Application of the Leray-Schauder Principle to the Analysis of a Nonlinear Integral Equation

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We study a nonlinear integral equation arising from the parametric closure for the third spatial moment in the Dieckmann–Law model of stationary biological communities.

$$\left(\overline{\omega} + b - \frac{\alpha}{2}\left(b - d - \frac{d'(b-d)}{Y}\right)\right) C = \\ = \frac{Y\overline{m}}{b-d} - \overline{\omega} + [\overline{m} * C] - \alpha\frac{b-d}{2Y}\left((C+2)[\overline{\omega} * C] + [\overline{\omega}C * C]\right), \tag{1}$$

where $\alpha \in [0;\ 1]$, $\overline{m} = bm$, $\overline{\omega} = d'\omega$, $Y = \int\limits_{\mathbb{R}} \big(C(x)+1\big) \cdot \overline{\omega}(x)\, dx$, and $*$ stands for the operation of convolution of functions.

We have studied the well-posedness of a problem related to a nonlinear integral equation derived by applying a parametric power-2 closure of the third spatial moment. Sufficient conditions for the existence and uniqueness of a solution of this equation were found in the case when the competition, ω and dispersal, m kernels are continuous. The following theorem on the existence of the solution was proved

Theorem 1. *Lets* $b > d \geq 0$, $d' > 0$, $\alpha \in (0;\ 1]$, $R < \dfrac{1}{\|\omega\|_C}$. *Then, for sufficiently small* d', *the equation 1 has a solution*

Also we find sufficient conditions under which the equation (1) has a unique solution.

Theorem 2. *Under the conditions of Theorem 1 there exist constants* $b > d \geqslant 0$ *and a small number* $d' > 0$ *such that the equation* (1) *has a unique solution.*

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References

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