Proton-Deuteron Scattering and Test of Time-Reversal Invariance

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Abstract. The integrated proton-deuteron scattering cross section $\tilde{\sigma}$ for transversely polarized protons ($P'_y$) and tensor polarized deuterons ($P_{xz}$) constitutes a null test signal for time-reversal invariance violating but P-parity conserving effects. This cross section will be measured at COSY. Using the generalized optical theorem and Glauber theory we study the null-test observable $\tilde{\sigma}$ for different types of T-odd P-even NN-interactions. The formalism includes full spin dependence of elementary pN-amplitudes and S- and D-components of the deuteron wave function.

1 Introduction

Time-invariance-violating (T-odd) P-parity conserving (P-even) (TVPC) interactions do not arise at the fundamental level within the standard model. This type of interaction can be generated by radiative corrections to the T-odd P-odd interaction discovered in the physics of kaons and B-mesons. However, in this case its intensity is too low to be observed in experiments at present \cite{1}. Thus, observation of TVPC effects would be considered as an indication of physics beyond the standard model.

As was shown in Ref. \cite{2}, the total polarized cross section $\tilde{\sigma}$ of the proton-deuteron scattering with vector polarization of the proton $p'_y$ and tensor polarization of the deuteron $P_{xz}$ constitutes a null-test observable for TVPC effects. The dedicated experiment is planned at COSY \cite{3} at proton beam energy 135 MeV. The first analysis of the TVPC null-test signal \cite{4} was done within the nonmesonic deuteron breakup channel $pd \rightarrow ppm$ estimated in the single scattering approximation. Recently we used the spin-dependent formalism \cite{5} of the Glauber theory to calculate the cross section $\tilde{\sigma}$ \cite{6} and "null-combinations" of some differential spin observables of the $pd$ elastic scattering \cite{7} which deviate from zero if the TVPC effects occur. The formalism includes full spin dependence of elementary pN-amplitudes and S- and D-components of the deuteron wave function. This formalism allows one to explain existing data on the non-polarized differential cross section and spin observables of the elastic $pd$ scattering at 135 MeV \cite{8}. Here we consider some qualitative arguments concerning the $\rho$-meson contribution to $\tilde{\sigma}$ and briefly explain the role of the deuteron D-wave.

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2 Elements of formalism

Time-reversal symmetry conserving and P-parity conserving (TCPC or T-even P-even) interactions lead to the following transition amplitude of the elastic $pd$ scattering at 0 degree \([9]\)

\[
e_{\beta}^{T}(0)_{\alpha\beta}^{TCPC} = g_{1} (\mathbf{e} \cdot \mathbf{e}^{\prime} - (\mathbf{m} \cdot \mathbf{e}^{\prime}) + g_{2} (\mathbf{m} \cdot \mathbf{e}^{\prime}) +
\]

\[
ig_{3} (\sigma \cdot [\mathbf{e} \times \mathbf{e}^{\prime}]) - (\sigma \cdot \mathbf{m} \cdot [\mathbf{e} \times \mathbf{e}^{\prime}]) + ig_{4} (\sigma \cdot \mathbf{m} \cdot [\mathbf{e} \times \mathbf{e}^{\prime}]),
\]

where $\mathbf{e}$ ($\mathbf{e}^{\prime}$) is the polarization vector of the initial (final) deuteron, $\mathbf{m}$ is the unit vector along the beam momentum, $\sigma$ is the Pauli matrix, $g_{i}$ ($i = 1, \ldots, 4$) are complex amplitudes. To the right-hand side of Eq.(1) one can add the TVPC (T-odd P-even) term in a very general form

\[
e_{\beta}^{T}(0)_{\alpha\beta}^{TVPC} = \tilde{g} [(\sigma \cdot [\mathbf{m} \times \mathbf{e}]) (\mathbf{m} \cdot \mathbf{e}^{\prime}) + (\sigma \cdot [\mathbf{m} \times \mathbf{e}^{\prime}]) (\mathbf{m} \cdot \mathbf{e})],
\]

where $\tilde{g}$ is the TVPC transition amplitude. The matrix elements of the operators (1), (2) are

\[
<\mu^{\prime}, \lambda^{\prime} = 1 | M^{TCPC} | \mu = 1, \lambda = 1 >= g_{1} + g_{4},
\]

\[
<\mu^{\prime}, \lambda^{\prime} = -1 | M^{TCPC} | \mu = 1, \lambda = 1 >= g_{1} - g_{4},
\]

\[
<\mu^{\prime}, \lambda^{\prime} = 0 | M^{TCPC} | \mu = 1, \lambda = 0 >= g_{2},
\]

\[
<\mu^{\prime}, \lambda^{\prime} = 0 | M^{TVPC} | \mu = 1, \lambda = 1 >= \sqrt{2} g_{3} + i \sqrt{2} \tilde{g},
\]

\[
<\mu^{\prime}, \lambda^{\prime} = 0 | M^{TVPC} | \mu = 1, \lambda = 0 >= \sqrt{2} g_{3} - i \sqrt{2} \tilde{g},
\]

where $\mu$ ($\mu^{\prime}$) and $\lambda$ ($\lambda^{\prime}$) are spin projections of the initial (final) proton and deuteron on the beam direction, respectively. All diagonal matrix elements of the $M^{TVPC}$ operator are zeros.

The total cross section of the $pd$ scattering has the form \([8]\)

\[
\sigma_{tot} = \sigma_{0} + \sigma_{1} \mathbf{p}^{\prime} \cdot \mathbf{p} + \sigma_{2} (\mathbf{p}^{\prime} \cdot \mathbf{m}) (\mathbf{p}^{\prime} \cdot \mathbf{m}) + \sigma_{3} P_{xz} + \sigma_{4} P_{y} P_{y},
\]

where $\mathbf{p}^{\prime}$ ($\mathbf{p}$) is the vector polarization of the initial proton (deuteron) and $P_{xz}$ and $P_{y}$ are the tensor polarizations of the deuteron. The OZ axis is directed along the proton beam momentum $\mathbf{m}$, $OY \uparrow \uparrow \mathbf{p}^{\prime}$, $OX \uparrow \uparrow [\mathbf{p}^{\prime} \times \mathbf{m}]$. In Eq. (8) the terms $\sigma_{i}$ with $i = 0, 1, 2, 3$ are non-zero only for T-even P-even interactions corresponding to Eq. (1) and the last term $\tilde{\sigma}$ constitutes a null-test signal of T-invariance violation with P-parity conservation. Using the generalized optical theorem we find $\tilde{\sigma} = -4 \sqrt{\pi} Im\frac{\tilde{g}}{\tilde{g}}$.

Hadronic amplitudes of $pN$ scattering are taken as \([5]\)

\[
M_{N}(\mathbf{p}, \mathbf{q}; \sigma, \sigma_{N}) = A_{N} + C_{N} \sigma \mathbf{n} + C_{N} \sigma_{N} \mathbf{n} + \sum_{l=n,q,k} B_{lN}^{i}(\mathbf{q})(\sigma) (\sigma_{N}) \hat{\mathbf{l}},
\]

where $\mathbf{q}$, $\mathbf{k}$ and $\mathbf{n}$ are defined as unit vectors along the vectors $\mathbf{q} = \mathbf{p} - \mathbf{p}^{\prime}$, $\mathbf{k} = \mathbf{p} + \mathbf{p}^{\prime}$ and $\mathbf{n} = [\mathbf{k} \times \mathbf{q}]$, respectively; $\mathbf{p}$ ($\mathbf{p}^{\prime}$) is the initial (final) proton momentum; $\sigma_{N}$ is the Pauli matrix acting on the spin state of the nucleon $N$. We consider the following terms of the TVPC NN interaction which were under discussion in Ref. \([4]\):

\[
t_{pN} = h_{N}/[(\sigma \cdot \mathbf{k})(\sigma_{N} \cdot \mathbf{q}) + (\sigma_{N} \cdot \mathbf{k})(\sigma \cdot \mathbf{q}) - (\sigma_{N} \cdot \sigma)(\mathbf{k} \cdot \mathbf{q})/m_{p}^{2} +
\]

\[
+ g_{N} [\sigma \times \sigma_{N}] \cdot [\mathbf{q} \times \mathbf{k}]/m_{p}^{2} + g_{N} [\sigma \times \sigma_{N}] \cdot [\mathbf{q} \times \mathbf{k}] [\tau \times \tau_{N}]/m_{p}^{2},
\]

Here $\sigma$ ($\sigma_{N}$) is the Pauli matrix acting on the spin state of the proton (nucleon $N = p, n$), $\tau$ ($\tau_{N}$) is the corresponding matrix acting on the isospin state; $m_{p}$ is the proton mass. In the framework of the phenomenological meson exchange interaction the term $g^{\prime}$ corresponds to $\rho$-meson exchange, and $h$-term provides the axial meson $h_{1}$ exchange.
2.1 $g'$-term

The $g'$ term contributes only to the charge exchange transitions, because the non-zero matrix elements of the isospin-operator connected with the $g'$ term in Eq. (10) are the following

$$<n, p|[\tau \times \tau_N]|p, n>= -i2, \quad <p, n|[\tau \times \tau_N]|n, p>= i2.$$  

(11)

The isospin matrix element of the C-odd isospin operator $T_3 = [\tau \times \tau_N]_3$ in Eq. (11) changes the sign under replacement $p \leftrightarrow n$. In contrast, the similar matrix elements for T-even P-even (strong) NN-interaction are equal one to other. This difference is one cause for the vanishing of the amplitude $\tilde{g}$ for the double scattering mechanism of the process $pd \rightarrow pd$. As was shown in Ref. [6], the $g'$-term gives zero contribution to $\tilde{g}$ within the Glauber model. Below we discuss this observation briefly.

The TVPC charge-exchange Glauber operator of the double scattering has a form [6]

$$O_{TVPC}^g = -\frac{1}{2}[M_{n\rightarrow np}(q_2)t_{np\rightarrow np}(q_1) + t_{np\rightarrow np}(q_2)M_{np\rightarrow np}(q_1)],$$  

(12)

where $q_1 = q/2 + q'$ is the transferred momentum in the first and $q_2 = q/2 - q'$ in the second collision and $q$ is the total transferred momentum. For the next step one has to calculate the matrix element of the operator (12) over the deuteron states $\psi(r)$ with the factor $\exp(iq'r)$ and integrate over $q'$. Under the sign of this integral the operator (12) is not changed after the substitution $q_1 \leftrightarrow q_2$ [5, 6]. Therefore, one may add to the right side of Eq. (12) the term $O_{TVPC}^g(1 \leftrightarrow 2)$ and divide the obtained sum by a factor of 2. In collinear kinematics ($q = 0$), this symmetry and linear dependence of $g'$-term on $[q \times k]$ lead to cancellation of the spin-independent term $A_N$ in the transition operator (12). The same is true for the $B_N$ terms in Eq. (9). Thus, only $C_N$ and $C_N'$ terms may contribute as [6]

$$O_{TVPC}^g = \frac{g}{\Pi} \left[ C_n(\sigma \cdot \hat{n}_1)(\sigma_n - \sigma_p) \cdot n_1 - C'_n(\sigma_n \cdot \hat{n}_1)(\sigma_p \cdot n_1) + C'_n n_1 \hat{n}_1 + C_p(\sigma \cdot \hat{n}_1)(\sigma_p - \sigma_n) \cdot n_1 - C'_p(\sigma_p \cdot \hat{n}_1)(\sigma_n \cdot n_1) + C'_p n_1 \hat{n}_1 \right],$$  

(13)

where $n_1 = [k \times q']$, $\hat{n}_1 = n_1/|n_1|$ and $q' = q_2 = -q_1$; $\Pi$ is a constant. In Eq. (13) the $C_N'$-terms do not contain the proton beam spin $\sigma$. We can show that this is a consequence of Eqs. (11). According to Eqs. (6), (7), it means that the contribution of the $C_N' \times g'$ term to the amplitude $\tilde{g}$ is zero. Furthermore, due to Eqs. (11) the remaining terms with $C_N$ in Eq. (13) contain the difference $\sigma_n - \sigma_p$, but not the sum. These terms can be rewritten as $V_p \sigma_p + V_n \sigma_n = 2(V_p + V_n)(\sigma_p + \sigma_n) \equiv 0$ [6]. Thus, the contribution of the operator (13) to the amplitude $\tilde{g}$ vanishes and this fact is directly connected with Eqs. (11).

Strong suppression of the contribution of the $\rho$-meson as compared to the axial $h_1$ meson was found numerically in the Faddeev calculations [10] of the null-test signal for the $nd$ scattering at 100 keV, but no explanation of this result was offered. We suppose that the cause for this suppression is the same spin-isospin structure of the scattering amplitude which leads to the vanishing $\rho$-meson contribution within the Glauber approach.

2.2 $h$- and $g$-terms

For the single scattering mechanism the amplitude $\tilde{g}$ vanishes within the Glauber theory. Using Eqs. (3)-(7) for the double scattering mechanism with $pn$-amplitudes (9) and (10) we find for the $h$- and $g$-terms in Eq.(10) that all T-even P-even amplitudes of the $pd$-scattering are zeros: $g_1 = g_2 = g_3 = g_4 = 0$. Furthermore, we find for the TVPC amplitude

$$\tilde{g} = \frac{i}{4\pi m_p} \int_0^\infty dq q^2[S_0^{(0)}(q) - 2\sqrt{2}S_2^{(1)}(q)][C'_n(q)(g_p - h_p) + C'_p(q)(g_n - h_n)],$$

(14)
where \( S_{0}^{(0)} = \int_{0}^{\infty} dr u_{r}(r) j_{0}(qr) \) and \( S_{2}^{(1)}(q) = 2 \int_{0}^{\infty} dr u_{r}(r) w_{r}(r) j_{2}(qr) \) are the elastic form factors of the deuteron, and \( u_{r}(r) \) and \( w_{r}(r) \) are the S-wave and D-wave form factors of the deuteron, respectively, [6]. We can show that the S-D wave interference, not considered in Ref.[6], considerably diminishes the null-test signal \( \tilde{\sigma} \) at the energies of the planned COSY experiment [3] \( \sim 100 \text{ MeV} \) as compared to the pure S-wave contribution and provides an enhancement at 700-800 MeV.

### 3 Summary

In contrast to Ref. [4] we show, using the optical theorem, that within the single scattering approximation the null-test observable \( \tilde{\sigma} \) is zero. Our result obtained within the Glauber theory is formulated by Eq. (14). Only the amplitude \( C_{N} \) appears in Eq.(14) whereas other T-even P-even \( pN \) amplitudes, which were found in Ref. [4] to contribute to the TVPC null-test signal, are absent in Eq. (14). Furthermore, we find the deuteron D-wave gives a valuable contribution to the null-test signal for the case of the \( h^{-} \) and \( g^{-} \)-type of interaction. The \( g' \)-term caused by the \( \rho^{-} \)-meson exchange in the TVPC NN-interaction makes a zero contribution to \( \tilde{\sigma} \) and this result is true in the case when both the S- and D-components of the deuteron wave function are taken into account. We discuss some symmetry arguments to clarify a cause for the vanishing contribution of the \( g' \)-term. The \( g' \)-optical potential [11] and the corresponding coupling constant of the \( \rho^{-} \)-meson to the nucleon \( g_{\rho} \) is widely used as a measure of intensity of the TVPC effects [12, 13]. Since the \( g' \)-term gives zero contribution to \( \tilde{\sigma} \) within the Glauber theory, this parameter cannot be applied straightforwardly for the nucleon-deuteron scattering as a scale of the TVPC interactions at large enough energies. However, the \( g' \)-term can give contribution to the null-test signal \( \tilde{\sigma} \) if this interaction is included into the deuteron bound state [6]. One-pion exchange is excluded from the TVPC NN-interaction [12], however, two-pion exchange probably contributes similarly to the P-violating \( pp \)-interaction [14]. Finally, TVPC NN forces can contribute to the electromagnetic p-d interaction due to the toroidal quadrupole form factor of the deuteron [15].

### References


