Acoustic Nonlinearity as a Mechanism for Liquid Drop Explosions in Drop-chain Fountains Generated by a Focused Ultrasound Beam

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Abstract—Ultrasonic atomization has been used in air humidifiers and is also involved in therapeutic applications of intense ultrasound such as boiling histotripsy. An as-yet unexplained phenomenon occurs when a focused ultrasound beam in water creates an acoustic fountain in the form of a drop chain, which explodes in less than a millisecond. In the present paper, we seek to develop a nonlinear theory to explain this phenomenon. We hypothesize that standing wave harmonics are generated inside the water drops due to acoustic nonlinearities, which, along with localized heat deposition in the drop center, may generate a superheated vapor bubble that causes the explosion.

Keywords—ultrasonic atomization; boiling histotripsy; acoustic fountain; acoustic nonlinearity; standing wave harmonics

I. INTRODUCTION: DROP-CHAIN FOUNTAINS

Ultrasound atomization, a process that occurs when an acoustic wave in liquid is directed toward an air interface, has been used commercially in air humidifiers and other such devices. Atomization was also recently proposed to explain tissue fractionation in boiling histotripsy. A focused ultrasound beam creates an acoustic fountain in water in the form of a drop chain. High-speed photography (Fig. 1) shows that for ultrasound intensities on the order of 100s of W/cm², one or several drops in such a fountain explode in less than a millisecond [1 – 4]. The goal of present paper is to develop a nonlinear theory for this process.

Fig. 1. Drop-chain fountain generated by a 2.165 MHz focused ultrasound transducer operating at 280 W/cm². A sequence of images corresponds to the times 2.6, 2.65, and 2.8 ms after the start of sonication.

In some drop-chain acoustic fountains, experimental observations show the appearance of a black spot in the center of the top drop in the acoustic fountain according to [3] (left) and [4] (right). We speculate that this black spot is caused by a bubble (or bubble cloud) in the drop formed through either boiling (as a result of strong dissipation) or cavitation (because of high tensile pressures).

II. METHODS

A. Theoretical model

Standing wave harmonics are generated inside an ultrasonically excited drop due to acoustic nonlinearities. The theoretical approach is based on a quadratic approximation of the acoustic wave equation inside a viscous liquid sphere. The equation is written in terms of acoustic pressure, and all quadratic nonlinear terms are captured in the right-hand side, $Q_{nl}$:

$$\Delta p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = Q_{nl}$$  \hspace{1cm} (1)

$$Q_{nl} = -\frac{\beta}{\rho c_0^4} \frac{\partial^2 \left( p^2 \right)}{\partial t^2} - \frac{\delta}{c_0^4} \frac{\partial^4 p}{\partial t^4}$$  \hspace{1cm} (2)

$$\left( \Delta + \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \right) \left( \frac{\rho \omega^2}{2} - \frac{p^2}{2 \rho c_0^2} \right)$$
Here $p$ is the acoustic pressure; $c_0$ is the speed of sound in a medium; $\rho_0$ is the medium density; $\beta$ is the coefficient of acoustic nonlinearity; $\delta$ is the dissipation factor; and $v$ is the vibrational velocity vector of an acoustical disturbance. The fluid motion is assumed to be spherically symmetric, with a finite acoustic pressure in the drop center and the pressure-release boundary condition at the drop surface:

$$\begin{align*}
    p(r = 0) &< \infty \\
    p(r = a) &= 0
\end{align*}$$

(3)

**B. Solution method**

The solution for the acoustic wave equation inside the water drop was developed as a series of harmonics with unknown coefficients $P_n$:

$$p = \sum_{n=1}^{\infty} \frac{P_n e^{-\alpha n} + P_n' e^{\alpha n}}{2} \sin(k_n r)$$

(4)

where $k_n = \pi n / a$ and $a_0 = k_0 c_0$. A further simplification was made by assuming slowly varying harmonics’ amplitudes $P_n$, which yielded an infinite system of coupled, first-order nonlinear differential equations for the amplitude coefficients:

$$\frac{dP_n}{dt} + \frac{8\alpha c_0}{P_n} P_n' = -i\beta n^2 \sum_{m=0}^{\infty} P_{2n-m} P_{2n+m} S_{2n+m-n}$$

(5)

Here $S_n = \text{Si}(\pi n)$, where $\text{Si}(.)$ is the sine integral. The system (5) was truncated and solved numerically with a finite-difference scheme.

**III. RESULTS**

Numerical solutions are shown in Fig. 3 for amplitudes of the harmonics $|P_n|/P_0$ in the center of the drop versus their number at different times, where $P_n$ is the amplitude coefficient of the $n$th harmonic and $P_0$ is the initial amplitude coefficient of the 1st harmonic.

Fig. 4 shows the numerical solutions for the time-dependent amplitudes $|P_n|/P_0$ of the first four harmonics in the center of the drop.

![Fig. 4. Time-dependent amplitudes $|P_n|/P_0$ of the first four harmonics in the center of the drop ($c_0/2a = 1$ MHz).](image)

When $P_0 = 1$ MPa, initial oscillations at the fundamental ultrasound frequency give rise to higher-order standing waves, resulting in strong distortion of the pressure wave, including shock formation, as shown in Fig. 5.

Acoustic heat deposition was calculated through the energy balance equation and used to determine the temperature distribution inside the drop and boiling initiation times. As the energy of the higher harmonics is localized close to the center of the drop, heating is found only in the very center of the drop. When $P_0 = 1$ MPa, the boiling temperature is reached at the drop center in approximately 3 ms, as shown in Fig. 6. This time is close to our experimental observations [2, 3] and those reported by Tomita [4], where the black spot, which we speculate to be a bubble (or bubble cloud) appears in the center of the top drop in the acoustic fountain.

![Fig. 5. Distortion of the pressure waveform in the drop center at the time of 1 ms.](image)

![Fig. 6. Temperature (°C) distribution near the drop center at different times.](image)
IV. DISCUSSION

This paper presents time-dependent pressure and temperature distributions calculated inside an ultrasonically excited liquid drop using the numerical solution of the system of coupled, first-order nonlinear differential equations for the amplitude coefficients of the acoustic pressure. Initial oscillations at the fundamental ultrasound frequency give rise to higher-order standing waves. This process results in strong distortion of the pressure waveform, including shock formation. The corresponding heating occurs several times more rapidly than predicted by linear acoustics to explain experimental observations of drop explosions while contributing to a better understanding of the ultrasonic atomization phenomenon.

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V. REFERENCES