where $\varepsilon$ is a guaranteed relative error, was found in [7].

In the current work, for the variation of the problem with an additional restriction that the coordinates of the input points are integer, an algorithm which finds an exact solution in $O(N(N^2 + qJ)(2MB + 1)^{qJ} + J^2 \log^2 N)$ time is constructed, where $B$ is the maximum absolute value of the coordinates of the input points and $M$ is the least common multiple for the numbers $M_1, \ldots, M_J$. In the case of the fixed dimension $q$ of the space and of the fixed number $J$ of required subsets, the proposed algorithm is pseudopolynomial and its time complexity is bounded by $O(N^3(MB)^{qJ})$.

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Modified Newton’s method to solve a transportation problem

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Let’s consider the transportation problem:

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \rightarrow \min, \quad (T)
\]

\[
\sum_{j=1}^{n} x_{ij} = a_i, \quad 1 \leq i \leq m, \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad 1 \leq j \leq n, \quad x_{ij} \geq 0.
\]

Let vector \( \bar{c} \in R^{mn} \) be denoted as \( \bar{c} = (c_{11}c_{12}...c_{1n}...c_{m1}c_{m2}...c_{mn})^T \), and \( \bar{x} \in R^{mn} \) be denoted as \( \bar{x} = (x_{11}x_{12}...x_{1n}...x_{m1}x_{m2}...x_{mn})^T \). Then a problem \((T)\) can be rewritten in the form of a standard LP problem:

\[
\bar{c}^T \bar{x} \rightarrow \min, \quad \bar{A} \bar{x} = \bar{b}, \quad \bar{x} \geq 0, \quad (\bar{P})
\]

where the \((m+n)\times mn\) matrix \( \bar{A} \) consists of zeros and ones so that \( \bar{A}_{ij} = 1 \) when \( i = 1...m, m+1...n; \ j = ((i-1)n+1)...in, 1+kn...n+kn; \ k = 0...m, \) and \( \bar{A}_{ij} = 0 \) otherwise. Here \( \bar{b} = (a_1...a_m, b_1...b_n)^T \).

It is well known from [1-3], that the problem \((\bar{P})\) reduces to the following unconstrained maximization problem:

\[
S(p, \beta, \hat{x}) = b^T p - \frac{1}{2} \| (\hat{x} + A^T p - \beta \bar{c})_+ \| ^2 \rightarrow \min, \quad (1)
\]

where \( \beta \) is a fixed scalar, and \( \hat{x} \) is a fixed point. Then if \( \beta \geq \beta_\ast \), where \( \beta_\ast \) is a threshold value, a projection of this fixed point on the solution set of the primal LP problem \((\bar{P})\) can be found by formula:

\[
\hat{x}_\ast = (\hat{x} + A^T p(\beta) - \beta \bar{c})_+ , \quad (2)
\]
where $p(\beta)$ is a solution of (1).

It was shown in [4] that the problem (1) can be effectively solved using the generalized Newton’s method. But the Hessian matrix for convex piecewise quadratic function $S(p, \beta, \hat{x})$ is undefined. That’s why it was used a generalized Hessian matrix which is $(m + n) \times (m + n)$ diagonal matrix:

$$\frac{\partial^2}{\partial p} S(p, \beta, \hat{x}) = -\bar{A}D^\sharp(z)\bar{A}^\top,$$

where $D^\sharp(z)$ is defined as $mn \times mn$ diagonal matrix having $z^i$ as a $i$th diagonal element which is equal to 1 if $(\hat{x} + \bar{A}^\top p - \beta \hat{c}^i) > 0$ and is equal to 0 otherwise, $i = 1...mn$.

When using the generalized Newton’s method the most time-consuming process is calculation of matrix (3). But due to the specific view of the matrix $\bar{A}$ corresponding to transportation problem the calculation of matrix (3) can be implemented much faster than for regular LP problem despite the increased dimension. The structure of the matrix $\bar{A}$ consists of four blocks, and the generalized Hessian matrix has accordingly the same structure so it can be rewritten quite easily at each iteration. That means we actually don’t need to recalculate the multiplication of matrices (3) at every step.

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Adaptive method for simultaneous untangling and optimization of 3d meshes

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Untangling and construction of optimal discrete deformations for real-life problems can be highly nonlinear and stiff variational problems. Solving them globally on large meshes can be prohibitively expensive. We present new adaptive untangling procedure based on dynamic extraction of subdomain with tangled mesh which allows for sharp reduce of computational cost for untangling. The idea is to extract regions where negative mesh jacobian values are present and to surround them by a buffer zones which allow for mesh untangling. During untangling procedure the number of inverted mesh elements is reduced by a factor of 20, after that the new subdomain is extracted and the process is repeated. We have found that even quite stiff untangling problems require just a few global iterations for solution.

In the example presented in Fig. 1 mesh is untangled in four iterations. In this test case initial structured mesh around complex body is constructed using algebraic generator and contains 165 thousands of “quadrature nodes” with negative jacobian of local mapping of hexahedral cells. Four adaptive untangling iterations are shown. Note that solving the same problems globally, without extraction of subdomains is order of magnitude more expensive. Simultaneous untangling and optimization means that during untangling procedure target mesh size in the boundary layer is taken into account which sharply reduces the cost of optimization following untangling phase.

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Fig. 1. Successive mesh fragment for adaptive untangling.
Images and Existence of Constrained Scalar and Vector Extrema

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By means of the Image Space Analysis (ISA), a new necessary and sufficient condition is established for the existence of vector extrema.

In the scalar case, the sufficient part is shown to shrink to a known one. First of all, we deliver a short introduction to ISA; despite its theoretical nature, ISA may suggest numerically viable tools; in this sense, a new result is outlined, which could be useful for improving numerical methods, like, e.g., for escaping from a local minimum or making a discontinuity point to disappear.

Subsequently, we deal with the existence condition. Some examples illustrate the results. Well known results are shown to be corollaries of the condition.

An application to the Bi-Level problems is then outlined. Some comments on perspectives end the talk.
On the $m$-Peripatetic Salesman Problem on random inputs

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The $m$-Peripatetic Salesman Problem ($m$-PSP) is a natural generalization of the classic Traveling Salesman Problem (TSP). It states as follows: given a complete undirected $n$-vertex graph $G = (V, E)$ with weight functions $w_i : E \rightarrow \mathbb{R}_+$, $i = 1, \ldots, m$, the problem is to find $m$ edge-disjoint Hamiltonian cycles $H_1, \ldots, H_m \subseteq E$ such that minimize or maximize their total weight:

$$
\sum_{i=1}^{m} w_i(H_i) = \sum_{i=1}^{m} \sum_{e \in H_i} w_i(e).$$

It is known that the problem is NP-hard. In paper [1] the asymptotically optimal approach was presented for the $m$-PSP with different weight functions on random inputs.

In this report we present an approach which under certain conditions gives asymptotically optimal algorithms for the $m$-PSP with identical weight functions on random inputs. This approach will be also correct for the $m$-PSP with different weight functions.

We assume that the weights of the edges are independent and identically distributed random reals, with distribution function $f(x)$ defined on $[a_n, b_n]$ or $[a_n, \infty)$, $0 < a_n \leq b_n$.

The approach consists of the following three steps.

**Step 1.** We uniformly split the initial complete $n$-vertex graph $G$ into subgraphs $G_1, \ldots, G_m$, so that each $G_i$ has $n$ vertices and about $\frac{n(n-1)}{2m}$ edges.

**procedure SPLIT(G):**

begin
  for $1 \leq i \leq m$ set $V(G_i) = V(G)$, $E(G_i) = \emptyset$;
  for each $e \in E(G)$:
    select at random with equal probabilities one of the sets $E(G_1), \ldots, E(G_m)$, let it be $E(G_i)$, add edge $e$ to $E(G_i)$.
end.
Step 2. We obtain subgraphs $\tilde{G}_1, \ldots, \tilde{G}_m$ deleting all edges in $G_i$, $1 \leq i \leq m$, which are heavier than $w^*$ that selected so as to retain only the most light edges in subgraphs, but to have enough edges in each $\tilde{G}_i$ for Step 3.

Step 3. In each subgraph $\tilde{G}_i$ we build a Hamiltonian cycle, using polynomial randomized algorithms, that w.h.p. (with probability $\to 1$ as $n \to \infty$) find a Hamiltonian cycle in a sparse random graph. We use algorithms $A_{GP}$ [2] and $A_{AV}$ [3].

Theorem 1. Let the weights of the input graph be i.i.d. random reals with uniform distribution function $\text{UNI}(x)$ defined on $[a_n, \beta_n]$, $0 < a_n \leq \beta_n$, or with shifted exponential distribution function $\text{Exp}(x) = 1 - \exp\left(-\frac{x-a_n}{\beta_n}\right)$, $0 < a_n \leq x$, defined on $[a_n, \infty)$, $0 < a_n$, with parameters $a_n, \beta_n$. Our approach gives asymptotically optimal solutions for the m-PSP, depending on the used algorithms [2,3]:

- $m = O(n^{0.5-\theta})$, $0 < \theta < 0.5$, and $\beta_n/a_n = o\left(\frac{n^\theta}{\sqrt{\ln n}}\right)$, if we use algorithm $A_{GP}$ in Step 3.

- $m = O(n^{1-\theta})$, $0 < \theta < 1$, and $\beta_n/a_n = o\left(\frac{n^\theta}{\ln n}\right)$, if we use algorithm $A_{AV}$ in Step 3.

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References


On Nash Point Search Algorithms for Three-Person Games (3PG)

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At OPTIMA-2014, we presented an approximate 3LP algorithm [1] for finding Nash points as solutions of 3PG given in the general setting. The goal of this talk is to determine the efficiency of 3LP algorithm for 3PG both in the general [2] and in the special [3] settings.

The general setting. Assume that player $l$, $l=1,2,3$, governs $n_l$ strategies, and a table $(a^{(l)}_{ijk})$ determines its payoff whenever the players select strategies $i$ ($1 \leq i \leq n_1$), $j$ ($1 \leq j \leq n_2$), and $k$ ($1 \leq k \leq n_3$), respectively. Thus, the finite 3PG in the mixed strategies is determined by the players’ payoff functions $f_l(x) = f_l(x_1, x_2, x_3) = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \sum_{k=1}^{n_3} a^{(l)}_{ijk} x_1 i x_2 j x_3 k$, $l = 1, 2, 3$, defined over the set $X = X^{(1)} \times X^{(2)} \times X^{(3)} \subset E^{n_1+n_2+n_3}$, where $X^{(l)} = \{x_l = (x_{1l}, x_{2l}, \ldots, x_{nl}) \in E^{n_l}: \sum_{r=1}^{n_l} x_{lr} = 1, x_{lr} \geq 0, r = 1, 2, \ldots, n_l\}$, $l = 1, 2, 3$.

The special setting. Let $A_{lm}$ be an $n_l \times n_m$-matrix, $l, m \in \{1, 2, 3\}$, $l \neq m$. Then the players’ payoff functions $f_l(x) = x_l^T \sum_{1 \leq m \leq 3, m \neq l} A_{lm} x_m$, $l = 1, 2, 3$, $x \in X$. Here and below, $\tau$ denotes the vector’s transpose.

It turns out that solving the 3PG is tantamount to the search of the global minimum (zero) of the Nash function $N(x_1, x_2, x_3) = \sum_{l=1}^{3} \delta_l(x)$, $x \in X$, where $\delta_l(x) = \max_{x_l \in X^{(l)}} f_l(x) - f_l(x)$, $l = 1, 2, 3$.

LP problems. Having fixed an arbitrary pair of strategy vectors $x_2 \in X^{(2)}$, $x_3 \in X^{(3)}$, we find an optimal solution $x_1^*$ of the following linear programming problem denoted as $\mathcal{P}_1(x_2, x_3)$:

**in the general case**

$$\sum_{i=1}^{n_1} \left( - \sum_{j=1}^{n_2} \sum_{k=1}^{n_3} a^{(l)}_{ijk} x_{2j} x_{3k} \right) x_{1i} + \alpha^{(1)}_1 + \alpha^{(1)}_2 \rightarrow \min,$$

$$\sum_{i=1}^{n_1} \left( \sum_{j=1}^{n_2} a^{(3)}_{ijk} x_{2j} \right) x_{1i} \leq \alpha^{(1)}_1, \quad k = 1, 2, \ldots, n_3,$$

$$\sum_{i=1}^{n_1} \left( \sum_{k=1}^{n_3} a^{(2)}_{ijk} x_{3k} \right) x_{1i} \leq \alpha^{(1)}_2, \quad j = 1, 2, \ldots, n_2,$$

$$\sum_{i=1}^{n_1} x_{1i} = 1, \quad x_{1i} \geq 0, \quad i = 1, 2, \ldots, n_1, \quad \alpha^{(1)}_1, \alpha^{(1)}_2 \in E^1;$$

**in the special case**
\[
\sum_{l=1}^{3} \left( -x_l^\tau \sum_{m=1, m \neq l}^{3} A_{lm}x_m \right) + \alpha_1^{(i)} + \alpha_2^{(i)} \rightarrow \min,
\]

where all \( n_m \) coordinates of vectors \( e_{nm} \) are equal to 1, \( m \in \{2, 3\} \). As a consequence, the inequality \( N(x_1^*, x_2, x_3) \leq N(x_1, x_2, x_3) \) holds for any strategy \( x \in X \). Similarly, having fixed the values of the other feasible pairs of strategy vectors \((x_1, x_3) \in X^{(1)} \times X^{(3)} \) and \((x_1, x_2) \in X^{(1)} \times X^{(2)} \), we detect the optimal solutions \( x_2^* \) and \( x_3^* \) for the corresponding linear programs \( \mathcal{P}_2(x_1, x_3) \) and \( \mathcal{P}_3(x_1, x_2) \).

**The 3LP algorithm.** Having chosen an arbitrary pair of (pure) strategy vectors, at each iteration of the algorithm, we solve consequently three LP problems thus obtaining a monotone non-increasing sequence \( N_q, q \geq 1 \). We stop when the difference \( N_q - N_{q-1} \) becomes small enough. If \( N_q \) equals zero (or reaches a small tolerance value), then we report a Nash point to be found. Otherwise we repeat the process starting with the next pair of strategies.

<table>
<thead>
<tr>
<th>Setting</th>
<th>Size, ( n_1 = n_2 = n_3 = n )</th>
<th>Start pairs</th>
<th>Time, minutes</th>
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</tr>
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</tr>
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<td></td>
<td>1000</td>
<td>352</td>
<td>2520</td>
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</tbody>
</table>

**References**

Approximation algorithms for the resource-constrained project scheduling problem

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We consider the resource constrained project scheduling problem with precedence and resource constraints (RCPSP). We are given a set of activities $J = \{1, \ldots, n\}$. The partial order on the set of activities is defined by a directed acyclic graph $G = (V, E)$. Each activity $j \in E$ is characterized by its processing time $p_j \in \mathbb{Z}^+$ and the resource requirement $r_{jk}(\tau)$ of the resource type $k$ on the time interval $[\tau - 1, \tau)$, $\tau = 1, \ldots, p_j$. For each constrained resource type $k$ is known its capacity $R^k_t$ during the time interval $[t - 1, t)$. All resources are renewable. Activities preemptions are not allowed. The objective is to find precedence and resource feasible start times for all activities $S = \{s_j\}$ such that the makespan of the project $C_{\text{max}}(S)$ is minimized. The model of the RCPSP can be described as follows:

$$ C_{\text{max}}(S) = \max_{j \in E} (s_j + p_j) \longrightarrow \min \{s_j\} \quad (1) $$

Subject to:

$$ s_i + p_i \leq s_j, \quad i \in \text{Pred}(j), \quad j \in E; \quad (2) $$

$$ \sum_{j \in U(t)} r_{jk}(t - s_j) \leq R^k_t, \quad k \in M, \quad t = 1, \ldots, T_k; \quad (3) $$

$$ s_j \in \mathbb{Z}^+, \quad j \in E, \quad (4) $$

where $\text{Pred}(j)$ is the set of immediate predecessor for activity $j \in U$, and $U(t) = \{j \mid s_j < t \leq s_j + p_j\}$ is the set of the activities which are being processed at the time instant $[t - 1, t)$ at the schedule $S$.

Since the problem described above is NP-hard, it is reasonable to design approximation algorithms with polynomial time complexity.

We offer few heuristic and metaheuristic algorithms for this problem. Authors developed deterministic [1] and stochastic [2] greedy algorithms, which both solve this problem with the time complexity dependant on the number of activities as $n \log n$.

We also present a genetic algorithm using two variants of crossover [3]. This crossovers are based on the most efficient use of limited resources. They are using heuristics, which account for the degree of importance of resources, which for any instance is derived from solving the relaxed problem with accumulative
resources. It is known that the relaxed problem is polynomially solvable. A polynomial time asymptotically optimal algorithm with an absolute error tending to zero for increasing number of activities was suggested by Gimadi [4]. We use this approximate algorithm to solve auxiliary relaxed problem. The algorithm uses the known representation of a work list, and two decoding procedures.

Quality of these algorithms has been analyzed in computational experiments using the standard data sets j30, j60, and j120 for the RCPSP from the PSPLIB-library, and numerical experiments demonstrated algorithm’s competitiveness. We have found the best solutions for a few instances from the dataset j120, and the best average deviation from the critical path lower bound for the datasets j60 (50000 and 500000 iterations) and j120 (500000 iterations).

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References

On Some Shape Optimization Problems for Thin Plates

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This work is devoted to the study of shape optimization problems for thin elastic plates. The results presented here are a natural continuation of [1–4]. Let $\Omega$ be a non-empty bounded domain in $\mathbb{R}^2$. Consider the following boundary-value problem: find $y$ such that

$$\Delta [D(x,u(x))\Delta y] = q, \quad x = (x_1,x_2) \in \Omega, \quad (1)$$

$$y|_{\Gamma_1} = \Delta y|_{\Gamma_1} = 0, \quad y|_{\Gamma_2} = \frac{\partial y}{\partial \nu}|_{\Gamma_2} = 0, \quad (2)$$

where $\partial \Omega = \Gamma_1 \cup \Gamma_2$, $\Gamma_1 \cap \Gamma_2 = \emptyset$, $D(x,u(x)) = e(x)u^\alpha(x)$, $\alpha > 0$, $u \in U$,

$$U = \{ u \in L^\infty(\Omega) : 0 < \underline{h} \leq u(x) \leq \overline{h}, \quad \int_\Omega \rho(x)u(x) \, dx \leq \overline{m} \}. \quad (3)$$

For a transversely loaded plate, the equation describing the equilibrium of the plate has the form (1), and the boundary conditions imposed at $y$ express the fact that the plate is supported on a part $\Gamma_1$ of its edge and clamped on the remaining part $\Gamma_2$. Let $y[u]$ denote a solution of (1), (2), corresponding to $u \in U$. Consider the following minimization problem:

$$F[u] = \int_\Omega f(x,y[u],\nabla y[u],\Delta y[u]) \, dx \rightarrow \min, \quad u \in U. \quad (4)$$

**Theorem 1.** Let the integral functional

$$L^2(\Omega) \times [L^2(\Omega)]^2 \times L^2(\Omega) \ni (y_1, y_2, y_3) \mapsto \int_\Omega f(x,y_1,y_2,y_3) \, dx$$

be lower semicontinuous with respect to the strong convergence of $(y_1,y_2)$ in $L^2(\Omega) \times [L^2(\Omega)]^2$ and the weak convergence of $y_3$ in $L^2(\Omega)$. Then there exists a solution to the problem (4) determined by (1)–(3).

Some special cases of this theorem were considered in [5].

The equation describing the buckling of a three-layered plate is as follows:

$$\Delta (e(x)u(x)\Delta y) = \lambda \Delta y. \quad (5)$$
Consider the optimization problem

$$\lambda_1[u] \rightarrow \max, \quad u \in \mathcal{U},$$

(6)

where $\lambda_1[u]$ denotes the lowest eigenvalue of (5), corresponding to $u \in \mathcal{U}$.

**Theorem 2.** There exists a solution $\hat{u}$ to the problem (6) determined by (2), (3), (5). Moreover, there exist an eigenfunction $\hat{y}$ associated with $\lambda_1[\hat{u}]$ and a number $\gamma$ such that the following pointwise conditions hold:

$$|\Delta \hat{y}(x)| < \gamma \iff \hat{u}(x) = \tilde{h},$$

$$|\Delta \hat{y}(x)| = \gamma \iff \tilde{h} < \hat{u}(x) < \hat{h},$$

$$|\Delta \hat{y}(x)| > \gamma \iff \hat{u}(x) = \hat{h}.$$ 

These results as well as another obtained ones will be presented in the talk in detail.

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**References**


Applied optimization problems: How to treat them

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Theory and practice of creation the algorithms and software for optimization problems has achieved considerable success in recent decades in Russia and abroad. There were developed a large number of reliable optimization software. It includes algorithms and tools, which allow to design effective technology, to monitor the computational process, to verify the solution and visualize calculation results.

However, the practical application of this wide range of software, unfortunately, is going too slow. In our opinion, the main reason is the complexity and poor formalization of mathematical simulation methods as well as the complex system of business relations, including information exchange and verification of the results of ”temporary creative collectives”, formed to solve specific applied problems.

The report discusses problems arising when one applies optimization tools to solve meaningful extremal problems. We propose the classification of the problems according to the criteria of dimension, complexity and costs and formulate the software requirements. The methods of estimation of models quality and checking their adequacy are considered. Suggested multimethod computing algorithms allow to increase the probability of success. Developed by authors software was applied for solving real problems from various scientific areas, such as mechanics, flight dynamics, cosmonavigation, electricity, robotics, economics, ecology, medicine, nanophysics, seismology and others.

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References

An Approach to Fractional Optimization

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The paper addresses the development of efficient methods for fractional programming problems [1] as follows

\[(\mathcal{P}) \quad f(x) := \sum_{i=1}^{m} \frac{\psi_i(x)}{\phi_i(x)} \downarrow \min_x, \; x \in S,\]

where \(\phi_i(x) > 0, \; \psi_i(x) > 0, \; i = 1, \ldots, m, \; \forall x \in S.\) This is a nonconvex problem with multiple local extremum which belongs to a class of global optimization. Together with problem \((\mathcal{P})\) we will also consider the following parametric optimization problem

\[(\mathcal{P}_\alpha) \quad \Phi_\alpha(x) := \sum_{i=1}^{m} [\psi_i(x) - \alpha_i \phi_i(x)] \downarrow \min_x, \; x \in S,\]

where \(\alpha = (\alpha_1, \ldots, \alpha_m)^\top \in \mathbb{R}^m\) is the vectorial parameter.

Let us introduce then the optimal value function \(V(\alpha)\) of Problem \((\mathcal{P}_\alpha)\) as follows

\[V(\alpha) := \inf_x \Phi_\alpha(x) \mid x \in S.\]

In addition, suppose that the following assumptions are fulfilled:

\[(a) \quad V(\alpha) > -\infty \; \forall \alpha \in \mathcal{K}, \text{ where } \mathcal{K} \text{ is a convex set from } \mathbb{R}^m;\]

\[(b) \quad \forall \alpha \in \mathcal{K} \subset \mathbb{R}^m \text{ there exists a solution } z = z(\alpha) \text{ to Problem } (\mathcal{P}_\alpha), \text{ i.e. } V(\alpha) = \sum_{i=1}^{m} [\psi_i(z) - \alpha_i \phi_i(z)].\]

Then it takes place the reduction (equivalence) theorem for the fractional programming problem with d.c. functions and the solution of the equation \(V(\alpha) = 0\) with the vector variable \(\alpha = (\alpha_1, \ldots, \alpha_m)^\top\) satisfying the following nonnegativity assumption

\[(\mathcal{H}(\alpha)) \quad \psi_i(x) - \alpha_i \phi_i(x) \geq 0 \; \forall x \in S, \; i = 1, \ldots, m.\]

**Theorem.** Suppose that in Problem \((\mathcal{P})\) the assumptions \((\mathcal{H}_1)\) are fulfilled. In addition, let there exist a vector \(\alpha_0 = (\alpha_{01}, \ldots, \alpha_{0m})^\top \in \mathcal{K} \subset \mathbb{R}^m\) for which the assumption \((\mathcal{H}(\alpha_0))\) is satisfied.
Besides, suppose that in Problem \((\mathcal{P}_{\alpha_0})\) the following equality holds

\[ V(\alpha_0) \triangleq \min_x \left\{ \sum_{i=1}^{m} [\psi_i(x) - \alpha_{0i}\phi_i(x)] \mid x \in S \right\} = 0. \]

Then any solution \(z = z(\alpha_0)\) to Problem \((\mathcal{P}_{\alpha_0})\) turns out to be a solution to Problem \((\mathcal{P})\), so that \(z \in \text{Sol}(\mathcal{P}_{\alpha_0}) \subset \text{Sol}(\mathcal{P})\).

This theorem opens the door to a justified use of the Dinkelbach’s approach for solving fractional programming problems with the goal function presented by a sum of fractions all given by d.c. functions.

Therefore, instead of solving Problem \((\mathcal{P})\) we propose to combine a solving Problem \((\mathcal{P}_\alpha)\) with a search with respect to parameter \((\alpha \in \mathbb{R}^m_+)\) in order to find a vector \((\alpha_0 \in \mathbb{R}^m_+)\) such that \(V(\alpha_0) = V(\mathcal{P}_{\alpha_0}) = 0\).

In this situation for every \(\alpha \in \mathbb{R}^m_+\) we must be able to find a global solution to \((\mathcal{P}_\alpha)\) and we can do it using the global search theory [2].

Finally, computational simulation testings have been carried out for some special test functions formed by linear and/or convex quadratic functions. First, the experiments have been performed on the low dimension’s examples from [3]. Afterwords, the approach has been tested on specially designed test problems up to dimension \(n = m = 100\). At the end, the test problems of dimension up to \(n = m = 200\) designed with the help of [4] have been also solved by the developed algorithm.

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REFERENCES

Determination of optimal forcing directions for synchronization of nonlinear oscillations

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The ability of oscillators to synchronize their oscillations is a fascinating phenomena with great variety of manifestations important in Physics and Life sciences (\cite{ref1, ref2}). In almost all fields of engineering this phenomena is taken advantage of to modify nonlinear oscillations using a periodic external signal.

Forced synchronization is traditionally understood as a modification of self-sustained oscillations to a weak periodic signal with approximately the same (or resonant) frequency. In this way even a very weak periodic external signal can ”entrain” nonlinear oscillator to oscillate at the same frequency as the forcing signal. This adjustment of self-sustained oscillations to weak periodic force is called frequency locking.

However, if the external signal is rather strong, then synchronization of self-sustained oscillations can occur even if their frequency is not close (nor resonant) to the forcing frequency. This effect is called suppression of self-sustained oscillations.

We treat the problem of forced frequency locking as a bifurcation control problem. Nonlinear oscillators are described as dynamical systems near fixed point oscillatory bifurcation, while external forcing is treated as a weak control, or perturbation.

We present a mathematical result of finding optimal forcing directions in order to synchronize oscillations to external signal. This result can be especially relevant for systems exhibiting complex dynamics that is sensitive to small variations of the initial point and/or system parameters. We show our results on illustrative examples.
Some Generalizations of Gradient-type Projection Method for Solving Quasi Variational Inequalities

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The theory and methods for solving variational inequalities are thoroughly treated in the scientific literature. An important generalization of variational inequalities are quasi-variational inequalities. If the convex set, which involved in the variational inequality, also depends upon the solution then the variational inequality is called the quasi variational inequality.

We study the following quasi variational inequality: find $x^* \in C(x^*)$ such that

$$\langle F(x^*), y - x^* \rangle \geq 0 \quad \forall y \in C(x^*),$$

where $C : H \to 2^H$ is set-valued mapping with nonempty convex and closed set $C(x) \subseteq H$ for all $x$ from Hilbert space $H$.

Note that the difficulty of problems with quasi variational inequalities is related to the fact that one must simultaneously solve a variational inequality and calculate a fixed point of a set-valued mapping. This explains why the literature on solution methods for quasi variational inequalities is not too extensive.

Consequently, there are numerous open questions.

Projection methods represent an important tool for finding the approximate solution of various types of variational inequalities. These methods have been extended and modified in various ways. The main idea in these techniques is to establish the equivalence between the quasi variational inequalities and some fixed point problem. We use this alternative equivalent formulation to suggest and analyze two variants (one-step and two-step) of iterative projection method for solving quasi variational inequalities.

Algorithm 1. Let $x_0 \in H$ be an arbitrary initial approximation of the solution. Suppose that, for a certain $k \geq 0$, the approximation $x_k \in C(x_k)$ has already been determined. Then the set $C(x_k)$ is defined. Find $x_{k+1}$ by

$$x_{k+1} = (1 - a_k)x_k + a_k P_{C(x_k)}[x_k - \alpha F(x_k)], \quad k = 0, 1, \ldots$$

where parameters $0 < a_k \leq 1$ and $\alpha > 0$ can be choosen on diferent ways.
Let us remark that fixed point formulation of problem (1) can be written as

\[
    u = P_{C(x)}[x - \alpha F(x)], \\
    x = P_{C(u)}[u - \alpha F(u)].
\]

This formulation enables to suggest and analyze the following two-step method for solving quasi-variational inequality (1).

**Algorithm 2.** For a given initial point \( x_0 \in H \), find the approximate solution \( x_{k+1} \) by the iterative schemes

\[
    u_k = (1 - b_k)x_k + b_k P_{C(x_k)}[x_k - \alpha F(x_k)],
\]

\[
    x_{k+1} = (1 - a_k)x_k + a_k P_{C(u_k)}[u_k - \alpha F(u_k)], \quad k = 0, 1, 2, \ldots
\]

where \( 0 < a_k \leq 1, \ 0 \leq b_k \leq 1 \) for all \( k \geq 0 \) and \( \alpha > 0 \) are parameters of method. This method is quite different than Koperlevich method. In case of \( b_k = 0 \) for all \( k \geq 0 \), this method is described by Algorithm 1.

Important class of methods for solving the quasi variational inequalities is also a class of continuous method, in which the process is described by differential equations. For the calculation of solutions of quasi-variational inequalities, we construct trajectories that start at an arbitrary point, and during the time converge to the set of solutions:

\[
    x'(t) = -a(t)x(t) + a(t) P_{C(x(t))}[x(t) - \alpha F(x(t))], \quad x(0) = x_0,
\]

where \( x_0 \) is given initial point in \( H \) and \( a(t) \) and \( \alpha \) are parameters of the method. For \( a(t) \equiv 1 \), this method becomes standard continuous gradient-type method.

We also establish sufficient conditions for the convergence of the proposed methods and estimate the rates of convergence.
Lower Bound on Restricted Isometry Constants for Tight Frames

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Tight frames are the key computational tool in such areas as compressed sensing [1] and other data communication technologies. The quality of a tight frame (represented by rectangular $m \times n$ matrix $A$ possessing the tightness property $AA^H = \frac{n}{m}I_m$) is characterized by the standard restricted isometry property (RIP)

$$
\exists \delta = \delta^{(m,n)}(k) \in (0,1) \ \forall v: \|v\|_0 = k \Rightarrow 1 - \delta \leq \frac{\|Av\|^2}{\|v\|^2} \leq 1 + \delta,
$$

where $\|v\|_0$ denotes the number of nonzero components in $v$. It is implied that $1 \ll m \ll n$, and for a possibly large $k$ it is desirable to have a possibly smaller restricted isometry constant $\delta$. The latter is obviously related to the common bound on condition numbers over all $m \times k$ submatrices $A_J = [a_{j(1)}, \ldots, a_{j(k)}]$ of $A$. Recall that the evaluation of the RIP constant is an NP-hard problem, see [1] and references therein. Our main result establishes the following interior bounds for the extreme eigenvalues of $A_J^H A_J$ over all index subsets $J = \{j(1), \ldots, j(k)\}$ holding for any tight frame:

$$
\min_{|J|=k} \lambda_{\min}(A_J^H A_J) \leq \rho^{(m,n)}_{\min}, \quad \max_{|J|=k} \lambda_{\max}(A_J^H A_J) \geq \rho^{(m,n)}_{\max},
$$

where $\rho^{(m,n)}_{\min}$ and $\rho^{(m,n)}_{\max}$ are the smallest and largest roots of the $k$th degree polynomial in $\lambda$ of the form $P_k^{(m-k,n-m-k)}(1 - \frac{2m}{n}\lambda)$, respectively. Here

$$
P_k^{(\alpha,\beta)}(x) = \sum_{q=0}^{k} \binom{k+\alpha}{q} \binom{k+\beta}{k-q} (x-1)^{k-q} (x+1)^q
$$

(1)

is the standard $k$th degree Jacobi polynomial [2]. The proof is based on the following determinant identity valid for any tight frame:

$$
\sum_{|J|=k} \det(A_J^H A_J - \lambda I_k) = \left(\frac{n}{2m}\right)^k P_k^{(m-k,n-m-k)}(1 - \frac{2m}{n}\lambda).
$$

(2)

Addressing the lower spectral bound, one can notice that if

$$
\lambda_* = \min_{|J|=k} \lambda_{\min}(A_J^H A_J),
$$

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then the left hand side of (2) is nonnegative for all \( \lambda \in [0, \lambda_*] \), and therefore its right hand side cannot be negative, which yields, by the simplicity of Jacobi polynomial roots,

\[
\lambda_* \leq \rho_{\min}^{(m,n)}.
\]

Similarly, one can obtain the bound for the largest eigenvalues of \( A_J^H A_J \).

Moreover, considering the quantity

\[
\tilde{\delta}(k, m, n) = \sqrt{\left( \frac{1}{m} - \frac{1}{n} \right) \left( k - \frac{1}{4} \right) \log \left( k - \frac{1}{4} \right)},
\]

one can verify numerically that for all \( 2 \leq k \leq m/2, \sqrt{n} \leq m \leq n/2, n \leq 256 \), and under a natural restriction \( \tilde{\delta}(k, m, n) \leq 1/\sqrt{2} \), it holds

\[
\min_{|J| = k} \lambda_{\min}(A_J^H A_J) \leq \rho_{\min}^{(m,n)} \approx 1 - \tilde{\delta}(k, m, n),
\]

\[
\max_{|J| = k} \lambda_{\max}(A_J^H A_J) \geq \rho_{\max}^{(m,n)} \approx 1 + \tilde{\delta}(k, m, n).
\]

This observation well agrees with the widely known probabilistic estimates of the restricted isometry in stochastic rectangular matrices, cf.[1]. It must be stressed that the presented result is completely deterministic.

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Generalized solutions of optimal control problems

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Not all the problems of the classical calculus of variations have a solution. The following problem is, perhaps, one of the simplest examples to confirm this thesis.

\begin{equation}
\begin{aligned}
\text{Minimize} & \quad \int_0^1 x^2(t) dt \\
\text{subject to} & \quad x(0) = 0, \quad x(1) = 1.
\end{aligned}
\end{equation}

Here, the minimum of integral is sought on the set of all smooth functions $x(t)$, whose values in points $t = 0, 1$ are fixed. A classic smooth solution to problem (1) does not exist. Indeed, there does not exist even a continuous solution: any minimizing sequence of arcs from example (1) pointwise converges to a discontinuous function $y(t)$ such that $y(t) = 0$ when $t \in [0, 1)$, and $y(1) = 1$.

Considering more general optimal control problems, the situation becomes even more complicated by the following reason. There are examples of differential control systems with terminal constraints, for which there does not exist even a single continuous admissible (feasible) arc. Indeed, consider the control system

\begin{equation}
\begin{aligned}
\dot{x} &= u, \quad \dot{y} = 1 - x, \quad u \geq 0, \quad t \in [0, 1], \\
x(0) &= 0, \quad x(1) = 1, \quad y(0) = 0, \quad y(1) = 0.
\end{aligned}
\end{equation}

It is clear that $x(t)$ is increasing, but $x(t) \leq 1$. Therefore, $y(t)$ is increasing as well. Thereby, in view of the terminal constraints, the compatibility of this system is possible only if $y(t) \equiv 0$. Hence, $x(t) = 1 \forall t \in (0, 1]$, and then at point $t = 0$, function $x(t)$ is discontinuous. By considering any functional over (2) we obviously come to a problem, in which continuous solutions do not exist.

The absence of classical solutions naturally gives rise to the so-called extension or relaxation of the problem of Variational Calculus or Optimal Control and leads to the notion of a generalized solution. Under the “extension” it is being understood the introducing into consideration the so-called generalized solutions: it is necessary to relax the notion of arc, that is to enlarge the class of admissible functions $x(t)$, so that in the enlarged class of arcs solutions to (1), (2) would already exist.
In this work, we construct an extension for general nonlinear control problem

\[
\begin{align*}
\text{Minimize} & \quad \phi(x_0, x_1) \\
\text{subject to} & \quad \dot{x} = f(x, u, t), \\
& \quad x_0 = x(0) \in A, \ x_1 = x(1) \in B, \\
& \quad u(t) \in U \text{ a.a. } t \in [0, 1],
\end{align*}
\]

where \(\phi, f\) – given smooth functions, \(A, B, U\) – given closed sets, and \(u(t)\) – a measurable function (for example, of class \(L^p([0, 1])\), \(p \geq 1\)).

Discontinuous trajectories can be expected only when the set \(U\) is unbounded. Indeed, on the one hand, it is clear that discontinuities occur when the arc derivative begins to take unbounded values (which is possible only when the set \(U\) is unbounded), see Example (1). On the other hand, in the case of bounded set \(U\), and under fairly general assumptions an extension of (3) into the class of absolutely continuous arcs is still feasible. Such an extension was proposed in [1] and is based on the notion of a generalized control.

In our investigation we combine the methods of [1] with the method of so-called Lebesgue discontinuous time variable change resulting the generalized impulsive control.

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On some clustering problems: NP-hardness and efficient algorithms with performance guarantees

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We consider some quadratic Euclidean clustering problems most closely related to data mining, machine learning, statistics, and computational geometry. The report purpose is review of some new results on the computational complexity of these problems, and on efficient algorithms with performance guarantees for their solutions.

Below is a list of considered problems.

Problem 1. Given a set \( Y = \{y_1, \ldots, y_N\} \) of points from \( \mathbb{R}^q \) and some positive integers \( M_1, \ldots, M_J \). Find a family \( \{C_1, \ldots, C_J\} \) of disjoint subsets of \( Y \) such that

\[
\sum_{j=1}^{J} \sum_{y \in C_j} \|y - \overline{y}(C_j)\|^2 \rightarrow \min,
\]

where \( \overline{y}(C_j) \) is the centroid (geometrical center) of the subset \( C_j \), under constraints \( |C_j| = M_j, j = 1, \ldots, J \), and \( \sum_{j=1}^{J} M_j \leq N \).

Problem 2. Given a set \( Y = \{y_1, \ldots, y_N\} \) of points from \( \mathbb{R}^q \) and a positive integer \( M \). Find a partition of \( Y \) into two non-empty clusters \( C \) and \( Y \setminus C \) such that

\[
|C| \sum_{y \in C} \|y - \overline{y}(C)\|^2 + |Y \setminus C| \sum_{y \in Y \setminus C} \|y\|^2 \rightarrow \min
\]

where \( \overline{y}(C) \) is the centroid of \( C \), subject to constraint \( |C| = M \).

Problem 3. Given a sequence \( Y = (y_1, \ldots, y_N) \) of points from \( \mathbb{R}^q \), and some positive integer numbers \( T_{\text{min}}, T_{\text{max}} \) and \( M \). Find a subset \( M = \{n_1, \ldots, n_M\} \) of \( N = \{1, \ldots, N\} \) such that

\[
\sum_{j \in M} \|y_j - \overline{y}(M)\|^2 \rightarrow \min,
\]

where \( \overline{y}(M) \) is the centroid of \( \{y_j \ | \ j \in M\} \), under constraints

\[
T_{\text{min}} \leq n_m - n_{m-1} \leq T_{\text{max}} \leq N, \ m = 2, \ldots, M,
\]

on the elements of \( (n_1, \ldots, n_M) \).
Problem 4. Given a sequence $\mathcal{Y} = (y_1, \ldots, y_N)$ of points from $\mathbb{R}^q$ and some positive integers $T_{\text{min}}, T_{\text{max}}, L,$ and $M$. Find nonempty disjoint subsets $\mathcal{M}_1, \ldots, \mathcal{M}_L$ of $\mathcal{N} = \{1, \ldots, N\}$ such that
\[
\sum_{l=1}^{L} \sum_{j \in \mathcal{M}_l} \|y_j - \overline{y}(\mathcal{M}_l)\|^2 + \sum_{i \in \mathcal{N} \setminus \mathcal{M}} \|y_i\|^2 \rightarrow \min,
\]
where $\mathcal{M} = \bigcup_{l=1}^{L} \mathcal{M}_l$, and $\overline{y}(\mathcal{M}_l)$ is the centroid of subset $\{y_j| j \in \mathcal{M}_l\}$, under the following constraints: (i) the cardinality of $\mathcal{M}$ is equal to $M$, (ii) concate- nation of elements of subsets $\mathcal{M}_1, \ldots, \mathcal{M}_L$ is an increasing sequence, provided that the elements of each subset are in ascending order, (iii) the inequalities (1) for the elements of $\mathcal{M} = \{n_1, \ldots, n_M\}$ are satisfied.

Problem 5. Given a sequence $\mathcal{Y} = (y_1, \ldots, y_N)$ of points from $\mathbb{R}^q$ and some positive integers $T_{\text{min}}, T_{\text{max}},$ and $L$. Find nonempty disjoint subsets $\mathcal{M}_1, \ldots, \mathcal{M}_L$ of $\mathcal{N} = \{1, \ldots, N\}$ such that the objective function (2) would be minimal, under the following constraints: (i) concatenation of elements of subsets $\mathcal{M}_1, \ldots, \mathcal{M}_L$ is an increasing sequence, provided that the elements of each subset are in ascending order, (ii) the inequalities (1) for the elements of $\mathcal{M} = \{n_1, \ldots, n_M\}$ are satisfied; (the cardinality of $\mathcal{M}$ assumed to be un- known).

Problem 6. Given a set $\mathcal{Y} = \{y_1, \ldots, y_N\}$ of points from $\mathbb{R}^q$, and positive integer $J > 1$. Find: a partition of $\mathcal{Y}$ into subsets $\mathcal{C}_1, \ldots, \mathcal{C}_J$ such that
\[
\sum_{j=1}^{J} \frac{1}{|\mathcal{C}_j|} \left\| \sum_{y \in \mathcal{C}_j} y \right\|^2 \rightarrow \min.
\]

The study of problems 1–3 and 5 was supported by the the Russian Foundation for Basic Research (projects no. 15-01-00462, no. 16-07-00168). The study of problems 4 and 6 was supported by the Russian Science Foundation (project no. 16-11-10041).
An approximation algorithm for one NP-hard problem of partitioning a sequence into clusters with restrictions on their cardinalities

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We consider the following strongly NP-hard [1] problem. Given a sequence $Y = (y_1, \ldots, y_N)$ of points from $\mathbb{R}^q$ and some positive integers $T_{\text{min}}, T_{\text{max}}, L$, and $M$. Find nonempty disjoint subsets $M_1, \ldots, M_L$ of $N = \{1, \ldots, N\}$ such that

$$F(M_1, \ldots, M_L) = \sum_{l=1}^{L} \sum_{j \in M_l} ||y_j - \bar{y}(M_l)||^2 + \sum_{i \in N \setminus M} ||y_i||^2 \rightarrow \min,$$ \hspace{1cm} (1)

where $M = \bigcup_{l=1}^{L} M_l$, and $\bar{y}(M_l) = \frac{1}{|M_l|} \sum_{j \in M_l} y_j \{y_j | j \in M_l\}$, under the following constraints: (1) the cardinality of $M$ is equal to $M$, (2) concatenation of elements of subsets $M_1, \ldots, M_L$ is an increasing sequence, provided that the elements of each subset are in ascending order, (3) the following inequalities for the elements of $M = \{n_1, \ldots, n_M\}$ are satisfied:

$$T_{\text{min}} \leq n_m - n_{m-1} \leq T_{\text{max}} \leq N, \hspace{0.5cm} m = 2, \ldots, M.$$

At present, for Problem 1, except for its particular case when $L = 1$ in (1), there are no efficient algorithms with guaranteed accuracy. For the mentioned case of Problem 1 the following results were obtained.

In [1], the variant of Problem 1 in which $T_{\text{min}}$ and $T_{\text{max}}$ are the parameters was analyzed. In the cited work it was shown that in the case when $L = 1$ this parameterized variant is a strongly NP-hard problem for any $T_{\text{min}} < T_{\text{max}}$. In the trivial case when $T_{\text{min}} = T_{\text{max}}$ the problem is solvable in polynomial time.

In [2], for the same case of Problem 1, when $L = 1$, a 2-approximation polynomial-time algorithm having $O(N^2(MN+q))$ running time was presented.

In addition, in [3,4], two subcases for the same case of problem when $L = 1$ were studied. In both subcases the dimension $q$ of the space was fixed. In [3],
for the subcase with integer inputs an exact pseudopolynomial algorithm was constructed. The time complexity of this algorithm is $O(MN^2(MD)^q)$, where $D$ is the maximum absolute coordinate value in the input set of points. For the subcase with real inputs in [4] a fully polynomial-time approximation scheme was proposed which, given a relative error $\varepsilon$, finds a $(1+\varepsilon)$-approximate solution of Problem 1 in $O(MN^3(1/\varepsilon)^{q/2})$ time.

The main result of this paper is an algorithm that allows to find a 2-approximate solution of Problem 1 in $O(LN^{L+1}(MN+q))$ time, which is polynomial if the number $L$ of clusters is fixed.

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References

An approximation algorithm for a problem of partitioning a sequence into clusters

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We consider the following strongly NP-hard \cite{1} Problem. Given a sequence $\mathcal{Y} = (y_1, \ldots, y_N)$ of points from $\mathbb{R}^q$ and some positive integers $T_{\text{min}}$, $T_{\text{max}}$, and $L$. Find nonempty disjoint subsets $\mathcal{M}_1, \ldots, \mathcal{M}_L$ of $\mathcal{N} = \{1, \ldots, N\}$ such that

$$\sum_{l=1}^{L} \sum_{j \in \mathcal{M}_l} \|y_j - \overline{y}(\mathcal{M}_l)\|^2 + \sum_{i \in \mathcal{N} \setminus \mathcal{M}} \|y_i\|^2 \to \min,$$

where $\mathcal{M} = \bigcup_{l=1}^{L} \mathcal{M}_l$, and $\overline{y}(\mathcal{M}_l)$ is the centroid of subset $\{y_j | j \in \mathcal{M}_l\}$, under the following constraints: (1) concatenation of elements of subsets $\mathcal{M}_1, \ldots, \mathcal{M}_L$ is an increasing sequence, provided that the elements of each subset are in ascending order, (2) the following inequalities for the elements of $\mathcal{M} = \{n_1, \ldots, n_M\}$ are satisfied: $T_{\text{min}} \leq n_m - n_{m-1} \leq T_{\text{max}} \leq N$, $m = 2, \ldots, M$; (the cardinality of $\mathcal{M}$ assumed to be unknown).

In \cite{2}, for the case of Problem, when $L = 1$, a 2-approximation polynomial-time algorithm having $O(N^2(N + q))$ running time was presented. In this work we present an algorithm that allows to find a 2-approximate solution of Problem in $O(LN^{L+1}(N + q))$ time, which is polynomial if the number $L$ of clusters is fixed.

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References


An approximation scheme for a balanced 2-clustering problem

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We consider the following strongly NP-hard [1]

Problem. Given a set $\mathcal{Y} = \{y_1, \ldots, y_N\}$ of points from $\mathbb{R}^q$ and a positive integer $M$. Find a partition of $\mathcal{Y}$ into two non-empty clusters $C$ and $\mathcal{Y} \setminus C$ such that

$$|C| \sum_{y \in C} \|y - \overline{y}(C)\|^2 + |\mathcal{Y} \setminus C| \sum_{y \in \mathcal{Y} \setminus C} \|y\|^2 \longrightarrow \min,$$

where $\overline{y}(C) = \frac{1}{|C|} \sum_{y \in C} y$ is the geometric center (centroid) of $C$, subject to constraint $|C| = M$.

In [2], an exact algorithm for the case of integer components of the input points was presented. If the dimension $q$ of the space is bounded by a constant, then this algorithm has a pseudopolynomial complexity.

In this work we present an approximation algorithm that allows to find a $(1 + \varepsilon)$-approximate solution in $O(qN^2(\sqrt{\frac{2q}{\varepsilon}} + 1)^q)$ time for a given relative error $\varepsilon$. If the space dimension is bounded by a constant this algorithm implements a fully polynomial-time approximation scheme.

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References

An approximation scheme for a problem of finding a subsequence

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We consider the following strongly NP-hard \cite{1} Problem.

\textbf{Problem.} Given a sequence $\mathcal{Y} = (y_1, \ldots, y_N)$ of points from $\mathbb{R}^q$, and some positive integer numbers $T_{\text{min}}, T_{\text{max}}$ and $M$. Find a subset $\mathcal{M} = \{n_1, \ldots, n_M\} \subseteq \{1, \ldots, N\}$ such that $\sum_{j \in \mathcal{M}} \|y_j - \bar{y}(\mathcal{M})\|^2 \rightarrow \min$, where $\bar{y}(\mathcal{M}) = \frac{1}{|\mathcal{M}|} \sum_{i \in \mathcal{M}} y_i$, under constraints $T_{\text{min}} \leq n_m - n_{m-1} \leq T_{\text{max}} \leq N$, $m = 2, \ldots, M$, on the elements of $(n_1, \ldots, n_M)$.

In \cite{2}, a 2-approximation polynomial algorithm having $O(N^2(N + q))$ running time was proposed. In \cite{3}, for the case of fixed space dimension and integer input points coordinates, an exact pseudopolyomial algorithm with $O(N^3(MD)^q)$-time complexity was presented, where $D$ is the maximum absolute coordinate value of the points in the input sequence.

In this work we present a FPTAS for the case of fixed space dimension with $O(MN^3(1/\varepsilon)^{q/2})$-time complexity for an arbitrary relative error $\varepsilon$.

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\textbf{References}


An algorithm of using the set of equivalence method for solving the multicriterial optimization problems

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An algorithm of step by step implementation the set of equivalence method for solving the multicriterial discrete optimization problems is described. Advantages of the method of finding the set of equivalence for solving this kind of problems [1, 2] are shown. The work of this method is illustrated on example of a set of key indicators of economic efficiency for generalized commercial enterprise and the elaborated corresponding mathematical model. In difference to the classical problem of finding the maximum profit for any business (where the profit is the only criterion), a multicriterial inverse [3, 4] optimization problem is considered. The solution of the formulated problem in a multidimensional pseudo-metric space is given.

The optimal sets spacial distributions for each criterion and the set of equivalence distribution (violet color) are shown on fig.1 and fig.2.

Fig. 1
REFERENCES


Approximability of the Euclidean Capacitated Vehicle Routing Problem

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The Capacitated Vehicle Routing Problem is the well known special case of Vehicle Routing Problem \cite{1}, which is widely adopted in operations research. In its simplest setting, the VRP can be treated as the combinatorial optimization problem aiming to design the cheapest collection of delivery \textit{routes} of the given capacity vehicle from single or multiple dedicated points (or depots) to a set of customers (clients) distributed in a given spatial region.

As the VRP, the CVRP is strongly NP-hard and APX-complete. Almost all known special cases of the CVRP (except the case when \( q \leq 2 \)) are also NP-hard even in Euclidean spaces of finite dimension.

Most approximation results for CVRP are obtained for the Euclidean plane. One of the first studies of two-dimensional Euclidean CVRP has been due to Haimovich and Rinnooy Kan \cite{2}, who presented several heuristics for this problem leading to the first PTAS for \( q = O(\log \log n) \).

In this paper, we extend the results obtained in \cite{2,3} to the case of any fixed dimension \( d > 1 \) and fixed number \( m \) of depots. Actually, we propose a new Efficient Polynomial Time Approximation Scheme (EPTAS) for the Euclidean CVRP, for which capacity \( q \), the number of depots \( m \), and dimension \( d > 1 \) are fixed. The algorithm proposed remains PTAS for the problem with fixed \( m \), \( d > 1 \), and \( q = O(\log \log n)^{1/d} \).

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REFERENCES

Approximation Schemes for the Generalized TSP in Grid Clusters

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The Generalized Traveling Salesman Problem (GTSP) is a generalization of the well known Traveling Salesman Problem (TSP). The main difference is that together with a weighted graph $G = (V, E, w)$ the instance is specified by a partition of the node set $V = V_1 \cup \ldots \cup V_k$ into disjunctive subsets or clusters. The goal is to find a minimum cost cycle such that each cluster is hit by exactly one node of this cycle.

We consider a geometric setting of the GTSP, in which a partition is specified by cells of the integer $1 \times 1$ grid (on the Euclidean plane). Even in this special setting, the GTSP remains intractable enclosing the classic Euclidean TSP on the plane. Recently [1], it was shown that this problem has $(1.5 + 8\sqrt{2} + \varepsilon)$-approximation algorithm with complexity bound depending on $n$ and $k$ polynomially, where $k$ is the number of clusters. We propose three approximation algorithms for the Euclidean GTSP on grid clusters. For any fixed $k$, all proposed algorithms are PTASs. Time complexities of the first two remain polynomial for $k = O(\log n)$ while the last one is a PTAS when $k = n - O(\log n)$. Although, the problem is polynomially solvable in the case of fixed $k$, our PTAS’s have rather small time complexity (wrt. $n$) and can be useful in tackling of Big Data.

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References

Complexity and Approximability of Geometrical Piercing Set Problem for Rectangles Intersecting a Diagonal Line

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The classic Hitting Set Problem (HSP) can be stated as follows. For a given hypergraph $G = (V, E)$, it is required to find a smallest vertex subset $H$ hitting all the hyperedges, i.e. such a subset $H$ that $H \cap e$ for any $e \in E$. In general case, this problem is equivalent to the well known Set Cover problem, it is NP-hard and hardly approximable. Nevertheless, there are many special cases of the HSP, which are polynomially solvable or can be approximated well. One of such special cases is called Piercing Set Problem (PSP). In PSP, a hypergraph is given implicitly by a collection of geometric shapes $S_1, \ldots, S_n$ located on the plane. Basically, in this case, we are aimed to find the smallest finite subset $H$ of the plane hitting all the shapes.

We consider a very special case of PSP, where all the shapes are axis-parallel rectangles intersecting a given straight line (diagonal). It is known that, even in this special setting, the PSP remains intractable while has a 4-approximate polynomial time algorithm.

Recently [1], it was shown that, for unit squares, the PSP has an exact algorithm with a rather huge but polynomial time-complexity. In the context of the hypothesis $P \neq NP$, this result seems very intriguing. Indeed, for the majority of known polynomially solvable problems, there are built exact algorithms with low-order polynomial complexity bounds.

We extend this result to construct polynomial and pseudo-polynomial exact and approximation algorithms for more several more wide subclasses of the PSP.

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Terminal control problem with fixed ends in dynamics: saddle-point technique

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In a Hilbert space, the problem of terminal control with linear dynamics and fixed ends of trajectory is considered. The integral objective functional has a quadratic form. In contrast to the traditional approach, the problem of terminal control is interpreted not as an optimization problem, but as a saddle-point problem. The solution to this problem is a saddle point of the Lagrange function with components in the form of controls, phase and conjugate trajectories.

We consider the simplest convex optimal control problem

\[
\begin{array}{l}
(x^*(\cdot), u^*(\cdot)) \in \text{Argmin} \left\{ \frac{1}{2} \int_{t_0}^{t_1} \left( \langle Q_1(t)x(t), x(t) \rangle + \langle Q_2(t)u(t), u(t) \rangle \right) dt ,
\right.

\left. \frac{d}{dt}x(t) = D(t)x(t) + B(t)u(t), \quad t_0 \leq t \leq t_1,
\right.

x(t_0) = x_0, \quad x(t_1) = x_1,
\end{array}
\]

where \( Q_1(\cdot), Q_2(\cdot) \) are continuous positive semidefinite symmetric \( n \times n, r \times r \)-matrices, \( D(\cdot), B(\cdot) \) are \( n \times n, n \times r \) continuous matrices; trajectory \( x(\cdot) \) is absolutely continuous function; set of admissible controls

\[ U = \{ u(\cdot) \in L_r^2[t_0, t_1] \mid \|u(\cdot)\|_{L^2} \leq \text{const} \}. \]

We need to find a control \( u^*(\cdot) \in U \) such that the corresponding trajectory \( x^*(\cdot) \) connects some starting point \( x_0 \) with a given point \( x_1 \in X_1 \) at the right end. Problem (1) is a convex programming problem formulated in Hilbert space. We introduce to (1) the Lagrangian

\[
\mathcal{L}(\psi; x(\cdot), u(\cdot)) = \frac{1}{2} \int_{t_0}^{t_1} \left( \langle Q_1(t)x(t), x(t) \rangle + \langle Q_2(t)u(t), u(t) \rangle \right) dt
\]

\[ + \int_{t_0}^{t_1} \langle \psi(t), D(t)x(t) + B(t)u(t) - \frac{d}{dt}x(t) \rangle dt \]

\[ \forall \psi(\cdot) \in \Psi_n^2[t_0, t_1], (x(\cdot), u(\cdot)) \in AC^n[t_0, t_1] \times U, x(t_0) = x_0, \ x(t_1) = x_1. \]

By linearization of the Lagrangian and using the saddle-point inequalities system, we get the dual problem to (1). By bringing together main elements
of primal and dual problems, we obtain a final system. The solution to this system is a saddle point of the Lagrangian, and some of its components form the desired solution [1],[2].

To solve the problem we build a controlled simple iteration method:

1) **predictive half-step**

\[
\frac{d}{dt} x^k(t) = D(t)x^k(t) + B(t)u^k(t), \quad x^k(t_0) = x_0,
\]

\[
\frac{d}{dt} \psi^k(t) + D^T(t)\psi^k(t) = -Q_1(t)x^k(t), \quad \psi^k(t_1) = 0,
\]

\[
\bar{u}^k(t) = \pi_I(u^k(t) - \alpha(Q_2(t)u^k(t) + B^T(t)\psi^k(t)));
\]

2) **basic half-step**

\[
\frac{d}{dt} \bar{x}^k(t) = D(t)\bar{x}^k(t) + B(t)\bar{u}^k(t), \quad \bar{x}^k(t_0) = x_0,
\]

\[
\frac{d}{dt} \bar{\psi}^k(t) + D^T(t)\bar{\psi}^k(t) = -Q_1(t)\bar{x}^k(t), \quad \bar{\psi}^k(t_1) = 0,
\]

\[
u^{k+1}(t) = \pi_U(u^k(t) - \alpha(Q_2(t)u^k(t) + B^T(t)\bar{\psi}^k(t))), \quad k = 0, 1, ...
\]

In linear-convex case, this approach could be interpreted as strengthening the Pontryagin maximum principle. It provides the convergence of computing process to solution of the problem in all components: the convergence in controls is weak, the convergence in phase and conjugate trajectories is strong.

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**References**


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Smoothing factor of the frontier transformation in the DEA models

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At present the DEA models are widely used for performance analysis of socio-economic systems. However, some investigators noted that inadequate results may arise in the DEA models. In our previous papers \cite{1,2} we discovered the reasons of such results. It was also proved that only terminal units give us necessary and sufficient set of units for smoothing the efficient frontier. In addition, a general algorithm was elaborated by our team for smoothing the frontier. This algorithm was based on using the notion of terminal units. Under the elaboration of the algorithm we stick to the following principles: a) all efficient units have to stay efficient after the frontier transformation; b) every inefficient unit will be projected on the efficient part of the frontier.

Moreover, we have to introduce the notion of smoothness of the frontier in order to compare original and transformed frontier. Our computational experiments using real-life data sets confirmed that smoothing factor represents adequate measure of the frontier transformation.

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References

Cosmonauts Training Scheduling Problem

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Nowadays, in Russia, spaceflight training scheduling is performed manually and without using any mathematical approaches. Due to that, fast changes of a training plan will cause a huge workload. We hope that the considered approaches and models will lead to reducing these workloads.

Commonly, the cosmonaut training planning is divided into the two stages: the volume planning and the timetabling. In the former one, for each cosmonaut a set of tasks is formed depending on requirements of their qualifications and forthcoming on-board experiments complexity conditions. For details, see at [1]. We study the second stage of the problem, i.e. timetabling. There is a set of crews. Each crew consists of a number of cosmonauts. Each cosmonaut has his own set of training tasks. Dates of the training start and finish are given. The goal is to form a training schedule for each cosmonaut.

From a mathematical point of view, the spaceflight training scheduling can be considered as a generalization of the resource-constrained project scheduling problem. This problem is NP-hard. In practice, a planning horizon is about 3 years. Each cosmonaut has an individual, aperiodical learning plan. So, the problem has a very large dimension and is hard to solve.

A mathematical model based on integer linear programming (ILP) is proposed. An alternative approach to the problem is to use Constraint Programming (CP) solvers. To use this approach, the problem has been reformulated as a Constraint Satisfaction Problem. The advantage of CP is its possibility to reduce the set of admissible values of variables. The important principle of CP consists of distinguishing constraint propagation and decision-making search.
Constraint propagation is a deductive activity which consists in deducing new constraints from existing constraints. The large number of constraints in our problem contributes to high efficiency of CP methods.

The calculations were performed using the solver IBM ILOG CPLEX ver. 12.6.2. A distinct advantage of CP is shown in presented Tab. 1. Here $|W|$ is a number of weeks in planning horizon, Var. is a number of variables, Constr. is a number of constraints in the problem.

Current implementation of the presented model in terms of CP allows us to form schedule with planning horizon equal to 1 year with less than 5 min. The main purpose of this research is to develop mathematical model and find approaches to solve it in order to implement Planner system in the near future.

**REFERENCES**

Dynamic programming approaches for single-track scheduling problem

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We consider a scheduling problem with the single railway track connecting two stations. Single railway tracks constitutes the major part of railway transport networks in many regions, and are very common in supply chains. Survey of the railway planning models and methods can be found in the publication of Lusby et al \cite{1}. Usually problems are considered in terms of scheduling theory as job-shop problems, and dynamic programming approach \cite{2} or heuristic method \cite{3} is applied.

We investigate the problem where trains travel between two stations, denote them as Station 1 and Station 2, which are connected by a single track. Each train travels either from Station 1 to Station 2 or from Station 2 to Station 1. The transportation commences at time $t = 0$. Define the set of all trains at both stations as $N$. Two models are considered: the model with the siding on a track and the model without a siding. For both models we propose the solution algorithms based on the dynamic programming method.

The model without a siding is described as follows. There are $\alpha$ different possible speeds of trains, where $\alpha$ is a rather small number. For example, we can divide trains into 3 groups according to the speeds: freight trains, passenger trains, express trains. Each train travels with one of constant $\alpha$ speeds, for each speed the train traversing time for the track is given. There must be a minimal safe distance between two trains simultaneously moving in one direction, so a minimal time interval between the departure of two trains is required, it depends on the speeds of successive trains. In addition, train movement on the track is possible only in restricted set of time intervals $V = \{t|t \in [u_1, v_1] \cup [u_2, v_2] \cup$
... ∪ [u_q, v_q]}, where q is a number of intervals. We consider the set of objective functions which can be represented with as special general form. For each train some additional parameters depending on objective function are given. In the most general case of the problem with α speeds of trains, given possible interval of movement V and λ sets of trains with specified departure order the solution algorithm constructs optimal schedule in \( O(q^2 \log q n^{2\alpha^2 + 2\alpha + 1} n^\lambda \log n) \) operations, where \( n = |N| \) is the number of trains on both stations at the initial moment. In some cases it is possible to significantly reduce the complexity, if \( \alpha = 1, V = \{t| t \in [0, \infty)\} \), and objective function is maximum lateness, then the complexity is \( O(n^2) \).

In the second model there is the siding on the track, which capacity is one train. On the track it is possible to cross two incoming trains only in the siding. All trains have equal constant speed, train traversing times for track segments separated by the siding are given. Minimal time interval is required between the departures of two trains from one station and between the arrival of two trains to the siding. For each train \( i \in N \) due date \( d_i \) or priority coefficient \( w_i \) are given. Two objective functions which have to be minimized are considered: maximum lateness and the weighted sum of arrival moments. For each objective function the complexity of solution algorithm is \( O(n^2) \).

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References

Application of splines for the evaluation of investment rate dynamics

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In the report the results of spline analysis of statistical data used in paper [1] are given. We used statistical data of this work, because the application of modern mathematical methods of analysis cannot only evaluate the dynamics of the parameters of the production function, but also to assess the dynamics of some macroeconomic indicators in the future. For decision the problem of dynamic sets smoothing the following specific functions applied: polynomial splines (linear, parabolic, cubic) with various defects, functions with piecewise-constant growth rate and functions with piecewise-constant values of the coefficient of elasticity.

Data analysis was accomplished using a computer programme that implements the algorithm for constructing functions with piecewise constant characteristics. On the base of spline-analysys it is shown that the production sector of the USA from 1899 to 1922 developed unevenly and owing to it the coefficient of elasticity of Cobb-Douglas production function was variable.

Approximation of theoretical dependence of the output-labour ratio on the capital-labour ratio by functions with piecewise constant elasticity allowed one to estimate time intervals in which this coefficient can be considered as the constant. Estimations of the values of the investment rate in 1899–1922 based on some hypotheses about the dynamics of the coefficient of disposals in this period were obtained. It is shown that statistical data used in paper [1] contain information that warns of a possible decline in production in US manufacturing in the 5-10 years (after 1922).

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References

Algorithm of decision of task of smoothing out dynamic rows by functions with piece-permanent parameters

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Actuality of tasks of retrospective analysis is determined by that understanding of history of development of process and its conformities to law directly influences on quality of prognoses. Therefore, a retrospective analysis must be preceded to the decision of task of prognostication. For correct authentication of the folded tendencies with the purpose of extrapolation, last different methods and receptions of smoothing out of dynamic rows are used in the future.

Development of this approach for the analysis of production dependencies is connected, in particular, with the researches, which were carried out under the supervision of the Prof. Yu.P. Ivanilov in the Computer Center of the Russian Academy of Sciences, by A.P. Abramov, V.A. Bessonov, V.A. Fadeyev, T.I. Gurova, K.H. Zoidov, V.V. Lebedev, K.S. Sviridenko and others in 1985–1996.

In the presented lecture, the algorithm of decision of task of smoothing out of dynamic rows functions is expounded with piece-permanent parameters [¹]. This algorithm will realize the method of progressing wave for the decision of some task of optimal management, to that the task of smoothing out is taken. Substantially, that here in the complement of the varied parameters the values of argument enter in the key points of the constructed function. Appropriate computer software allows you to build the following specific functions: polynomial splines (linear, parabolic, cubic) with various defects, functions with piecewise-constant growth rate and functions with piecewise-constant values of the coefficient of elasticity.

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References

Relation between Mangasarian-Fromovitz condition
and some other constraint qualifications in nonlinear
programming

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Constraint qualifications (CQs) play an important role in nonlinear
programming since they allow to provide the validity of the Kuhn-Tucker
necessary optimality condition and to construct numerical algorithms for finding optimal
points.

Consider the set \( C = \{ y \in \mathbb{R}^m \mid h_i(y) \leq 0, \ i \in I, h_i(y) = 0, \ i \in I_0 \} \) of
feasible points in nonlinear programming problem, where \( I = \{ 1, ..., s \} \), \( I_0 = \{ s + 1, ..., p \} \), \( h_i(y) \), \( i = 1, 2, ..., p \) are continuously differentiable functions from
\( \mathbb{R}^m \) to \( \mathbb{R} \).

The most well-known CQ is the Mangasarian-Fromovitz condition (MFCQ)
[1], which holds at a point \( y \in C \) if \( \Lambda_0(y) = \{ 0 \} \) where

\[
\Lambda_0(y) = \{ \lambda \in \mathbb{R}^p \mid \sum_{i=1}^{p} \lambda_i \nabla h_i(y) = 0, \ \lambda_i \geq 0 \ \text{and} \ \lambda_i h_i(y) = 0 \ \text{for} \ i \in I \}.
\]

In spite of important place of MFCQ in optimization theory there are problems where MFCQ does not hold though some other CQs can be fulfilled. One of such constraint qualifications is the constant rank condition (CRCQ) by Janin
[2]. It is known that MFCQ and CRCQ are independent of each other, i.e. there are examples where MFCQ is satisfied while CRCQ is not and vice versa. The relation between MFCQ and CRCQ was recently studied in the work [3]
by Shu Lu where interesting results were obtained. As demonstrated in this
work, if CRCQ holds at a point \( y \in C \) then by removing some constraints and
transforming some constraints of inequality type into equalities one can obtain
a set which locally does not differ from \( C \) but for which MFCQ holds at this
point.

One of the generalizations of MFCQ is the relaxed Mangasarian-Fromovitz
constraint qualification (RMFCQ) which has been proposed in [4] (see also [5,6]). Later this condition has been introduced in [7] under the name CRSC (constant
rank of the subspace component condition).

Let \( I(y) = \{ i \in I \mid h_i(y) = 0 \} \) and \( I^a(y) = \{ i \in I(y) \mid \exists \lambda \in \Lambda_0(y) \text{such that} \lambda_i > 0 \} \).
Definition 1. RMFCQ holds at a point \( y^0 \in C \) if there exists some neighbourhood \( V(y^0) \) of this point such that 
\[
\text{rank}\{\nabla h_i(y), \ i \in I_0 \cup I^a(y^0)\} = \text{rank}\{\nabla h_i(y^0), \ i \in I_0 \cup I^a(y^0)\}
\]
for all \( y \in V(y^0) \).

RMFCQ is weaker not only with respect to MFCQ but also it is weaker with respect to such constraint qualifications as CRCQ and the relaxed constant rank condition [8], the conditions CPLD and RCPLD [7].

The following theorem generalizes the result by Shu Lu [3] and demonstrates the relationship between RMFCQ and MFCQ.

**Theorem 1.** Let the RMFCQ holds at a point \( y^0 \in C \). Then there exist index subsets \( J^a \subset I^a(y^0) \) and \( J_0 \subset I_0 \) such that the set \( C \) locally coincides with the set \( C^# = \{ y \in \mathbb{R}^m \mid h_i(y) \leq 0, \ i \in I \setminus I^a(y^0), h_i(y) = 0, \ i \in J_0 \cup J^a \} \) in a neighborhood of \( y^0 \) and the Mangasarian-Fromovitz constraint qualification holds at \( y^0 \in C^# \).

References

Pareto Frontier Visualization in Developing the Control Rules for Angara River Basin

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Visualization of the Pareto frontier is an efficient tool for decision support in multiobjective environmental decision problems with many objectives (4 to 9 objectives). The Interactive Decision Maps (IDM) technique implements visualization of the Pareto frontier by approximating the Edgeworth-Pareto Hull (EPH), i.e. the maximal set, which Pareto frontier coincides with the Pareto frontier of the feasible objective set, and by subsequent visualization of its Pareto frontier by displaying the decision maps, i.e. overlapped bi-objective slices of the EPH while the value of a third objective is changed [1]. To visualize the Pareto frontier for 4 to 9 objectives, the interactive and animated decision maps are used. Graphic information on the Pareto frontier helps the decision maker or the negotiators to specify the preferred non-dominated objective point (feasible goal point) consciously. Then, the associated decision is provided.

Approximating the EPH is the most complicated step of the IDM technique. In the 1990s, effective methods for polyhedral approximating the convex EPH were developed. In the 2000s, hybrid methods that integrate classic gradient-based optimization methods and random search techniques with evolutionary multiobjective techniques were developed for non-linear non-convex problems with up to 9 objectives. In this paper, application of the IDM technique in the framework of the development of control rules for the Angara River reservoir system is described.

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Multi-criteria approach to the analysis of the efficiency of optimization algorithms

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It’s well-known that the performance of optimization methods heavily depends on a selection of method’s parameters values. Moreover usually there is trade-off between various criteria, e.g. running time and obtained optimum. In this paper we propose the multi-criteria approach to analyze the impact of the method parameters on the efficiency of an optimization process.

We evaluate the proposed approach on the problem of minimization of the potential energy of the 2D-crystal lattice considered in [1]. We study the coordinate descent method with the following parameters: Inc — increase of the step in the case of success and Dec — decrease of the step in the case of failure.

The following approach has been proposed:

1. Choose basic criteria. In our example we choose AverValue — the average value of the objective function at the found solution obtained after the given number of trials and the total running time required to complete these iterations. The first important observation is that the number or trials should be sufficiently large to guarantee the stability of the criteria value. Fig. 1 shows that for the small number of trials the average value vary significantly from one trial to another and the stabilizes after approximately 150 trials. Further tests were carried out for a given number of trials \( N = 150 \).

2. Building a grid of two parameters optimization algorithm on a given field of study with a certain step. On this grid, we have carried out tests and build the Pareto front with the rejection of dominated values.

![Fig. 1. Average values over trials](image)

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Fig. 2 shows the results for *AverValue* and *Time* criteria, on the left — all points, on the right — only non-dominated points. White point represents an ideal result (all criteria achieves minimum values). Color of the points indicates their proximity to the ideal.

Now it seems clear, that turquoise points are the best ones, but this we reserve to a decision maker.

![Graph showing results for AverValue and Time criteria.](image)

Fig. 2. Total 42 results obtained, non-dominated 12 ones

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**References**

On optimal control of dynamical systems described by differential inclusions

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The systems modelled by the differential equations with multiple-valued right parts are considered in [1–6] and in other works. Some questions of existence and stability of differential inclusions are studied in [1–3]. The stability conditions and stabilization of the systems described by the differential inclusions containing control in the right parts is obtained in [4] by means of Lyapunov functions method.

In this report the system of movement of object from the start point to the final point with the an exit in the vertical direction is considered. The specified system is described by differential inclusions, and the multiple-valued components in the right parts of the equations is connected with need of the accounting of resistance of the rarefied environment.

The problem of optimal control consists in a choice of parameters of draft and values of time so that object motion from the initial point to final with intermediate achievement of height was carried out with the minimal fuel consumption.

By means of results of works [1–3] the stability analysis of the nominal motion determined by criterion of optimality is performed. The algorithms of optimal trajectory search are offered for object transition to the purpose in three-dimensional space. The set of programs realized in Matlab environment is developed. This set of programs contains of modules for data input and for a graphic illustration of trajectories. The program calculates draft force and velocities of the control object. We consider also the modifications of models described in [5, 6].

The offered algorithms and the set of programs can be used for selection of optimal parameters of the motion of transport systems in the conditions of incomplete information.

REFERENCES


On a gradient constraint problem for scalar conservation laws

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We consider a Dirichlet-Neumann boundary problem in a bounded domain for scalar conservation laws. Using an idea from [1], we propose an informal solution concept by considering an elliptic approximation to the problem (see Chapter 3 of [2]). We are not yet able to prove existence or uniqueness of the solution satisfying the proposed solution concept, but, under appropriate assumptions, we prove that a corresponding weak limit satisfies the considered equation in the interior of the domain. The basic tool is the compensated compactness method. In the case when the flux is continuously differentiable with respect to all the variables, Remark 1 of [3] implies that a weak limit of the elliptic approximation satisfies the Kruzhkov admissibility conditions in the interior of the domain.

We also provide numerical examples to justify, in the special situation, one of the limiting assumptions of the theoretical framework.

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The estimation of the company’s market capitalization based on production models taking into account the deficit of current assets

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Economic downturn in a number of the states of the eurozone (Italy, Spain, Portugal, Greece), and also decrease in growth rates in a number of the countries with economy of the catching-up type (for example Russia and China) is led to decrease in demand in the world markets. The interruptions in sales of products lead to the current assets deficit and requirement of their replenishment by credits. Pledge when crediting current assets is capitalization of the company which fall leads to increase of interest rates and lowering of the credit rating of the company. Therefore an important task is development of the tool which allows to estimate capitalization of the company on the basis of indicators of its activities. Such tool can be created based on a class of mathematical models of production taking into account current assets deficit which is developed by authors since the end of the 90th years and consistently describes schemes of functioning of production in the conditions of unstable demand at various stages of development of the Russian economy [1-2]. The modern version of model considers restriction of trade infrastructure and formalized in the form of Bellman’s equation for which the solution in an explicit form found [2]. The benefit of the model is that it allows calculating the average indicators of activities comparable to data of the official reporting of the companies (IFRS standard). The model allows estimating capitalization of the company depending on indicators of its activities, an external economic environment and size of a discount of the company’s income that characterizes appeal of the company to investors. In terms of model the analysis of influence on dynamics of company’s creditworthiness indicator (which is understood as the capitalization relation to current assets), a discount rate of company’s income is carried out that allows estimating compliance of expectations of the market to real indicators of the company. Below are presented the results of estimation of creditworthiness indicator for the FCA, Italy and MCC, China in case of the level of a discount rate of income equals to average rate on long-term loans (dashed lines) and the level of a discount rate reflecting the expectations of the market which have developed by results of the biddings on the stock exchange (firm lines) (Fig.1). For the FCA company the essential growth of a share price can be explained with expansion of a dealer network of the company on the American market due
to absorption of Daimler Crysler though indicators of activities of the company correspond to much lower capitalization. For MCC real capitalization grows in a case, but much more slowly, than share market value that it is possible to explain with strong long-term state support of the company. The authors were supported by the RSF, project N16-11-10246.

Fig. 1

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Identification of a dynamic model of Russian economy with two kinds of capital

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Paper [1] proposed an economic model with two kinds of capital - old and new. It is assumed that the old capital $A(t)$, created during the Soviet time, from 2008 is only eliminated (see [2, 1] for explanations),

$$\dot{A} = -\mu_t A(t), \quad A(t_0) = A_0,$$

and the new capital $B(t)$ is increased due to investments and is eliminated as a result of aging.

$$\dot{B} = J(t) - \mu_t A(t), \quad B(t_0) = B_0.$$  (2)

This two kinds of capital and labour $L(t)$

$$\dot{L} = \gamma t L(t), \quad L(t_0) = L_0.$$  (3)

are three factors of the Cobb-Douglas production function for GDP $Y(t)$.

$$Y(t) = Y_0 (A(t)/A_0)^\alpha (B(t)/B_0)^\beta (L(t)/L_0)^\lambda.$$  (4)

For closing our model we can use the same heuristic equations for the GDP components (export $E(t)$, import $I(t)$, investment $J(t)$) as in [2] and the balance equation for “consumption” $Q(t)$:

$$E(t) = \varepsilon t Y(t)/p(t),$$  (5)

$$I(t) = \iota (Y(t) - p(t)E(t))/q(t),$$  (6)

$$J(t) = \xi (Y(t) + q(t)I(t))/s(t),$$  (7)

$$Q(t) = Y(t) - s(t)J(t) + q(t)I(t) - p(t)E(t).$$  (8)

It is proposed an indirect parameter identification method for a Russian economic model by statistical time series of macroeconomic indicators of Russia 2008-2014. The procedure of identification of the model includes a parallel computing on cluster supercomputer as in [3-8].

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Improved computing realization of atmospheric general circulation model

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The climate model of the Computing Center of RAS includes an atmospheric unit, based on the Atmospheric General Circulation Model (AGCM) with parameterization of a number of subgrid processes, global ocean model and a model of the evolution of sea ice [1, 2]. Proposed modified method of AGCM parallelization with calculating the contribution of physics and dynamics, respectively, on two groups of processors with the same input data leads to a more efficient and flexible scheme of calculation. Model version with a finer spatial resolution and ocean general circulation model is developed. AGCM is the software package which simulates many physical processes [3, 4]. There are two major program components: AGCM Dynamics block, which calculates the fluid flow described by the primitive equations by finite differences, and AGCM Physics block which computes the effect of processes not resolved by the model grid (such as solar and heat radiative fluxes, internal sub-grid scale adiabatic processes, moist and convection processes). The results obtained by AGCM Physics are supplied to AGCM Dynamics as forcing for the flow and thermodynamics calculations. The AGCM code uses a three dimensional staggered grid for velocity and thermodynamic variables (temperature, pressure, water vapor mixing ratio, etc.). The AGCM Dynamics itself consists of two main components: a spectral filtering part and the actual finite difference calculations. The filtering operation is needed at each time step in regions close to the poles to ensure the effective grid size there satisfies the stability requirement for explicit time difference schemes when a fixed time step is used throughout the entire spherical finite-difference grid. [5]. Processors domain decomposition in the two-dimensional horizontal plane grid is used in a parallel implementation of the model. This choice is based on the fact that vertical physical processes strongly link grid points and that the number of grid points in the vertical direction is usually small. That makes parallelization less efficient in the vertical direction. Each subdomain of this grid is a rectangular area that contains all points of the grid in the vertical direction. Two types of interprocessor communications are mainly in this case [5]. Data exchanges are needed between logically adjacent processors (nodes) in the calculation of finite differences and remote data exchanges are needed, in particular, to carry out the operation of spectral filtering.
The analysis shows that the results for the modified scheme give satisfactory results and it is possible to use it. Scalar program calculation time takes 33 Modified method advantages are clear for experiments with relatively large amount of processors, when many interprocessors data exchange exist in Dynamics block, but Physics block does not have this problem. This effect explains the slowing down of original method calculations [7]. Number of processors, distributed for Dynamics block in the modified method is the same for all experiments, and number of processors, distributed for Physics block increase with the more detailed description of physical processes in the model experiments.

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References

Ellipsoidal estimation of the attraction domain for affine systems with constrained control resource

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We consider a class of single-input affine control systems that can be represented in the canonical form [1]

\[
\dot{x}_1 = x_2, \ldots, \dot{x}_{n-1} = x_n, \dot{x}_n = f_0(x) + f_1(x)u,
\]

where \( x \in D_x \subseteq \mathbb{R}^n \), \( f_0(x) \) and \( f_1(x) \neq 0 \) are continuous functions, and \( u \) is a scalar control. Applying feedback \( u = -(\sigma(x) + f_0(x))/f_1(x) \), where \( \sigma(x) = c^T x \), \( c^T = [c_1, \ldots, c_n] \), \( c_i > 0 \), we obtain a linear closed-loop system. While \( c_i \)'s can always be selected such that the latter system is globally asymptotically stable, the original system (1) cannot be linearized in the entire domain because of a constrained control resource. Hence, the origin is only locally stable, and we arrive at the problem of finding (an estimate of) the attraction domain of a nonlinear closed-loop system.

Assuming that the control \( u \) is subject to the constraint \( |u| \leq \bar{u} \) and defining the feedback in the entire domain \( D_x \) to be

\[
u(x) = -\text{sat}_{\bar{u}}[(\sigma(x) + f_0(x))/f_1(x)],
\]

where \( \text{sat}_{\bar{u}}[\cdot] \) is the saturation function, we set the following problem: given a system of form (1) closed by feedback (2), find the “best” ellipsoidal estimate \( \Omega(P) = \{x : x^T P x \leq 1, P > 0\} \subseteq D_x \) of the attraction domain of the zero solution. Under the “best” estimate, we mean, e.g., the ellipsoid for which the trace of the corresponding matrix \( P \) is minimal.

With regard to (2), the closed-loop system can be written as

\[
\dot{x}_1 = x_2, \ldots, \dot{x}_{n-1} = x_n, \dot{x}_n = -\Phi(\sigma, x).
\]

Along with (3), we introduce the linear nonstationary system \( \dot{x} = A_{\beta(t)} x \):

\[
\dot{x}_1 = x_2, \ldots, \dot{x}_{n-1} = x_n, \dot{x}_n = -\beta(t)\sigma(x),
\]

System (4) is said to be absolutely stable in a sector \([\beta_0, 1]\) if its zero solution is asymptotically stable for any functions \( \beta(t) \) satisfying the condition \( 0 < \beta_0 \leq \beta(t) \leq 1 \) [2]. If, \( \forall x \in \mathbb{R}^n \), \( \Phi(\sigma, x) \) lay in the same sector, i.e.,

\[
0 < \beta_0 \leq \Phi(\sigma, x)/\sigma \leq 1,
\]
then system (3) would also be absolutely stable [2]. If (5) does not hold in \( R^n \)
(like in our case), for an estimate of the attraction domain, one can take an
invariant ellipsoid of system (4) in which condition (5) holds [3]. The following
theorem reduces finding of such an ellipsoid to solving a system of linear matrix
inequalities (LMIs).

**Theorem.** Let \( \Omega(P) \subseteq D_x \) and \( \min_{x \in \Omega(P)} U(x) \geq U_0 > 0 \), where \( U(x) = |f_1(x)|\bar{u} - |f_0(x)| \). Let \( P > 0 \) be a solution of the LMI system

\[
PA_1 + \frac{1}{2}A_1^T P \leq 0, \quad PA_{\beta_0} + \frac{1}{2}A_{\beta_0}^T P \leq 0, \quad P \geq c^T \beta_0^2 / U_0^2, \quad (6)
\]

for some \( 0 < \beta_0 \leq 1 \), where \( A_1 \) and \( A_{\beta_0} \) are constant matrices obtained from \( A_{\beta(t)} \) by the substitution of 1 and \( \beta_0 \) for \( \beta(t) \). Then, \( \Omega(P) \) is an invariant ellipsoid belonging to the attraction domain of system (3).

To ensure the inclusion \( \Omega(P) \subseteq D_x \) and fulfillment of the inequality
\( \min_{x \in \Omega(P)} U(x) \geq U_0 \), it is proposed to approximate \( D_x \) by a family of do-
mains \( \Pi(U_0) \subseteq D_x, 0 < U_0 \leq |f_1(0)|\bar{u} - |f_0(0)| \), where \( \Pi(U_0) \) is a domain with
a boundary composed of first- and second-order surfaces satisfying the condi-
tion \( \min_{x \in \Pi(U_0)} U(x) = U_0 \). The inscribing of an ellipsoid into such a domain
reduces to solving an LMI system \( l_i(P) \leq 0, \ i = 1, \ldots, m \).

Then, having solved the latter LMI system jointly with (6) by means of, say,
Matlab procedure \texttt{mincx}, which finds \( P \) with the minimal trace, one gets the
best ellipsoidal estimate for given \( \beta_0 \) and \( U_0 \) with the corresponding performance
index \( F(U_0, \beta_0) = \text{trace} \ P(U_0, \beta_0) \). By minimizing this function of two variables,
one gets the desired ellipsoidal estimate.

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Synchronization, self-organization and self-optimization in nonlinear dynamical systems

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Synchronization effects in linear and quasi-linear dynamical systems are well studied [1]. Similar effects in essentially nonlinear systems can be detected in numerical experiments with dynamical models [2]. The synchronization and self-organization effects study in strongly nonlinear dynamical systems with deterministic chaos is of special interest [3]. In this study effective research tool of the solutions becomes an interactive search mode [4].

Synchronization effects in multidimensional nonlinear dynamic systems in mechanics and physics associated with the phenomenon of self-organization in more complex systems - the economy and society. The phenomenon of self-organization can be interpreted as the natural spontaneous dynamical system transition to optimal condition.

We discuss an alternative approach when the objective function to determine the system parameters is the product of deterministic chaos. Cases highlighted when the self-organization can be associated with a self-optimization. The objective function in this case is formed by natural way.

REFERENCES

ARIMA-GARCH models of RTS and MICEX futures dynamics

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In Russia, the most liquid futures contracts on stock indices are futures on RTS and MICEX indices. To assess the market and credit risk of derivatives portfolio the Monte Carlo method based on the simulation of multiple future scenarios of initial uncertainty factors is commonly used. On the basis of simulated future factor values for each scenario corresponding derivatives prices are calculated.

In the basic theory of index futures pricing based on the concept of perfect market in the absence of arbitrage opportunities the ”Cost of Carry” model is used to assess the price of the stock index futures. The implementation of the ”Cost of Carry” model for the futures on RTS and MICEX indices showed that the difference between the calculated and actual futures prices may be significant.

This report describes causes of price differences, presents the application results of existing stock index futures pricing models to futures on RTS and MICEX indices, and offers a number of new index futures pricing models based on a modification of ”Cost of Carry” model, that can be applied within the framework of the Monte Carlo method for the derivatives portfolio risk assessment. Proposed models are based on ARIMA-GARCH processes with different explanatory factors and conditional distribution functions. The comparative statistical analysis of proposed models based on Akaike, Schwarz and Hannan-Quinn information criteria is presented.

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On alternative duality and symmetric lexicographical correction of improper linear programs

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Consider the dual pair of linear programs

\[
\begin{align*}
\text{max}\{ (c, x) : Ax \leq b, \; x \geq 0 \}, & \quad (1) \\
\text{min}\{ (b, y) : A^T y \geq c, \; y \geq 0 \}, & \quad (2)
\end{align*}
\]

where \( A = (a_{ij})_{m \times n}, \; c \in \mathbb{R}^n \) and \( b \in \mathbb{R}^m \) are given, \( x \in \mathbb{R}^n \) is the vector of primal variables, \( y \in \mathbb{R}^m \) is the vector of dual variables, \((\cdot, \cdot)\) means the scalar product. Assume that programs (1)–(2) may be improper \cite{1} (i.e. one of these programs or both of them may be infeasible). Suppose that initial constraints systems are divided onto series of subsystems arranged by their permissibility for relaxation. It means that initial data in (1)–(2) is parted onto blocks like this:

\[
\begin{array}{c|c|c}
\hline
\mathcal{c} & \mathcal{x} \\
\hline
A & b & y \\
\hline
\end{array}
\quad \sim 
\quad \begin{array}{c|c|c}
\mathcal{c}_0 & \mathcal{c}_1 & \ldots & \mathcal{c}_{n_0} \\
\hline
x_0 & x_1 & \ldots & x_{n_0} \\
\hline
\end{array}
\quad \sim 
\quad \begin{array}{c|c|c}
\mathcal{c} & \mathcal{x} \\
\hline
A_0 & b_0 & y_0 \\
\hline
A_1 & b_1 & y_1 \\
\hline
\vdots & \vdots & \vdots \\
\hline
A_{m_0} & b_{m_0} & y_{m_0} \\
\hline
\end{array}
\]

To determine the generalize (relaxed) solution for (1)–(2) write out the dual pair of symmetric relaxed programs

\[
\begin{align*}
\text{max}\{ (c - \Delta c, x) : A_s x \leq b_s + \Delta b_s \; (s = 0, \ldots, m_0), \; x \geq 0 \}, & \quad (3) \\
\text{min}\{ (b + \Delta b, y) : B_s^T y \geq c_s - \Delta c_s \; (s = 0, \ldots, n_0), \; y \geq 0 \}, & \quad (4)
\end{align*}
\]

where \( \Delta c = [\Delta c_0, \ldots, \Delta c_{n_0}] \in \mathbb{R}^n \) and \( \Delta b = [\Delta b_0, \ldots, \Delta b_{m_0}] \in \mathbb{R}^m \) are the parameters of relaxation. The vector \( \Delta \mathcal{c} = [\Delta \mathcal{c}_0, \ldots, \Delta \mathcal{c}_{n_0}] \) is called an optimal lexicographical relaxation of objective vector in (3)–(4) iff

\[
\begin{align*}
\Delta \mathcal{c}_0 &= \text{arg min}\{ \| \Delta c_0 \| : B_0^T y \geq c_0 - \Delta c_0, \; y \geq 0 \}, \quad \ldots, \\
\Delta \mathcal{c}_{n_0} &= \text{arg min}\{ \| \Delta c_{n_0} \| : B_{n_0}^T y \geq c_{n_0} - \Delta c_{n_0}, \\
&\quad B_s^T y \geq c_s - \Delta \mathcal{c}_s \; (s = 0, \ldots, n_0 - 1), \; y \geq 0 \}.
\end{align*}
\]
Analogously, the vector $\Delta \tilde{b} = [\Delta \tilde{b}_0, \ldots, \Delta \tilde{b}_{m_0}]$ is called an optimal lexicographical relaxation of rhs-vector in (3)–(4) iff

$$\Delta \tilde{b}_0 = \arg \min \{\|\Delta b_0\| : A_0 x \leq b_0 + \Delta b_0, \ x \geq 0\}, \ldots,$$

$$\Delta \tilde{b}_{m_0} = \arg \min \{\|\Delta b_{m_0}\| : A_{m_0} x \leq b_{m_0} + \Delta b_{m_0},$$

$$A_s x \leq b_s + \Delta \tilde{b}_s \ (s = 0, \ldots, m_0 - 1), \ x \geq 0\}.$$

In this report it is described the connection between these vectors and the solutions of the dual pair of convex programs

$$P : \max_{x \geq 0} \Phi_\sigma(x), \ \Phi_\sigma(x) = (c, x) - \sum_{s=1}^{n_0} \frac{\beta_s}{2} \|x_s\|^2 - \sum_{s=1}^{m_0} \frac{1}{2\alpha_s} \|(A_s x - b_s)^+\|^2,$$

$$D : \min_{y \geq 0} \Psi_\sigma(y), \ \Psi_\sigma(y) = (b, y) + \sum_{s=1}^{n_0} \frac{1}{2\beta_s} \|(B^T_s y - c_s)^+\|^2 + \sum_{s=1}^{m_0} \frac{\alpha_s}{2} \|y_s\|^2,$$

generated as maxmin and minmax of the regularized Lagrangian

$$\mathcal{L}_\sigma(x, y) = (c, x) - (y, Ax - b) - \sum_{s=1}^{n_0} \frac{\beta_s}{2} \|x_s\|^2 + \sum_{s=1}^{m_0} \frac{\alpha_s}{2} \|y_s\|^2.$$

This work extends previous results [2] of authors onto more general case.
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**References**


Consider the nonlinear programming (NLP) problem with inequality constraints

\[ \min \varphi(x) \text{ subject to } g_1(x) \leq 0, \ldots, g_m(x) \leq 0, \]  

where \( \varphi \) and \( g_j, j = 1, \ldots, m \) are smooth functions from \( \mathbb{R}^n \) to \( \mathbb{R} \).

The Lagrangian for the problem (1) is defined as

\[ \mathcal{L}(x, \lambda) = \varphi(x) + \sum_{j=1}^{m} \lambda_j g_j(x), \]

where \( \lambda = (\lambda_1, \ldots, \lambda_m)^T \) is a vector of Lagrange multipliers. The Kuhn-Tucker (KK) conditions of optimality are satisfied at \( x^* \) with some \( \lambda^* \in \mathbb{R}^m \) if

\[ \mathcal{L}'_x(x^*, \lambda^*) = \varphi'(x^*) + \sum_{j=1}^{m} \lambda_j^* g_j'(x^*) = 0 \]  

\[ \lambda_j^* g_j(x^*) = 0, \lambda_j^* \geq 0, g_j(x^*) \leq 0, j = 1, \ldots, m. \]  

There are various methods for solving the NLP problem based on the KK system. However, these approaches usually require some regularity constraints qualification, second-order sufficient conditions (SOSC) and strict complementarity conditions (SCC). Moreover we have to take into account feasibility of constraints, i.e. \( g_j(x^*) \leq 0, j = 1, \ldots, m \). For the quadratic convergence of the Newton method for the system (2)–(3) it is necessary SOSC and SCC as well. There are numerous of NLP problems for which these assumptions are failed.

Our approach is based on the construction of \( p \)-regularity theory (see [1,2]) and on transforming the inequality constraints into equalities. Namely, by introducing slack variables \( s_1, \ldots, s_m \) we get the equivalent equality constrained problem

\[ \min \varphi(x) \text{ subject to } g_1(x) + s_1^2 = 0, \ldots, g_m(x) + s_m^2 = 0. \]  

The necessary optimality conditions for this problem are

\[ F(x, s, \lambda) = (L'(x, s, \lambda))^T = (L'_x(x, s, \lambda), L'_s(x, s, \lambda), L'_\lambda(x, s, \lambda))^T = 0, \]
where \( L(x, s, \lambda) = \varphi(x) + \sum_{j=1}^{m} \lambda_j (g_j(x) + s^2) \).

To solve the system (6) by Newton method we require that matrix \( F'(x^*, s^*, \lambda^*) \) is not singular. If the SCC does not hold at \( x^* \), i.e. there exists an index \( j \) such that \( \lambda_j^* = 0 \) and \( s_j^* = 0 \) then \( F'(x^*, s^*, \lambda^*) \) is singular. Our goal is to apply the \( p \)-factor method for \( p = 2 \) to solve this singular optimization problem. The main scheme of \( p \)-factor method (\( p = 2 \)) for the system (6) is

\[
    z^{k+1} = z^k - \left\{ F'(z^k) + P_2 F''(z^k) h \right\}^{-1} \cdot \left( F(x^k) + P_2 F'(z^k) h \right),
\]

where \( z = (x, s, \lambda)^T \) and \( P_2 \) is a matrix of the orthoprojection onto \((\text{Im} F'(z^*))^\perp\).

**Definition.** The mapping \( F: \mathbb{R}^{n+2m} \to \mathbb{R}^{n+2m} \) is called 2-regular at \( z^* \) along an element \( h \in \mathbb{R}^{n+2m} \), if a matrix \( F'(z^*) + P_2 F''(z^*) h \) is nonsingular.

For our optimization problem the following result will be hold.

**Theorem 1.** Let \( f, g \in C^3(\mathbb{R}^{n}) \). Assume that for \( x^* \in \mathbb{R}^{n} \), there exists a Lagrange multiplier \( \lambda^* \in \mathbb{R}^{m} \) satisfying (2)-(3) and SOSC holds. Then Lagrange optimality system (6) is either regular or 2-regular at \( z^* = (x^*, s^*, \lambda^*)^T \) along some vector \( h \in \mathbb{R}^{n+2m} \).

Moreover for the scheme (7) the following convergence rate is valid: \( \|z^{k+1} - z^*\| \leq C \|z^k - z^*\|^2 \), \( k = 0, 1, \ldots \).

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**References**


Optimization actuators movements of robotic system with perturbation effect

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Consider the problem of synthesis of optimal control of a robotic system with an electromechanical actuator and transmission screw-nut, which is described in the following state-space equations:

\[
\begin{aligned}
\ddot{x} &= \frac{k_{em} \cdot r_1}{J_r + m \cdot r_1 \cdot r_2} \cdot I_a + \dot{y} \\
\dot{I}_a &= -\frac{R}{L} \cdot I_a - \frac{k_{em}}{r_1 \cdot L} \cdot \dot{x} + \frac{1}{L} \cdot u + \frac{k_{em}}{r_1 \cdot L} \cdot \dot{y}
\end{aligned}
\]  

or in vector-matrix form \( \dot{X} = A \cdot X + B \cdot u + A \cdot G \cdot Y \), where \( X = [x_1 \ x_2 \ x_3]^T \) — the state vector;

The generalized scheme of vibration-proof robot system is shown in figure 1, where 1 - the object of vibration protection; 2 - base; 3, 4 - accelerometers on the subject and the base; 5 - the gauge of the relative movement; 6 - control; 7 - the electric motor; 8 - power amplifier; 9 - spindle; 10 - a nut. Shown in the figure 2 block "Object of control", describes a system of equations.

Figure 1: Driving robotic system with an electromechanical drive.

Figure 2: Generalized system of vibration protection scheme with optimal control.

We define the structure of the optimal discrete controller in the form of a matrix relation \( u[i] = -F \cdot X[i] \), where \( F = [f_1 \ f_2 \ f_3] \) — Matrix of the feedback factor in the variable state. In fact, the optimal controller acts
as a feedback role of the state of the system. To solve the optimal control problem of synthesis will form a vector $F$ controlled variables $F = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$ =

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = D \cdot X,$$

where $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ —matrix of communications coordinates state and controlled variables. As a criterion of quality will take the integral of quadratic forms of controlled variables and manipulated species:

$$J = \int_0^{\infty} (q_1 \cdot z_1^2(t) + q_2 \cdot z_2^2(t) + r \cdot u^2(t)) dt \rightarrow \text{min}$$

where $q_1,q_2,r$ —The weighting coefficients of the squares of the two controlled variables and manipulated respectively, that characterize the contribution of each term in the criterion function. The weights should be chosen empirically, based on the results of mathematical modeling. For discrete systems [1] criterion function (3) in the form:

$$J = \sum_{i=0}^{\infty} (X^T [i] D^T \cdot Q \cdot D \cdot X [i] + r \cdot u [i]) \rightarrow \text{min}$$

where $Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$ —matrix of weighting factors under controlled variables.

Thus, we have formulated the task of management is to minimize the amplitude of the movement speed of the object (and hence the amplitude of its acceleration) and to limit the level of movement of the object relative to the base, thus limiting the available-resource management. Feedback gain matrix is given by $F = (r + B^T_{\Delta} \cdot P \cdot B_{\Delta})^{-1} \cdot B^T_{\Delta} \cdot P \cdot A_{\Delta}$, where $P$ — positive definite square auxiliary matrix dimension. The matrix $P$ satisfies the discrete Riccati equation [2]

$$P = D^T \cdot Q \cdot D + A^T_{\Delta} \cdot P \cdot A_{\Delta} - A^T_{\Delta} \cdot P \cdot B_{\Delta} (r + B^T_{\Delta} \cdot P \cdot B_{\Delta})^{-1} \cdot B^T_{\Delta} \cdot P \cdot A_{\Delta}$$

This equation has a solution, if the pair $(A, B)$ is completely controllable and has a unique positive definite solution in the form of a symmetric matrix $P$, and such optimum closed system is definitely stable, since all eigenvalues of the matrix $(A - B \cdot F)$in modulus less than one. Solving Riccati equation numerically and highlighting positive definite matrix $F$, we find control $u [i]$ in form $u [i] = -f_1 \cdot \dot{x} [i] - f_2 \cdot z [i] - f_3 \cdot I [i]$, where the matrix of the feedback
factor has the form \( \mathbf{F} = [553, 7196, 587, 3993, 0, 9638] \). The modeling of the robotic system when disturbance action on the part of the base with a frequency \( f = 1 \text{Hz} \). At the same time suppressing the disturbing factor of the vibration exposure in amplitude was \( A_x/A_y = 0.08 \). Relative movement of the object is limited and can be held at an acceptable level. The level of the steady relative movement can be reduced by increasing the weight \( q_2 \) in the criterion function (3). However, it should be borne in mind that in this case will increase the amplitude of the acceleration on the object.

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New Parallel Multi-Memetic MEC-based Algorithm for Loosely Coupled Systems

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This work presents a new parallel adaptive population-based algorithm for solving global unconstrained optimization problems, which utilizes a multi-memetic approach. The algorithm is based on the concept of Mind Evolutionary Computation (MEC) \cite{1}. In turn, multi-memetic approach implies the hybridization of an arbitrary global optimization technique with several local optimization methods (memes).

The proposed method can be easily considered as a multi-population algorithm since it works with a certain number of sub-populations $S = \{s_i, i \in [1 : K]\}$ which is determined by the number of available computational cores $K$. Each sub-population then evolves independently from one another which leads to minimization of communicational expenses between nodes. Such a feature makes it promising to use this algorithm on grid-systems composed of personal computers. In addition, each sub-population implements its own evolution strategy as well as different values of algorithm’s free parameters.

At the initialization stage, a certain number of initial points (individuals) is generated in accordance with $LP_\tau$ sequence \cite{2}. This set of individuals is divided into sub-populations by the value of objective function. Individuals within a particular sub-population then evolves following modified memetic MEC algorithm. Two memes were used in this work, namely the Nelder-Mead method and the Gauss-Seidel method \cite{3}.

In addition, the algorithm implies an adaptive strategy in order to a of solving global optimization problems based on the population Mind Evolutionary Computation (MEC) \cite{1}. The algorithm can be parallelized easily and can run on loosely-coupled computing systems with distributed memory. This distinctive feature makes it promising to use on grid-systems composed of personal computers (desktop grids) \cite{2}. Since desktop grids are widely spread nowadays due to relatively low cost the development of algorithms for this kind of systems is an important task.

MEC simulates some aspects of human behavior in the society. Each individual in this algorithm is regarded as an intelligent agent, operating in the arbitrary group of people. In order to achieve a high score, an individual has to study under the most successful individuals of his group. The same principle is used in the intergroup competition.
Canonical algorithm was modified by authors to prevent the preliminary convergence. Operation of decomposition of the search region was added at the very begining of the algorithm. Besides, hybridization of MEC with memetic algorithm was proposed.

A memetic algorithm was introduced as a metaheuristic method that is based on a certain combination of any population algorithm and one or more local optimization methods (memes). Overall scheme of the memetic algorithm is as follows. Group initialization \( S = (s_i, i \in [1 : |S|]) \), where \( s_i \) is the agent of a population and \( |s| \) is the total number of agents, takes place at the first iteration of the algorithm \( (t = 0) \). Then all operators of the population algorithm are applied to every agent of the population \( S \). On the basis of new positions of every agent \( s_i \), local search is carried out, that is memes are launched from these positions. After the local search the quality of new positions of agents is estimated. Then the iteration counter is incremented \( (t = t + 1) \) and the termination conditions are tested.

Presented scheme of the memetic algorithm is flexible enough and allows one to modify it in many different ways and, in particular, to create a multi-memetic algorithm. In this case the swarm of available memes is created \( M = (m_j, j \in [1 : |M|]) \); the most efficient of them then improves the current position of the agent \( s_i \). The key problems when developing multi-memetic algorithms are selecting the most efficient meme and choosing best strategies to control memes application.

Described algorithm was implemented by the authors as well as all memes with a use of Wolfram Language. The application has a unified interface which enables a usage of various local optimization methods and separate module for controlling different adaptive strategies.

Performance investigation of the proposed parallel multi-memetic algorithm and its implementation was carried out using eight-dimensional benchmark functions [4]. Numerical experiments were based on the concept of multi-start with \( k = 100 \) launches. Averaged best obtained value of objective function \( \bar{F} \), standard deviation \( \sigma \) and mean number of iterations \( \bar{\lambda} \) were used as criteria for performance evaluation. Obtained results demostranted that proposed algorithm and its software implementation are capable of determining a global minimum of multiextremal functions with a high probability.

Further studies will be devoted to the development of new adaptive strategies which would also take user’s expertise into account.


In this paper, the model of rhythmical production [1–3] in discrete time is developed for the case of continuous time.

Let $p(t)$, $u(t)$, $x(t)$, $0 \leq t \leq T$, be continuous function representing the rate of supply of raw materials for production of some product, piece-wise continuous function representing the rate of elaborating of the raw materials and continuous function representing the quantity of raw materials in the stock of the volume $V$ accordingly. The balance of raw materials in the stock may be described by the equation

$$x(t) = x(0) + \int_0^t p(\tau)d\tau - \int_0^t u(\tau)d\tau, \quad 0 \leq t \leq T. \quad (1)$$

The total amount $P(t) = x(0) + \int_0^t p(\tau)d\tau$ of raw materials to be elaborated and the total amount $v(t) = \int_0^t u(\tau)d\tau$ of elaborated raw materials are to satisfy the inequality

$$0 \leq x(t) = P(t) - v(t) \leq V. \quad (2)$$

A piece-wise continuous function $u(t)$, $0 \leq t \leq T$, satisfying the inequality (2) and the equality

$$\int_0^T u(\tau)d\tau = Q, \quad Q \in [P(T) - V, P(T)], \quad (3)$$

is called to be feasible control. A feasible control minimizing the functional

$$J[u] = \int_0^T f(u(\tau))d\tau, \quad (4)$$

the function $f(u)$ of the class $C^1$ being strictly convex, is called to be optimal control.

**Theorem 1.** If $0 \leq P(t) - \frac{Q}{T}t \leq V$, $0 \leq t \leq T$, then $u^0(t) \equiv \frac{Q}{T}, 0 \leq t \leq T$, is optimal control.

**Theorem 2.** Let $u^0(t)$ and $x^0(t)$, $0 \leq t \leq T$, be optimal control and optimal trajectory accordingly. If $0 < x^0(t) < V$ at some closed interval $[t_1, t_2]$ then $u^0(t) \equiv \text{const}$ for $t \in [t_1, t_2].
Theorem 3. Let $u^0(t), 0 \leq t \leq T,$ be optimal control having a discontinuity at a point $t^*$:

a) if $u^0(t^*-0) < u^0(t^*+0),$ then $x^0(t^*) = 0$;

b) if $u^0(t^*-0) > u^0(t^*+0),$ then $x^0(t^*) = V.$

Corollary 1. Let $u^0(t), x^0(t), 0 \leq t \leq T,$ be optimal control and optimal trajectory accordingly, and the equality

$$x^0(t^*) = 0, \quad (x^0(t) = V),$$  \hspace{1cm} (5)

is held at some point $\overline{t} \in (0, T).$ Then $u^0(\overline{t}-0) = u^0(\overline{t}+0)$ and $\dot{x}^0(\overline{t}) = 0.$

Corollary 2. If $0 < x(0) < V,$ and $0 < x(T) < V,$ the optimal trajectory consists of some number of arcs $0 < x^0(t) < V,$ $\tau_i < t < t_i, \ i = 1, m, \tau_1 = 0, \ t_m = T,$ corresponding to constant values of optimal control $u^0(t) \equiv c_i,$ linked by pieces of bounds $x^0(t) \equiv V$ or $x^0(t) \equiv 0,$ determined at the closed intervals $[t_i, \tau_{i+1}], \ i = 1, m-1.$ All arcs are tangent to according pieces of bounds at the points $\tau_i$ and $t_i.$ Moreover, if $x^0(t) \equiv V$ ($x^0(t) \equiv 0$) at the closed interval $[t_i, \tau_{i+1}],$ then $c_i \geq c_{i+1}$ ($c_i \leq c_{i+1}$).

References


Iterative Equilibrium Searching in Piecewise Linear Exchange Model

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The exchange model with piecewise linear separable concave utility functions is considered. This consideration extends the author’s original approach to the equilibrium problem in a linear exchange model and its variations. The conceptual base of this approach is the scheme of polyhedral complementarity. It has no analogs and made it possible to obtain the finite algorithms for some variations of the exchange model. Especially simple algorithms arise for linear exchange model with fixed budgets (Fisher’s model). This is due to monotonicity property inherent in the models and potentiality of arising mappings. The algorithms can be interpreted as a procedure similar to the simplex-method of linear programming.

It is natural to study applicability of the approach for more general models. The considered piecewise linear version of the model is reduced to a special exchange model with upper bounds on variables and the modified conditions of the goods’ balances.

For such a model the monotonicity property is violated. But it remains, if upper bounds are substituted by financial limits on purchases. A version of the polyhedral complementarity algorithm for this type of models was developed by author earlier.

This is the idea of proposed iterative algorithm for initial problem. On each step of the process the approximating model is formed. The equilibrium price vector of this model is used to get the next approximation. This approach has been proved earlier for general case of linear exchange model.

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The solution of an applied problem with mixed constraints

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We consider a dynamic model of functioning for a group of gas deposits with interacting wells [1]. The optimum control problem is put and solved over a finite horizon with mixed constraints.

Problem 1. About maximizing cumulative production for a group of gas deposits,

We want to maximize the functional

$$\sum_{i=1}^{n} \int_{0}^{T} q_i(t)N_i(t) \, dt$$

(1)

in differential relations

$$\dot{q}_i = -\frac{q_i^0}{V_i} q_i(t)N_i(t), \quad i = 1, 2, \ldots, n,$$

(2)

with initial conditions

$$q_i^0 > 0, \quad i = 1, 2, \ldots, n$$

(3)

and under restrictions on controls

$$0 \leq N_i \leq \bar{N}_i, \quad i = 1, 2, \ldots, n,$$

(4)

$$\sum_{i=1}^{n} q_i(t)N_i(t) \leq \bar{Q},$$

(5)

Controls $N_i(i = 1, 2, \ldots, n)$ belong to the set of measurable functions. The right end of the phase trajectory is free.

The variables have the following designations: $N_i$ is a fund of well production at the $i$-th deposit; $\bar{N}_i$ is the upper limit of the well production fund; $q_i$ is the average well production rate at $i$-th deposit; $V_i$ denotes recoverable reserves in the $i$-th deposit; $\bar{Q}$ is the total ”shelf” of deposits. The group consists of $n$ deposits. It is assumed that the values $\frac{q_i^0}{V_i} \bar{N}_i$ are different and all gas deposits are sorted by values $\frac{q_i^0}{V_i} \bar{N}_i$ in ascending order. These values are interpreted as the percentage of gas extraction from the deposits.
The existence of an optimal control solution follows, for example, from the theorem, given in [2, §4.2]. To solve problem 1 we use the proposition formulated by the Nobel laureate in Economics K. Arrow[3].

The optimal exploitation policy for deposits is as follows. First, at time \( t = 0 \) the first deposit is chosen for the exploitation. If the capacity of the first deposit is not enough to make gas production at the level of the total value of "shelf" deposits, i.e. \( q_0^1N_1 \leq Q \), then the second deposit is put into exploitation. If the capacity of the first deposit is enough to carry out the extraction of gas at the level of the total value of "shelf" deposits, i.e. \( q_0^1N_1 > Q \), then the first deposit development continues with production at the level of \( \bar{Q} \). In this case, some of these wells are disabled. The average production rate of wells at first deposit reduces over time. For the same reasons gas production at other active wells can fall.

New unused wells are included in the exploitation to compensate for the fall in gas production. At time \( t = \tau_1 \) all wells from the first deposit are included into gas production. The second deposit enters into the exploitation to compensate for the loss of total gas production. All wells from the first and second deposits are included in the gas production at time \( t = \tau_2 \). The compensation mechanism continues till the moment \( t = T_{max} \). All wells in all deposits are involved in the gas production at time \( t > T_{max} \).

**References**

Exact penalization and global optimality conditions in nonconvex optimization

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Consider the optimization problem:

\[
(P) : \begin{cases}
    f_0(x) \downarrow \min \quad x \in S \subset \mathbb{R}^n, \\
    f_i(x) \leq 0, \quad i \in I := \{1, \ldots, m\},
\end{cases}
\]

where all \( f_i = g_i(x) - h_i(x), \quad i \in I \cup \{0\} \) with smooth convex functions \( g_i(\cdot), \quad h_i(\cdot), \quad g_i, h_i : \mathbb{R}^n \to \mathbb{R}, \quad i \in I \cup \{0\} \).

Let introduce the \( l_\infty \)-penalty function \cite{1}–\cite{3}

\[
W(x) := \max\{0, f_1(x), \ldots, f_m(x)\} = \max\{0, f_i(x), i \in I\}.
\]

Further, consider the penalized problem as follows \((\sigma > 0)\)

\[
(P_\sigma) : \quad \Theta_\sigma(x) := f_0(x) + \sigma W(x) \downarrow \min \quad x \in S.
\]

As well-known \cite{1}–\cite{3}, if \( z \in \text{Sol}(P_\sigma) \), and \( z \in D := \{x \in S : f_i(x) \leq 0, \quad i \in I\} \), then \( z \in \text{Sol}(P) \). In addition, if \( z \in \text{Sol}(P) \), then under supplementary conditions \cite{1}–\cite{3} for some \( \sigma_* > 0, \quad \sigma_* \geq \|\lambda_z\|_1 \) (where \( \lambda_z \) is the KKT-multiplier corresponding to \( z \)), the inclusion \( z \in \text{Sol}(P_\sigma) \) holds. Moreover, \( \text{Sol}(P) = \text{Sol}(P_\sigma) \), so that Problems \((P)\) and \((P_\sigma)\) turn out to be equivalent \( \forall \sigma \geq \sigma_* \).

It can be readily seen that the penalized function \( \Theta_\sigma(\cdot) \) is a d.c. one, since the functions \( f_i(\cdot), \quad i \in I \cup \{0\} \), are as such. Actually, since \( \sigma > 0 \),

\[
\Theta_\sigma(x) = G_\sigma(x) - H_\sigma(x), \quad H_\sigma(x) := h_0(x) + \sigma \sum_{i \in I} h_i(x),
\]

\[
G_\sigma(x) := \Theta_\sigma(x) + H_\sigma(x) = g_0(x) + \sigma \max\left\{ \sum_{i=1}^m h_i(x); \max_{i \in I} g_i(x) + \sum_{j \neq i} h_i(x) \right\},
\]

it is clear that \( G_\sigma(\cdot) \) and \( H_\sigma(\cdot) \) are convex functions. For \( z \in S \) denote \( \zeta := \Theta_\sigma(z) \).

**Theorem 1.** It \( z \in \text{Sol}(P_\sigma) \), then

\[
\forall (y, \beta) : \quad H_\sigma(y) = \beta - \zeta,
\]

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the following inequality holds

\[ G_\sigma(x) - \beta \geq \langle \nabla h_0(y) + \sigma \sum_{i \in I} \nabla h_i(y), x - y \rangle \quad \forall x \in S. \quad (6) \]

So, Theorem 1 reduces nonconvex (d.c.) Problem \( (P_{\sigma}) \) to a solving the family of convex linearized problems of the form

\( (P_{\sigma} L(y)) : \quad G_\sigma(x) - \langle \nabla H_\sigma(y), x \rangle \downarrow \min \quad x \in S, \)

depending on the parameters \((y, \beta)\) fulfilling the equation (5).

If for such a pair \((\hat{y}, \hat{\beta})\) and some \(u \in S\) the inequality (6) is violated, i.e. \(G_\sigma(u) < \beta + \langle \nabla H_\sigma(y), u - y \rangle\), then due to convexity of \(H_\sigma(\cdot)\) we obtain with the help of (5) that \(G_\sigma(u) < \beta + H_\sigma(u) - H_\sigma(y) = H_\sigma(u) + \zeta\). It implies that \(\Theta_\sigma(u) = G_\sigma(u) - H_\sigma(u) < \zeta := \Theta_\sigma(z)\), so that \(u \in S\) is better than \(z\), i.e. \(z \notin (P_{\sigma})\).

It means that Global Optimality Conditions (5), (6) of Theorem 1 possesses the constructive (algorithmic) property allowing to construct local and global search methods for solving Problem \( (P_{\sigma}) \) [4].

In particular, they enable us to escape a local pit of \( (P_{\sigma}) \) and to reach a global solution. Moreover, the Global Optimality Conditions to Problem \( (P_{\sigma}) \) provide for KKT-conditions at \(z\) for the original Problem \( (P) \). So, the Global Optimality Conditions (5), (6) of Theorem 1 is connected with classical optimization theory [1]–[3].

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Protection and safety for people optimization in emergency situations with radiation leakage

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In cases of accidents at the nuclear power plants and terror attacks with “dirty bombs” International Commission on Radiological Protection (ICRP) recommends the principles of cost optimization for the residual dose of ionizing radiation as a corroboration of measures on liquidation of radioactive contamination effects and radiological protection. The optimal value of this dose is one, which balances the harm from exposure and intervention costs [1, pp.86-87].

An optimization of intervention costs requires a corroboration of the criterion that is adequate to the nature of this goal. The possible criteria are quite diverse. The most common criteria are are the following:

Maximization of intervention benefits:

\[
B = \text{value of averted harm} - \text{cost of intervention}
\]  \hspace{1cm} (1)

Minimization of intervention costs:

\[
Z = \text{loss from residual dose} + \text{cost of intervention}
\]  \hspace{1cm} (2)

Minimization of intervention costs rate:

\[
K = \frac{\text{cost of intervention}}{\text{value of averted harm}}
\]  \hspace{1cm} (3)

Maximization of specific cost-effectiveness:

\[
V = \frac{\text{value of averted harm}}{\text{cost of intervention}}
\]  \hspace{1cm} (4)

In cases of certain characteristics of the criterion function from the residual dose, its optimum estimate can be determined by means of the methods of unconditional optimization:

\[
\frac{d_{\text{criterion}}}{d_{\text{residual dose}}} = 0
\]  \hspace{1cm} (5)

In some studies, these criteria are assumed to be equivalent [2]. However, there is an obvious identity only for the 1st and 2d criteria and the 3rd and the 4th ones.
The solutions of the optimization problem for such intervention measures as a decontamination and a decontamination with temporary resettlement of the population indicate that the optimum estimates of the residual doses are higher with the last two criteria, compared with the first ones. This result indicates that the criteria focused on improving the specific cost-effectiveness underestimate its social effects, measured as the health and life loss due to radiation and the results of their decrease.

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REFERENCES

Healthcare expenditure optimization subject to health burden in Russian regions

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Increase in the health care effectiveness in Russia is directly related to the rational distribution of financial, material and human resources between the regions, that is impossible without reliable estimations of the health burden due to morbidity and premature mortality. However, health loss estimations in the regions of Russia by means of developed methods is limited due to insufficient completeness of statistical data on morbidity and mortality in Russian regions. The study was dedicated to the development of procedure for determining the health burden from premature mortality and regional healthcare systems effectiveness evaluation, based on existing morbidity and mortality databases in Russian regions.

According to the proposed approach, the years of life lost due to premature mortality have been determined as the difference between the threshold level of life expectancy at birth and the average age of death from specific causes. For example, in the global burden of disease studies (indicators DALY, DALE, HALE, etc.) threshold level of life expectancy is 92 years [2]. In order to ensure comparability of estimates over time and between different areas, there were calculated age-standardized indicators of the average age of death [1].

There were calculated estimates of years of life lost due to premature mortality from the basic reasons (certain infectious and parasitic diseases, neoplasms, diseases of the circulatory system, diseases of the respiratory system, diseases of the digestive system) in Russian regions for the period 2006 – 2013 years.

Based on a comparison of the obtained health loss indicators from premature mortality and healthcare expenditure there were corroborated conclusions in relation to the cost-effectiveness of resource allocations in regional healthcare systems. According to the proposed approach, Russian regions were divided into four areas, corresponding to different levels of costs and health burden: high loss and high costs (area I), low loss and high costs (area II), low loss and low costs (area III), high loss and low costs (IV). It was used a median as a threshold for separating high and low levels of costs and health burden. In the regions situated in an area of high rates of losses and high costs (area I) activities aimed at strengthening the population health cannot provide social welfare. In regions with low losses and high costs (area II) it is advisable to carry out an additional analysis of the situation in order to clarify the reasons for low losses.
These regions can be considered as a potential source of resources for the other regions of the Russian Federation, if the results of additional analysis confirm a surplus in financing, that does not have a significant impact on public health. In regions with high losses and low level of investment in health there is a problem of insufficient funding (area IV).

The proposed approach to the definition of loss due to premature mortality can be used for evaluation of the current effectiveness of regional healthcare systems, as well as for monitoring the economic impact of measures aimed at reducing the health burden in regions of the country.

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REFERENCES

The classical linear assignment problem (AP) can be formulated as follows: given an input cost matrix $C = (c_{ij})$, the problem is to find a permutation $\pi \in S_n$ of size $n$, such that

$$\sum_{i=1}^{n} c_{i\pi(i)} \to \min (\max).$$

This is a polynomially solvable problem. It is known that a permutation can be represented as a decomposition into disjoint cycles. Modifying the AP by adding different conditions on the number of cycles in the permutation, one may obtain different hard routing problems. An obvious example, the additional condition for the permutation $\pi$ to be a cycle leads to the classical NP-hard Traveling salesman problem (TSP). The assignment problem with given number $m$ of cycles in the permutation $\pi$ is known as the NP-hard $m$-Cycles Cover Problem.

Another natural modification of the linear AP is a multi-index assignment problem (MIAP). It is applied in the practical problems of communication, logistics, production, and economics. Researchers study variations of MIAP with additional constraints: axial or planar statements, kinds of permutations, etc. The requirements for the permutations in MIAPS to be single cycles also give different hard routing problems.

In this work the following results are presented.

1. In 2000 A. I. Serdyukov introduced the $m$-index axial assignment problem on single-cycle permutations of size $n$. It turned out that for $m > 2$ and even $n$ the problem does not have any feasible solutions. So the question of solvability of this problem in case of odd $n$ was studied in [3]. It was shown that for every $2 < m \leq 8$, there is a number $n_m$ such that the axial $m$-index problem is solvable for odd $n > n_m$.

2. Another studied problem is the 3-index $m$-layered planar assignment problem on single-cycle permutations of size $n$ or the $m$-Peripatetic salesmen problem (m-PSP). This is an NP-hard problem. In [1, 2] the approximation algorithms for the m-PSP are created for the m-PSP with different and identical weight functions on random inputs. The conditions under which the algorithms
are asymptotically optimal are obtained in the cases when the input data (elements of $m \times n \times n$ matrix) are independent and identically distributed random reals with uniform distribution on $[a_n, b_n], 0 < a_n < b_n$, shifted exponential distribution on $[a_n, \infty), 0 < a_n$, or any distribution function that dominates the uniform or shifted exponential distribution functions.

3. In [4] the approach to solving the mentioned above m-Cycles Cover Problem was developed. The approach transforms an approximation TSP algorithm into an approximation m-CCP algorithm. In this report the extended range of successful transformations with proven performance guarantees for the obtained solutions is presented.

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Automatic Differentiation in Python

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Nowadays, in many applications one should deal with the problem of exact derivative calculation. One of the possible solutions is to use automatic differentiation, which may be implemented in Python by means of the following automatic differentiation tools, which are the object of this research: PyADOL-C, PyCppAD, CasADi, Computation Graph Toolkit (CGT), Theano, or AD. These tools are analyzed to highlight the advantages of using each of them.

Because of the fact that in many applications the computation of derivatives is the main task, it is important to use an efficient tool, which has maximal runtime speed. Moreover, it should have such precision of gradient calculation that is almost equal to machine one. Therefore, performance and precision each of the tools should be assessed. For these purposes a cluster optimization problem, which can be described as follows, was chosen. We are looking for a geometrical structure of the cluster with identical atoms, the interaction between which is described by Lennard-Jones pair potentials:

$$v(\rho) = \rho^{-12} - 2\rho^{-6},$$  \hspace{1cm} (1)

by using the following objective function:

$$E(x) = \sum_{i<j} v(\rho(x_i - x_j)),$$  \hspace{1cm} (2)

The cluster database with 1610 different configurations was used to test each of these tools as follows. First, the average time of gradient calculation was found to assess performance. Second, by using predefined clusters, the norm of the gradient was calculated for each of these clusters to assess precision. It should be noted here that despite the usage of the dataset with optimal cluster configurations, the precise optimum was found by using optimize.minimize function of the SciPy package with the L-BFGS method. It was necessary due to rounding that took place when the configurations were stored to the dataset.

It was shown that the PyADOL-C and PyCppAD tools have much better performance for big clusters than the other ones. The precision of these two tools was assessed by calculating the difference between gradient norms, which were obtained at different optimal configurations. It was concluded that PyCppAD
has the best performance among others, while having almost the same precision as the second-best performing tool - PyADOL-C.

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Markets of energy resources play an important role in economies of many countries. We consider a problem of social welfare optimization for such markets with account of production costs, consumers’ utilities and costs of transmission capacities’ increments. In general the problem of transport system optimization is NP-hard (see Guisewite, Pardalos, 1990). Below we determine conditions for submodularity and for supermodularity of the social welfare function on the set of transmitting lines. These properties provide a possibility to apply the known efficient optimization methods (see Khachaturov, 1989).

Consider a market including several local markets and a network transmission system. Let $N$ denote the set of nodes and $L \subseteq N \times N$ be the set of edges. Every node $i \in N$ corresponds to a perfectly competitive market. Demand function $D_i(p)$ and supply function $S_i(p)$ characterize respectively consumers and producers in the market. The demand function relates to the consumption utility: $U_i(q) = \int_0^q (D_i - 1_i(v)) dv$. The supply function $S_i(p)$ determines the optimal production volume: $S_i(p) = \arg\max_v (pv - c_i(v))$, where $c_i(v)$ is the minimal production cost of volume $v$ at node $i$. The total profit of producers at node $i$ under price $p$ is $\Pr_i(p) = \int_0^p S_i(p) dp$. Every line $(i, j) \in L$ is characterized by initial transmission capacity $Q_{0ij}$, unit transmission cost $e_{ij}$, cost function of the transmission capacity increment including fixed costs $e_{ij}^f$ and variable costs $e_{ij}^v$, $e_{ij}^v$ is a monotonous convex function of increment $(Q_{ij} - Q_{0ij})^\dagger$. Let $q_{ij}$ denote the flow from market $i$ to market $j$, $q_{ij} = -q_{ji}$. Denote $Z(i)$ the set of nodes connected with node $i$. Under any fixed flows of the good $\vec{q} = (q_{ij}, (i, j) \in L)$ and production volumes $\vec{v} = (v_i, i \in N)$, the total social welfare for the network market is

$$W(\vec{q}, \vec{v}) = \sum_{i \in N} [U_i(v_i + \sum_{l \in Z(i)} q_{li}) - c_i(v_i)] - \sum_{(i, j) \in L, i < j} E_{ij}(q_{ij}),$$

$$E_{ij}(q_{ij}) = \begin{cases} e_{ij}^f + e_{ij}^v (|q_{ij}| - Q_{ij}^0) + e_{ij}^t |q_{ij}|, & \text{if } |q_{ij}| > Q_{ij}^0, \\ e_{ij}^t |q_{ij}|, & \text{if } |q_{ij}| \leq Q_{ij}^0. \end{cases}$$

The welfare optimization problem under consideration is

$$\max_{\vec{q}, \vec{v}} W(\vec{q}, \vec{v}) \quad (1)$$
For any $\overline{L} \subseteq L$, consider a problem (1) with fixed set $\overline{L}$ of expanded lines.

Let $\widehat{W}(\overline{L})$ denote the maximal welfare in the latter problem. Then problem (1) reduces to $\max_{\overline{L} \subseteq L} \widehat{W}(\overline{L})$. Below we also consider problem (1) without construction costs and under constraint: $|q_{ij}| \leq Q_{ij}, (i, j) \in L$. Let $\tilde{p}_i(\overline{Q}), i \in N$, denote the equilibrium prices corresponding to the solution of this problem.

A function $w(L), L \subseteq L$, is submodular (resp. supermodular) on $L$, if for any $L_1, L_2 \subseteq L$ $w(L_1) + w(L_2) \geq (\leq) w(L_1 + L_2) + w(L_1 \cap L_2)$.

**Theorem 1.** For a chain-type market with $n$ nodes, let the initial prices $p_i(\overline{Q}^0), i = 1, ..., n$, monotonously decrease in $i$. Then, for any $\overline{Q} \geq \overline{Q}^0$, $p_i(\overline{Q}) \geq p_{i+1}(\overline{Q}), i = 1, ..., n - 1$, and function $\widehat{W}(\overline{L})$ is supermodular. The complexity of search for the optimal set $\overline{L}^*$ under $\overline{Q}^0 = 0$ does not exceed $\frac{(n-1)n}{2}$.

Consider a star-type market where $N = \{0, 1, ..., n\}, L = \{(0, i), i = 1, ..., n\}, \overline{Q}^0$, $p_i(\overline{Q}^0) < p_0(\overline{Q}^0)$ for $i \in I_1 = \{2, ..., m\}, p_i(\overline{Q}^0) > p_0(\overline{Q}^0)$ for $i \in I_2 = \{m + 1, i, ..., n\}$. For $M \subseteq L$, let $(\overline{Q}^0||\overline{Q}^0_M)$ denote vector $\overline{Q}$ such that $Q_l = Q^0_l$ for $l \notin M$, $Q_l = \infty$ for $l \in M$.

**Theorem 2.** Let $\forall i \in I_1 \ p_i(\overline{Q}^0||\overline{Q}^0_{I_1}) < p_0(\overline{Q}^0||\overline{Q}^0_{I_1})$ and $\forall i \in I_2 \ p_i(\overline{Q}^0||\overline{Q}^0_{I_2}) > p_0(\overline{Q}^0||\overline{Q}^0_{I_2})$. Then the social welfare function $\widehat{W}(L_1 \cup L_2)$ is submodular in $L_1 \subseteq I_1$ under a fixed set $L_2 \subseteq I_2$, and is also submodular in $L_2 \subseteq I_2$ under a fixed set $L_1 \subseteq I_1$. Besides that, for any $L_1, l \in I_1 \setminus L_1$, the welfare function increment $\widehat{W}(L_1 \cup L_2) - \widehat{W}(L_1, L_2)$ monotonously increases in the set $L_2$, and for any $L_2, l \in I_2 \setminus L_2$, the increment $\widehat{W}(L_1, l \cup L_2) - \widehat{W}(L_1, L_2)$ monotonously increases in the set $L_1$.

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The computational technique for approximation of nonlinear functional differential equations of pointwise type

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Functional differential equations of pointwise type (FDEPT) attracted the attention of researchers is not only an interesting mathematical object, but also as a tool for investigating of applied problems in various scientific fields. An important application is the possibility of a reduction of infinite ordinary differential equations ODE to small dimension FDEPT infinite ODE (see., eg., [1]—[2]). However, this approach can not be efficiently applied in practice due to the underdevelopment of numerical methods for solving initial boundary value problems for such systems.

It is considered the system

$$F_i(x(g(t)), \dot{x}(g(t))) = 0, \quad i = 1, n, \quad t \in [t_0, t_1],$$

where $F_i : R^n \times R^n \rightarrow R^1$.

The values of the phase variables derivatives are determined beyond the basic range $t \in [t_N, t_K]$, $t_N \leq t_0$, $t_K \geq t_1$.

$$x^L_i = h^L_i(t), \quad t \in [t_N, t_0] \quad \text{and} \quad x^R_i = h^R_i(t), \quad t \in [t_1, t_K], \quad i = 1, n.$$

The boundary conditions are given by functionals

$$K_j(g(x(\tau_j)), \dot{g}(x(\tau_j))) = 0, \quad j \in [1, l], \quad \tau_j \in [t_0, t_1].$$

A more detailed description of the problem statement can be found in this paper [3].

To approximate the original problem on the fixed grid of nodes at the time the trajectories are approached by using spline functions. The coefficients of these functions are selected by finding the minimum of residual functional

$$I(x(t)) = \sum_{i=1}^{n} v_i^N \int_{t_N}^{t_0} (\dot{x}_i(g(t)) - h^L_i(t))^2 dt +$$
\[ \sum_{i=1}^{n} v_i^N \int_{t_1}^{t_2} (\dot{x}_i(g(t)) - h_i^R(t))^2 dt + \]
\[ \sum_{j=1}^{l} K_j^2 (g(x(\tau_j)), \dot{g}(x(\tau_j))) + K_0 (g(x(\tau_0)), \dot{g}(x(\tau_0))) \rightarrow \min, \]

where \( v_i^N, i = 1, n \) and \( v_j^K, j = 1, l \) are the weighting coefficients for residuals and boundary conditions.

It has not been possible the using of well-known types of splines ("natural", with fixed boundary conditions, Akima splines, etc.) for achieving a good estimation accuracy of the trajectories. Therefore we developed the algorithm of building "controlled" splines: it is selected the appropriate values for the boundary conditions - the second derivatives at the endpoints of the grid and searched spline coefficients at the same time. The proposed algorithms are implemented within the software OPTCON-F [3].

The results of computational experiments demonstrated the effectiveness of the proposed technology for research of considerable problems.

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REFERENCES

Algorithms for global optimization based on Curvilinear Search

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Applied optimization problems are as a rule multiextreme. Existing approaches are resultative for limited class of problems or they impose strict restrictions on the task setting. Currently, it is actual the development of new effective approaches to solve the problems of global optimization.

We propose to employ curvilinear search technique for solving multiextremal optimization problems. This approach based on using of random initial and auxiliary points for the constructing of curves covering the range of permissible values.

The search of the smallest value of objective function is carried out along these lines on each iteration [1]. We can choose the method of generating the auxiliary points and selection of one-dimensional search algorithm for constructing the different modification of the algorithm.

The proposed technique was applied for solving some problems from different areas, for example, problems from Mongolian industry (it is research competitive companies which share the bread market of the city Ulaanbataar) and nanophysics (it is simulated the quantum logic operations in system of quantum dots).
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REFERENCES

Determining parameters of hydrological model

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A one-dimensional model of vertical water transfer in soil is considered. We assume that soil is an isothermal porous homogeneous medium. Then a water transfer can be described by one-dimensional nonlinear parabolic equation. Following initial-boundary value problem is considered:

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left( D(\theta) \frac{\partial \theta}{\partial z} \right) - \frac{\partial K}{\partial z}, \quad (z,t) \in Q,
\]

\[
\theta(z,0) = \varphi(z), \quad z \in (0,L),
\]

\[
\theta(L,t) = \psi(t), \quad t \in (0,T),
\]

\[
\frac{\partial \theta}{\partial t} \bigg|_{z=0} = \left( D(\theta) \frac{\partial \theta}{\partial z} - K \right) \bigg|_{z=0} + R(t) - E(t), \quad t \in (0,T),
\]

\[
\theta_{\min} \leq \theta(0,t) \leq \theta_{\max}, \quad t \in (0,T).
\]

Here, \( z \) is the coordinate; \( t \) is time; \( \theta(z,t) \) is humidity at point \((z,t)\); \( Q = (0,L) \times (0,T) \); \( \varphi(z) \) and \( \psi(t) \) are given functions; \( D(\theta) \) is the coefficient of diffusion; \( K(\theta) \) is the hydraulic conductivity; \( R(t) \) is precipitation; \( E(t) \) is evaporation; \( \theta_{\min} \) and \( \theta_{\max} \) are minimal and maximal values of humidity respectively.

Diffusion coefficient \( D(\theta) \) and hydraulic conductivity \( K(\theta) \) are determined by widely used formulas of van Genuchten [1]-[2]:

\[
K(\theta) = K_0 S^{0.5} [1 - (1 - S^{1/m})^m]^2,
\]

\[
D(\theta) = K_0 \frac{1 - m}{\alpha m (\theta_{\max} - \theta_{\min})} S^{0.5 - 1/m} \times
\]

\[
\times [(1 - S^{1/m})^{-m} + (1 - S^{1/m})^m - 2],
\]

where \( S = \frac{\theta - \theta_{\min}}{\theta_{\max} - \theta_{\min}} \), and \( K_0, \alpha, m \) are some parameters.

We call this problem a direct problem. Discrete analogue of the direct problem has a form:
\[
\Phi_0^n = -\left( \frac{1}{\tau} + \frac{2}{h} D_{1/2}^{n-1} \right) \theta_0^n + \frac{2}{h} D_{1/2}^{n-1} \theta_i^n + \\
+ \frac{1}{\tau} \theta_0^{n-1} + \frac{2}{h} \left( -K_{1/2}^{n-1} + R^n - E^n \right) = 0,
\]
\[
\theta_{\min} \leq \theta_0^n \leq \theta_{\max}, \quad 1 \leq n \leq N,
\]
\[
\Phi_i^n = \frac{1}{h^2} D_{i-1/2}^{n-1} \theta_i^{n-1} + \frac{1}{h^2} D_{i+1/2}^{n-1} \theta_i^{n+1} - \\
- \left\{ \frac{1}{\tau} + \frac{1}{h^2} \left( D_i^{n-1} + D_i^{n-1/2} \right) \right\} \theta_i^n + \\
+ \left\{ \frac{\theta_i^{n-1}}{\tau} + \frac{1}{h} \left( K_{i-1/2}^{n-1} - K_i^{n-1} \right) \right\} = 0,
\]
\[
1 \leq i \leq I - 1, \quad 1 \leq n \leq N,
\]
\[
\Phi_i^I = \theta_i^n - \psi^n = 0, \quad 1 \leq n \leq N, \quad \theta_0^I = \varphi_i, \quad 0 \leq i \leq I.
\]

As a rule, parameters \(K_0, \alpha, m, \theta_{\min}, \theta_{\max}\) are hard-determined through experiments. Earlier in [3] some of these parameters were determined as a result of solution of an optimal control problem, in which the objective function is mean square deviation of simulated soil moisture from some prescribed values. Numerical solution was determined by gradient method. The gradient of objective function was calculated applying fast automatic differentiation formulas [4]. But the process of numerical optimization was difficult. Here, we reformulate optimal control problem and introduce new objective function. Obtained numerical results are discussed and analyzed.

References

On a variant of dual simplex-like algorithm for linear semi-definite programming problem

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Consider the linear semi-definite programming problem

\[
\min C \cdot X, \quad A_i \cdot X = b^i, \quad 1 \leq i \leq m, \quad X \succeq 0,
\]

where \(C, X\) and \(A_i, 1 \leq i \leq m\), are symmetric matrices of order \(n\), the inequality \(X \succeq 0\) indicates that \(X\) must be a semi-definite matrix. The operator \(\cdot\) denotes the Frobenius inner product between two matrices. The dual problem to (1) is

\[
\max b^T u, \quad \sum_{i=1}^m u^i A_i + V = C, \quad V \succeq 0, b = [b^1, \ldots, b^m], u = [u^1, \ldots, u^m].
\]

In [1] the simplex-like algorithm for solving (1) had been proposed. Here the variant of this method intended for solving the dual problem (2) is considered. Similar to the primal algorithm the dual one is based on some approach for solving the system of optimality conditions for (1)

\[
X \cdot V = 0, \quad A_i \cdot X = b^i, \quad V = C - \sum_{i=1}^m u^i A_i,
\]

where \(X \succeq 0, V \succeq 0, 1 \leq i \leq m\). These conditions can be rewritten in vector form, using direct sums of columns of matrices. It is proved that the dual algorithm generates the sequence of extreme points \(\{u_k\}\), which belongs to feasible set of (2) and converges to the solution of (2).

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References

Control of Phase Boundary Evolution in Metal Solidification for New Thermodynamic Parameters of the Metal

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The problem of controlling the phase boundary evolution in the course of solidification of metals with different thermodynamic properties is studied. This problem models the solidification of molten metal in casting. According to numerous studies of this process, for a product of high quality to be obtained in a given setup, it is desirable that the shape of the phase boundary be as close to a plane as possible and that its speed be close to a prescribed one. The underlying mathematical model of the process is based on a three-dimensional nonstationary two-phase initialboundary value problem of the Stefan type.

The solidification of the metal can be described as follows. Molten metal is poured into a mold. The cooling of the mold and the metal inside it occurs due to the interaction of the object with its surroundings. For this purpose is used a special setup consisting of a melting furnace, inside which the object moves, and a cooler which is a large tank filled with liquid aluminum. The mold with molten metal is being immersed into the coolant. Liquid aluminum has a relatively low temperature and thus proceeds the crystallization of metal. On the other hand, the object gains heat from the furnace walls, which prevents the solidification process from proceeding too fast.

The velocity of the mold relative to the furnace is a parameter that has a large effect on the evolution of the solidification front in the metal. We use it as a control function. To find a control function satisfying the imposed technological requirements, we formulate an optimal control problem. This problem consists in choosing a regime of metal cooling and solidification in which the solidification front has a preset shape and moves at a speed close to the preset one.

In this work, the problem of controlling the phase boundary evolution in
solidification is considered for a new material to be solidified. The goal of this research is to analyze the influence exerted on the solution of the optimal control problem by the thermodynamic parameters of the material under study.

The study was performed for a new improved setup model in which the ceiling is heated up to a rather high temperature. Specifically, this model describes the situation when several molds are lined up in the furnace near each other (as occurs in an actual setup). It was assumed that some of the lateral walls of the mold (on the sides where there are no vertical furnace walls) are made of a heat-insulated material that loses its thermal insulation property in liquid aluminum.

For the new material under study, research was carried out concerning the grids to be used. The time and space grids were chosen so as to achieve the required accuracy of the numerical results. The time grid was nonuniform. A finer time grid was used at the beginning of the cooling process, since the largest variations in the temperature field were observed over this time period. Because of the new thermodynamic parameters, the time step was smaller (roughly by 25%) than that used with the old parameters.

The control functions are determined by optimal control problems, which are solved numerically with the help of gradient optimization methods. The gradient of the cost function is exactly computed by applying the fast automatic differentiation technique.

Based on the studies performed in this work, we conclude that the thermodynamic properties of materials have a large effect on the solidification process and that the approach proposed for the control of the phase boundary evolution in solidification is effective and can be applied to materials with various thermodynamic properties.

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Endogenous Club Formation in a Uni-dimensional World with the Possibility to Stay Alone

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We consider a game-theoretic model of jurisdiction formation where a continuum of agents vary by a uni-dimensional parameter. The agents may form communities to enjoy a club good. Its cost depends both on the agent’s characteristic and the club composition. The benefits are substantially small and agents may choose to stay alone. We show that even in a simple model no Nash-stable club collection may exist.

THE MODEL

A continuum of agents live on a unit interval $[0, 1]$. Their population density $f$ is piecewise continuous and non-zero. If a connected jurisdiction $S = [a, b]$ is formed then each member gets some benefit $V$. The cost, however, is distributed unevenly. Denote by $P(S)$ the population of $S$, i.e., $P(S) = \int_a^b f(x) \, dx$ and by $m(S)$ the median of $S$, i.e., such point $m$ that $\int_a^m f(x) \, dx = \int_m^b f(x) \, dx$. Then an agent with characteristic $z$ bears cost $c(z, S) = \frac{g}{P(S)} + \rho|m(S) - z|$ while joining coalition $S$. This function is defined even if $z$ is not a member of $S$. The rationale behind such a cost is the following: the cost $g$ of providing the good is distributed evenly; the good is situated at $m(S)$ since this point minimizes the average distance from the agents, and each agent pays $\rho$ for a unit of distance...
Definition 1. A collection is a finite set of connected jurisdictions $S_1 = [a_1, b_1], \ldots, S_n = [a_n, b_n]$ with pairwise disjoint interiors. W.l.o.g. we may assume that $0 \leq a_1 < b_1 \leq a_2 < b_2 \leq \cdots \leq a_n < b_n \leq 1$.

Definition 2. A collection is Nash stable if no agent wishes to change their affiliation. Namely, three inequalities must hold:

(i) Individual rationality: $c(z, S_k) \leq V$ for any $k$ and $z \in S_k$;

(ii) Stability for members: $c(z, S_k) \leq c(z, S_l)$ for any $k \neq l$ and $z \in S_k$;

(iii) Stability for non-members: if $z$ does not belong to any $S_k$ then $c(z, S_k) \geq V$ for all $k$.

ANALYSIS

One may easily obtain the following nice result.

Theorem 1. Any Nash stable collection must satisfy the border indifference property (BIP): a border agent must be indifferent between the two options. Namely, if $b_k = a_{k+1}$ then $c(b_k, S_k) = c(a_{k+1}, S_{k+1})$ and if $b_k < a_{k+1}$ then $c(b_k, S_k) = 0$ and $c(a_{k+1}, S_{k+1}) = 0$. (Similarly, if $a_1 > 0$ then $c(a_1, S_1) = 0$ and if $b_n < 1$ then $c(b_n, S_n) = 0$.)

But even border indifference is not guaranteed.

Theorem 2. For certain values of the parameters no non-trivial Nash stable collection while some jurisdiction is beneficial for all members.

To prove Theorem 2, we construct and carefully analyze a very sophisticated example with $V = 0.1115$, $\rho = \frac{2}{9}$ and the following density:

\[
f(x) = \begin{cases} 
45, & x \in \left[\frac{3}{9}, \frac{4}{9}\right] \cup \left[\frac{6}{9}, \frac{7}{9}\right]; \\
36, & x \in \left(\frac{4}{9}, \frac{5}{9}\right]; \\
27, & \text{otherwise}.
\end{cases}
\]

We show that $S = \left[\frac{3}{9}, \frac{7}{9}\right]$ is profitable for all members but nevertheless no nontrivial collection with BIP exist.

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REFERENCES

Classical optimal design on annulus

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We optimize a distribution of two isotropic materials that occupy an annulus in two or three dimensions, heated by a uniform heat source, aiming to maximize the total energy. In elasticity, the problem models the maximization of the torsional rigidity of a cylindrical rod with annular cross section made of two homogeneously distributed isotropic elastic materials [4]. More precisely, we consider the conductivity problem in an annulus \( \Omega \subseteq \mathbb{R}^d \):

\[
-\div(A \nabla u) = f \\
u \in H^1_0(\Omega),
\]

where the conductivity matrix \( A \) is of the form \( A = \chi \alpha I + (1 - \chi) \beta I \), with a characteristic function \( \chi \) representing the region occupied by the first phase. The optimal design problem deals with maximization of the energy functional \( I(\chi) = \int_{\Omega} fu \, dx \), over the set of all measurable characteristic functions \( \chi \) satisfying the condition \( \int_{\Omega} \chi \, dx = q_\alpha \), which prescribes the amounts of given phases.

Commonly, optimal design problems do not have solutions (such solutions are called classical), so one considers proper relaxation of the original problem. Relaxation by the homogenization method consists in introducing generalized materials, which are mixtures of original materials on the micro-scale.

We shall consider the problem with a constant right-hand side \( f \). The interesting result is that on a simply connected open set \( \Omega \), with smooth connected boundary, the classical solution appears only if \( \Omega \) is a ball [2], which was originally showed in [5] but with an additional smoothness assumption on interface between phases. Moreover, even on a ball, if maximization is replaced by minimization, the optimal design is not classical [6]. For the problem of minimizing
energy functional on a ball and arbitrary right-hand side the explicit calculation of optimal microstructure is presented in [3], while the multiple state case is treated in [1], and in both situations no classical solution occurs.

If $\Omega$ is a ball, in order to maximize the energy the better conductor should be placed inside a smaller (concentric) ball, whose radius can easily be calculated from the constraint on given amounts of materials. By analysing the optimality conditions [5,7], we are able to show that in the case of annulus, the solution is also unique, classical and radial. Depending on the amounts of given materials, we find two possible optimal configurations. If the amount of the first phase is less than some critical value, then the better conductor should be placed in an outer annulus. Otherwise, the optimal configuration consists of an annulus with the better conductor, surrounded by two annuli of the worse conductor.

The same holds true in two and three dimensions. The precise solution can be determined by solving a system of nonlinear equations, which can be done only numerically.

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REFERENCES

At first glance, the problems of unconstrained optimization and asymptotic stability represent quite separate fields of research. However they have much in common, both in the problem formulation and the techniques exploited. The unconstrained minimization problem is formulated as follows: Given a function to be minimized, design an algorithm (in the form of either difference or differential equation) and prove its convergence to a minimum point. Conversely, the analysis of asymptotic stability of an equilibrium point of a difference or differential equation is traditionally performed via use of Lyapunov’s direct method. Namely, a Lyapunov function is constructed such that it decreases monotonically on the trajectories of the system. In other words, the given data and the design goals in both problems change places. We summarize this concept in the table below:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Optimization</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>Function</td>
<td>Equation</td>
</tr>
<tr>
<td>Technique</td>
<td>Design a method</td>
<td>Construct a Lyapunov function</td>
</tr>
<tr>
<td>Goal</td>
<td>Prove convergence</td>
<td>Prove stability</td>
</tr>
</tbody>
</table>

These links between the two fields enable their mutual enrichment both in terms of techniques and results. First, in the optimization theory, one can exploit the diversity of Lyapunov functions beyond the standard candidates (objective function or the distance to a minimum point). We will demonstrate this possibility for the heavy-ball method in unconstrained minimization. On the other hand, in the asymptotic stability analysis, it was traditional just to prove stability, while the rate of convergence was of a lesser interest; moreover, it cannot be assessed with the commonly used technique based on LaSalle’s invariance principle. Needless to say that in optimization, finding such estimates is the key issue. Also, in optimization problems it is often the case that the
minimum point is not unique or may not exist. Similar nonstandard assumptions, the non-uniqueness of the equilibrium are of interest in the analysis of difference and differential equations.

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